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Symmetry in the insulator–quantum-Hall–insulator transitions observed in a Ge/SiGe quantum well

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We examine the magnetic-field-driven insulator–quantum-Hall–insulator transitions of the two-dimensional hole gas in a Ge/SiGe quantum well. We observe direct transitions between low and high magnetic-field insulators and the $\nu=1$ quantum Hall state. With increasing magnetic field, the transitions from insulating to quantum Hall and quantum Hall to insulating are very similar with respect to their transport properties. We address the temperature dependence around the transitions and show that the characteristic energy scale for the high-field transition is larger. [S0163-1829(97)51648-1]

The quantum-Hall (QH) effect is an excellent system to study transitions between insulating and metallic behaviors in two dimensions. In the integer QH effect the basic physics is governed by the Landau levels (LL). The system becomes insulating at $T=0$ when the energy of the lowest LL crosses and exceeds the Fermi energy. In this case we have a transition from the QH state, characterized by the filling factor $\nu=1$, to an insulator. This was observed by Paalanen *et al.*[1] in low mobility samples. For much cleaner samples with very high mobilities, the transition to an insulating behavior can occur from much lower fractional QH states like $\nu=1/5$ as observed by Jiang *et al.*[2] This transition was interpreted as evidence for the formation of a Wigner crystal. In the zero magnetic-field case a similar transition was observed by Kravchenko *et al.*[3] but as a function of density.

Another intriguing phenomenon has been observed recently by Jiang *et al.*[4] and others:[5,6] they observed a direct transition from a low magnetic-field (B) insulating phase to the $\nu=2$ state. Later Shahar *et al.*[7] observed a transition to the $\nu=1$ state in a two-dimensional electron system and Song *et al.*[8] saw direct transitions between a low B insulator and the $\nu=3$, $\nu=2$ and $\nu=1$ quantum-Hall states, depending on density. The general framework for understanding these insulator-to-QH states transitions can be found in the pioneering work of Kivelson, Lee, and Zhang[9] (KLZ) in terms of a global phase diagram (GPD). The theory of KLZ is successful in explaining transitions in the integer and fractional QH regime but it fails in accounting for the direct transitions to higher-order QH states such as the $\nu=3$ state.[8] It is therefore essential to study these transitions in detail in order to improve our understanding of the GPD.

The purpose of this work is to give a better understanding of the insulator–QH–insulator transitions by concentrating on transitions involving only the $\nu=1$ QH state. In particular, the similarities between the low- and high-B field transitions will be demonstrated. We will further present a detailed study of the temperature (T) dependence of the resistivity around both, the low-B field transition, which separates a low-B insulator from the spin split $\nu=1$ QH state, and the high-B field transition that separates the QH state $\nu=1$ from the insulating high-B field phase.

This paper is organized as follows. After discussing the experimental details we will focus on the diagonal resistivity, which allows us to identify the low- and high-B field transition points. We study the T dependence of the slopes of the diagonal resistivity at the transitions. Our main result is that there is a clear similarity between both transitions, essentially differing only by their energy scales. We conclude with a discussion of our results.

The results of this work were obtained from a two-dimensional hole system (2DHS) in a strained Ge layer. The 2DHS is contained in the Ge layer which is under compressive strain. The sample studied was grown by MBE technique and consists of a graded buffer $Si_{1-x}Ge_x$ layer grown on a Si (100) substrate, followed by a uniform buffer $Si_{0.4}Ge_{0.6}$ layer and a 150-Å-thick Ge layer sandwiched in between $Si_{0.4}Ge_{0.6}$ layers where Boron modulation doping is placed. The 2DHS has a mass of $\sim 0.1m_e$, which is density dependent.[8,10]

We could vary the density and mobility by means of a metallic front gate. A MOSFET structure was made by depositing an insulating layer between the metal and the cap layer. A standard Hall bar was processed with Ti/Al gate and Al/Au alloyed Ohmic contacts with a 50-μm-wide channel and 600-μm apart voltage probes. By applying a gate voltage between 0 V and 6 V we could vary the density between $n=0.7-6\times10^{11}$ cm^{-2} and the mobility between $0.3-20\times10^3$ cm^2/Vs. In our whole gate voltage range we observed no gate leakage.

