PHYSICAL REVIEW B VOLUME 56, NUMBER 22 1 DECEMBER 1997-II

Possibility of periodically reentrant superconductivity in ferromagnet/superconductor layered structures

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(Received 22 August 1997)

We develop the theory of the proximity effect in layered ferromagnetic metal/superconductor (F/S) structures taking into account finite transparency of the F/S interface as well as depression of the Cooper pairing and diffusionlike motion of conduction electrons by a strong exchange field of the ferromagnet. It is shown that the oscillatory dependence of the critical temperature on the F-layer thickness is due to a periodic modulation of the F/S boundary transparency by the oscillations of pair amplitude within the F layer. It is possible not only in the F/S multilayers, but in the F/S bilayers as well. The phenomena of reentrant and periodically reentrant superconductivity in the F/S contacts and superlattices are predicted. The competition between the "0" phase and "π" phase types of superconductivity in the F/S multilayers is also discussed.
[S0163-1829(97)51546-3]

The analysis of experiments[1–5] on ferromagnet/superconductor (F/S) multilayers testifies that there is various qualitative behavior of the critical temperature T_c dependence on the ferromagnetic layer thickness d_f for the same F/S structures. In particular, in the experiments on systems Fe/V (Ref. 1) and Gd/Nb (Ref. 2) the fast initial T_c decrease and the subsequent output on a plateau were observed with increasing d_f. In other experiments on the same systems (Refs. 3, 4, and 5, accordingly) the oscillatory behavior $T_c(d_f)$ have preceded the same output on a plateau. The theoretical interpretation of oscillations $T_c(d_f)$ in Refs. 6 and 7 was reduced to the periodic "switching" from the traditional "0" phase type superconductivity to the "π" phase type, in which the sign of the order parameter Δ under the transition through F layers changes. However, as it will be shown below, the theories[6,7] are valid only in the case of high transparency of the F/S boundary and are limited to the extremely dirty ferromagnetic metal case, when $2I\tau_f \ll 1$, where I is a exchange field and τ_f^{-1} is a frequency of the electrons scattering on nonmagnetic impurities. Therefore two different types of experimental $T_c(d_f)$ dependences cannot be described in the context of the existing theory.[6,7] Moreover, recently it was revealed that the oscillation $T_c(d_f)$ takes place in the trilayered structure Fe/Nb/Fe,[8] where the "π" phase type superconductivity is impossible. Hence the question about the origin of nonmonotonic dependence $T_c(d_f)$ in the F/S systems remains unsolved and a theory adequately describing an available set of the experimental data is required.

In this paper we present the complete theory of the proximity effect in the layered F/S structures taking into account finite transparency of the F/S boundaries. We also take into account the depression of the Cooper pairing by a strong exchange field I and a competition between a diffusionlike and spin-wave-like motion of quasiparticles in a ferromagnet. The present theory allows us not only to explain available experimental data, but also to predict a number of effects, the pronounced peculiarity of which might be periodically reentrant superconductivity appearance. The oscillatory behavior of $T_c(d_f)$ can be possible not only in the multilayers, but also in F/S bilayers and $F/S/F$ trilayers.

First let us consider a planar contact between a ferromagnetic metal occupying the region $-d_f<z<0$ and a superconductor occupying the region $0<z<d_s$. In a vicinity of the second order phase transition point the critical temperature T_c of the F/S contact is determined from the Gor'kov integral equation for the order parameter $\Delta(z)$ of a nonuniform system. For convenience we write it down in the terms of anomalous Usadel function $F_{\alpha\beta}(z,\omega)$ (see, for example, Refs. 6 and 7)

$$\Delta(z)=\frac{1}{2}\lambda(z)\pi T{\sum_{\omega}}' \sum_{\alpha\neq\beta} F_{\alpha\beta}(z,\omega), \quad (1)$$

$$F_{\alpha\beta}(z,\omega)=\frac{1}{\pi N(z)}\int_{-d_f}^{d_s} H_{\alpha\beta}(z,z',\omega)\Delta(z')dz', \quad (2)$$

