Variation of the Josephson current with carrier concentration in the barrier

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A phenomenological investigation of the dc Josephson current in superconductor– (S-) normal-metal– (N) [semiconductor– (Sm-)] superconductor junctions is carried out as a function of the carrier concentration in the barrier. The occurrence of a nonmonotonic dependence of the Josephson current on the carrier concentration is predicted within some limits. This study suggests that in S-N- (Sm-) S structures there is an optimum carrier concentration range which gives the maximum value of the Josephson current I_C and the junction parameter $I_C R_N$. [S0163-1829(97)01226-5]

I. INTRODUCTION

Junctions based on the Josephson effect[1] are of great interest for a wide range of electronic applications and for the study of fundamental properties of superconductors.[2] Of crucial importance for the realization of these structures is the nature of the barrier and its interfaces with superconducting electrodes. Metallic barriers have advantages over insulators for some applications. Moreover, the boundary between a superconductor (S) and a normal metal (N) [semiconductor (Sm)] (Refs. 2 and 3) also has particular interest for fundamental physics. In these interfaces basic phenomena such as the proximity effect[4] and Andreev reflection occur.[5] Several investigations have been performed on junctions, which employ both low-critical-temperature superconductors[2,3,6–9] (LTCS's) and high-critical-temperature superconductors[10–13] (HTCS's). In some structures, the presence of S/N interfaces seems to be unavoidable. For instance, this can occur in HTCS grain boundaries (GB's), where GB's do not seem to act as simple insulating barriers.[14] It can also take place in LTCS nominal S-insulator- (I-) S junctions because of the existence of high-transparency transmission for a small fraction of electrons.

In this paper a phenomenological analysis of the Josephson properties of S-N- (Sm-) S structures is carried out. The carrier concentration N_S in the barrier can be varied over a wide range ($N_S = 10^{17}$–10^{22} cm^{-3}) (the semiconductor is usually heavily doped and can be regarded as a low-carrier-density metal). In multilayered structures, for instance, the barrier layers are usually produced by sputtering or by laser ablation from targets in which N_S is controllably adjusted.[12] The first motivation of this analysis originates from the experimental problem of determining a suitable carrier concentration to optimize the Josephson current I_C and the junction parameter $V_C = I_C R_N$ (R_N is the normal resistance). As far as applications are concerned, the tuning of the parameters of individual junctions to optimum operation values is very attractive. This problem also exists in the context of HTCS Josephson trilayers, where various doping levels of the barrier material are commonly explored.[12] Moreover, helpful information can be obtained on gated Josephson devices, where an electric field in opportune conditions can produce variations of the carrier concentration and the properties of the junction. This could also be applied to structures based on HTCS grain-boundary junctions, if one assumes a correspondence between some properties of the grain boundaries and the carrier densities[15] in the GB barrier.

We study the Josephson current I_C as a function of the carrier concentration N_S and temperature T within the framework of the proximity effect. Our analysis can be applied to S-N- (Sm-) S junctions, provided that the system is in the dirty limit and the S/N (Sm) interfaces are characterized by a low transparency, common in many types of junctions.[7,8,14] Once the correlation between the properties of the barrier and the carrier concentration is established,[3] a crucial point is to study the effect of the modifications of the S/N interfaces on the Josephson current I_C when the carrier concentration N_S in the barrier is changed. This is the main aim of the present paper. If we change the number of carriers by employing, for example, barriers that have been doped to different percentages, the Josephson current will vary. This behavior can be quantitatively described through the variation of some parameters typical of the proximity effect,[4,6,7] such as the coherence length in the barrier $\xi_N = (hD/2\pi k_B T)^{1/2}$ and the boundary resistance (D being the diffusion constant).[4] These depend on the Fermi velocities and, as a consequence, on N_S. Variations of N_S influence both the mismatch of electronic properties between the electrodes and the barrier[16] and the boundary conditions for the order parameter at the S/N (Sm) interface.

