Diffusion-constant renormalization in the weakly disordered t-J model: Recovery of a metal-insulator transition in a generic disorder model

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We examine the metallic phase of the t-J model in the presence of weak site disorder using the $1/N$ expansion to treat the strong electron correlations. To next leading order in $1/N$ we calculate the quasiparticle interactions including diffusive effects. We then investigate the effect of these quasiparticle interactions on the diffusion constant perturbatively as a first step towards a full scaling theory. We find that in $2+\epsilon$ dimensions a bona-fide Anderson transition is recovered. [S0163-1829(97)06339-X]

I. INTRODUCTION

The interplay between strong electron correlations and disorder remains one of the open questions in condensed matter. At present the most comprehensive treatments start from the weak disorder problem and introduced correlations into the microscopic theory that describes the Anderson transition. This type of approach, as introduced by Finkelshtein[1] and reviewed recently in,[2] examines correlation effects through their renormalization of the diffusion constant and in turn incorporates disorder effects on the Fermi-liquid parameters that describe the electron correlations. The resulting renormalization-group equations predict metal-insulator transitions when symmetry-breaking interactions (magnetic field, magnetic impurities, and spin-orbit scattering) are present. However, one type of model in which the situation remains unclear are the ''generic'' models with no symmetry-breaking interactions. In this case it is found that the Landau parameter corresponding to spin-triplet particle-hole interactions (the paramagnon channel) diverges under disorder scaling implying a preemptive magnetic transition close to two dimensions.

Another line of approach is to start from the strongly correlated electron system and turn on the disorder, avoiding the initial invocation of Fermi liquid parameters and instead dealing with actual model parameters. In the context of heavy fermion systems, a number of authors have considered this approach either using the Gutzwiller approach[3] or by enforcing the local occupation constraints using auxilliary bosons.[4] Disorder of either the site or bond type is then introduced[5] and its effects on the mean-field theory are calculated. While the thermodynamics of the heavy fermion state is largely unaffected by disorder,[5] transport properties show different behavior [either by treating the disorder within coherent potential approximation[6] (CPA) or by summing maximally crossed graphs[7]]. Kondo insulators also have been profitably investigated this way.[8–10]

In the context of one-band models Zimanyi and Abrahams[11] studied the t-J model with site disorder present using slave bosons at the mean-field level to incorporate occupation restrictions in the hopping term, but treated the antiferromagnetic term as an electron interaction that scaled under disorder using the Finkelshtein approach. The site disorder was coupled to the hopping via the charge susceptibility, which led to a broadening of the Mott-Hubbard region.

Overall, the nature of the ground state of the pure t-J model is still not settled. The simplest mean-field theories that yield a metallic ground state already incorporate some aspects of the antiferromagnetic interaction via a renormalization of the electron hopping term, thus yielding large Fermi surfaces consistent with photoemission results on high-T_c materials. It makes sense therefore to examine the effect of weak disorder within such a formalism, although to really make good contact with the comprehensive approaches applied to weakly interacting disordered systems[2] one has to proceed beyond mean-field level. This is because these approaches study the renormalization of parameters such as the diffusion constant, which itself is only obtained by studying diffusive corrections to the particle-hole propagator.

Moreover, to obtain the effect of correlations on the diffusion constant we need to derive the effective interaction between the weakly disordered quasiparticles. This definitely requires proceeding to the $1/N$ Gaussian level and summing up diffusive corrections to the boson propagators. Previously we studied the infinite-U Hubbard model within this very approach,[12] calculating the free energy and quasiparticle lifetime, by specifically including such diffusion pole graphs into the fluctuating boson propagators that appear at the Gaussian $1/N$ level, although there we did not actually derive the quasiparticle interaction.

Our purpose in the present paper is to extend our previous work to the t-J model in the presence of weak site disorder, again summing diffusion pole graphs that determine the boson propagators, but this time specifically in order to obtain the quasiparticle interaction. We then use this quasiparticle interaction to renormalize the diffusion propagator itself following the diagrammatic procedure of Castellani *et al.*,[13] which, as these authors showed, is completely equivalent to the Finkelshtein renormalization-group approach. Although we do not attempt the full scaling theory here, we will show that a generic disorder model such as the t-J model does recover a bona-fide metal-insulator transition in the present approach.

