# Optical anisotropic-dielectric response of mercuric iodide 

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#### Abstract

Anisotropic optical properties of mercuric iodide $\left(\mathrm{HgI}_{2}\right)$ were studied by variable-angle spectroscopic ellipsometry (VASE). Angular-dependent polarized reflectance and transmittance at three special optical-axis configurations, concerning the uniaxial anisotropic nature of the crystal, were derived to facilitate the VASE analysis. Two surface orientations of this tetragonal crystal were selected, i.e., an $a$-plane and a $c$-plane sample. Room-temperature multiple-angle spectroscopic ellipsometry measurements from both samples with three different optical configurations along with polarized transmission measurements were jointly analyzed by the VASE analysis through multiple-sample, multiple-model methods. Anisotropic dielectric functions of single-crystal $\operatorname{HgI}_{2}, \varepsilon_{\perp}(\omega)$ and $\varepsilon_{\|}(\omega)$, for optical electric-field vector oriented perpendicular and parallel to the $c$ axis, respectively, were obtained in the range $1.24-5.1 \mathrm{eV}$. Different absorption energy-band edges, at room temperature, were observed from the ordinary and extraordinary dielectric responses at 2.25 and 2.43 eV , respectively. This is consistent with the results related to the optical transitions between the conduction band and the heavy- and light-hole valence band indicated by theoretical studies. A surface model related to the surface roughness and defects of $\mathrm{HgI}_{2}$ was established and characterized by the VASE analysis. [S0163-1829(97)05539-2]


## I. INTRODUCTION

Crystal mercuric iodide $\left(\mathrm{HgI}_{2}\right)$ is an important technological material for room-temperature radiation (i.e., gamma and x ray) detectors. ${ }^{1-5}$ It also presents some interesting scientific issues regarding its tetragonal crystal structures. The optical anisotropic behavior was observed via the reflectance and absorption spectra of $\mathrm{HgI}_{2}$ crystal when the optical electric-field vectors $\mathbf{E}$ oriented parallel and perpendicular to the $c$ axis. ${ }^{6-11}$ The imaginary part of the dielectric functions $\varepsilon_{2 \perp}(\omega)(\mathbf{E} \perp c)$ were deduced from the reflectivity spectra by means of Kramers-Kronig (KK) relations under several assumptions. ${ }^{10}$ However, the optical anisotropic dielectric response of this crystal, such as dielectric functions ( $\varepsilon=\varepsilon_{1}$ $+\varepsilon_{2}$ ), have not been determined directly by precise optical measurements. Since $\mathrm{HgI}_{2}$ crystal is tetragonal, it must be described by two dielectric-response functions, $\varepsilon_{\perp}(\omega)$ and $\varepsilon_{\| \|}(\omega)$, for optical electric-field vector oriented perpendicular (ordinary) and parallel (extraordinary) to the $c$ axis, respectively. One frequently encountered difficulty is to calculate and analyze the optical measurement results in dealing with the uniaxial anisotropic nature of the crystal.

In this paper, we report the determination of roomtemperature anisotropic dielectric functions $\varepsilon_{\perp}(\omega)$ and $\varepsilon_{\|}(\omega)$ of $\mathrm{HgI}_{2}$ crystal in the range $1.24-5.1 \mathrm{eV}$, by variableangle spectroscopic ellipsometry (VASE). The optical functions were extracted from simultaneous analysis of VASE measurements of two $\mathrm{HgI}_{2}$ crystal samples: one $c$-plane surface, to which the $c$ axis of the crystal is perpendicular;
and another $a$-plane surface, to which the $c$ axis is parallel. An analytical angular-dependent polarized reflectance and transmittance at three special optical-axis configurations, for the uniaxial anisotropic crystal were derived and a surface model concerning the roughness and possible near-surface defects was established for the VASE analyses. Roomtemperature energy-band absorption edges of the $\mathrm{HgI}_{2}$ crystal from the ordinary and extraordinary dielectric response will be discussed and presented.

## II. EXPERIMENTAL DETAILS

## A. Sample preparations

Two $\mathrm{HgI}_{2}$ surfaces from this tetragonal crystal were prepared for the VASE measurements: a $c$-plane surface, to which the $c$ axis of the crystal is perpendicular, and an $a$-plane surface, to which the $c$ axis is parallel. Three special optical configurations were chosen for the VASE measurements: a $c$-plane surface, i.e., the $c$ axis is normal to the sample surface, which we term here as $c-p$ configuration; an $a$-plane surface with $c$ axis is in the plane of sample surface and parallel to the plane of incidence, $a E p$ configuration; and an $a$-plane surface with the $c$ axis perpendicular to the plane of incidence, aEs configuration.