Depending on the nature of the barrier and the S/N interface, two different situations can occur. The Josephson current I_C dependence on N_S can be either monotonic or nonmonotonic. In the latter case, we found the existence of an optimal doping of the barrier, which gives the maximum values of I_C and V_C. In the next section, the main formalism will be presented, while the results and conditions that determine the two different behaviors will be discussed in Sec. III.

II. MAIN FORMALISM AND LIMITS OF VALIDITY

We use the model developed by Kupryanov and co-workers[7] to investigate the properties of S/N bilayers on the basis of the Usadel equations in the ''dirty'' limit. This approach has the advantage of taking into account a varying boundary resistance and describing, in some detail, the nature of the prefactor of the exponential dependence of I_C on the ratio L/ξ_N (L being the barrier thickness). The order parameter in a S/N bilayer can be expressed through two dimensionless parameters[7]

$$\gamma=\rho_S\xi_S/\rho_N\xi_N\,,\qquad \gamma_B=R_B/\rho_N\xi_N\,,$$

where $\rho_{N,S}$ and $\xi_{N,S}$ are the normal-state resistivities of the junction materials and their coherence lengths in N and S, respectively, while R_B is the specific resistance of the S/N boundary. γ_B is a measure of the coupling between the two slabs of the bilayer: The higher the value of the resistance, the weaker the coupling between N and S is. The tunnel regime at the S/N interface is obtained for $\gamma_B \gg 1$. Direct information on the spatial variation of the order parameter is given by γ. For $\gamma \ll 1$ the effects on the superconductor due to the proximity of the normal metal are small, in contrast to the limit $\gamma \gg 1$. In this case many quasiparticles diffuse from N to S.[7] Both the parameters γ and γ_B are related to N_S, due to the dependence of $\rho_{S,N}$ and $\xi_{S,N}$ on the carrier concentration N_S.

On the basis of such a general formalism, it is possible to obtain the expression for V_C and therefore I_C:[7]

$$e\gamma_B V_C/(2\pi T_C)=2(T/T_C)^{1/2}\sum_{\omega>0}\,[\Delta^2/(\Delta^2+\omega^2)]\times\sqrt{\pi T/\omega}\,\sinh^{-1}[L/\xi_N^*\sqrt{\omega/(\pi T_C)}],\tag{1}$$

where $\omega=(2n+1)\pi T$ and ξ_N^* is the coherence length in N at T_C. Equation (1) is valid in the limit of low transmission probability at the S/N interface for $\gamma_B \gg \max(1,\gamma)$. It is applicable to short and long bridges, provided that $\gamma_B^{-1} \ll L/\xi_N^* \ll \gamma_B$ and $\gamma_B \gg (T_C/T)^{1/2}$. This situation occurs in junctions that employ interfaces with a HTCS electrode or, more often, of Schottky nature.[10,11] The limits of the model's validity allow an extension of the traditional theories, valid for long bridges, to short bridges.[2,3] The above conditions do not allow the extension of the proposed approach to very low temperatures.

In the present investigation, the effect of γ is neglected, as a first approximation. For high values of the boundary resistance ($\gamma_B \gg 1$), the critical current I_C depends very weakly on γ. Nevertheless, the qualitative results we find are not affected if the dependence of I_C on γ is taken into account.

III. RESULTS AND DISCUSSION

Equation (1) allows the calculation of the dependence of the critical current I_C on T/T_C for SNS sandwiches ($\gamma_B \gg 1$) for different values of the L/ξ_N^* ratio. In this case, the normal junction resistance is mainly due to the interface resistance

$$R_N \approx 2\gamma_B\rho_N\xi_N^* \quad S^{-1} \approx 2R_B S^{-1},$$

where S is the cross section of the junction.

Therefore I_C depends on the carrier concentration N_S through ρ_N and ξ_N. Once the temperature is set in Eq. (1), I_C can be straightforwardly obtained as a function of N_S. We assume that the coherence length for $N_S^{1/3} \approx 10^7$ cm^{-1} has the same value as the barrier thickness L ($L/\xi_N^*=1$) in the calculations below. This choice is arbitrary.

