Defects in two- and three-dimensional soft lattices: Application to vortices in layered superconductors

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(Received 30 May 1997)

It is shown that within continuum theory and linear elasticity the interaction of point defects in two- and three-dimensional lattices with isotropic long-range interaction is ideally screened; i.e., the elastic relaxation of the background lattice exactly compensates the long-range interaction between the defects. A small residual short-range interaction results from the discreteness of the lattice and from the nonlinear elastic displacements. The self-energy of a point defect is also strongly reduced by the lattice relaxation. These general results are applied to the lattices of Abrikosov vortex lines and of point vortices (''pancakes'') in layered superconductors. In the anisotropic lattice of pancakes this screening is not complete. In particular, screening changes the sign of the interlayer interaction, and while without screening the perpendicular magnetic field of a pancake decreases exponentially with increasing distance, the screened field decreases only algebraically due to the long-range elastic displacements of the pancake lattice around a vacancy or interstitial. [S0163-1829(97)05138-2]

I. INTRODUCTION

The vortex lattice in type-II superconductors is characterized by a repulsive interaction between the Abrikosov vortex lines of range λ, the magnetic penetration depth, which is typically much larger than the lattice spacing a.[1] One consequence of this long-range interaction is that the shear modulus c_{66} of the vortex lattice is much smaller than the uniaxial compression modulus c_{11} or the bulk modulus $c_{11}-c_{66} \geqslant (32\pi/\sqrt{3})(\lambda/a)^2 c_{66}$. Another consequence of the long-range interaction is that structural defects in such a soft lattice have very small energy as compared to the binding energy of a vortex in the lattice. This means, e.g., that the energy of a point defect (an interstitial or vacancy) is strongly reduced by the relaxation of the surrounding lattice.[2] This property applies both to two-dimensional (2D) lattices like lattices of vortex lines or of the short vortices in thin films, and to three-dimensional (3D) lattices like atomic lattices with long-range forces or the lattice of ''pancake vortices''[3] in layered superconductors, which interact by a very long-range logarithmic potential.[3–9]

In the present paper it is shown that within continuum approximation and linear elasticity, the compensation of the two energies of the unrelaxed point defect and of the lattice relaxation is nearly complete. This compensation concerns both the self-energy of each point defect and the interaction energy between two (or more) point defects. The relaxation of the surrounding lattice reduces the defect energy the more the longer the range of the lattice interaction is. This finding applies to isotropic 2D and 3D lattices, and with some restrictions also to anisotropic lattices. For example, in the 3D lattice of pancake vortices in layered superconductors, the energy of a pancake-antipancake pair (interstitial and vacancy) positioned in the same layer and of distance ρ is proportional to $\ln(\rho/\xi)$ *before* the pancake lattice is allowed to relax (or in the absence of this background lattice). But *after* lattice relaxation this factor is reduced to about $\ln(a/\xi)$, where ξ is the in-plane coherence length or radius of the vortex core and $a \ll \rho$ the in-plane lattice distance.[10] The resulting screened interaction does only weakly depend on the defect spacing ρ but depends mainly on the lattice spacing a; it is thus of order of the disregarded energy terms which account for the discrete lattice structure and for the nonlinear elastic displacements of the nearest neighbors. The strong reduction of the interaction and self-energies of point defects in lattices with long-range interaction is thus not changed qualitatively if one goes beyond the continuum approximation (which holds best for smooth long-range interaction potentials) and if one includes the nonlinearity of the elastic lattice deformation. In general these corrections to the defect energy have to be calculated numerically for given lattice symmetry and defect position.

The long-range interaction of the lattice means that within continuum theory the elasticity of the medium is nonlocal. This elastic nonlocality is fully accounted for in our theory. Remarkably, within linear elastic continuum theory, the spatially decreasing displacement field $\mathbf{u}(\mathbf{r})$ around a point defect turns out to be universal, i.e., independent of the specific shape of the isotropic interaction potential if its range goes over several lattice spacings. Namely, one has $\mathbf{u}=\pm\mathbf{r}/(2\pi r^2 n_2)$ for isotropic 2D lattices and $\mathbf{u}=\pm\mathbf{r}/(4\pi r^3 n_3)$ for isotropic 3D lattices, where $n_2 \approx a^{-2}$ and $n_3 \approx a^{-3}$ are the 2D and 3D densities and the plus and minus signs apply to the interstitial and vacancy, respectively. These displacement fields mean that during the relaxation of an infinite lattice around a vacancy or interstitial, exactly *one* atomic volume $1/n_2$ or $1/n_3$ passes through any circle (2D) or sphere (3D) centered at this point defect. The number of lattice points in a circle or sphere of arbitrary radius remains thus unchanged if a lattice point is removed or added. Indeed, these two displacement fields mean a pure shear strain, which leaves the density constant.