Sliced and polished $c$-plane and $a$-plane $\mathrm{HgI}_{2}$ samples were subjected to a chemical etching to ensure clean and fresh surface before the VASE measurements. In the etching process, the samples were immersed in a $10 \% \mathrm{KI}$ (by weight) solution for about 2 min . After the KI etching, the samples
were immediately rinsed with deionized water for $\sim 2-3 \mathrm{~min}$. This KI etching treatment removes a surface layer of about $60 \mu \mathrm{~m}$ thickness (i.e., a $\sim 30 \mu \mathrm{~m} / \mathrm{min}$ etching rate). ${ }^{12}$ The fresh ruby red $\mathrm{HgI}_{2}$ surfaces were blown in dry air to remove the excess water. VASE measurements were performed immediately on the new fresh surfaces.

## B. VASE and polarized transmission measurements

The spectroscopic ellipsometry (SE) is designed to accurately determine the values of $\tan (\psi)$ and $\cos (\Delta)$, which are the amplitude and projected phase of the complex reflectance ratio,

$$
\begin{equation*}
\rho=r_{p} / r_{s}=\tan (\psi) e^{i \Delta} \tag{1}
\end{equation*}
$$

where $r_{p}$ and $r_{s}$ are the reflectance coefficients of light polarized parallel to $(p)$ or perpendicular to $(s)$ the plane of incidence. The $\psi$ and $\Delta$ are sensitive to changes of the surface conditions, overlayer thicknesses, dielectric functions, and other parameters of the sample. ${ }^{13,14}$

The pseudodielectric function $\langle\varepsilon\rangle$ is obtained from the ellipsometrically measured values of $\rho$, in a two-phase model (ambient/substrate):

$$
\begin{equation*}
\langle\varepsilon\rangle=\left\langle\varepsilon_{1}\right\rangle+i\left\langle\varepsilon_{2}\right\rangle=\varepsilon_{a}\left[\left(\frac{1-\rho}{1+\rho}\right)^{2} \sin ^{2} \Phi \tan ^{2} \Phi+\sin ^{2} \Phi\right], \tag{2}
\end{equation*}
$$

regardless of the possible presence of surface overlayers. ${ }^{13}$ The $\varepsilon_{a}$ in Eq. (2) represents the ambient dielectric function (i.e., $\varepsilon_{a}=1$ in vacuum). Since the $\mathrm{HgI}_{2}$ is a tetragonal crystal, it exhibits uniaxial anisotropy and must be described by two dielectric-response functions $\varepsilon_{\perp}(\omega)$ and $\varepsilon_{\|}(\omega)$, representing optical electric-field vector-oriented perpendicular (ordinary) and parallel (extraordinary) to the $c$ axis of the crystal, respectively.

Ellipsometric measurement of the complex ratio $\rho$ at a single wavelength and angle of incidence provides two quantities ( $\psi$ and $\Delta$ ), which can be used to determine two parameters describing the sample, e.g., dielectric function ( $\varepsilon=\varepsilon_{1}$ $+i \varepsilon_{2}$ ) of an isotropic bulk sample with no overlayer complications. Spectroscopic (multiple wavelengths) ellipsometric measurements greatly increase the number of determined parameters of the sample, especially when measurements were made at more than one angle of incidence. Thus, VASE makes possible the detailed evaluation of crystals with tetragonal structures.

Multiple angle SE measurements of the two $\mathrm{HgI}_{2}$ samples at the three special optical configurations, mentioned above, were taken in the range $1.24-5.1 \mathrm{eV}$ with an increment of 0.02 eV , at five different angles of incidence ranging from $67.5^{\circ}$ to $77.5^{\circ}$ with an increment of $2.5^{\circ}$. Polarized transmission measurements in the same spectral range with the same increment were also made from an $a$-plane $\mathrm{HgI}_{2}$ sample with the two optical configurations, i.e., $\mathbf{E} \perp\langle c\rangle$ and $\mathbf{E} \|\langle c\rangle$, respectively. The VASE and polarized transmission measurements were made using a variable-angle spectroscopic ellipsometer, which was equipped with a beam-chopped, rotating analyzer to increase the stray light rejection and signal-tonoise ratio.