where $\lambda(z>0)=\lambda_s$ and $\lambda(z<0)=\lambda_f$ are dimensionless parameters of the electron-electron interaction, $N(z)$ is the density of states at the Fermi level, α and β are the spin indices and the prime on the summation denotes the cutoff at the Debye frequency ω_D; T is the temperature, $\omega=\pi T(2n+1)$ is the Fermi frequency. It can be shown by the diagram technique (see Ref. 9 and Fig. 1) that in the presence of the exchange field and conduction electrons scattering on nonmagnetic impurities the correlator $H_{\alpha\beta}(z,z',\omega)$ is a solution of the other integral equation

$$H_{\alpha\beta}(z,z',\omega)=K_{\alpha\beta}(z,z',\omega) + \int_{-d_f}^{d_s} \frac{K_{\alpha\beta}(z,z_1,\omega)H_{\alpha\beta}(z_1,z',\omega)}{2\pi N(z_1)\tau(z_1)}dz_1; \quad (3)$$

see for a comparison Ref. 10. Here we use the following notations:

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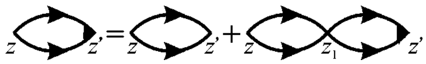

FIG. 1. Diagrammatic representation of Eq. (2) for vertex part $H_{\alpha\beta}(z,z',\omega)$. Solid lines correspond to the normal state Green functions averaged on the nonmagnetic impurities configurations.

$$K_{\alpha\beta}(z,z',\omega)=\frac{1}{(2\pi)^2}\int G_{\alpha\alpha}(\mathbf{p},z,z',\omega)G_{\beta\beta}\times(\mathbf{p},z,z',-\omega)d^2p, \quad (4)$$

where $G_{\alpha\alpha}(\mathbf{p},z,z',\omega)$ is the Green function of conduction electrons in a normal phase, **p** is the two-dimensional momentum in the plane of contact, and $\tau^{-1}(z)$ is the impurity scattering rate which stepwiselike changes its value with transition through the F/S boundary $z=0$ as well as $N(z)$. Solving the problem with a potential barrier at the F/S interface for value $K_{\alpha\beta}(z,z',\omega)$ (see, for example, Ref. 10), it can be shown that the integral equation set (2), (3) is reduced to a boundary problem for function $F_{\alpha\beta}(z,\omega)$ in the dirty metal case, when the mean free path $l_j=v_j\tau_j$ $(j=f,s)$ becomes the least among all characteristic scales. This boundary problem contains the diffusionlike type differential equation

$$\left[|\omega|+iI(z)g_{\alpha\beta}\ \mathrm{sgn}\ \omega-\frac{1}{2}D_{\alpha\beta}(z)\frac{\partial^2}{\partial z^2}\right]F_{\alpha\beta}(z,\omega)=\Delta(z) \quad (5)$$

and the boundary conditions relating a flux of the function $F_{\alpha\beta}(z,\omega)$ with its jump on the flat F/S interface $z=0$

$$D_{s\alpha\beta}\frac{\partial F_{\alpha\beta}(z,\omega)}{\partial z}\bigg|_{z=+0}=\frac{\sigma_s v_s}{4}[F_{\alpha\beta}(+0,\omega)-F_{\alpha\beta}(-0,\omega)];$$

$$D_{f\alpha\beta}\frac{\partial F_{\alpha\beta}(z,\omega)}{\partial z}\bigg|_{z=-0}=\frac{\sigma_f v_f}{4}[F_{\alpha\beta}(+0,\omega)-F_{\alpha\beta}(-0,\omega)]; \quad (6)$$

where $g_{\uparrow\downarrow}=-g_{\downarrow\uparrow}=1$, and the F/S contact transparencies σ_f and σ_s are connected by the detailed balance relation $\sigma_s v_s N_s=\sigma_f v_f N_f$.[11] The presence of the ferromagnet spin stiffness leads to the complex diffusion coefficient,[12] i.e.,

$$D_{\alpha\beta}(z)=\frac{D(z)}{1+2i\tau(z)I(z)g_{\alpha\beta}\ \mathrm{sgn}\ \omega};\quad D(z)=D_j=v_j l_j/3, \quad (7)$$

where $I(z<0)=I$ and $I(z>0)=0$. Equations (5) and (6) are right on the condition $l_j<\xi_j$ ($\xi_j=\mathrm{Re}\sqrt{D/2\pi T+2iI_j}$ is the coherence length) and in addition the parameter l_f should be less than the ferromagnet spin stiffness length $a_f=v_f/2I$.