Similar displacement fields $u \propto 1/r$ and $u \propto 1/r^2$ follow from linear elasticity theory for the radially symmetric strain

around any pointlike perturbation in a 2D or 3D medium. The remarkable new findings of the present theory are that the continuum approximation yields also the *amplitude* of this strain, if the strain is caused by a vacancy or interstitial, and that the specific form of the interaction potential drops out from the result. The displacement field within this approximation is, therefore, a *topological property* of the point defect, as is also the displacement field around a dislocation.

While the strong reduction of defect energies by the lattice relaxation applies to 2D and 3D lattices of any objects with long-range interaction, a further related effect is restricted to lattices of magnetic vortices. In vortex lattices of superconductors the magnetic field $\mathbf{B}(\mathbf{r})$ within the London theory (see below) is the linear superposition of isolated vortex fields. If one takes out one vortex line, then the relaxation of the remaining vortices generates an additional magnetic field which in linear elastic continuum approximation exactly equals the field of the removed vortex. The depression of $\mathbf{B}(\mathbf{r})$ near the vacancy is thus ''filled'' or ''repaired'' by the lattice relaxation. The same repairing or screening occurs near interstitial vortex lines and near point defects in the 3D lattice of pancake vortices,[10] but in the latter case the screening is not perfect. After relaxation of the background vortex lattice, the resulting $\mathbf{B}(\mathbf{r})$ is thus almost constant, like in the absence of the point defect. Only the small, spatially periodic variation of $\mathbf{B}(\mathbf{r})$, caused by the discreteness of the vortex lattice, is slightly perturbed near the vacancy.[2] But in the anisotropic 3D pancake lattice the perturbation field $B_1(\mathbf{r})$, in addition to this local perturbation, has a weak long-range tail decreasing as $1/r^3$.

The outline of this paper is as follows. General expressions for the total, interaction, and self-energies of 3D and 2D lattices, and in particular of the 3D pancake-vortex lattice, are given in Sec. II. The continuum approximation of elasticity theory is defined in Sec. III. Within continuum approximation, in Sec. IV we derive the universal displacement field around a vacancy or interstitial and show that the magnetic field of a removed or added vortex line is ideally screened and that the self energy of a point defect is reduced to terms of order of the nearest neighbor interaction. In Sec. V the interaction between point defects is found to be reduced even further; in isotropic lattices this interaction is completely screened within continuum approximation. These general results are applied and extended to the highly anisotropic 3D lattice of pancake vortices in layered high-T_c superconductors in Sec. VI, where the screening of the magnetic field and of the interaction of pancake vacancies or interstitials is not complete. The results are summarized in Sec. VII.

II. 2D AND 3D VORTEX LATTICES

The triangular lattice of Abrikosov vortices in type-II superconductors with spacing $a=(2\Phi_0/\sqrt{3}\overline{B})^{1/2}$ exhibits strongly overlapping vortex fields if the average induction $\overline{B}=\langle B(\mathbf{r})\rangle$ or vortex density $n_2=\overline{B}/\Phi_0$ is not too small, namely, in the induction range $B_{c1}<\overline{B}<0.2B_{c2}$ corresponding to vortex spacings $2\lambda>a>6\xi$. Here $B_{c1}\approx\Phi_0\ln\kappa/(4\pi\lambda^2)$ and $B_{c2}=\Phi_0/(2\pi\xi^2)\approx(2\kappa^2/\ln\kappa)B_{c1}$ are the lower and upper critical fields, $\Phi_0=h/2e=2\times10^{-15}$ T m^2 is the flux quantum, λ the magnetic penetration depth, ξ the Ginzburg-Landau (GL) coherence length, and $\kappa=\lambda/\xi$ the GL parameter. For the London theory to apply we have to assume $\kappa\gg1$ and $b=\overline{B}/B_{c2}<0.2$. In London approximation, the magnetic field of an isolated vortex line along z is $B_v(\rho)=(\Phi_0/2\pi\lambda^2)K_0(\rho/\lambda)$, $\rho^2=x^2+y^2$, and the interaction between two parallel vortex lines is $V_2(\rho)=B(\rho)\Phi_0/\mu_0=(\Phi_0^2/2\pi\lambda^2\mu_0)K_0(\rho/\lambda)$. The modified Bessel function $K_0(x)$ has the limits $K_0(x)\approx\ln(1/x)$ for $x\ll1$ and $K_0(x)\approx(\pi/2x)^{1/2}e^{-x}$ for $x\gg1$. The superposition of isolated vortex fields in an ideal lattice yields $B(\mathbf{r})=\overline{B}+\delta B(\mathbf{r})$ where the periodic modulation typically is much smaller than the average $\overline{B}$, with variance $\sigma=\langle(\delta B)^2\rangle^{1/2}\leqslant0.0609\Phi_0/\lambda^2\approx0.765B_{c1}/\ln\kappa$.[11] In the same induction range the compression modulus of the vortex lattice is $c_{11}\approx\overline{B^2}/\mu_0$ and the shear modulus $c_{66}\approx\overline{B}\Phi_0/(16\pi\lambda^2\mu_0)\approx c_{11}/(8b\kappa^2)\ll c_{11}$ with $b=\overline{B}/B_{c2}$. The finite radius of the vortex core $r_c\approx\sqrt{2}\xi$ removes the logarithmic singularities of $B_v(\rho)$ and $V_2(\rho)$ by replacing in $K_0(\rho/\lambda)$ the distance ρ by $(\rho^2+2\xi^2)^{1/2}$ as shown by Clem.[12] The finite vortex core reduces the field variance σ.[13,14] In the following the bar on the average induction $\overline{B}$ will be omitted for convenience.