The measurements were performed in a He^{-3} refrigerator at T ranging from 300 mK to 7 K, using an ac lock-in technique with an excitation current of 0.2 nA. The results were reproducible with a current of 0.05 nA, within experimental accuracy. dc measurements were performed to check for consistency. The results in this work are obtained at a fixed gate voltage of $V_G=5.2$ V. The measured density was 0.87×10^{11} cm^{-2}.

In Fig. 1 we show the diagonal resistivity, ρ_{xx}, as a function of B, for various T ranging from 0.3 to 4.2 K. The Hall

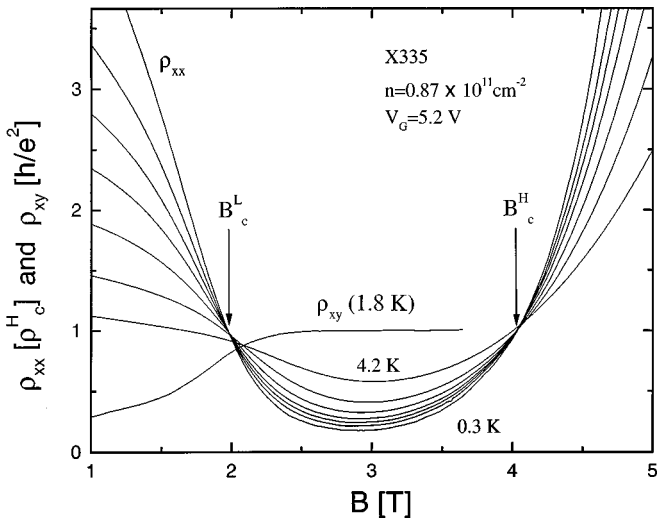

FIG. 1. Diagonal and Hall resistivity as a function of magnetic field. $\rho_c^H = 2.2h/e^2$. The temperatures are 0.3, 0.55, 0.75, 0.9, 1.2, 2.4, and 4.2 K.

resistivity is plotted at 1.8 K. Three regions can clearly be distinguished, which we now describe in order of increasing B.

The first region is a low-B field insulating phase characterized by an increasing resistivity with decreasing T and a linear Hall resistivity as B tends to zero. The second region is the $\nu=1$ QH state. At B_c^L the two regions are clearly separated by a T-independent ρ_{xx} (at low temperatures). The $\nu=1$ QH state is characterized by a well-developed plateau in the Hall resistivity, with the expected value of $\rho_{xy}=h/e^2$, and a decreasing ρ_{xx} with decreasing T. Finally, the last region is similar to the first and is also characterized by a diverging diagonal resistivity when decreasing T. Here again we have a well-defined transition point, with a T-independent resistivity, $\rho_c^H=\rho_{xx}(B_c^H)$, for low enough temperatures.

The transition points can be reliably extracted by plotting, as we do in Fig. 2, the resistivities as a function of T for

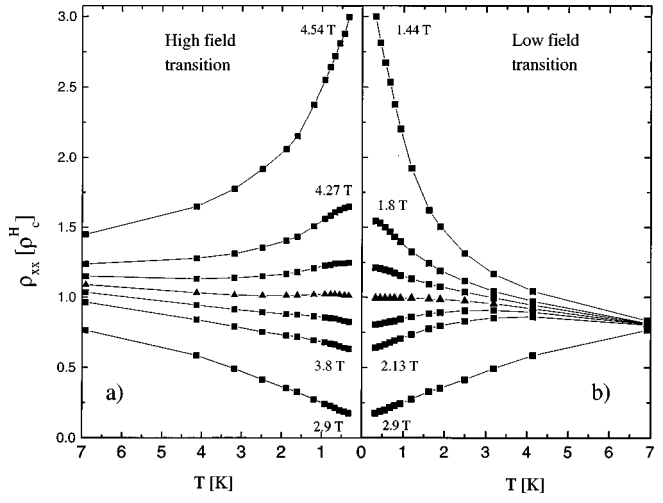

FIG. 2. Temperature dependence of the resistivities around the low- and high-field transitions. (a) The magnetic fields corresponding to the central resistivity curves are 3.94, 4.04, and 4.14 T and in (b) they are 2.05, 1.975, and 1.9 T.

different values of B. In Fig. 2(a) we present the high-B field transition and observe that the plot corresponding to the magnetic field $B_c^H=4.04$ T has almost no T dependence. The resistivity remains constant within 1% between 0.3 K and 3.2 K. For higher magnetic fields the resistivity diverges with decreasing T whereas for lower magnetic fields the dependence is opposite. The low-B field transition, presented in Fig. 2(a), is very similar. Extracting the magnetic field corresponding to the T-independent behavior below 1.8 K, we obtain a critical B for the high-B field transition of $B_c^L=1.975$ T.