II. FORMULATION

In this section we formulate the t-J model in the presence of disorder following our previous line of approach to the

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infinite-U Hubbard model.[12] Our formulation reduces in the pure case to the $1/N$ treatment of the t-J model by Wang, Bang, and Kotlia.[14] The main difference with our previous infinite-U work[12] is then in the inclusion of the antiferromagnetic coupling. At the mean-field level this leads to an extra contribution to the quasifermion hopping rate, while at the $1/N$ level it contributes to separate radial and angular bond fluctuations in addition to the already present hopping and constraint field fluctuations. These extra Gaussian level contributions are related to density and chirality fluctuations in the strongly correlated Fermi liquid. Our starting point is the Lagrangian for the t-J model,

$$L(\tau)=\sum_i [f_{i\sigma}^\dagger(d/d\tau-\mu)f_{i\sigma}+b_i^\dagger d/d\tau b_i + i\lambda_i(f_{i\sigma}^+f_{i\sigma}+b_i^+b_i-N/2)] \quad (1)$$

$$+\sum_{i,j} [N/J|\Delta_{ij}|^2-f_{i\sigma}^\dagger f_{j\sigma}(\Delta_{ij}-t/Nb_j^\dagger b_i) + \text{H.c.}], \quad (2)$$

where $f_{i\sigma}$ destroys a spin carrying fermion at site i, b_i destroys a charge carrying boson at site i, while the infinite-U constraint is enforced by the Lagrange multiplier λ. The complex bosonic field Δ_{ij} denotes the valence bond variable, which in the large-N approach is introduced via a Hubbard-Stratonovich transformation to decouple the exchange term in the t-J model. The hopping rate, spin exchange, and chemical potential are as usual denoted by t, J, and μ, respectively. The partition function in this representation is then given by a functional integral over the fermion and auxilliary fields

$$Z=\int D\Delta^\dagger D\Delta\ Db^\dagger Db\ Df^\dagger Df\ D\lambda\ \exp\left[-\int L(\tau)d\tau\right]. \quad (3)$$

Added to the above Lagrangian is a term

$$L_i=\sum_{r_i,R_a} u(r_i-R_a)f_{i\sigma}^\dagger f_{i\sigma} \quad (4)$$

denoting the effect of a set of random impurities located at sites R_a. The impurity potential couples to the local quasiparticle density as in other auxilliary boson treatments.[5–7,11] As far as the formal aspects are concerned, we then follow standard large-degeneracy treatments and integrate over the fermion fields yielding a effective boson Lagrangian. This is then expanded around its saddle point, yielding a mean-field component plus a Gaussian correction.

The mean-field equations thus obtained are given in terms of the saddle-point results through the single-particle Green's function, which is still in a site representation since the disorder average is yet to be performed. In keeping with the standard diffusion pole approaches, we keep only the lowest-order Born approximation to the single-particle Green's function. Since the disorder is in such cases assumed to be weak, we can easily show that the averages over the single-particle Green's function in the mean-field equations yield corrections of order $(\epsilon_F\tau)^{-1}$ relative to the mean-field equations in the pure case.[14] This feature is consistent with the results of nondiffusion pole CPA-type approaches[5,6] for weak disorder.

The mean-field results are then identical to the pure case[14] to leading order in the disorder. In particular, the λ field shifts the chemical potential so as to satisfy the Luttinger counting rule, the mass is renormalized by the singlet exchange energy and the effects of the occupation restriction, while the average boson field b_0^2 is proportional to the hole doping δ. The explicit form taken by the mean-field equations is

$$\Delta=J\sum_k \cos(k_x)f(E_k),\quad \lambda=4t\Delta/J,\quad b_0^2=N\delta/2, \quad (5)$$

where the quasiparticle dispersion is given by $E_k=-2(\Delta+tb_0^2/N)[\cos(k_x)+\cos(k_y)]-\mu+\lambda$. Thus the quasiparticles experience a mean-field mass enhancement $m^*/m=t/[2(\Delta+t\delta/N)]$ that is large only if the ratio t/J is large; otherwise in the zero doping limit it stays modest.

III. DIFFUSIVE CORRECTIONS TO THE POLARIZATION PARTS AT 1/*N* ORDER

Our interest lies in going beyond mean-field level, which means that we require the weak-disorder contribution to the free energy at next leading (Gaussian) order in $1/N$. This means following the usual prescription of calculating the fluctuations of the boson operators around their mean-field values and keeping only the quadratic component in the effective action. The Gaussian component of the partition function then has the disorder average performed within the individual Bose propagators as in the previous infinite-U paper.[12] In this procedure each term in the expansion of the partition function in powers of $1/N$ has the disorder average performed individually with only the dominant low-energy diffusive corrections kept at each order. The upshot is that translational invariance is restored with the effective action given by

$$L_B=\tfrac{1}{2}\sum_{\alpha,\beta}\sum_{q,\omega_n}\phi_\alpha(q,i\omega_n)D_{\alpha\beta}^{-1}(q,i\omega_n)\phi_\beta(-q,-i\omega_n), \quad (6)$$