At larger reduced inductions $b>0.2$, the increasing overlap of the vortex cores reduces the field variance σ and the shear modulus c_{66},[15] as has been shown recently quantitatively by numerical solution of the GL equations.[13] For $b>0.2$ the energy of the vortex system is no longer composed of pair interactions. But an effective pair interaction $V_2(\rho)$ may be derived[16] which reproduces the correct nonlocal elastic energy of the vortex lattice[17] to a good approximation in the entire induction range $0\leqslant b\leqslant1$. This effective interaction has the range $\lambda'=\lambda/(1-b)^{1/2}$ and saturates at $\rho\leqslant\xi'=\xi/(2-2b)^{1/2}$.

The small shear modulus, small induction variance, and small energy of defects as discussed in Sec. I, are all due to the long-range interaction $\lambda\gg a$. In thin films with thickness $d\ll\lambda$ the interaction range between vortices becomes even larger, $\lambda_{\rm film}=2\lambda/d$,[2,18] reducing the shear modulus c_{66} even more.[19] In the limit of very thin superconducting films the interaction between the short vortices with a pointlike core becomes logarithmic, acting mainly via the stray field outside the film. The same logarithmic interaction acts beween the pancake vortices in layered superconductors when the Josephson coupling between the superconducting layers may be disregarded. The 3D magnetic interaction $V_3(r)$ between such pancake vortices is highly anisotropic, being repulsive for pancakes in the same layer but attractive and much weaker for pancakes in different layers.[3] This type of interaction arranges the pancakes to parallel stacks, which behave like the highly flexible (low tension) vortex lines described by the anisotropic London theory.[15]

In general, the energy of a 2D or 3D arrangement of N lines or points with pair interaction takes the form

$$F=\frac{1}{2}\sum_{\mu=1}^{N}\sum_{\nu=1}^{N}V(\mathbf{r}_\mu-\mathbf{r}_\nu), \tag{1}$$

where for 2D systems the pair potential is $V=V_2(\mathbf{r}_2)$ with $\mathbf{r}_2=\vec{\rho}=(x,y,0)$ (lines along z) and for 3D systems $V=V_3(\mathbf{r}_3)$ with $\mathbf{r}_3=(x,y,z)$. The 2D potential V_2 contains a factor L, the specimen length along z. The sum (1) comprises both the interaction energies (terms $\mu\neq\nu$) and the self-energies (terms $\mu=\nu$), which may thus be written as

$$U_{\rm self}=\frac{1}{2}V(0). \tag{2}$$

In particular, for a lattice of magnetically interacting pancake vortices one has for $s\ll\lambda$,

$$V_3(\mathbf{r})=\int\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})e^{i\mathbf{kr}}, \tag{3}$$

$$\tilde{V}_3(\mathbf{k})=\frac{\Phi_0^2s^2}{\mu_0}\frac{k^2/q^2}{1+k^2\lambda^2}, \tag{4}$$

where $\mathbf{k}=(k_x,k_y,k_z)$, $\mathbf{q}=(k_x,k_y,0)$, $k=|\mathbf{k}|$, $q=|\mathbf{q}|$, s is the layer spacing, and $\lambda=\lambda_{ab}$ is the penetration depth for supercurrents flowing in the layers (in the ab or xy plane). The integral (4) is taken over $q\leq 1/\xi$ (cutoff due to the finite vortex core) and over $-\pi/s\leq k_z\leq\pi/s$ due to the periodicity along z. In the self- and interaction energies of the pancakes, a logarithmic cutoff at the specimen radius R is required, which is conveniently introduced by choosing the integration boundaries $1/R\leq q\leq 1/\xi$ in Eq. (4). The self-energy of a pancake thus becomes

$$U_{\rm self}=\frac{\Phi_0^2s}{4\pi\lambda^2\mu_0}\ln\frac{R}{\xi}>0. \tag{5}$$

The interaction of two pancakes in the same layer is

$$V_3(x,y,0)=\frac{\Phi_0^2s}{2\pi\lambda^2\mu_0}\ln\frac{R}{\rho'}\geq 0, \tag{6}$$

where $\rho'=(x^2+y^2+\xi^2)^{1/2}$ was introduced to obtain the correct limit $V(0)=2U_{\rm self}$ (5). As mentioned above, Clem's[12] improved GL calculation replaces ξ^2 by $2\xi^2$ in ρ', which is also confirmed by new computations.[13,14] Two pancakes in different planes $z=0$ and $z=z_m$ at horizontal distance $r_2=\rho=(x^2+y^2)^{1/2}\gg\lambda$ interact by the attractive potential

$$V_3(x,y,z_n)=-\frac{\Phi_0^2s^2}{4\pi\lambda^3\mu_0}\exp\left(-\frac{|z_m|}{\lambda}\right)\ln\frac{R}{\rho}<0. \tag{7}$$

