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Effect of the magnetic permeability on photonic band gaps

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We study a class of photonic crystals in which both the dielectric constant ϵ and the magnetic permeability μ vary within the crystal. We find that in the case where both ϵ and μ have their maximum values in the same material, the photonic band gaps tend to disappear. However, in the opposite case where ϵ and μ have their maximum values in different materials, the gaps become wider with maximum gap over midgap frequency as high as 0.8. We also discuss the possibility of designing such photonic crystals with ferrites. [S0163-1829(97)06227-9]

Recently, there has been growing interest in the development of easily fabricated photonic band gap (PBG) materials;[1–22] these are periodic dielectric materials exhibiting frequency regions where electromagnetic (EM) waves cannot propagate. The interest in PBG materials arises from their possible applications in several scientific and technical areas such as filters, optical switches, cavities, design of more efficient lasers, etc.[1,2] Most of the research effort has been concentrated in the development of two-dimensional (2D) and three-dimensional (3D) PBG materials consisting of positive and frequency independent dielectrics[1–19] because, in that case, one can neglect the possible problems related to the absorption.[15,20] There are some more recent works on PBG materials constructed from metals[21,22] that suggest that those metallic structures may be advantageous in low-frequency regions where the metals become almost perfect reflectors.

For several proposed applications, such as efficient broadband antennas, we need PBG with wide stop bands. One possibility is to use photonic crystals with overlapping stop bands.[23] In this process several different PBG materials are stacked together, but the performance may be inferior to a single PBG crystal.

Here, we theoretically study photonic crystals with wide PBG constructed from ferrites where both the dielectric constant ϵ and the magnetic permeability μ change along the photonic crystal. As a first step, we ignore any anisotropy in ϵ and μ. In that case, we should solve the following differential equation:

$$\nabla\times[\epsilon^{-1}\nabla\times(\mu^{-1}\mathbf{B})]=\frac{\omega^2}{c^2}\,\mathbf{B}, \tag{1}$$

where the magnetic induction $\mathbf{B}$ should obey $\nabla\mathbf{B}=0$. It is interesting to point out that Eq. (1) is equivalent to

$$\nabla\times[\mu^{-1}\nabla\times(\epsilon^{-1}\mathbf{D})]=\frac{\omega^2}{c^2}\,\mathbf{D}, \tag{2}$$

where the displacement $\mathbf{D}$ should obey $\nabla\mathbf{D}=0$. This implies that by exchanging the values of ϵ and μ the results should remain the same.

Following previous work by Ho *et al.*,[3,4] we expand $\mathbf{B}$ in plane waves,

$$\mathbf{B}=\sum_{\mathbf{G}}\sum_{\lambda=1}^{2} h_{\mathbf{G},\lambda}\hat{e}_\lambda e^{i(\mathbf{k}+\mathbf{G})\mathbf{r}}, \tag{3}$$

where $\mathbf{k}$ is a wave vector in the Brillouin zone of the lattice, $\mathbf{G}$ is a reciprocal lattice vector, and $\hat{e}_1,\hat{e}_2$ are unit vectors perpendicular to $\mathbf{k}+\mathbf{G}$ because of the transverse nature of $\mathbf{B}$.

Substituting into Eq. (1) we obtain the following matrix equations:

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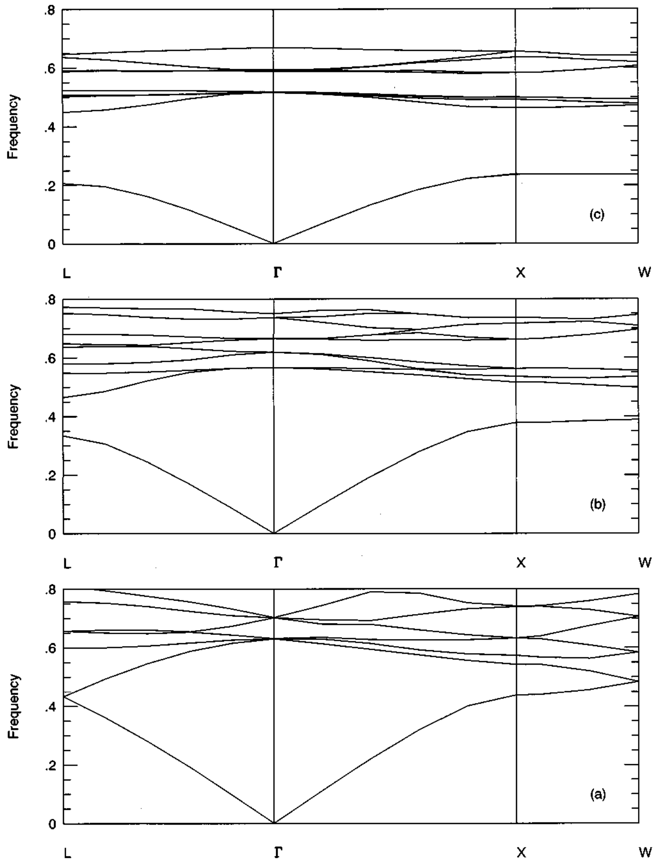

FIG. 1. The band structure of EM waves propagating in a diamond lattice consisting of spheres with filling ratio $f=0.34$. The ratio of the velocity of light is $r_v=1/4$ in all the cases. The ratio of the magnetic permeability is 4 (a), 1 (b), and 0.25 (c).

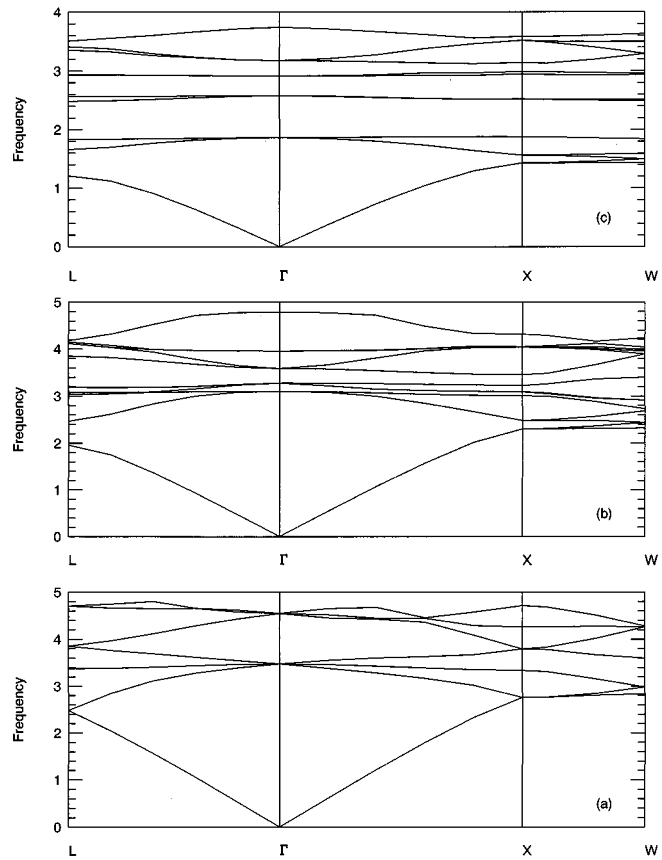

FIG. 2. The band structure of EM waves propagating in a fcc lattice consisting of spheres with filling ratio $f=0.86$. The ratio of the velocity of light is $r_v=4$ in all the cases. The ratio of the magnetic permeability is 0.25 (a), 1 (b), and 4 (c).

$$\sum_{\mathbf{G}',\lambda'} M^{\lambda,\lambda'}_{\mathbf{G},\mathbf{G}'} h_{\mathbf{G}',\lambda'} = (\omega^2/c^2) h_{\mathbf{G},\lambda}, \tag{4}$$

where

$$M^{\lambda,\lambda'}_{\mathbf{G},\mathbf{G}'} = \sum_{\mathbf{G}''} \epsilon^{-1}_{\mathbf{G}-\mathbf{G}''} \mu^{-1}_{\mathbf{G}''-\mathbf{G}'} (\mathbf{k}+\mathbf{G})[(\mathbf{k}+\mathbf{G}'')\hat{e}_{\lambda'}]\cdot \hat{e}_{\lambda} \tag{5}$$