FIG. 1. Surface model for VASE analysis.
A surface model was assumed, for the VASE analysis, including two top graded overlayers, a top surface rough layer, and a subsurface layer representing the distribution of possible surface defects, as shown in Fig. 1. In this model, each surface layer contains more than one constituent. The Bruggeman effective-medium approximation ${ }^{15}$ (EMA) is employed to calculate the effective optical constants of the mixed layer. It can be expressed as

$$
\begin{equation*}
f_{A} \frac{\varepsilon_{A}-\varepsilon}{\varepsilon_{A}+2 \varepsilon}+f_{B} \frac{\varepsilon_{B}-\varepsilon}{\varepsilon_{B}+2 \varepsilon}=0 \tag{3}
\end{equation*}
$$

where $\varepsilon$ is the effective dielectric function of the mixed layer. The values of $\varepsilon_{A}$ and $\varepsilon_{B}$ are the dielectric functions of the material $A$ and $B$, respectively, and $f_{A}$ and $f_{B}$ are the relative volume fractions. This is based on the assumption of a homogeneous mixture and a random-aggregate spherical microstructure (e.g., a $\mathrm{HgI}_{2}$ layer with voids). Notice that since $\mathrm{HgI}_{2}$ is anisotropic, the top two EMA layers, $d_{1}$ and $d_{2}$, are treated as anisotropic layers, in which $\varepsilon_{\perp}$ and $\varepsilon_{\|}$ satisfy Eq. (3), respectively. A detailed application of this surface model is presented later in this paper.

VASE data from the three different reflection geometries, from both $c$-plane and $a$-plane $\mathrm{HgI}_{2}$ samples, and the polarized transmission data from the $a$-plane surface were numerically fitted simultaneously through a multiple-sample and multiple-model regression analysis ${ }^{16}$ of the assumed surface model, by varying the anisotropic optical constants of the crystal as well as the surface roughness layer thicknesses and the voids volume percentage until the calculated and measured values match as closely as possible. This is done by minimizing the $\chi^{2}$ error function defined as

$$
\begin{align*}
\chi^{2}= & \sum_{i, j}\left\{\left[\frac{\psi\left(h \nu_{i}, \Phi_{j}\right)-\psi^{C}\left(h \nu_{i}, \Phi_{j}\right)}{\sigma_{i \psi}}\right]^{2}\right. \\
& \left.+\left[\frac{\Delta\left(h \nu_{i}, \Phi_{j}\right)-\Delta^{C}\left(h \nu_{i}, \Phi_{j}\right)}{\sigma_{i \Delta}}\right]^{2}\right\} \tag{4}
\end{align*}
$$

where $h \nu$ is the photon energy, $\Phi$ is the external angle of incidence, $\sigma$ is the measured standard deviation of the measurement, and $i$ and $j$ are used to sum over all the photon energies and external angles of incidence, respectively. In all three configurations, the interface reflection Jones matrix $r$ is diagonal. ${ }^{17}$

TABLE I. Fresnel's equations for anisotropic interfaces; the physical meaning of angles of $\theta_{1}, \theta_{2 e}$, and $\alpha$ is illustrated in Fig. 2.