The energy $F_{\uparrow\downarrow}$ of a pancake-antipancake pair in the same layer and with separation $\rho\geq\xi$ does not depend on the specimen radius R,

$$F_{\uparrow\downarrow}=2U_{\rm self}-V_3(r_2,0)=\frac{\Phi_0^2s}{2\pi\lambda^2\mu_0}\ln\frac{\rho}{\xi}\geq 0. \tag{8}$$

Summing the 3D potential (3) over all $z_m=ms$ (m integer) from $m=1$ to $m=L/s\gg 1$, one obtains the 2D interaction V_2 between straight parallel vortex lines of length $L\gg s$,

$$V_2(\rho)=\int\frac{d^2q}{8\pi^3}\tilde{V}_2(q)e^{i\mathbf{qr_2}}=\frac{\Phi_0^2L}{4\pi\lambda^2\mu_0}K_0\left(\frac{\rho'}{\lambda}\right), \tag{9}$$

$$\tilde{V}_2(q)=\frac{L}{s^2}\tilde{V}_3(\mathbf{q},0)=\frac{\Phi_0^2L}{\mu_0}\frac{1}{1+q^2\lambda^2}. \tag{10}$$

Note that formulas (5)–(10) were derived from the 3D potential (3) with Eq. (4) inserted, using the boundaries $R^{-1}\leq q\leq\xi^{-1}$ and $|k_z|\leq\pi/s$. For the following calculations of energies and displacement fields of lattice defects we will need only Eq. (1) with a general potential $V(\mathbf{r})$, which in some of our results even drop out.

III. ELASTIC CONTINUUM APPROXIMATION

The continuum approximation of the elasticity theory of a lattice disregards the specific structure of the lattice. Formally this means, e.g., that a sum over reciprocal lattice vectors $\mathbf{K}$ is replaced by its main term $\mathbf{K}=0$, $\Sigma_{\mathbf{K}}f(\mathbf{k}+\mathbf{K})\approx f(\mathbf{k})$, where $\mathbf{k}$ is the vector of a periodic strain field. This approximation is good if the function $f(\mathbf{k})$ decreases rapidly within the first Brillouin zone (BZ), e.g., when the energy of compressional waves in a lattice with long-range interaction is calculated. In calculations of the energies of shear and tilt waves, the continuum approximation (by taking the limit of small k) is more intricate, requiring the integration of a function $f(\mathbf{k}+\mathbf{k}')$ over $\mathbf{k}'$.

In this section the elastic moduli of three- and two-dimensional lattices are derived from the interaction potential $V(\mathbf{r})$ in Eq. (1). The calculation follows the lines of the derivation of the elastic energy of the vortex lattice in isotropic superconductors in Ref. (17b), which was extended to anisotropic superconductors in Refs. 20, see also the review[15].

To derive the linear elastic energy $F_{\rm elast}$ of a distorted lattice one defines the discrete field of displacements $\mathbf{u}_\mu=\mathbf{r}_\mu-\mathbf{R}_\mu$, where $\mathbf{R}_\mu$ are the ideal lattice positions. First I consider the general 3D lattice with three displacement components $\mathbf{u}_\mu=(u_{\mu x},u_{\mu y},u_{\mu z})$. These results may be applied to the lattice of pancake vortices in layered superconductors and to lattices of curved or straight vortex lines by putting all z components $u_{\mu z}=0$. This is so since the pancakes can move only within the superconducting layers (planes $z=z_m=ms$, m integer) and since the deformation of a line lattice is described by a two-component displacement field $\mathbf{u}_\mu=(u_{\mu x},u_{\mu y})$. The Fourier transform of the discrete displacement field $\mathbf{u}_\mu$ (and of the force field $\mathbf{f}_\mu$, see below) defined on a 3D lattice with density n_3 is introduced as

$$\mathbf{u}_\mu=\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde{\mathbf{u}}(\mathbf{k})e^{i\mathbf{kR}_\mu},$$

$$\tilde{\mathbf{u}}(\mathbf{k})=\frac{1}{n_3}\sum_\mu\mathbf{u}_\mu e^{-i\mathbf{kR}_\mu}. \tag{11}$$