The received boundary problem (1), (5), and (6) for the proximity effect in the F/S contact differs from the former one[6,7] in two respects. Firstly, the boundary conditions, which were used in Refs. 6 and 7, consist of a continuity of the Usadel function at the F/S interface $(z=0)$. It is a special case of Eqs. (6) which corresponds to the high transmission limit $\sigma_j\gg l_j/\xi_j$ or to the neglect of the $F_{\alpha\beta}(z,\omega)$ flux through the F/S boundary. However, the σ_j value determines the rate at which electrons are transferred between F and S layers (see Ref. 11) and it strongly depends on the F/S boundary manufacturing conditions and the sample preparation technique. Because of this, the σ_j value remains a fitting parameter of theory or has to be measured experimentally. Secondly, the depression of diffusionlike motion of conduction electrons by the exchange field I of ferromagnet leads to the occurrence of an imaginary part in the effective diffusion coefficient $D_f^*=D_f/(1+2iI\tau_f)$.[12] Due to this the quasiparticles motion in the ferromagnet has the mixed diffusionlike and spin-wave-like character. The spin-wave contribution dominates with the increase of parameter $2I\tau_f$ and the penetration depth of the pair amplitude into a ferromagnet becomes larger than a period of its oscillations, as will be shown below.

Strong pairbreaking by the exchange field I ($I\gg\pi T_{cs}$ where T_{cs} is the critical temperature of the isolated S layer) is the main mechanism of the superconductivity destruction in the F/S systems. For the sake of simplicity in the following calculations we neglect the order parameter induced in a F layer $\Delta(z<0)\simeq 0(\lambda_f\simeq 0)$ and we search the solutions of Eqs. (1), (5), and (6) in the form excluding the electrons flux through external boundaries of the contact, i.e., $F_s(z,\omega)\propto\cos k_s(z-d_s)$ for $z>0$ and $F_f(z,\omega)\propto\cos k_f(z+d_f)$ for $z<0$. We also assume that k_s and k_f do not depend on the frequency ω. Here we use the symmetry of the Usadel function $F_{\alpha\beta}(\omega)=F^*_{\beta\alpha}(\omega)=F_{\beta\alpha}(-\omega)$ and, turning to positive frequencies ω in Eq. (1), we drop spin indices for convenience. Then we receive the closed set of equations for the reduced temperature $t=T_c/T_{cs}$ of the F/S contact superconducting transition

$$\ln t=\Psi\left(\frac{1}{2}\right)-\mathrm{Re}\ \Psi\left(\frac{1}{2}+\frac{D_s k_s^2}{4\pi T_{cs}t}\right),$$

$$D_s k_s\tan k_s d_s=\frac{\sigma_s v_s}{4-(\sigma_f v_f/D_f^* k_f)\cot k_f d_f},$$

$$k_f^2=-\frac{2iI}{D_f^*}=-\frac{2iI(1+2iI\tau_f)}{D_f}, \quad (8)$$

where $\Psi(x)$ is the digamma function. As follows from Eq. (8), pair-breaking parameter $D_s k_s^2$ and critical temperature T_c strongly depend on the layer's thickness, the F/S boundary transparency, and a relation between the Fermi level exchange splitting $2I$ and the collision's frequency of electrons τ_f^{-1} with nonmagnetic impurities in the ferromagnet.