| Scattering geometry | Polarization | $\mathrm{r}_{12}$ | $\mathrm{t}_{12}$ |
| :---: | :---: | :---: | :---: |
| 1:c axis normal to boundary (C-P) | p polarization | $\mathrm{r}_{12 \mathrm{p}}=\frac{\sqrt{\varepsilon_{2 \mathrm{~s}}}}{\sqrt{\varepsilon_{2 \mathrm{~s}}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos (\alpha)}$ | $\mathrm{t}_{12 \mathrm{p}}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{2 \mathrm{~s}}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos (\alpha)}$ |
|  |  | $\begin{gathered} \sqrt{\varepsilon_{2 \mathrm{~s}}}=\frac{\sqrt{\varepsilon}}{\sqrt{\left[\sqrt{\varepsilon_{2 \\|}} \cos \left(\theta_{2 \mathrm{e}}\right)\right.},} \\ \tan (\alpha)=-\left(\frac{\varepsilon_{2 \perp}}{\varepsilon_{2 \\|}}\right) \tan \left(\theta_{2 \mathrm{e}}\right), \end{gathered}$ | $\begin{aligned} & =\frac{\sqrt{\varepsilon_{1}} \sin \left(\theta_{1}\right)}{\left.\sin \left(\theta_{2 \mathrm{e}}\right)\right]^{2}} \cos \left(\theta_{2 \mathrm{e}}-\alpha\right), \\ & \sqrt{\sqrt{\varepsilon_{2 \perp}}\left[1-\left(\frac{\varepsilon_{1}}{\varepsilon_{2 \\|}}\right)\left(\sin \left(\theta_{1}\right)\right)^{2}\right]^{1 / 2}} . \end{aligned}$ |
|  | $s$ polarization | $\mathrm{r}_{12 \mathrm{~s}}=\frac{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{2 \perp}}}{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{2 \mathrm{o}}\right)}$ | $\mathrm{t}_{12 \mathrm{~s}}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{1} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{21}}} \cos \left(\theta_{2 \mathrm{oj}}\right)}$ |
|  |  | $\sin \left(\theta_{2}\right.$ | $\frac{\sin \left(\theta_{1}\right)}{\varepsilon_{2 \perp}}$ |
| 2: caxis in boundary II to plane of incidence (aEp) | p polarization | $\mathrm{r}_{12 \mathrm{p}}=\frac{\sqrt{\varepsilon_{2 s}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{1}} \cos (\alpha)}{\sqrt{\varepsilon_{2 s}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos (\alpha)}$ | $\mathrm{t}_{12 \mathrm{p}}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{2 \mathrm{~s}}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos (\alpha)}$ |
|  |  | $\begin{aligned} & \sqrt{\varepsilon_{2 \mathrm{~s}}}=\frac{\sqrt{\varepsilon_{2}}}{\sqrt{\left[\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{2 \mathrm{e}}\right)\right.}} \\ & \tan (\alpha)=-\left(\frac{\varepsilon_{2 \\|}}{\varepsilon_{2 \perp}}\right) \tan \left(\theta_{2 \mathrm{e}}\right) \end{aligned}$ | $\begin{aligned} & \overline{\left.n_{\\|} \sin \left(\theta_{2 e}\right)\right]^{2}} \cos \left(\theta_{2 \mathrm{e}}-\alpha\right), \\ & \tan \left(\theta_{2 \mathrm{e}}\right)=\frac{\sqrt{\varepsilon_{1}} \sin \left(\theta_{1}\right)}{\sqrt{\varepsilon_{2 \\|}} \cos \left(\theta_{2 \mathrm{o}}\right)} . \end{aligned}$ |
|  | s polarization | $\mathrm{r}_{12 \mathrm{~s}}=\frac{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{20}\right)}{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{2 \mathrm{o}}\right)}$ | $\mathrm{t}_{12 s}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{1} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{21}} \cos \left(\theta_{2 \sigma}\right)}}$ |
|  |  | $\sin \left(\theta_{2}\right.$ | $\frac{\sin \left(\theta_{1}\right)}{\overline{\varepsilon_{2 \perp}}}$ |
| 3: c axis in boundary $\perp$ to plane of incidence (aEs) | p polarization | $\mathrm{r}_{12 \mathrm{p}}=\frac{\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{1}} \cos \left(\theta_{2 \mathrm{o}}\right)}{\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos \left(\theta_{2 \mathrm{o}}\right)}$ | $\mathrm{t}_{12 \mathrm{p}}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{2 \perp}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{1}} \cos \left(\theta_{2 \mathrm{o}}\right)}$ |
|  |  | $\sin \left(\theta_{2 \mathrm{o}}\right)=\frac{\sqrt{\varepsilon_{1}} \sin \left(\theta_{1}\right)}{\sqrt{\varepsilon_{2 \perp}}}$ |  |
|  | s polarization | $\mathrm{r}_{12 \mathrm{~s}}=\frac{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)-\sqrt{\varepsilon_{2 \\|}} \cos \left(\theta_{2 \mathrm{e}}\right)}{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{22}} \cos \left(\theta_{2 \mathrm{e}}\right)}$ | $\mathrm{t}_{12 \mathrm{~s}}=\frac{2 \sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)}{\sqrt{\varepsilon_{1}} \cos \left(\theta_{1}\right)+\sqrt{\varepsilon_{2 \\|}} \cos \left(\theta_{2 \mathrm{e}}\right)}$ |
|  |  | $\sin \left(\theta_{2 \mathrm{e}}\right)=\frac{\sqrt{\varepsilon_{1} \sin \left(\theta_{1}\right)}}{\sqrt{\varepsilon_{2 \\|}}}$ |  |

## III. RESULTS

## A. Fresnel's equations for anisotropic interfaces

VASE analysis employs the coefficients of reflectance and transmittance as functions of spectral optical dielectric response of each media and the angle of incidence at the boundary of the two media. For isotropic materials, they are well known as Fresnel's equations. For anisotropic media,
this issue has been theoretically studied and applied to certain cases. ${ }^{18}$ However, the complete analytical expressions including reflection and transmittance are not readily available. In Ref. 17, the study was limited to reflection SE measurements on a bulk anisotropic material. Therefore, only reflection coefficients were derived and used in the ellipsometry analysis. In our studies, reflection VASE and polarized transmission data from anisotropic samples were analyzed


FIG. 2. Refraction by a uniaxial anisotropic medium. The optical axis is in the plane of incidence and perpendicular to the sample surface.
simultaneously in a multiple-model, multiple-sample approach; hence, reflectance and transmittance equations are needed for our VASE analysis. Furthermore, those equations are also needed in studying the anisotropic materials by using transmission VASE measurements. ${ }^{13}$ In this section, we present the Fresnel's equations for anisotropic interfaces in the three special optical configurations in Table I.