The integral (11) is over the first Brillouin zone (BZ) of the lattice; as a consequence, $\tilde{\mathbf{u}}(\mathbf{k})=(\tilde{u}_x,\tilde{u}_y,\tilde{u}_z)$ is periodic in $\mathbf{k}$ space. Using Eqs. (1), (3), (11), and the relation

$$\sum_\mu e^{i\mathbf{kR}_\mu}=8\pi^3n_3\sum_{\mathbf{K}}\delta(\mathbf{k}-\mathbf{K}), \tag{12}$$

where the sums are over the real and reciprocal lattices, the expansion of the energy F (1) up to quadratic terms in the displacements $\mathbf{u}_\mu$ yields the linear elastic energy

$$F_{\rm elast}=\frac{1}{2}\int_{\rm BZ}\frac{d^3k}{8\pi^3}\Phi_{\alpha\beta}(\mathbf{k})\tilde{\mathbf{u}}_\alpha(\mathbf{k})\tilde{\mathbf{u}}_\beta(-\mathbf{k}) \tag{13}$$

with the elastic matrix

$$\Phi_{\alpha\beta}(\mathbf{k})=n_3^2\sum_{\mathbf{K}}\,[(\mathbf{K}+\mathbf{k})_\alpha(\mathbf{K}+\mathbf{k})_\beta\tilde{V}_3(\mathbf{K}+\mathbf{k}) - \mathbf{K}_\alpha\mathbf{K}_\beta\tilde{V}_3(\mathbf{K})]. \tag{14}$$

In general, the indices α,β denote the x,y,z components, but for vortex lattices only the x,y components enter. The definition of $\Phi_{\alpha\beta}(\mathbf{k})$ here differs by a factor of the density n_3 (or n_2) from the definition in Ref. 17.

The lattice elastic matrix (14) is periodic in $\mathbf{k}$ space, i.e., one has $\Phi_{\alpha\beta}(\mathbf{k}+\mathbf{K})=\Phi_{\alpha\beta}(\mathbf{k})$ for any vector $\mathbf{K}$ of the reciprocal lattice. The elastic moduli are obtained by comparing this lattice result with the elastic matrix of a continuum. For the uniaxial continuum of vortex points or vortex lines one has

$$\Phi_{\alpha\beta}(\mathbf{k})=(c_{11}-c_{66})k_\alpha k_\beta+\delta_{\alpha\beta}(c_{66}q^2+c_{44}k_z^2), \tag{15}$$

where α,β now denote x,y, $\delta_{\alpha\beta}=1$ for $\alpha=\beta$, $\delta_{\alpha\beta}=0$ for $\alpha\neq\beta$, $q^2=k_x^2+k_y^2$, and c_{11}, c_{66}, and c_{44} are the moduli for uniaxial compression and for shear and tilt.

The $\Phi_{\alpha\beta}(\mathbf{k})$, Eqs. (14) and (15), should coincide when $\mathbf{k}$ is in the central region of the BZ. To compare Eqs. (14) and (15) one thus has to perform the limit of small wave vectors $\mathbf{k}$ in the lattice result (14). This step, called continuum approximation, yields the elastic moduli of the lattice. The bulk modulus $c_b=c_{11}-c_{66}$ is relatively easily obtained by omitting all terms $\mathbf{K}=0$ in Eq. (15). This yields $(c_{11}-c_{66})k_\alpha k_\beta=n_3^2\tilde{V}_3(\mathbf{k})k_\alpha k_\beta$, and thus $c_{11}-c_{66}=n_3^2\tilde{V}_3(\mathbf{k})$. The bulk modulus thus in general depends on the wave vector $\mathbf{k}$ of the compressional strain, which means that the elastic response is nonlocal. If the potential $V_3(r)$ is of short range, its Fourier transform $\tilde{V}_3(\mathbf{k})$ only weakly depends on $\mathbf{k}$, and the elasticity is approximately local. The continuum approximation (16) of the general expression (14) is, however, best if $V(\mathbf{r})$ is of long range and thus $\tilde{V}(\mathbf{k})$ strongly decreases inside the first BZ. In this case the shear modulus is small, $c_{66}\ll c_{11}$, therefore $c_b=c_{11}-c_{66}\approx c_{11}$ and

$$c_{11}(\mathbf{k})\approx c_b(\mathbf{k})\approx n_3^2\tilde{V}_3(\mathbf{k}). \tag{16}$$

In particular, for the 3D lattice of pancake vortices one finds from Eqs. (4) and (16) the compressional modulus

$$c_{11}(\mathbf{k})=\frac{B^2}{\mu_0}\,\frac{k^2/q^2}{1+k^2\lambda^2}. \tag{17}$$

This pancake result coincides with the $c_{11}(\mathbf{k})$ of the anisotropic London theory[15] in the limit of infinite anisotropy, i.e., for penetration depths $\lambda_c\gg\lambda_{ab}=\lambda$.