The spatial changes of the Usadel function in a F layer are characterized by a wave number $k_f=1/\xi_f'-i/\xi_f''$, where ξ_f' responds for an oscillation period, and ξ_f'' responds for the penetration depth of the Cooper pairs in the ferromagnet. In a dirty limit ($l_f\ll\xi_f',\xi_f''$) these two lengths differ only by small corrections of the order $I\tau_f$

$$\xi_f'=\sqrt{\frac{D_f}{I}}(1-I\tau_f);\quad \xi_f''=\sqrt{\frac{D_f}{I}}(1+I\tau_f);\quad 2I\tau_f\ll 1, \quad (9)$$

as it is seen from (8). Owing to a strong attenuation, the oscillations of the function $F_f(z,\omega)$ are not displayed and in the last equation in (8) $\tan k_f d_f\approx -i$ at $d_f\gg\xi_f'$, that corresponds to an output of the pair-breaking factor $D_s k_s^2$ and the

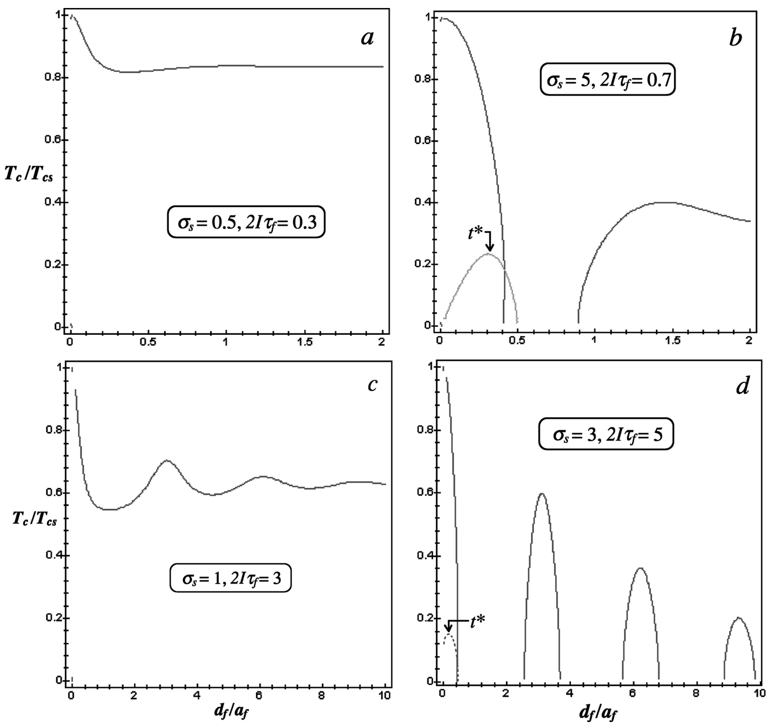

FIG. 2. The dependence of reduced transition temperature for F/S bilayer on the reduced F layer thickness d_f/a_f for various values of parameters ($N_s v_s = N_f v_f$, $d_s = 500$ Å and $\xi_{so} = 400$ Å), where ξ_{so} is the BCS coherence length: (a) critical temperature flattens out; (b) reentrant superconductivity; (c) critical temperature oscillations; and (d) periodically reentrant superconductivity. The tricritical points line (t^*) is shown by the dashed line.

dependence $T_c(d_f)$ on a constant at small transparencies σ_j, as shown in Fig. 2(a). This behavior was observed in Gd/Nb bilayers.[2] At the sufficiently large transparency of F/S boundary and the strong exchange field I the phenomenon of the reentrant superconductivity is possible, as shown in Fig. 2(b).

In the opposite limit $2I\tau_f > 1$ (this is possible since $I \sim 10^3$ K for Gd and Fe) the diffusion description of the conduction electrons motion in F layer employed above fails (see Ref. 12), whereas the traditional condition of the dirty limit $\pi T \tau_f \ll 1$ can be held. In this case the asymptotic behavior of $F_f(z,\omega)$ is described by a wave number k_f which is slightly distinct from that in Eqs. (8), namely

$$k_f^2 \approx -\frac{2iI(1+2iI\tau_f)}{v_f l_f}; \quad \xi_f' \simeq a_f = \frac{v_f}{2I}; \quad \xi_f'' \simeq l_f; \quad 2I\tau_f \gg 1. \tag{10}$$

It follows from joint solution of Eqs. (2)–(4) and (8).