Those equations were derived based on Maxwell's equations and the properties of uniaxial anisotropic media (crystals). In an isotropic medium, the electric-field $\mathbf{E}$ and displacement field vector $\mathbf{D}$ are in the same direction and always normal to the wave propagation and the velocity of a plane wave is a constant independent of the direction of propagation in the medium. Therefore, the refractive indices of an isotropic medium is a sphere corresponding to a constant frequency. These are no longer true in anisotropic media, except for certain particular directions. For uniaxial anisotropic crystals, the electric-field vector $\mathbf{E}$ and displacement field vector $\mathbf{D}$ are in general not in the same direction. So does the wave propagation direction represented by the Poynting vector $\mathbf{S}$ and the direction of the wave vector $\mathbf{k}$, as shown in Fig. 2. The angle of incidence $\theta_{1}$, and other angles $\theta_{2 e}$ and $\alpha$, used in the Fresnel's equations listed in Table I are illustrated in Fig. 2.

## B. Analysis and discussion

Figure 3 shows pseudodielectric response $\langle\varepsilon\rangle$ of an $a$-plane single-crystal $\mathrm{HgI}_{2}$ surface, measured by VASE with $a E p$ and $a E s$ configuration, respectively, at five different angles of incidence ranging from $67.5^{\circ}$ to $77.5^{\circ}$ with an increment of $2.5^{\circ}$. The obviously different pseudodielectric responses from the $a E p$ and $a E s$ geometries, and angular dispersions of $\left\langle\varepsilon_{1}\right\rangle$ and $\left\langle\varepsilon_{2}\right\rangle$ in both situations provide strong evidence of the uniaxial anisotropic nature of $\mathrm{HgI}_{2}$ crystal. The $c$ axis of this $a$-plane sample was at first assumed parallel to one of its cleaved edges. This assumption was proved to be correct in the later VASE analysis.

Polarized transmission measurements were made on the $a$-plane surface with optical electric-field vector $\mathbf{E}$ oriented parallel $(\mathbf{E} \|\langle c\rangle)$ and perpendicular $(\mathbf{E} \perp\langle c\rangle)$ to the $c$ axis, respectively, in a range from 1.24 to 2.5 eV with an increment of 0.02 eV , as shown in Fig. 4.

The $\mathrm{HgI}_{2}$ surface was modeled as two graded layers containing voids, labeled as $d_{1}$ and $d_{2}$ in thickness, as shown in


FIG. 3. Pseudodielectric response $\langle\varepsilon\rangle$ of an $a$-plane singlecrystal $\mathrm{HgI}_{2}$ surface, measured by VASE with $a E p$ and $a E s$ configuration, respectively, at five different angles of incidence ranging from $67.5^{\circ}$ to $77.5^{\circ}$ with an increment of $2.5^{\circ}$.

Fig. 1. The top layer $\left(d_{1}\right)$ is mainly used to describe the physical roughness of the sample surface, while the sublayer $\left(d_{2}\right)$ is designed to reflect the possible subsurface defects. In the top layer $\left(d_{1}\right)$, the volume voids percentage was linearly graded from $50 \%$ to $x \%$, while in the sublayer $\left(d_{2}\right)$ the voids percentage was linearly graded from $x \%$ to $0 \%$. The linear grading was achieved by dividing one layer into five thin slices with evenly incremented $x$ values. The effective optical constants of the mixed graded layers used in the VASE analysis were calculated using the Bruggeman EMA. During the VASE regression analysis, the coupled optical constants of the $\mathrm{HgI}_{2}$ in both bulk and the EMA top layers, $\varepsilon_{1 \perp}, \varepsilon_{1 \|}$, $\varepsilon_{2 \perp}, \varepsilon_{2 \|}$, and the thicknesses of the top ( $d_{1}$ ) and subsurface $\left(d_{2}\right)$ layers, plus the voids percentage $x$ in the subsurface layer were varied to fit all the VASE data from two samples in $c-p, a E p$, and $a E s$ configurations and the polarized transmission data, simultaneously. Each sample was described by its own surface model, the numerical fitting was based on the assumption that the optical constants of the $c$-plane and $a$-plane samples are the same and they were


FIG. 4. Polarized transmission of $\mathrm{HgI}_{2}$.
a-plane sample (N16-12):



(b)

| graded $\mathrm{Hgl}_{2} /$ Voids $(50-\mathrm{x} \%)$ | $88.9 \AA$ |
| :---: | ---: |
| graded $\mathrm{Hgl}_{2}$ Voids $(\mathrm{x} \%-0) \quad 998.2 \AA$ |  |
| $\mathrm{Hgl}_{2}$ Substrate |  |

(a)

FIG. 5. VASE analysis of the model fit from the $a$-plane $\mathrm{HgI}_{2}$. The solid line represents the best fit and the dotted or dashed lines are the experimental VASE and transmission data, in (a) and (b), respectively. The sketch at the lower right is the surface model for the analysis.
c-plane sample (N15-21):



FIG. 6. VASE analysis of the model fit from the $c$-plane $\mathrm{HgI}_{2}$. This $c$-plane sample was analyzed simultaneously with the $a$-plane sample shown in Fig. 5 via multiple sample VASE analysis.