For the 2D lattices of straight vortex lines or of the pancake vortices in a film or layer of 2D density $n_2=B/\Phi_0$, one writes Eq. (16) in the form $c_{11}(\mathbf{q})\approx n_2^2\tilde{V}_2(\mathbf{q})/L$ where $\mathbf{q}=(q_x,q_y)=(k_x,k_y,0)$ and L is the vortex length. This gives for the 2D lattice of stiff vortex lines $c_{11}(\mathbf{q})=(B^2/\mu_0)(1+q^2\lambda^2)^{-1}$. The bulk modulus for the lattice of flexible vortex lines in isotropic superconductors, $c_{11}(\mathbf{k})=(B^2/\mu_0)(1+k^2\lambda^2)^{-1}$ with $\mathbf{k}=(k_x,k_y,k_z)$, cannot be derived in this way since for this system the total energy cannot be written as a sum of isotropic *scalar* pair interactions. The correct interaction between the vortex segments $d\mathbf{r}_\mu$, $d\mathbf{r}_\nu$ of arbitrarily curved vortex lines in isotropic superconductors is *vectorial* containing a scalar product, $d\mathbf{r}_\mu d\mathbf{r}_\nu V(|\mathbf{r}_\mu-\mathbf{r}_\nu|)$. This vectorial interaction originates from the coherence of each vortex line, which formally corresponds to a strong Josephson coupling between the vortex segments on the same line. For anisotropic London superconductors the interaction of the line segments is *tensorial* and anisotropic, $d\mathbf{r}_\mu^\alpha d\mathbf{r}_\nu^\beta V_{\alpha\beta}(\mathbf{r}_\mu-\mathbf{r}_\nu)$.[15,21] For non-$s$-wave superconductors the vortex interaction is expected to be more complicated.

The tilt modulus c_{44} follows from Eqs. (14) and (15) as $c_{44}(\mathbf{k})=\Phi_{xx}(k_x,k_y,k_z)/k_z^2$. For the pancake lattice this yields

$$c_{44}(\mathbf{k})=\frac{B\Phi_0}{8\pi\lambda^4k_z^2\mu_0}\ln\left(1+\frac{k_z^2}{\lambda^{-2}+q_0^2}\right), \tag{18}$$

where $q_0\approx q_{\rm BZ}$ is a cutoff radius of the order of the BZ radius $q_{\rm BZ}=(4\pi B/\Phi_0)^{1/2}\approx\pi/a$. Note that in this decoupling limit c_{44} depends only on k_z and is the linear superposition of all vortex contributions, i.e., it is proportional to B apart from the weak logarithmic dependence on B when $4\pi B/\Phi_0>1/\lambda^2$. This pancake result for $c_{44}(\mathbf{k})$, like that for $c_{11}(\mathbf{k})$ (17), coincides with the anisotropic London result in the limit $\lambda_c\gg\lambda_{ab}$.

The shear modulus of the triangular pancake and vortex lattices is not dispersive, $c_{66}=B\Phi_0/(16\pi\lambda^2\mu_0)$ for $q\ll q_{\rm BZ}$ as mentioned in Sec. I. For triangular lattices with arbitrary rotationally symmetric interaction $V_2(r)$ one has

$$c_{66}=\frac{n_2}{16}\sum_\mu\,[R_\mu^2V_2''(R_\mu)+3R_\mu V_2'(R_\mu)] \tag{19}$$

with $V'=dV/dr$ and $V''=d^2V/dr^2$. This lattice sum may be converted into a reciprocal lattice sum. However, for the vortex interaction $\tilde{V}_2(q)\propto(1+q^2\lambda^2)^{-1}$, this sum diverges logarithmically. One either has to introduce a convergence factor, e.g., $V_2(q)\to V_2(q)e^{-\xi q}$ and take the limit $\xi\to0$, or one may use the general expression[17]

$$c_{66}=\frac{n_2^2}{16}\left(\sum_{\mathbf{Q}}\,f(Q)-\int\frac{d^2q}{4\pi^2n_2}f(q)\right),$$

$$f(q)=q^2\tilde{V}_2''(q)+3q\tilde{V}_2'(q), \tag{20}$$

with 2D reciprocal lattice vectors $\mathbf{Q}$. The integral in Eq. (20) exactly vanishes when $V_2(q)q^2\to0$ for $q\to\infty$, as can be seen by partial integration. For the London potential one has $V_2(q)q^2\to$const for $q\to\infty$, and this integral gives the dominating contribution to c_{66}.[17]

IV. SINGLE VACANCY OR INTERSTITIAL

If one lattice point is removed from a lattice, the lattice relaxes towards this vacancy. In this section the lattice displacements $\mathbf{u}_\nu$, the defect energy, and (for magnetic vortices) the magnetic field of the vacancy, are calculated within linear elastic continuum theory. The corresponding results for an added lattice point (interstitial) are obtained from the vacancy expressions by changing the signs of the displacement field and of the energy terms. Additional energy terms which depend on the specific lattice structure and on the position of the point defect within the lattice, require numerical computations and are not given here.