where the fluctuating boson fields are represented here by a vector $\phi=(r,\lambda,R^\nu,A^\nu)$ where $\nu=x,y$ distinguishes the two spatial components of the radial and angular gauge fields R,A. The inverse boson propagators $D(q,i\omega)$ are then composed of a static part P and a dynamic polarization part Π so that

$$D^{-1}(q,i\omega_n)=2NP(q)+N\Pi(q,i\omega_n). \quad (7)$$

The static components have nonzero values $P_{11}=\epsilon_q$, $P_{21}=P_{12}=ib_0^2/N$, and $P_{33}=P_{44}=P_{55}=P_{66}=\Delta^2/J$. In deriving P_{11} we have made use of the mean-field equations to eliminate the average over the leading-order Green's function in favor of the "holon" dispersion relation $\epsilon_q=2t(\Delta/J)[2-\cos(q_x)-\cos(q_y)]$. This component an important role in the pure case where it leads to a different collective mode associated with separation of the charge degrees of freedom. Also in the pure case this mode leads to such non-Fermi-liquid features as a broad background in the single-particle spectrum[14] and at two-loop order[15] a positive spin-exchange Landau parameter F_0^α for large mass enhancements.

Following the methods of the previous paper,[12] we obtain, after performing the disorder average, the polarization matrix

$$\Pi_{\alpha,\beta}(q,i\omega_n)=\Pi^0_{\alpha,\beta}(q,i\omega_n)+\Pi^1_{\alpha,\beta}(q,i\omega_n), \qquad (8)$$

where the first term is merely the polarization matrix without any diffusive insertions, namely,

$$\Pi^0_{\alpha,\beta}(q,i\omega)=T\sum_{k,\omega_n}\Lambda_\alpha(k_+,k_-)\Lambda_\beta(k_+,k_-)\times G(k_-,i\omega_n)G(k_+,i\omega+i\omega_n), \qquad (9)$$

where the momentum-dependent vertices arising in the slave boson formulation of the t-J model are given by (following Wang, Bang, and Kotliar[14])

$$\Lambda(k_+,k_-)=[tb_0(\cos(k_{+x})+\cos(k_{+y})+\cos(k_{-x})+\cos(k_{-y})]/N,i,-2\Delta\cos k_\nu,2\Delta\sin k_\nu) \qquad (10)$$

and we have introduced the notation $k_+=k+q/2$, $k_-=k-q/2$. The x and y components of the gauge fields are again distinguished by the index ν. The Green's functions appearing in the expression for Π_0 are the mean-field ones $G(k,\omega)^{-1}=i\omega-E_k-i\ \mathrm{sgn}(\omega)/2\tau$, where the quasiparticle lifetime is defined by $1/2\tau=n_iu(0)^2m^*\rho_0$. Here the quasiparticle density of states is defined by $m^*\rho_0$, with the concentration of impurities given by n_i. In the low-frequency, low-wave-vector limit Π^0 takes, for weak disorder, the form of a wave-vector average over the Fermi surface of the momentum-dependent vertices appearing in the Lagrangian, i.e., $\Pi^0_{\alpha,\beta}=-\Sigma_k\Lambda_\alpha(k,k)\Lambda_\beta(k,k)\delta(E_k)$. Such averages can almost all be expressed in terms of the residual kinetic energy at the Fermi surface ϵ_0 via the relation

$$\sum_k\delta(E_k)[\cos(k_x)+\cos(k_y)]^n=m^*\rho_0\epsilon_0, \qquad (11)$$

with ϵ_0 itself defined as $\epsilon_0=-[\cos(k_x)+\cos(k_y)]-m^*E_k$. We find that the nonzero components are given by $\Pi^0_{22}=m^*\rho_0$, $\Pi^0_{12}=\Pi^0_{21}=-(2itb_0/\sqrt{N})m^*\rho_0\epsilon_0$, $\Pi^0_{11}=-(4t^2b_0^2/N)m^*\rho_0\epsilon_0^2$, $\Pi^0_{13}=\Pi^0_{14}=-(2tb_0/\sqrt{N})\Delta\epsilon_0^2m^*\rho_0$, and $\Pi^0_{23}=\Pi^0_{24}=-i\Delta\epsilon_0m^*\rho_0$.

The remaining radial gauge fields have nondiffusive components given by $\Pi^0_{33}=\Pi^0_{44}=-4\Delta^2m^*I_2$ and $\Pi^0_{34}=\Pi^0_{43}=-2\Delta^2m^*[\rho_0\epsilon_0^2-2I_2]$, where I_2 denotes the Fermi surface average $I_2=\Sigma_k[\cos(k_x)]^n\delta(\cos(k_x)+\cos(k_y)+\epsilon_0)$. Thus, as far as the nondiffusive contributions to the Bose propagators are concerned, the disorder leaves their low-energy form unaffected, i.e., they are identical to their zero k,ω limits in the pure problem.