After the replacement $D_f \Rightarrow 3D_f$ the set of Eq. (8) becomes suitable for the description of the dependence $T_c(d_f)$ in the F/S contacts with relatively pure layers of ferromagnet and/or with a strong exchange field. The Usadel function, the pair-breaking factor $D_s k_s^2$, and critical temperature oscillate as the thickness d_f increases with a period determined by the spin stiffness length a_f. These oscillations are damped if $d_f > 2l_f$ ($\gg a_f$) and the dependence $T_c(d_f)$ flattens out, as shown in Fig. 2(c). The similar behavior of $T_c(d_f)$ was observed in trilayered contact Fe/Nb/Fe.[8] Note that Eqs. (8) can be used for $F/S/F$ trilayers with a substitution $d_s \Rightarrow d_s/2$. It is interesting that for rather large meanings of parameters σ_s and $2I\tau_f$ the superconductivity of the F/S contact has periodically reentrant character for low temperatures, as shown in Fig. 2(d). The number of maxima of the dependence $T_c(d_f)$ is defined by the number of halfwaves of function $\cos k_f(z+d_f)$ fitted on the F layer thickness. Every time, when this number becomes an integer, there is effective ''self-locking'' of the F/S boundary and critical temperature passes through the maximum. The minima points correspond to odd numbers of wave quarters within F layer.

Hence the physical reason of oscillatory behavior of the function $T_c(d_f)$ in F/S systems is the periodic compensation of the exchange field paramagnetic effect by oscillations of the pair amplitude inside the F layer. It leads to the periodic modulation of the S/F boundary transparency σ_s for $2I\tau_f > 1$ according to expressions (8) and Figs. 2(c) and 2(d). The nonmagnetic scattering presence leads to the fact that this exchange field compensation becomes incomplete, and modulations σ_s and T_c are damped. Thus the superconductivity in F/S bilayers has a combined character. In S layers the BCS type of pairing is realized with zero pairs momentum. In F layers pairing conditions correspond to the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) mechanism[13] with nonzero pairs momentum $k \approx a_f^{-1}$ and, hence, with pair amplitude oscillations with period πa_f.

It is easy to show that Eqs. (8) for the critical temperature of the F/S contacts are generalized by replacement $d_j \Rightarrow d_j/2$ in the case of the ''0'' phase type F/S superlattices with a period $d_f + d_s$. However, in the F/S multilayer's the possible competition between ''0'' and ''π'' phase types of superconductivity[6,7] should be taken into account.

In the case of the ''π'' phase type superconductivity the critical temperature T_c is found from Eqs. (8), where function $\cot x$ has to be replaced by $-\tan x$ on the right-hand side of the second equation in addition to the above-mentioned replacement $d_j \Rightarrow d_j/2$. As shown in Fig. 3(a), the weak transparency σ_s and small parameter $2I\tau_f$ lead to a very insignificant difference between ''0'' phase type and ''π'' phase type solutions for T_c in the ''switching'' area. Therefore the experimentalists have probably observed on the multilayers Fe/V (Ref. 1) and Gd/Nb (Ref. 2) fast initial decrease T_c and the subsequent output on a plateau with an increase of d_f instead of oscillations. If the parameters σ_s and $2I\tau_f$ are