A. Displacement field around point defects in isotropic 3D and 2D lattices

From the energy sum (1) the forces $\mathbf{f}_\mu$ exerted on the lattice points $\mathbf{r}_\mu$ by a vacancy positioned at $\mathbf{r}=0$ are $\mathbf{f}_\mu=\nabla V(\mathbf{r}_\mu)$. Within linear elasticity theory, the forces at the displaced lattice positions $\mathbf{r}_\mu=\mathbf{R}_\mu-\mathbf{u}_\mu$ are replaced by the forces at the undisplaced ideal lattice position $\mathbf{R}_\mu$. The force field and its Fourier transform in a 3D lattice are then, with Eqs. (11) and (12),

$$\mathbf{f}_\mu=\nabla V_3(\mathbf{R}_\mu)=\int \frac{d^3k}{8\pi^3}\widetilde{V}_3(\mathbf{k})i\mathbf{k}e^{i\mathbf{k}\mathbf{R}_\mu}, \tag{21}$$

$$\widetilde{\mathbf{f}}(\mathbf{k})=\int \frac{d^3k'}{8\pi^3}\widetilde{V}_3(\mathbf{k}')i\mathbf{k}'\frac{1}{n_3}\sum_\mu e^{i(\mathbf{k}'-\mathbf{k})\mathbf{R}_\mu}$$

$$=\sum_{\mathbf{K}} i(\mathbf{k}+\mathbf{K})\widetilde{\mathbf{V}}_3(\mathbf{k}+\mathbf{K})\approx i\mathbf{k}\widetilde{V}_3(\mathbf{k}). \tag{22}$$

The last step of keeping only the dominating term $\mathbf{K}=0$ in the sum (22) again means a continuum approximation, which is allowed if the Fourier transformed potential $\widetilde{V}_3(\mathbf{k})$ decreases rapidly within the first BZ.

The forces $\mathbf{f}_\mu$ (21) cause displacements $\mathbf{u}_\mu$ which follow from elastic equilibrium. From the linear elastic energy (13) and the definition (11) of $\widetilde{\mathbf{f}}(\mathbf{k})$ one obtains

$$\widetilde{\mathbf{f}}_\alpha(k)=\Phi_{\alpha\beta}(\mathbf{k})\widetilde{\mathbf{u}}_{\boldsymbol{\beta}}(\mathbf{k})/n_3,$$

$$\widetilde{\mathbf{u}}_\alpha(\mathbf{k})=\Phi^{-1}_{\alpha\beta}(\mathbf{k})\widetilde{\mathbf{f}}_\beta(\mathbf{k})\cdot n_3, \tag{23}$$

where $\Phi^{-1}_{\alpha\beta}(\mathbf{k})$ is the inverted matrix defined by $\Phi^{-1}_{\alpha\beta}\Phi_{\beta\gamma}=\delta_{\alpha\gamma}$. In an isotropic elastic medium the longitudinal force field $\widetilde{\mathbf{f}}\|\widetilde{\mathbf{k}}$ (22) causes a longitudinal displacement field $\widetilde{\mathbf{u}}\|\widetilde{\mathbf{k}}$ given by $\widetilde{\mathbf{u}}(\mathbf{k})=\widetilde{\mathbf{f}}(\mathbf{k})/[k^2c_{11}(\mathbf{k})]$ where $c_{11}(\mathbf{k})$ is the uniaxial compression modulus, which in general is dispersive. Inserting here $c_{11}(\mathbf{k})=n_3^2V_3(\mathbf{k})$ (16) and $\widetilde{\mathbf{f}}(\mathbf{k})=i\mathbf{k}\widetilde{V}_3(\mathbf{k})$ (22), we obtain the displacement field caused by a vacancy in a 3D isotropic lattice within linear elastic continuum approximation,

$$\widetilde{\mathbf{u}}(\mathbf{k})=\frac{i\mathbf{k}\widetilde{V}_3(\mathbf{k})n_3}{k^2\widetilde{V}_3(\mathbf{k})n_3^2}=\frac{i\mathbf{k}}{k^2n_3}. \tag{24}$$

Similarly one finds for 2D lattices

$$\widetilde{\mathbf{u}}(\mathbf{q})=\frac{i\mathbf{q}\widetilde{V}_2(\mathbf{q})n_2}{q^2\widetilde{V}_2(\mathbf{q})n_2^2}=\frac{i\mathbf{q}}{q^2n_2}. \tag{25}$$

Fourier transforming this back using Eq. (11), one obtains the lattice displacements caused by a vacancy in isotropic 3D and 2D lattices,

$$\mathbf{u}^{3\mathrm{D}}_\mu=\frac{-\mathbf{R}_\mu}{4\pi R^3_\mu n_3},\quad \mathbf{u}^{2\mathrm{D}}_\mu=\frac{-\mathbf{R}_\mu}{2\pi R^2_\mu n_2}. \tag{26}$$

These radially symmetric displacements caused by a vacancy in an infinite 3D or 2D lattice describe a pure shear strain $\tau_{3\mathrm{D}}=3/(4\pi r^3n_3)$ and $\tau_{2\mathrm{D}}=1/(\pi r^2n_2)$. The amplitude of the displacements are such that exactly *one* unit cell volume $1/n_3$, or area $1/n_2$, moves through a sphere or circle of arbitrary radius. Therefore, when one lattice point is removed, the subsequent elastic relaxation of the lattice is such that the number of lattice points remains constant in any finite volume or area surrounding this vacancy. In regions which do not contain this vacancy, the number of lattice points also remains constant, since the vacancy causes a pure shear strain which keeps the lattice density unchanged. This general result is modified when the removed lattice point is rebuilt into the ideal lattice in the sense of a Frenkel defect, cf. Sec. VI C.