The same cannot be said of the diffusive corrections where the nonanalytic dependence on the impurity concentration is crucial. The diffusive corrections take the form of the regular ladder insertions into the polarization parts (shown in Fig. 1)

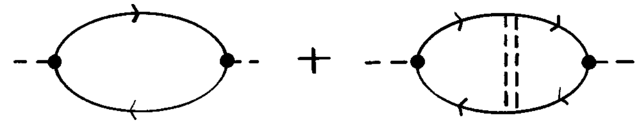

FIG. 1. Polarization parts for the boson propagators appearing at the one-loop level. The first graph represents the nondiffusive contribution in terms of the mean-field propagators $G(k,\omega)$ (full lines) and the momentum-dependent vertices $\Lambda(k_+,k_-)$ (shown here as dots). The second graph gives the diffusive correction in terms of the vertices $T_\alpha(q,\omega_n,\omega_m)$ (combinations of Green's functions and momentum-dependent vertices) and the zeroth-order diffusive propagator $S(q,\omega)$ shown as a double-dotted line.

$$\Pi^1_{\alpha,\beta}(q,i\omega_n)=T\sum_{\omega_n}n_iu_0^2T_\alpha(q,\omega_m,\omega_m)S(q,i\omega_n,i\omega_m)T_\beta(q,\omega_m,\omega_n), \qquad (12)$$

where the vertices are given by

$$T_\alpha(q,\omega_m,\omega_n)=\sum_k\Lambda_\alpha(k_+,k_-)G(k_-,i\omega_m)\times G(k_+,i\omega_n+i\omega_m). \qquad (13)$$

These have, in the low-wave-vector, low-energy limit, contributions again that depend on Fermi surface averages. For the case of the hopping and constraint fields and in the case of the radial gauge fluctuations we obtain

$$T_\alpha(q,\omega_m,\omega_n)=2\pi\tau\sum_k\Lambda_\alpha(k,k)\delta(E_k), \qquad (14)$$

which for the various nonvanishing components yields $T_\alpha=2\pi\tau m^*\rho_0(2tb_0/N,i,\Delta\epsilon_0,\Delta\epsilon_0)$. The angular gauge fields do not contribute to T as do the other fields, since they involve averages of $\sin(k_{x,y})$ over the Fermi surface and hence do not contribute in the low-k,ω limit of interest to us. The angular gauge components still have nondiffusive components, but these factorize out of the density correlation function since the off-diagonal terms mixing the angular components with the radial gauge fields and the constraining gauge fields vanish for the same reasons as given above. Hence we can say that as far as the diffusion pole resummation approach is concerned, the chiral fluctuations present in the pure case do not participate in the interactions between the diffusive modes.

Returning to Fig. 1, we note that the internal parts of the polarization bubbles correspond to the regular diffusion pole ladder terms, which, as in the work of Huang and Rasul, yields the diffusion propagator

$$S(q,\omega_m,\omega_n+\omega_m)=\frac{\theta(\omega_m+\omega_n)\theta(-\omega_m)+\theta(\omega_m)\theta(-\omega_m-\omega_n)}{\tau[|\omega_n|+Dq^2]}, \qquad (15)$$

which constitutes an infinite sum of ladder terms. The diffusion constant appearing here is defined as $D=4(\Delta+t\delta/N)^2\langle\sin(k_x)^2\rangle_{\mathrm{FS}}$, which involves, as one would expect, a Fermi surface average of the quasiparticle velocity. Since for low frequencies the vertices are independent of energy, the frequency sum in Eq. (12) yields the expression $T\Sigma_{\omega_n} S(q,\omega_m,\omega_n+\omega_m)=|\omega_m|/2\pi\tau(|\omega_n|+Dq^2)$.

Inserting the above low-energy and wave-vector expressions yields, after adding together the diffusive and nondiffusive components, the expressions for the polarization parts

$$\Pi_{22}(q,\omega_n)=F(q,\omega_n)=\frac{m^*\rho_0 Dq^2}{|\omega_n|+Dq^2}, \tag{16}$$

with the other constraining field polarization components related to Π_{22} by $\Pi_{11}(q,\omega_n)=-(4t^2b_0^2\epsilon_0^2)/NF(q,\omega_n)$ and $\Pi_{12}(q,\omega_n)=\Pi_{21}(q,\omega_n)=-(2itb_0\epsilon_0)/\sqrt{N}F(q,\omega_n)$. For the combined constraint-radial gauge components we obtain $\Pi_{1\nu}(q,\omega_n)=-(2tb_0\Delta\epsilon_0^2)/\sqrt{N}F(q,\omega_n)$ and $\Pi_{2\nu}(q,\omega_n)=i\Delta\epsilon_0 F(q,\omega_n)$, while for the remaining purely radial gauge components we find