and $\epsilon_{\mathbf{G}-\mathbf{G}''}$, $\mu_{\mathbf{G}''-\mathbf{G}'}$ are the Fourier transforms of $\epsilon(\mathbf{r})$ and $\mu(\mathbf{r})$. Results reported in this paper were obtained with matrix sizes of the order of 400. Such matrix sizes could create errors of up to 10%, especially in cases where the ratios of ϵ or μ are big. However, we calculated some of the high error cases using even higher matrix sizes (as high as 1600) and we found the same conclusions.

Figure 1 shows the band structure of a diamond lattice consisting of spheres with filling ratio 0.34 (touching spheres). In all the cases, the ratio of the velocities of light, $r_v=c_i/c_o$ is 1/4 (c_i and c_o are the velocities inside and outside of the spheres, respectively). For ratio of dielectric constants, $r_\epsilon=\epsilon_i/\epsilon_o=16$ and $\mu=1$ everywhere, there is a full band gap between the second and the third bands with gap over midgap frequency, $\Delta\omega/\omega_g=0.18$. For $r_\epsilon=4$ and ratio of the magnetic permeabilities, $r_\mu=\mu_i/\mu_o=4$, the gap disappears. However, for $r_\epsilon=64$ and $r_\mu=1/4$, the first gap becomes wider and a second gap appears between the eight and ninth bands. $\Delta\omega/\omega_g$ is 0.62 and 0.11 for the first and the second gap, respectively.

We find that the gaps become wider when ϵ and μ have their maximum values in different materials. This argument is further supported from the results shown in Fig. 2. In this figure, we show the band structure of EM waves propagating in a fcc lattice consisting of overlapping spheres with filling ratio $f=0.86$. In all the cases, the ratio of the velocities of light, $r_v=c_i/c_0=4$. For $r_\epsilon=1/16$ and $r_\mu=1$, it is well known[3,4] that there is no gap between the second and the third bands because these bands are degenerate at the W point [see Fig. 2(c)]. However, there is a higher, smaller gap between the eight and ninth bands with $\Delta\omega/\omega_g=0.015$.[24] For $r_\epsilon=1/4$ and $r_\mu=1/4$, there is no gap while for $r_\epsilon=1/64$ and $r_\mu=4$, there are three gaps in the frequency range that we examine. The first gap is between the fifth and sixth bands, the second appears between the eighth and ninth bands, and the third appears between the tenth and eleventh bands, with $\Delta\omega/\omega_g=0.28,0.12,0.04$, respectively.

Figure 3 shows the gap over the midgap frequency $\Delta\omega/\omega_g$ of the first gap as a function of the dielectric constant ratio r_ϵ for EM wave propagating in a diamond lattice consisting of spheres with filling ratio $f=0.34$. Results for three different ratios of the magnetic permeability ($r_\mu=0.25,1,4$ are shown. In all the cases, there is a critical value of r_ϵ

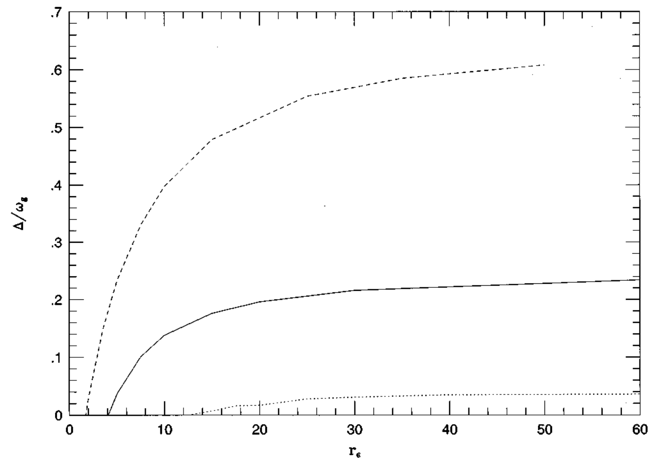

FIG. 3. The gap over the midgap frequency as a function of the ratio of the dielectric constants, $r_\epsilon = \epsilon_i/\epsilon_o$, for EM waves propagating in a diamond lattice consisting of spheres with filling ratio $f=0.34$. Solid, dotted, and dashed curves correspond to ratios of the magnetic permeability, $r_\mu = \mu_i/\mu_o = 1,4,0.25$.