In order to evaluate I_C, the dependences of ξ_N and ρ_N on N_S have to be established within some approximations. The role of R_B has to be pointed out. According to traditional proximity theories,[4,6,7,17] $\xi_N \propto v_F \propto N_S^{1/3}$ for a three-dimensional N barrier and $\xi_N \propto v_F \propto N_S^{1/2}$ for a two-dimensional N barrier.

For ρ_N the free electron (gas) approximation ($\rho_N \propto 1/N_S$) can be used.[18] The case in which the mobility μ depends on N_S (Ref. 18) is also considered. Despite the fact that the microscopic picture is rather complicated, different effects of $\xi_N(N_S)$ and $\rho_N(N_S,\mu)$ can be simply identified and correlated to the results. Once the limits are chosen, the calculations are straightforward. Results will be presented in the next two subsections. In the latter the possibility that the specific resistance R_B can be related to N_S will be taken into account.

A. R_B independent of N_S

Let us first consider the case of R_B substantially independent of N_S in the limit of $\rho_N \propto 1/N_S$. In the case of a three-dimensional N layer, I_C, calculated from Eq. (1), is plotted as a function of the carrier concentration N_S in Figs. 1(a) and 1(b) in a low- ($0.2<T/T_C<0.5$) and in a high- ($0.9<T/T_C<1$) temperature range, respectively (T_C being the critical temperature of S). The behavior at intermediate temperatures is analogous. A nonmonotonic dependence of the Josephson current I_C on N_S is evident. In this case, it has been assumed that N_S is almost independent of temperature (for $T<T_C$), as occurring, for example, in materials such as Nb-doped $SrTiO_3$, where no carrier freeze-out is observed. At all temperatures, the carrier concentration can be tuned in order to optimize I_C and V_C of a Josephson junction. The maximum of I_C seems to occur at a carrier concentration of the order of $N_S=10^{19}$ cm^{-3}. Such a value weakly depends on T and is almost independent of the barrier material, unless the scattering time is very different from the typical values.[4,18] For carrier concentrations dependent on temperature T [$N_S \propto T^{3/2}\exp(-E_g/k_BT)$, where E_g is the energy difference between the conduction and the valence band],[18,19] $I_C(T)$ qualitatively exhibits a nonmonotonic behavior also as a function of T. This result is in agreement with the predictions of the microscopic approach by Itskovich and Shekhter.[17] In the case of semiconductinglike behavior $\rho_N \propto N_S^{-\beta}$ (with $0<\beta<2/3$), a monotonic behavior of I_C is substantially found.[2,3] The same situation occurs if we extend the same proximity approach to the two-dimensional case. In this work, effects due to localization and interference[21] are neglected and could be suitably taken into account in a more general frame.

B. R_B dependent on N_S

In a more general approach than the one just described, the R_B variation induced by N_S through the boundary conditions should be taken into account. As is known from classical theories, if we produce a mismatch of the Fermi velocities at an interface, an increase of the interface resistance will