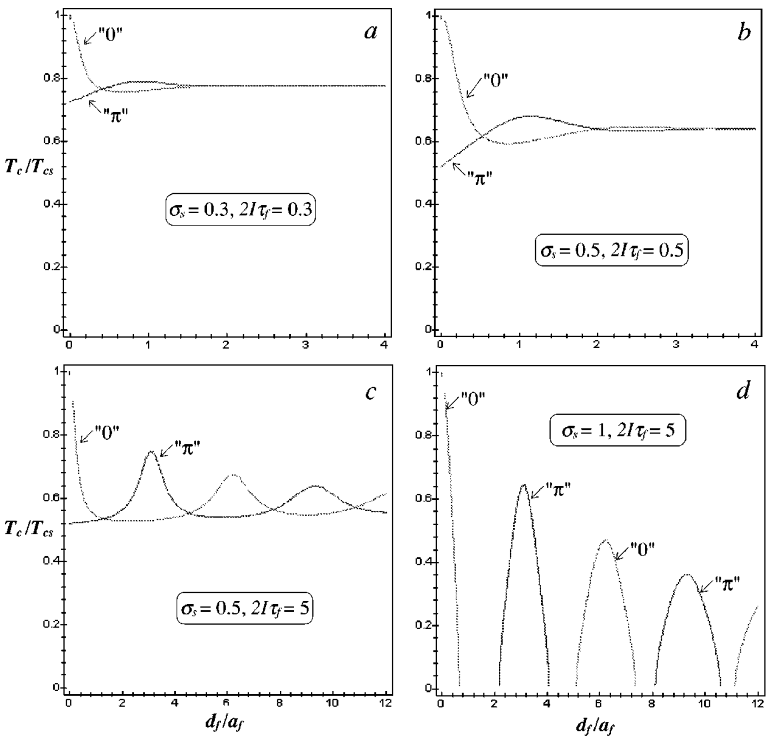

FIG. 3. The phase diagrams $T_c/T_{cs}(d_f/a_f)$ for F/S superlattices ($N_s v_s = N_f v_f$, $d_s = 500$ Å and $\xi_{so} = 400$ Å): (a) critical temperature flattens out; (b) local maximum of critical temperature; (c) critical temperature oscillations with a competition ''0'' phase type and ''π'' phase type of superconductivity; and (d) periodically reentrant superconductivity with alternation of ''0'' phase type and ''π'' phase type of peaks. The marks ''0'' and ''π'' near the curves correspond to ''0'' phase type and ''π'' phase type of superconductivity, respectively.

slightly larger than the preceding ones, there is an individual burst (local maximum) of critical temperature $T_c(d_f)$ as a result of the transition from ''0'' phase type branch of function $T_c(d_f)$ to ''π'' phase type one [see Fig. 3(b)]. That behavior of $T_c(d_f)$ is likely to have been observed in Refs. 4, 5 on the same Gd/Nb multilayers. Hence the difference between the results in Refs. 2, 4, and 5 may be explained by distinctions in the Gd/Nb boundary transparency and the Gd layer's purity, which may arise at the F/S multilayer's formation. At the large value of transparency ($\sigma_s = 5$, $2I\tau_f = 0.2$) on the phase diagram $T_c(d_f)$ two superconductivity regions corresponding to the ''0'' and ''π'' types are in the neighborhood and are separated by a nonsuperconducting region of thicknesses d_f. Thus there occurs the reentrant superconductivity in the F/S multilayers that resembles the one in Fig. 1(b).

The oscillations of $T_c(d_f)$ are essential for the higher values of $2I\tau_f$. Their period in the F/S multilayers for either ''0'' phase type of superconductivity or ''π'' phase type separately is twice more than that in appropriate F/S contact. The competition of the ''0'' and ''π'' states shown in Figs. 3(c) and 3(d) leads to their alternation on the phase diagrams and to formal coinciding of the oscillations period in superlattices and contacts. The availability of $T_c(d_f)$ oscillations does not mean an imperative presence in the ''π'' phase superconductivity in F/S multilayers, but the experiment here plays a crucial role. Thus to be certain that a ''π'' phase state is present on the $T_c(d_f)$ experimental phase diagram of F/S multilayers it is necessary to take parallel measurements on F/S contacts. This is important, since ''π'' phase superconductivity with the modulation of the order parameter and pair amplitude is the one-dimensional LOFF state[13] over the whole F/S superlattice in contrast to the ''0'' phase one in which the LOFF state is realized in F layers only.

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