Since the main idea in this work is to concentrate on properties related to transitions into insulating phases we chose a system with a strong T dependence in these phases, in particular in the low-B field phase. This implies the choice of a system with strong disorder, which leads to a high value of the minimum diagonal resistivity in the QH state, even at 0.3 K. Therefore, we are unable to directly compare our result $\rho_c^H=2.2h/e^2$ with the value h/e^2 obtained in the work of Shahar *et al.*,[11] which demonstrated the universality of the resistivity at B_c^H. They had only included in their study samples for which the minimum diagonal resistivity in the QH state was vanishing at low T. We indeed observe a systematic increase of ρ_c^H with increasing V_G.

We can now turn to the main result of this work, concerning the similarities between both transitions. Extracting the resistivities from Fig. 2, we first note that

$$\rho_c^L=\rho_c^H\pm 3\%, \tag{1}$$

where $\rho_c^L=\rho_{xx}(B_c^L)$. It is interesting to mention that a similar approximate relation (1) holds for the $\nu=2$ QH state to high and low-B field transitions.[4–6] These experiments were performed in systems where the $\nu=1$ state was not resolved. The main difference between our system and conventional $Al_xGa_{1-x}As/GaAs$ systems[4–6] is a comparatively higher Zeeman energy of our Ge samples. This leads to the observation of a well-developed spin resolved $\nu=1$ plateau. Since relation (1) applies only at the transition points, we will now concentrate on the behavior around the transition points and study the similarity between both transitions as a function of B and T. We will first focus on the B dependence and then turn to the T dependence.

The best way to compare the B dependence is to overlap both transitions as a function of the filling factor ν, which is dimensionless. We therefore convert our diagonal resistivity data of Fig. 1 as a function of ν, which is obtained by measuring the density from the Hall resistance. The result is represented in Fig. 3. We note that the two transitions are almost indistinguishable. The main difference is the effective temperature of the two transitions, which can be associated to a characteristic energy scale of each transition. The high value of $\nu_c^H=0.87$ can be expected due to the strong overlap of the density of states of higher Landau levels.

In the following we present our temperature dependence analysis at the transitions. There are several ways for studying this temperature dependence. The prevailing studies[4,5] assume a scaling behavior around the critical point and try to collapse all data on one curve. Because this study assumes a scaling behavior it is difficult to extract any information other than a scaling exponent. We will use an approach

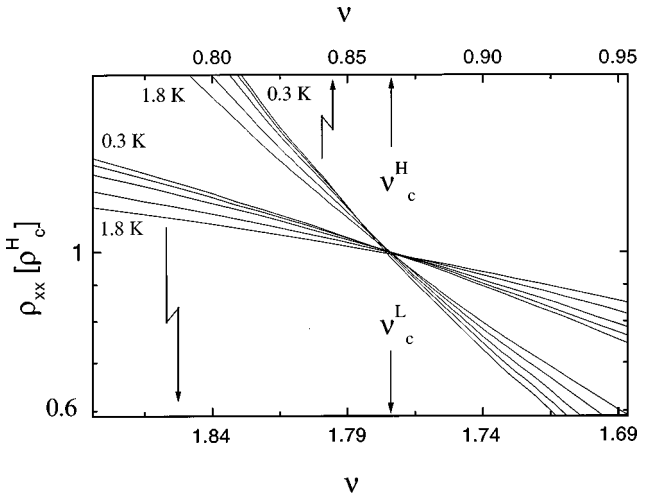

FIG. 3. Resistivities, on a log scale, as a function of filling factor. The group of curves with steeper slopes corresponds to the high-B field transition with critical filling factor $\nu_c^H = 0.87$ obtained from B_c^H. The critical filling factor $\nu_c^L = 1.77$ of the low-B field transition corresponds to B_c^L. The bottom ν scale is reversed.