$$\Pi_{33}(q,\omega_n)=-4\Delta^2 m^* I_2+\Delta^2\epsilon_0^2 m^*\rho_0\frac{|\omega_n|}{|\omega_n|+Dq^2}=\Pi_{44}(q,\omega_n), \tag{17}$$

$$\Pi_{34}(q,\omega_n)=\Pi_{43}(q,\omega_n)=\Pi_{33}(q,\omega_n)+8\Delta^2 m^* I_2-2\Delta^2\epsilon^2 m^*\rho. \tag{18}$$

These polarization parts are then required as input to the charge-density correlation functions as well as the effective interaction.

IV. CHARGE SUSCEPTIBILITY AND QUASIPARTICLE INTERACTION

Having obtained all the required matrix elements, we return to the explicit definition of the charge susceptibility following Wang, Bang, and Katliar,[14]

$$\chi(q,i\omega)=\langle n(q,i\omega_n)n(-q,-i\omega_n)\rangle=4Nb_0^2 D_{rr}(q,i\omega), \tag{19}$$

which becomes

$$\chi(q,i\omega_n)=\frac{Nm^*\rho_0 Dq^2}{|\omega_n|+[1+f(q)]Dq^2}, \tag{20}$$

where the factor $f(q)$ plays the role of a wave-vector-dependent Landau parameter and is defined by

$$f(q)=2t|\epsilon_0|-2\epsilon_0^2 J+\epsilon_q/2b_0^2. \tag{21}$$

In the limit of low wave vector this reduces to the extended singlet Landau parameter F_0^s found in the pure case.[14] Taking first the limit of zero frequency yields the compressibility

$$\frac{dn}{d\mu}=\chi(q\to 0,0)=Nm^*\rho_0/(1+F_0^s), \tag{22}$$

with $F_0^s=2t|\epsilon_0|-2|\epsilon_0|^2 J$. For finite frequencies we find a diffusionlike pole at a frequency

$$\omega=iDq^2[1+F_0^s+\epsilon_q/2b_0^2], \tag{23}$$

which at low wave vectors corresponds to a diffusion pole with a diffusion constant modified such that $D\to D[1+F_0^s]$. This result extends the previous infinite-U result[12] to include antiferromagnetic interactions.

In order to study the renormalization of the diffusion pole parameters the central ingredient in our discussion becomes the interaction between the weakly disordered quasiparticles. As the disorder averages have already been performed within the boson propagators (entering the Gaussian $1/N$ corrections to the effective action), it is straightforward to make use of the translationally invariant effective action L_B and to follow the derivation of the effective interaction in the pure case. Formally, the effective interaction is that of Ref. 14, namely,

$$v(k,k',q,\omega)=-\Sigma_{\alpha,\beta}\Lambda_\alpha(k_+,k_-)D_{\alpha,\beta}(q,\omega)\Lambda_\beta(k'_+,k'_-); \tag{24}$$

however, a significant simplification may be made by returning to the effective Lagrangian level and defining new radial gauge fields $R_+=(R_x+R_y)/\sqrt{2}$ and $R_-=(R_x-R_y)/\sqrt{2}$. It is then found that within the present diffusion pole treatment the only radial field coupling to the constraint fields is the s-wave field R_+. There still remains the d-wave field R_-, but the propagator for this boson field is nondiffusive. In addition, we shall see that as far as the one-loop corrections to the diffusion constant are concerned, only the s-wave component is relevant and the effective interaction in this channel does have diffusive components.

Anyway, following the same approach as in Ref. 14 it is straightforward to drive effective interaction in the s-wave channel. Placing the incoming wave vectors on the Fermi surface we obtain

$$v(q,\omega)=-\Lambda S^{-1}\Lambda=\frac{f(q)}{1+\frac{f(q)m^*\rho_0 Dq^2}{|\omega|+Dq^2}}. \tag{25}$$

At very low q this shows the expected Landau-like behavior consistent with the charge susceptibility. However, at larger values of the wave vector the explicit q dependence of $f(q)$ dominates over the Landau part leading to a form basically identical to that obtained with dynamically screened long-range Coulomb potential $v(q,\omega)=(|\omega|+Dq^2)/2m^*\rho_0 Dq^2$.