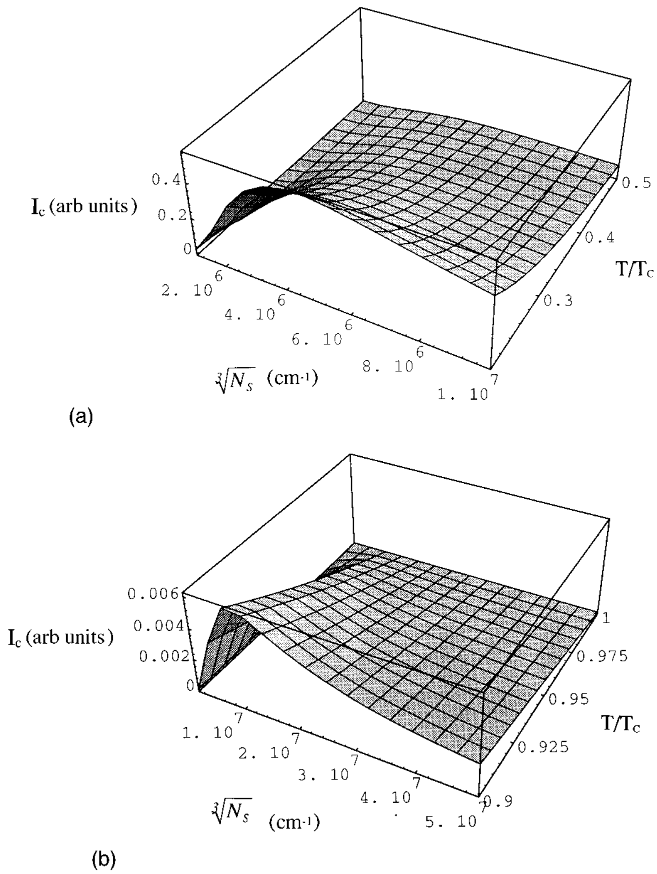

FIG. 1. I_C, calculated according to Eq. (1) in the limit for $\rho_N \propto N_S^1$, is reported as a function of the carrier concentration N_S in a low- ($0.2<T/T_C<0.5$) (a) and in a high- ($0.9<T/T_C<1$) (b) temperature range, respectively. A nonmonotone behavior of the Josephson current I_C on N_S is evident.

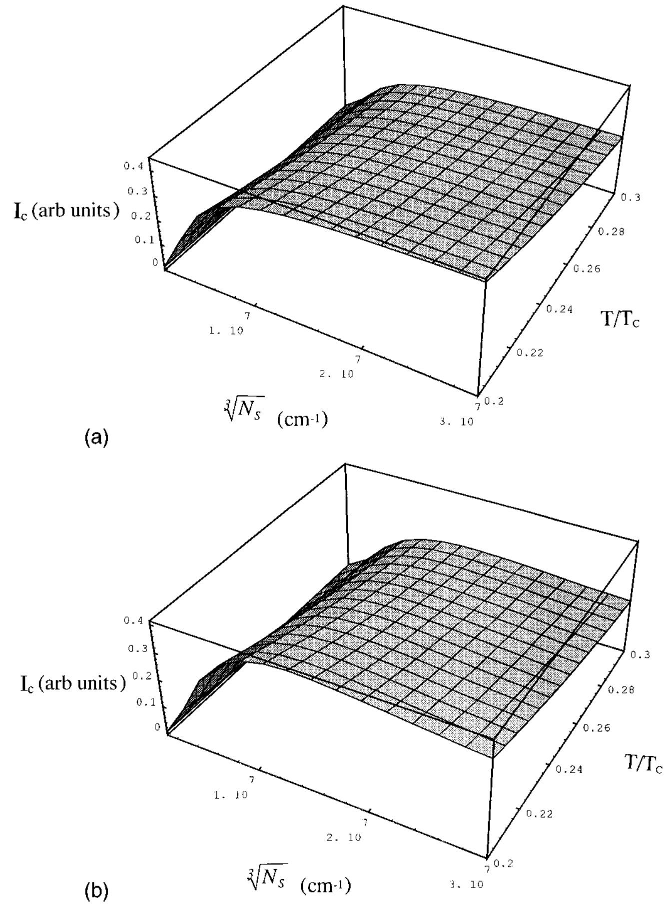

FIG. 2. I_C dependence on N_S for $\rho_N \propto N_S^{2/3}$: (a) for R_B independent of N_S and (b) for R_B according to Eq. (2). The temperature range is $0.2<T/T_C<0.3$ (k_{FS} has been assumed of the order of 10^{21} cm^{-3}). A nonmonotone behavior of I_C on N_S is evident only in the latter case.

occur.[16] The experimental variation of R_N as a function of doping or of a gate voltage (V_G) could be a sign of a R_B dependence on N_S in the case where other effects can be ruled out[20] and $\gamma_B \gg 1$. This last condition guarantees that $R_N \propto R_B$.