FIG. 7. Anisotropic dielectric functions of $\mathrm{HgI}_{2}$ with ordinary $(\mathbf{E} \perp c)$ and extraordinary $(\mathbf{E} \| c)$ responses, respectively.
coupled together through a multiple-sample VASE analysis. The multiple-sample method allows us to analyze several independent VASE data simultaneously, and couple some of the common optical constants together during the numerical fitting. This method greatly increases the ratio of SE determined quantities to fitting parameters, so that reduces the possible coupling between fitting variables.

As an illustration, we show the results of multiple-sample VASE analysis in Figs. 5 ( $a$ plane) and 6 ( $c$ plane). For the $a$-plane sample, four angles of incidence of VASE data of each optical configuration (i.e., $a E p$ and $a E s$, respectively) were fitted, while five angles of incidence VASE data from the $c$-plane sample were used for analysis. As we can see, the model fit (the solid line) and the experimental data (the dotted or dashed lines) are matched very closely. The VASE analysis shows that this $a$-plane sample (N16-12) has a top roughness layer of $\sim 88.9 \AA$ followed by a subsurface layer of $\sim 998.2 \AA$ with $x=10.9 \%$, while the $c$-plane $\mathrm{HgI}_{2}$ sample resulted in thicknesses of 80.6 and $983.1 \AA$ with $x$ $=12.1 \%$, for the top and subsurface layer, respectively. We believe the surface roughness layers described by EMA, especially the subsurface layer, are related to surface defects and surface aging of $\mathrm{HgI}_{2}$ crystal. A detailed study of the surface aging of $\mathrm{HgI}_{2}$ crystal is reported in a separate paper. ${ }^{19}$


FIG. 8. The Kramers-Kronig fit of the dielectric functions of $\mathrm{HgI}_{2} . \varepsilon_{\perp}$ and $\varepsilon_{\|}$refer to ordinary and extraordinary responses of dielectric functions, respectively.

TABLE II. Parameter values of the KK fit of the dielectric functions of $\mathrm{HgI}_{2}$.

|  | $\varepsilon_{1}^{\text {offset }}$ | $E_{i}(\mathrm{eV})$ | $A_{i}$ |
| :--- | :--- | :--- | :--- |
| $E_{\perp}$ | 3.516 | $E_{1}=5.3228, E_{2}=1.1079$ | $A_{1}=2.9958, A_{2}=0.3634$ |
| $E_{\\|}$ | 3.422 | $E_{1}=5.4225, E_{2}=0.9998$ | $A_{1}=5.6906, A_{2}=0.68639$ |

The surface model with two top graded EMA layers, as shown in Fig. 1, was decided by comparisons of regression analyses on several different surface models, such as bulk only, one top EMA layer with graded or without graded $x$ values, etc. The minimized $\chi^{2}$ error function, defined in Eq.

TABLE III. Anisotropic optical properties of $\mathrm{HgI}_{2}$.