It should be noted that this form holds for the larger wave-vector regime $q>k_F/\sqrt{\delta}$ instead of as in the long-range Coulomb case, the long-wavelength limit. It may be possible at very small hole dopings for this crossover to be consistent with the small values of wave vector required for the diffusion pole treatment to be valid, but we have always then to remember the crossover to the Fermi-liquid form at very low q. Still we might expect this to lead to a modification of the full scaling behavior as the hole doping is varied.

V. RENORMALIZATION AND SCALING BEHAVIOR OF THE DIFFUSION CONSTANT

The basic element in the diagrammatic renormalization procedure is to obtain corrections to the leading-order diffusion propagator $S(q,\omega)$ [defined in Eq. (15)] connecting the quasiparticle and quasihole lines on opposite side of the Fermi surface. We follow the procedure of Castellani *et al.*[13] and consider the corrections shown in Figs. 2(a)–2(d), which yield the weak-disorder corrections to the diffusion propagator to lowest order in the inverse of the diffusion constant. The details of the calculations follow closely those given in the paper by Castellani *et al.* so we shall not repeat them. We would however, stress the following points.

(a) In evaluating these graphs a collection of mean-field level particle and hole propagators appear, each of which has an imaginary part proportional to the impurity concentration and includes a dependence on external and internal wave vectors.

(b) In the weak-disorder limit of interest to us the imaginary parts constrain these wave vectors to lie on the Fermi surface. In all the cases shown in Fig. 2 this means that only the isotropic s-wave component of the quasiparticle interaction is relevant. Hence, in the following results only $v(q,\omega)$ and not its nondiffusive d-wave partner appears in the self-energy corrections to the diffusion propagator.

We then proceed to list the self-energy corrections. The first three graphs [Figs. 2(a)–2(c)] yield a total contribution

$$\Sigma_{abc}=-2\pi N(0)\tau^2\sum_{q''}\sum_{|\omega_q|}\frac{v(q'',\omega_{q''})}{|\omega_{q''}|+Dq''^2}[\Omega+|\omega_{q''}|+D(q^2+q''^2)] \tag{26}$$

(where the frequency sum is limited to less than that of the incoming hole) along with similar graphs involving self-energy corrections on the hole lines. It should be noted that these graphs involve the quasiparticle interaction $v(q,\omega)$ together with two powers of a vertex renormalization factor $1/\tau[|\omega_{q''}+Dq''^2]$ (indicated by dotted lines dressing each interaction vertex). These factors, which are responsible for the dominant singularities from these graphs, appear whenever the intermediate quasiparticle states in the self-energy graphs are on opposite sides of the Fermi surface. Hence, in these diagrams, the intermediate states (the continuous lines either side of the dressed vertices) are holelike.

However, the actual graph largely responsible for the diffusion constant renormalization [shown in Fig. 2(d)] involves no such vertex factors since the intermediate lines exchange a full diffusion propagator. They can only do this if they remain on the same side of the Fermi surface as the original incoming lines. They then yield a contribution

$$\Sigma_d=2\pi N(0)\tau^2\sum_{q''}\sum_{\omega_{q''}>-\epsilon}\frac{v(q'',\omega_{q''})}{|\omega_{q''}+\Omega|+D(q^2+q''^2)}, \tag{27}$$

which again is accompanied by a time-reversed partner. The above self-energy corrections are then expanded for small external frequencies and wave vectors with the result that, following the renormalization scheme of Castellani *et al.*,[13] the diffusion propagator becomes

$$S(q,\omega)=\frac{\chi^2/\tau}{z|\Omega|+D'q^2}, \tag{28}$$

where the renormalized diffusion constant D' is given by

$$D'=D\left[1-\frac{8}{d}\sum_{q,\omega_q>|\Omega|}\frac{v(q,\omega_q)Dq^2}{(\omega_q+Dq^2)^3}\right], \tag{29}$$

which yields logarithmic corrections to the diffusion constant. Also present are a frequency renormalization factor z