which is more general in the sense that it does not suppose any scaling form. Following Wang *et al.*,[6] this approach consists simply of taking the slopes of the diagonal resistivity at the transition, i.e., near the T-independent point. In order to obtain a dimensionless result we evaluate

$$\alpha_0^{-1} = \left(\frac{\partial \ln(\rho_{xx})}{\partial \nu} \right) \Bigg|_{\nu_c},$$

which is simply the slope of the plots in Fig. 3. In addition to giving a dimensionless result, this method has the advantage that $\ln(\rho_{xx})$ can be well approximated by a linear function of ν, as was demonstrated by Shahar *et al.*[12] for the high-B field transition. A new result of this work is that for the low-B field transition we also observe a similar linear dependence. This linear dependence of the slopes allows us to extract the slopes in a well-defined and straightforward way for the different temperatures. It is interesting to note that a similar linear dependence is also observed as a function of density in the zero B metal-insulator transition.[13]

In Fig. 4 we plot the dimensionless inverse slopes, α_0, on a ln-ln scale. It is evident from the data that it is not possible to extract any power law consistently for neither the low- nor the high-B field transitions. It is possible, however, to extract some interesting information from Fig. 4 concerning the different underlying energy scales. When the slopes of the low-B field transition are divided by three we can overlap the low-B field transition and the high-B field transition for the lowest temperatures. This is consistent with a lower characteristic energy scale for the low-B field transition.

We further tried to fit the T dependence following Shahar *et al.*[12] The suggestion is that α_0 follows a linear T dependence. We observe in Fig. 5(a) a good agreement with this behavior for the high-field transition. In the low-field transition case, Fig. 5(b), the data start to deviate from this behavior at 1.7 K. This deviation can also be understood in terms of the lower effective temperature scale in the low-field transition, which drives the system faster into a different behavior. The same ratio 3 between both transitions is obtained for the zero-temperature extrapolated value of α_0, but a ratio of 6 for the linear temperature coefficient.

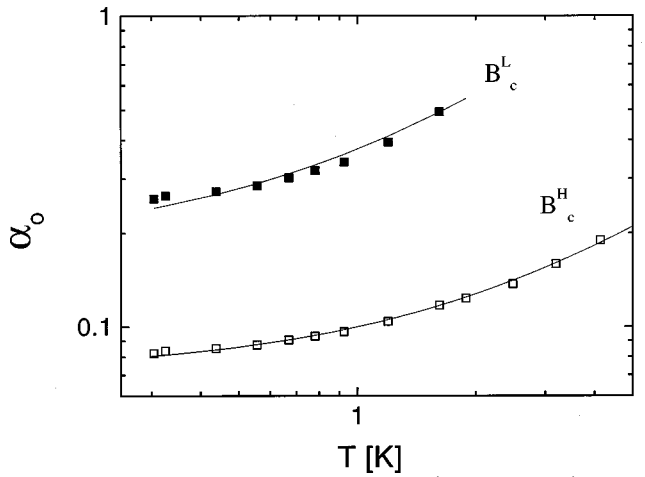

FIG. 4. α_0 for the low-B field transition (solid symbols) and the high-B field transition (open symbols). The lines are the fits obtained from Fig. 5.

Coming back to Fig. 2, similar conclusions can be drawn concerning the different energy scales. For the high-B field transition, ρ_{xx} remains constant within 1% over a wide temperature range, i.e., from our base temperature, $T = 0.3$ K, to approximately 3.2 K, whereas for the low-B field transition ρ_{xx} remains constant only up to 1.7 K. At 7 K the relative deviation to the 0.3 K value is 7% for the high-B field transition but 20% for the low-B field transition. We note here a different qualitative behavior: at the low-B field transition ρ_{xx} *decreases* with higher temperatures whereas at the high-B field transition ρ_{xx} *increases* with higher temperature.

In the following we discuss our results in light of existing theories. Theories of disordered systems predict overall localized states in two dimensions and at zero B.[14] However, when the quantization due to LL becomes important at high fields it is expected that extended states exist at the center of LL's.[15]

There are several theoretical and numerical results dealing with the crossover from the localized zero B field state to the