| eV | $\varepsilon_{1 \perp}$ | $\varepsilon_{2 \perp}$ | $\varepsilon_{1 \\|}$ | $\varepsilon_{2 \\|}$ | $\alpha_{\perp}\left(10^{3} \mathrm{~cm}^{-1}\right)$ | $\alpha_{\\|}\left(10^{3} \mathrm{~cm}^{-1}\right)$ |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| 5.100 | 1.792 | 2.838 | 2.825 | 4.050 | 457.14 | 531.38 |
| 5.000 | 1.622 | 3.011 | 3.009 | 4.334 | 480.50 | 539.56 |
| 4.900 | 1.445 | 3.191 | 3.201 | 4.629 | 503.79 | 547.14 |
| 4.800 | 1.190 | 3.437 | 3.461 | 5.052 | 538.17 | 561.37 |
| 4.700 | 1.066 | 3.711 | 3.642 | 5.148 | 563.11 | 549.77 |
| 4.600 | 0.895 | 4.143 | 3.958 | 5.396 | 602.88 | 545.11 |
| 4.500 | 0.772 | 4.645 | 4.343 | 5.495 | 639.85 | 526.10 |
| 4.400 | 0.657 | 5.239 | 4.685 | 5.582 | 678.03 | 508.66 |
| 4.300 | 0.760 | 5.968 | 5.047 | 5.580 | 706.60 | 485.03 |
| 4.200 | 1.144 | 6.818 | 5.399 | 5.501 | 722.99 | 457.36 |
| 4.100 | 1.854 | 7.483 | 5.705 | 5.338 | 711.01 | 426.61 |
| 4.000 | 2.614 | 7.839 | 5.967 | 5.281 | 681.38 | 405.58 |
| 3.900 | 3.317 | 8.040 | 6.269 | 5.257 | 648.34 | 386.56 |
| 3.800 | 3.996 | 8.224 | 6.711 | 5.214 | 617.89 | 364.13 |
| 3.700 | 4.702 | 8.400 | 7.158 | 5.010 | 588.43 | 333.23 |
| 3.600 | 5.867 | 8.387 | 7.749 | 4.442 | 539.25 | 280.57 |
| 3.500 | 6.853 | 7.940 | 7.953 | 3.777 | 478.26 | 231.45 |
| 3.400 | 7.434 | 7.327 | 7.924 | 3.196 | 422.35 | 191.90 |
| 3.300 | 7.808 | 6.916 | 7.741 | 2.795 | 383.00 | 165.42 |
| 3.200 | 8.251 | 6.748 | 7.647 | 2.651 | 355.86 | 153.22 |
| 3.100 | 8.983 | 6.369 | 7.748 | 2.420 | 316.46 | 134.99 |
| 3.000 | 9.538 | 5.706 | 7.699 | 2.167 | 269.94 | 117.62 |
| 2.900 | 10.060 | 4.856 | 7.683 | 1.946 | 219.02 | 102.36 |
| 2.800 | 10.164 | 3.841 | 7.636 | 1.684 | 168.08 | 85.98 |
| 2.700 | 9.907 | 3.036 | 7.551 | 1.506 | 130.48 | 74.62 |
| 2.600 | 9.635 | 2.584 | 7.508 | 1.383 | 108.71 | 66.21 |
| 2.500 | 9.487 | 2.300 | 7.554 | 1.310 | 93.93 | 60.15 |
| 2.400 | 9.595 | 2.090 | 7.960 | 1.200 | 81.57 | 51.59 |
| 2.300 | 9.655 | 1.513 | 7.892 | 0.000 | 56.59 | 0.00 |
| 2.200 | 10.088 | 0.395 | 7.309 | 0.000 | 13.87 | 0.00 |
| 2.100 | 8.902 | 0.000 | 6.967 | 0.000 | 0.00 | 0.00 |
| 2.000 | 8.385 | 0.000 | 6.719 | 0.000 | 0.00 | 0.00 |
| 1.900 | 8.021 | 0.000 | 6.522 | 0.000 | 0.00 | 0.00 |
| 1.800 | 7.729 | 0.000 | 6.328 | 0.000 | 0.00 | 0.00 |
| 1.700 | 7.497 | 0.000 | 6.152 | 0.000 | 0.00 | 0.00 |
| 1.600 | 7.210 | 0.000 | 5.963 | 0.000 | 0.00 | 0.00 |
| 1.500 | 6.987 | 0.000 | 5.857 | 0.000 | 0.00 | 0.00 |
| 1.400 | 6.735 | 0.000 | 5.695 | 0.000 | 0.00 | 0.00 |
| 1.300 | 6.503 | 0.000 | 5.488 | 0.000 | 0.00 | 0.00 |
| 1.200 | 6.259 | 0.000 | 5.291 | 0.000 | 0.00 | 0.00 |
|  |  |  |  |  |  |  |

(4), of the proposed surface model was significantly lower than other models. On the other hand, as shown in Fig. 3, the nonzero $\left\langle\varepsilon_{2}\right\rangle$ values under 2.25 eV (the lower absorption edge observed by the polarized transmission), also suggests a top surface layer structure.