We assume that $R_B(N_S)$ can be approximated as

$$R_B(N_S)=R_B[1-\alpha+R(\alpha)|k_{FS}(N_S')-k_{FN}(N_S)|^2/|k_{FS}(N_S') + k_{FN}(N_S)|^2], \tag{2}$$

where α is the transmission probability at the interface, $R(\alpha)$ a coefficient depending on α, and k_{FS} (N_S') and k_{FN} (N_S) the Fermi momentum in the S and N layers, respectively. The classical dependence of the Fermi momentum on the carrier concentration ($k_{FN} \propto N_S^{1/3}$) is assumed. Equation (2) is a phenomenological extension of the microscopic expression in Ref. 16, obtained for a completely transmissive interface. We notice that for $\alpha=1$, $R(\alpha)=1$ is in agreement with established theories,[16] while $R(\alpha)$ vanishes for α approaching 0. The term depending on k_{FS} and k_{FN} takes into account, within certain approximations, the mismatch of the electronic properties of the materials forming the interface.

In Figs. 2(a) and 2(b), $I_C(N_S)$ is reported in the two cases of R_B independent of N_S (a) and of R_B dependent on N_S according to Eq. (2) (b), respectively. This is calculated for $0.2<T/T_C<0.3$ in the approximation of low values of the transmission probability α and for $\rho_N \propto N_S^{2/3}$ (k_{FS} has been assumed of the order of 10^{21} cm^{-3}). Only in the latter case is a nonmonotonic behavior found. This means that a maximum I_C value can occur also in junctions employing semiconductinglike barriers provided that the boundary resistance depends on N_S.

The monotonic or nonmonotonic behavior of I_C vs N_S can be explained by the competition of the various factors, mainly related to the nature of the barrier. In this context, we expect that both a decrease of the resistivity ρ_N of the barrier (as N_S increases) and an enhancement of the boundary resistance at the S/N interface can reduce I_C [Eqs. (1) and (2)]. These effects tend to counterbalance the influence of the increase of the coherence in N (ξ_N). The conditions for I_C vs N_S to be nonmonotonic are more favorable in the case of a weak dependence of I_C on $\xi_N(N_S)$ (i.e., for high N_S values). Furthermore, when R_B depends on N_S, the position of the maximum $I_C(N_S)$ can change. This last will approximately correspond to the value of the carrier concentration in S, which minimizes the boundary resistance.

For two-dimensional barriers, under the previous hypotheses and in the limit $\rho_N \propto 1/N_S$, a nonmonotonic I_C vs N_S relation similar to Fig. 2(b) is also obtained.

The same ideas could be also applied to HTCS grain-boundary junctions, as long as we suppose that changes in the microstructure of the GB could modify properties of the GB interface (such as transmission probability) and the carrier concentration in the ''barrier.'' Within this framework the variations of the V_C value as a function of the morphology of the GB (which can be correlated to critical current density J_C, normal conductance, etc.) would also be related to changes of N_S.

IV. CONCLUSIONS

The effect of a variable carrier concentration in the barrier on the dc Josephson current in superconductor–normal-metal–superconductor structures is studied within a classical proximity effect framework. The I_C vs N_S relation can be both monotonic and nonmonotonic, depending on the nature of the barrier and its interfaces with the electrodes. These results partially enlighten some aspects of the problem of optimizing the critical current by changing the carrier concentration in the barrier, once the electrode materials are given. A junction employing Nb or $YBa_2Cu_3O_7$ as electrodes and Nb- or Ta-doped $SrTiO_3$ as a barrier could turn out to be an interesting system[21,13] to study the dependence of I_C and V_C on N_S.

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[1] B. J. Josephson, Phys. Lett. **1**, 251 (1962); Rev. Mod. Phys. **36**, 216 (1964).

[2] A. Barone and G. Paternò, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982); K. K. Likharev, in *The New Superconducting Electronics*, edited by H. Weinstock and R. W. Raltson (Kluwer, Dordrecht, 1993), p. 429.