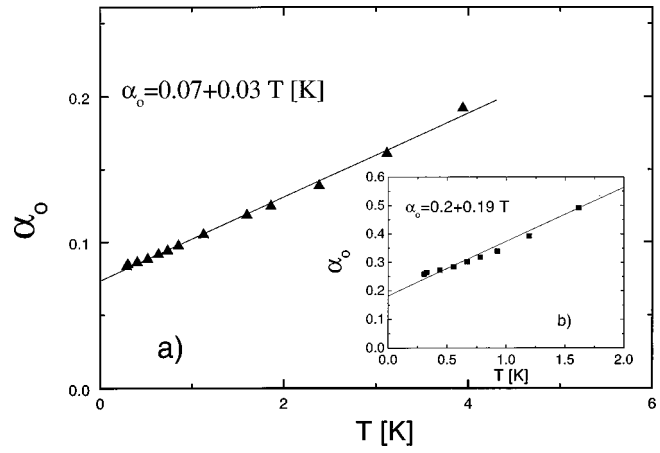

FIG. 5. α_0 on a linear graph as a function of temperature for the high-field transition. The inset shows the low-field transition up to 1.7 K. The straight lines are linear fits to the data.

delocalized high-B field state. The original argument of Khmel'nitzkii and Laughlin[16] describes the crossover as follows: the energies of the extended states originally at the center of a Landau level at high fields, float up with decreasing magnetic field. This is commonly referred to as the floating up picture. When the magnetic field is decreased the Fermi energy crosses the up-floating lowest energy level of extended states and yields an insulator. In this framework, the low-field transition is also induced by the crossing of a LL, in analogy to the high-field transition. This picture has gained recent support by numerical and theoretical results.[17] Pruisken[18] describes the crossover in terms of a field theory, recovering the localized case as the simplectic limit and the delocalized case as the unitary limit. An alternate description is given by Liu *et al.*[19] Their numerical results suggest that in the center of each LL there is a localization-delocalization transition with increasing B field.

The similarity between both transitions is consistent with the floating up picture as they would both be described by the LL level crossing of the Fermi energy. However, the floating picture does not predict a substantial different energy scale for both transitions, as opposed to the inter-LL localization-delocalization picture. All theories described above assume a single-particle picture, but at low fields other energy scales like interparticle interactions become relatively more important and could alter the simple one-particle physics.

In conclusion, we analyzed the insulator–quantum-Hall-state $\nu=1$ insulator transitions. The transitions are characterized by a temperature-independent diagonal resistivity. The main result arising from this study is that both transitions are very similar in terms of the magnetic field and temperature dependence so that it seems reasonable to assume a similar mechanism for both transitions.

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[1] M.A. Paalanen *et al.*, Solid State Commun. **50**, 841 (1984).
[2] H.W. Jiang *et al.*, Phys. Rev. Lett. **65**, 633 (1990).
[3] S.V. Kravchenko *et al.*, Phys. Rev. B **50**, 8039 (1994).
[4] H.W. Jiang *et al.*, Phys. Rev. Lett. **71**, 1439 (1993).
[5] R.J.F. Hughes *et al.*, J. Phys.: Condens. Matter **6**, 4763 (1994).
[6] T. Wang *et al.*, Phys. Rev. Lett. **72**, 709 (1994).
[7] D. Shahar, D.C. Tsui, and J.E. Cunningham, Phys. Rev. B **52**, 14 372 (1995).
[8] S.H. Song *et al.*, Phys. Rev. Lett. **78**, 2200 (1997).
[9] S. Kivelson, D.H. Lee, and S.C. Zhang, Phys. Rev. B **46**, 2223 (1992).
[10] Y.H. Xie *et al.*, Appl. Phys. Lett. **63**, 2263 (1993).
[11] D. Shahar *et al.*, Phys. Rev. Lett. **74**, 4511 (1995).
[12] D. Shahar *et al.*, cond-mat/9706045 (unpublished).
[13] D. Simonian, S.V. Kravchenko, and M.P. Sarachik, Phys. Rev. B **55**, R13 421 (1997).
[14] E. Abrahams *et al.*, Phys. Rev. Lett. **42**, 673 (1979).
[15] For a general overview of the problem we refer to the following recent review by M. Janssen *et al.*, *Introduction to the Theory of the Integer Quantum Hall Effect*, edited by J. Hajdu (VCH, Weinheim, 1994).
[16] D.E. Khmel'nitzkii, JETP Lett. **38**, 552 (1983); R.B. Laughlin, Phys. Rev. Lett. **52**, 2304 (1984).
[17] F.D.M. Haldane and Kun Yang, Phys. Rev. Lett. **78**, 298 (1997) and references therein.
[18] A.M.M. Pruisken, Physica A **181**, 1 (1992).
[19] D.Z. Liu, X.C. Xie, and Q. Niu, Phys. Rev. Lett. **76**, 975 (1996).