Figure 7 shows the room-temperature (RT) anisotropic dielectric functions of $\mathrm{HgI}_{2}$ crystal in a range of 1.24-5.1 eV , as results of the VASE analysis. It is noticeable that the absorption edges of the ordinary and extraordinary dielectric response $\varepsilon_{2}$ are different: $\sim 2.25 \mathrm{eV}$ for the ordinary and $\sim 2.43 \mathrm{eV}$ for the extraordinary response. This is consistent with the results related to the optical transitions between the conduction band and the heavy- and light-hole valence band indicated by theoretical studies. ${ }^{20}$

KK transformation relation was employed to check the consistency of the anisotropic dielectric functions. The KK transformation reflects the nature of relation between the real and imaginary part of the dielectric function $\varepsilon=\varepsilon_{1}+i \varepsilon_{2}$, and can be written as

$$
\begin{equation*}
\varepsilon_{1}(\hbar \omega)=1+\frac{2}{\pi} p \int_{0}^{\infty} \frac{x \varepsilon_{2}(x)}{x^{2}-(\hbar \omega)^{2}} d x \tag{5}
\end{equation*}
$$

where $\hbar \omega$ is the photon energy. By the KK transformation, the real part of the dielectric function $\varepsilon_{1}$ can be obtained through the imaginary part $\varepsilon_{2}$. However, the KK transformation integrates the entire spectral range, while our VASE measurements are limited in the range of $1.24-5.1 \mathrm{eV}$. Two nonbroadening oscillators were employed to cover the unmeasured spectral range. The modified KK transformation is written as

$$
\begin{align*}
\varepsilon_{1}^{\mathrm{KK}}(\hbar \omega)= & \varepsilon_{1}^{\mathrm{offset}}+\sum_{i=1}^{2} \frac{A_{i}}{(\hbar \omega)^{2}-E_{i}^{2}} \\
& +\frac{2}{\pi} p \int_{1.24 \mathrm{eV}}^{5.1 \mathrm{eV}} \frac{x \varepsilon_{2}^{\mathrm{meas}}(x)}{x^{2}-(\hbar \omega)^{2}} d x \tag{6}
\end{align*}
$$

where $A_{i}$ and $E_{i}$ are the amplitude and center energy for the $i$ th oscillator, respectively. An $\varepsilon_{1}^{\text {offset }}$ was used to replace the unit value in the KK relation. Thus, values of $\varepsilon_{2 \perp}$ (ordinary) and $\varepsilon_{2 \|}$ (extraordinary) obtained through VASE analysis were used to calculate $\varepsilon_{1 \perp}$ (ordinary) and $\varepsilon_{1\| \|}$ (extraordinary) via the KK relation Eq. (6), respectively. The calculated values were compared with the VASE determined $\varepsilon_{1 \perp}$ and $\varepsilon_{1 \|}$, through a regression analysis by varying the values of $A_{i}$, $E_{i}$, and $\varepsilon_{1}^{\text {offset }}$ until calculated and measured values match as
closely as possible. The results of the KK fit are shown in Fig. 8 and Table II. It is obvious that a good KK fit was obtained by applying one oscillator on the higher-energy side and one on the lower-energy side for each set of $\varepsilon$, as indicated by the arrows in Fig. 8. The lower-energy-side oscillator is often not needed because of the simple nonabsorbing behavior below the material band gap. However, one oscillator with low-center-energy is needed in this case because of the strong zone-center phonon absorption of $\mathrm{HgI}_{2}$ crystal. ${ }^{21}$

Selected values of $\varepsilon_{\perp}(\omega)$ and $\varepsilon_{\|}(\omega)$, shown in Fig. 7, are listed in Table III. Notice that the listed values at 1.2 eV were extrapolated from Fig. 7. Also listed are polarized absorption coefficients of $\mathrm{HgI}_{2}, \alpha_{\perp}$ and $\alpha_{\|}$, for optical electricfield vector $\mathbf{E}$ oriented perpendicular and parallel to the $c$ axis, respectively. The absorption coefficients were calculated through $\langle\alpha\rangle=4 \pi\langle k\rangle / \lambda$, where $\langle k\rangle$ is the extinction coefficients of the refractive indices of $\mathrm{HgI}_{2}$.

## IV. CONCLUSIONS

In summary, we have studied the anisotropic optical properties of $\mathrm{HgI}_{2}$ crystal, by variable angle spectroscopic ellipsometry. Polarized reflectance and transmittance at three special optical-axis configurations, concerning the uniaxial anisotropic nature of the crystal, were derived as functions of angles of incidence to facilitate the VASE analysis. Two surface orientations of the tetragonal crystal were chosen, i.e., an $a$-plane and a $c$-plane surface, for the study. Roomtemperature anisotropic dielectric functions of $\mathrm{HgI}_{2}$ crystal, both ordinary $\varepsilon_{\perp}(\omega)$ and extraordinary $\varepsilon_{\|}(\omega)$, in the range of $1.24-5.1 \mathrm{eV}$, were extracted by the VASE measurements and multiple-sample and multiple-model analysis. Both dielectric functions satisfy the Kramers-Kronig relation, respectively. Different energy-band absorption edges were observed, at room temperature, from the ordinary and extraordinary dielectric responses at 2.25 and 2.43 eV , respectively. They are related with optical transitions between the conduction band and the heavy- and light-hole valence band.

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