[3] I. Giaever and H. Zeller, Phys. Rev. B **1**, 4278 (1970); A. Barone and M. Russo, Phys. Lett. **49A**, 45 (1974); V. Z. Kresin, Phys. Rev. B **34**, 7587 (1986); J. Seto and T. Van Duzer, in *Proceedings of the 13th Conference on Low Temperature Physics*, edited by W. O. Sullivan, K. Timmerans, and A. Hammel (Plenum, New York, 1972), Vol. 3, p. 328; Z. Ivanov and T. Claeson, Jpn. J. Appl. Phys. Suppl. **26**, 1617 (1987); T. Nishino, E. Yamada, and U. Kawabe, Phys. Rev. B **33**, 2042 (1986).

[4] P. G. de Gennes, Rev. Mod. Phys. **36**, 225 (1964).

[5] A. F. Andreev, Zh. Eksp. Teor. Fiz. **48**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].

[6] V. Z. Kresin, Phys. Rev. B **28**, 1294 (1983).

[7] M. Yu. Kupryanov and V. F. Luckichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)]; A. A. Golubov and M. Yu. Kupryanov, *ibid.* **96**, 1420 (1989) [**69**, 805 (1989)].

[8] F. Tafuri, A. Di Chiara, F. Fontana, F. Lombardi, and G. Peluso, Phys. Rev. B **53**, 11 770 (1996).

[9] A. Yoshida, H. Tamura, H. Takauchi, T. Hato, and N. Yokoyama, IEEE Trans. Appl. Supercond. **AS-5**, 2892 (1995).

[10] M. Yu. Kupryanov and K. K. Likharev, Usp. Fiz. Nauk. **160**, 49 (1990) [Sov. Phys. Usp. **33**, 340 (1990)].

[11] A. Di Chiara, F. Fontana, G. Peluso, and F. Tafuri, Phys. Rev. B **44**, 12 026 (1991).

[12] J. B. Bamer, C. T. Rogers, A. Inam, R. Ramesh, and S. Bersey, Appl. Phys. Lett. **59**, 742 (1991); M. A. Verhoven, G. J. Gerritsma, H. Rogalla, and A. A. Golubov, *ibid.* **69**, 848 (1996); T. Tomio, H. Miki, H. Tabata, T. Kawai, and S. Kawai, J. Appl. Phys. **76**, 5886 (1994).

[13] D. K. Chin and T. Van Duzer, Appl. Phys. Lett. **58**, 753 (1991).

[14] D. Dimos, P. Chaudari, and J. Mannhart, Phys. Rev. B **41**, 4038 (1990); B. H. Moeckly, D. K. Lathorp, and R. A. Buhrman, *ibid.* **47**, 400 (1993).

[15] Z. G. Ivanov, E. A. Stepantsov, A. Ya. Tzalenchuk, R. I. Shekter, and T. Claeson, IEEE Trans. Appl. Supercond. **AS-5**, 2925 (1995); B. Mayer, J. Mannhart, and H. Hilkenkamp, Appl. Phys. Lett. **68**, 3031 (1996).

[16] E. L. Wolf, *Principles of Electron Tunneling Spectroscopy* (Clarendon, New York, 1985); G. B. Arnold, Phys. Rev. B **23**, 1171 (1981); E. L. Wolf and G. B. Arnold, Phys. Rep. **91**, 31 (1982).

[17] I. F. Itskovich and R. I. Shekhter, Fiz. Nizk. Temp. **7**, 863 (1981) [Sov. J. Low Temp. Phys. **7**, 418 (1981)].

[18] N. W. Aschroft and N. D. Mermin, *Solid State Physics* (HRW International, Amsterdam, 1976).

[19] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (Wiley, New York, 1981).

[20] A. Chrestin, T. Matsuyama, and U. Merkt, Phys. Rev. B **49**, 468 (1994); G. E. Rittenhouse and J. M. Graybeal, *ibid.* **49**, 1182 (1994).

[21] M. Gurvitch, H. L. Stormer, R. C. Dynes, J. M. Graybeal, and D. C. Jacobson, Bull. Am. Phys. Soc. **31**, 438 (1986); G. Deutscher and R. W. Simon, J. Appl. Phys. **69**, 4137 (1991).