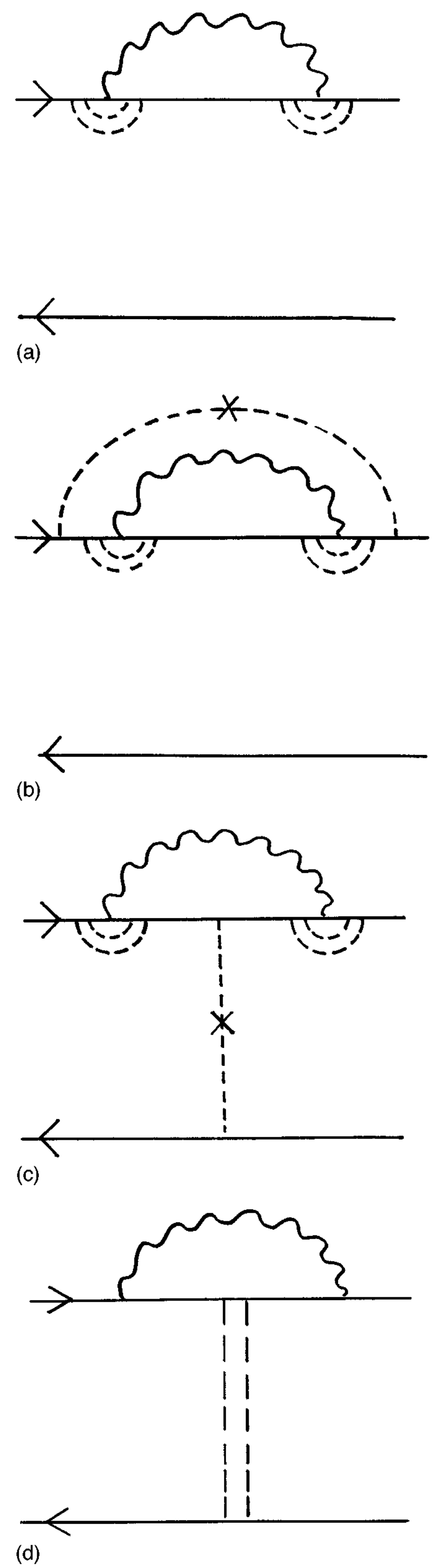

FIG. 2. Self-energy corrections to the diffusion propagator. The notation is the same as in Fig. 1, except for the single dotted lines (representing a single potential scatterer) and the wavy lines, which denote the quasiparticle interaction $v(q,\omega)$. The quasiparticle interaction lines are themselves dressed by diffusive vertices when they connect quasiparticles on opposite sides of the Fermi surface [as in (a)–(c)]. The dominant diffusion propagator correction (d) does not get dressed in this way.

and a wave-function renormalization χ, each of which involves logarithmic factors. In evaluating D' we have to distinguish the frequency ranges according to the filling factor. For the lowest-frequency interval the logarithmic contribution depends on the Landau parameter appropriate for the pure system F_0^s that we defined earlier, whereas at high frequencies the ''long-range'' form of $v(q,\omega)$ takes over and the universal form of the scaling theory (i.e., independent of F_0^s) is obtained. Before evaluating the integrals explicitly we have to define, following the standard weak-disorder scaling treatments, the renormalized ''resistance'' $t=1/4\pi^2 D' m^* \rho_0$, so that to leading logarithmic order the high-energy form is $\Delta t=-t^2 \ln(1/|\Omega|\tau)$, while at lower energies the dependence on the Landau parameter F_0^s is restored,

$$\Delta t=-t^2\left[1-\frac{\ln(F_0^s)}{F_0^s-1}\right]\ln\left(\frac{1}{|\Omega|\tau}\right). \tag{30}$$

Thus, for the very-low-energy region, in two dimensions, the scaling equation for the resistivity becomes, to leading order in t,

$$\frac{dt}{d(\ln|\Omega|)}=t^2\left[1-\frac{\ln F_0^s}{F_0^s-1}\right], \tag{31}$$

which implies for all values of the Landau parameter F_0^s a resistivity that scales to infinity, with a resulting metal-insulator transition in two dimensions. In $2+\epsilon$ dimensions, we may, following Castellani *et al.*,[13] redefine a dimensionless resistance $t=\lambda^{\epsilon/(2+\epsilon)}/4\pi^2 D' m^* \rho_0$, which, to zeroth order in ϵ, adds a term $-\epsilon t/2$ to the right-hand side of the above scaling equation. This leads to a non-Gaussian fixed point and implies a metal-insulator transition in $2+\epsilon$ dimensions.

Ultimately, the reason for our recovering a metal-insulator transition lies in the fact that the $1/N$ expansion introduces the density fluctuations at next leading order in $1/N$; hence only F_0^s appears. Whatever the behavior of this parameter itself under scaling, the sign of the right-hand side of the scaling equation remains unchanged and the metal-insulator transition survives.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have investigated the interplay between strong correlation and weak disorder for the t-J model. We introduce a weak-site-disorder term into the t-J model and, by combining the $1/N$ expansion approach to the strong correlations with the standard diffusion pole treatment of the disorder, we derived the effective quasiparticle interaction between the weakly disordered quasiparticles. We then used this as input into the standard diagrammatic treatment of the weak-disorder corrections to the effective diffusion constant and studied the overall consequences of the large-N approach to strong correlations as regards the scaling theory of the metal-insulator transition.