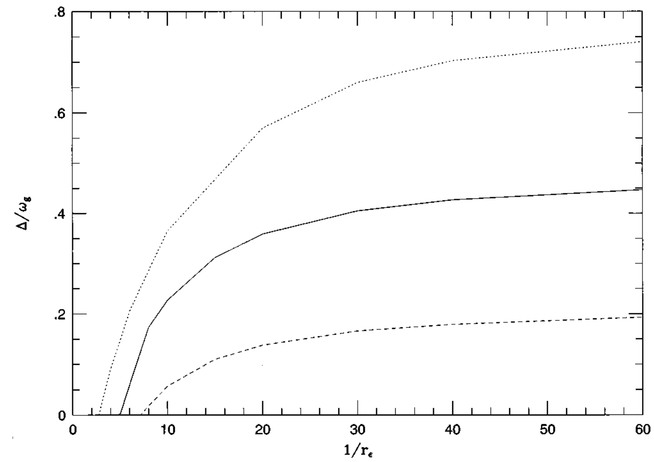

FIG. 4. The gap over the midgap frequency as a function of $1/r_\epsilon = \epsilon_o/\epsilon_i$, for EM waves propagating in a diamond lattice consisting of spheres with filling ratio $f=0.81$. Solid, dotted, and dashed curves correspond to ratios of the magnetic permeability, $r_\mu = \mu_i/\mu_o = 1,4,0.25$.

where the gap opens up while at high values of r_ϵ, $\Delta\omega/\omega_g$ saturates to a constant value. The critical value of r_ϵ is 12.5, 4.5, 1.8 and the saturated value of $\Delta\omega/\omega_g$ is 0.04, 0.23, and 0.62 for $r_\mu = 4,1,0.25$, respectively.

A similar plot is shown in Fig. 4 for EM waves propagating in a diamond lattice consisting of overlapping spheres with $f=0.81$. In this case, the critical value of r_ϵ is 0.14, 0.22, 0.4 while the saturated value of $\Delta\omega/\omega_g$ is 0.2, 0.45, 0.75 for $r_\mu = 0.25,1,4$, respectively.

Ferrites have a permeability tensor whose elements can be easily controlled through a dc magnetic bias field.[25] Such materials have been used as substrates in frequency-selective surfaces (FSS). The resonant frequency of the FSS may be varied by changing the applied magnetic field.[25] Very large values of the permeability μ can be achieved in ferrites at microwave frequencies generally below 100 MHz with low loss. However, at higher frequencies the permeability drops and the loss increases. The frequency below which the material has high permeability and low loss is approximately inversely proportional to the low-frequency permeability[26] for several soft ferrite materials. For example, the $K1$ and $K12$ classes of nickel-zinc ferrite have low-frequency permeabilities of 80 and 20 but μ decreases (accompanied by an increase in the loss) above 20 and 100 MHz, respectively, for these ferrites.[26]

Ferrites may be used in novel photonic crystals with another high dielectric material at such frequencies. The permeability of ferrites is strongly dependent on the applied magnetic field and with temperature. This makes possible the novel tuning of the photonic band gaps with applied magnetic fields and temperature, a particularly attractive feature for sensor applications.

Using the plane-wave expansion method, we calculated the band structure of electromagnetic waves propagating in photonic crystals. They are constructed from two different materials. We allow both the dielectric constant ϵ and the magnetic permeability μ to be varied along the photonic crystal. We found that the photonic band gaps become wider when ϵ and μ have their maximum values in different materials.

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[1] See J. Opt. Soc. Am. B **10**, 208 (1993), special issue on development and applications of materials exhibiting photonic band gaps.

[2] See *Photonic Band Gaps and Localization*, Proceedings of the NATO ARW, edited by C. M. Soukoulis (Plenum, New York, 1993).

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