We found that, compared with our previous work on the infinite-U Hubbard model, that the extra antiferromagnetic bond fluctuations contributed radial parts that coupled to the density fluctuations, while the angular bond fluctuations associated with chiral effects in the pure system did not survive the dressing by diffusive effects. As a result, the effective interaction could be decomposed into an s-wave part, which participated in the diagrammatic corrections to the diffusion constant, together with a d-wave part, which did not. In fact, of the two components, only the s-wave part was dressed by diffusive corrections.

Nevertheless, the correction to the diffusion constant obtained by following the diagrammatic procedure of Ref. 13 [equivalent to the renormalization-group (RG) procedure of Finkelshtein] was found to imply scaling towards an Anderson transition in $2+\epsilon$ dimensions. It may of course be argued that this is because the present $1/N$ approach unduly favors a rigorous systematic expansion procedure over the inclusion of physically relevant spin fluctuations; however, arbitrarily these are usually included. In response to this criticism we believe that (a) the t-J model already includes magnetic effects (antiferromagnetic), which in the $1/N$ treatment for the pure model lead to a renormalized bandwidth and chiral fluctuations, and (b) it is important to isolate precisely those effects in a strongly correlated electron system that can be related directly to model parameters. In addition, we have shown explicitly the decoupling of these chiral fluctuations in disordered systems together with the different effects disorder has on s-wave and d-wave interactions. Moreover, it is hardly justified to apply the full RG procedure to a purportedly ferromagnetic spin Landau parameter when the two-loop calculations for the pure t-J model with large mass enhancements[15] indicate that F_0^a is positive. Ultimately, however, the physical justification for our procedure must be found in those systems that do not appear close to a ferromagnetic transition.

The present work has several possible extensions. First, the full scaling calculation (following Ref. 13) should be carried out. The only drawback to this being done at present lies in the treatment of the logarithmic disorder corrections to the effective interaction. The Fermi-liquid parameters in Ref. 13 for large and small momentum transfer are renormalized separately, whereas in the present approach a single effective interaction valid for all momentum transfers is obtained. It is not at present clear how to compensate for this difference.

Another possible extension concerns the d-wave effective interaction. This is particularly important for anisotropic superconductors where disorder is present. It has been shown by Rojo and Balseiro,[16] who carried out a self-consistent disorder treatment of d-wave operators valid over the entire superconductor-insulator transition, that d-wave pairing indeed survives the Anderson transition and is consistent with localized one-particle states. It would be of interest to see within the present approach how the quasiparticle interactions inherent in the t-J model affect this picture.

Overall, our major conclusion is that for the ''generic'' strongly correlated model such as the t-J model the $1/N$ expansion ''rescues'' the metal-insulator transition by placing, in a rigorous fashion, a microscopic emphasis on density fluctuations over ferromagnetic spin fluctuations at the $1/N$ level. We found that in $2+\epsilon$ dimensions, the scaling equations recover the expected weak localization fixed point in terms of the bare parameters of the t-J model rather than arbitrary Landau parameters.

[1] A. M. Finkelshtein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1985)].
[2] D. Belitz and T. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).
[3] K. Ueda and T. M. Rice, Phys. Rev. B **34**, 6420 (1986).
[4] D. M. Newns and N. Read, Adv. Phys. **36**, 999 (1988).
[5] Z. Tesanovich, Phys. Rev. B **34**, 5212 (1986).
[6] R. Freytag and J. Keller, Z. Phys. B **80**, 241 (1990).
[7] J. W. Rasul, Phys. Rev. B **44**, 11 802 (1991).
[8] P. Schlottmann, Phys. Rev. B **46**, 998 (1992); R. Sollie and P. Schlottmann, J. Appl. Phys. **69**, 5478 (1991); **70**, 5803 (1991).
[9] J. W. Rasul, Phys. Rev. B **51**, 2576 (1995).
[10] Zheng-zhong Li, Wang Xu, Chen Chen, and Ming-wen Xiao, Phys. Rev. B **50**, 11 332 (1994).
[11] G. Zimanyi and E. Abrahams, Phys. Rev. Lett. **64**, 2719 (1990).
[12] W.-N. Huang and J. W. Rasul, J. Phys. C **5**, 8877 (1993); Solid State Commun. **87**, 821 (1993).
[13] C. Castellani, C. Dicastro, M. Ma, and P. A. Lee, Phys. Rev. B **30**, 527 (1984).
[14] Z. Wang, Y. Bang, and G. Kotliar, Phys. Rev. Lett. **67**, 2733 (1991).
[15] J. Rasul, Phys. Rev. B **50**, 2133 (1994); **51**, 2632 (1995).
[16] A. G. Rojo and C. A. Balseiro (unpublished).