A remarkable feature of the displacments (24)–(26) is that the specific form of the interaction potential $V(r)$ has dropped out. One may say that the nonlocality (dispersion) of the elastic response is compensated by the nonlocal character of the force field. The range of the potential $V(r)$ enters only in so far as the continuum approximation in Eqs. (16) and (22) requires that $V(r)$ is smooth and acts over at least a few lattice spacings, so that its Fourier transform decreases rapidly within the first BZ. Furthermore, $V(r)$ has to be such that the lattice is stable, i.e., all elastic moduli have to be positive. This means in particular that $c_{11}(\mathbf{k})=n^2\widetilde{V}(\mathbf{k})$ (16) has to be positive for all $\mathbf{k}$, else spontaneous nucleation of compressional waves would occur.

B. Magnetic field of the displaced vortices

A further feature of the displacement field (26) is its ''repairing'' or ''screening'' character. I show this for the particular case of parallel vortex lines. If the magnetic field $B(x,y)$ (along z) of the vortex arrangement is the linear superposition of isolated vortex fields $B_v(\mathbf{r})$,

$$B(\mathbf{r})=\sum_\mu B_v(\mathbf{r}-\mathbf{r}_{\boldsymbol{\mu}}),\quad B_v(\mathbf{r})=\int\frac{d^2q}{4\pi^2}\widetilde{B}(\mathbf{q})e^{i\mathbf{q}\mathbf{r}}, \tag{27}$$

e.g., with $\widetilde{B}(\mathbf{q})=\Phi_0/(1+q^2\lambda^2)$, then the field perturbation $B_1(\mathbf{r})$ caused by the vortex displacements $\mathbf{u}_\mu$ around a vacancy in the vortex lattice may be written as

$$B_1(\mathbf{r})=\sum_\mu B_v(\mathbf{r}-\mathbf{R}_\mu-\mathbf{u}_\mu)-\sum_\mu B_v(\mathbf{r}-\mathbf{R}_\mu)$$

$$=\int\frac{d^2q}{4\pi^2}\widetilde{B}(\mathbf{k})e^{i\mathbf{q}\mathbf{r}}\sum_\mu e^{-i\mathbf{q}\mathbf{R}_\mu}(e^{-i\mathbf{q}\mathbf{u}_\mu}-1). \tag{28}$$

Expanding the factor $e^{-i\mathbf{q}\mathbf{u}_\mu}-1\approx -i\mathbf{q}\mathbf{u}_\mu$ up to the linear terms and using definition (11) of $\tilde{\mathbf{u}}(\mathbf{k})$ in its 2D form, we obtain

$$B_1(\mathbf{r})=-n_2\int \frac{d^2q}{4\pi^2}\tilde{B}(\mathbf{k})i\mathbf{q}\tilde{\mathbf{u}}(\mathbf{q})e^{i\mathbf{q}\mathbf{r}}. \tag{29}$$

Inserting here $\tilde{\mathbf{u}}(\mathbf{q})=i\mathbf{q}/(q^2n_2)$ from Eq. (25) we find that the perturbation $B_1(\mathbf{r})$ coincides with $B_v(\mathbf{r})$ (27). This means that the linear correction to the magnetic field caused by the vortex displacements around a removed vortex *exactly equals the field of the removed vortex line*, irrespective of the particular form of $B_v(\mathbf{r})$ or $\tilde{B}(\mathbf{q})$.

Within linear continuum approximation the vortex displacements are, therefore, such that they ''repair'' or ''screen'' the field change caused by the removed vortex. This universal result is independent of the particular shapes of both the interaction potential $V_2(\mathbf{r})$ and of the vortex field $B_v(\mathbf{r})$, and it is independent of the vortex density $n_2=B/\Phi_0$. From its derivation it is clear that this screening also applies to 3D systems if the considered field is the linear superposition of individual fields. The screening even applies when these individual fields are not rotationally symmetric and not monotonic, provided the displacements are given by Eqs. (24)–(26).

Going beyond the continuum approximation one finds (cf. Ref. 2) that the small periodic variation of $B(\mathbf{r})$ is modified such that at the position of the vacancy there is now a minimum in $B(\mathbf{r})$ rather than a maximum, but the amplitude of the small variation $\delta B(\mathbf{r})=B(\mathbf{r})-\overline{B}$ stays approximately constant near the vacancy.

C. Energy of a point defect

Inserting the displacement field (24) into $F_{\rm elast}$ (13) and changing the sign (since the work done by the forces is $-2F_{\rm elast}$), one obtains the elastic energy of the lattice deformation around a vacancy. Using the property $k_\alpha\Phi_{\alpha\beta}k_\beta=k^4c_{11}(\mathbf{k})$ for the isotropic elastic medium and $c_{11}=n_3^2\tilde{V}_3(\mathbf{k})$ (16) one finds

$$F_{\rm elast}=-\frac{1}{2}\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k}). \tag{30}$$

This expression looks identical to the negative self energy $-U_{\rm self}$ (2), (3) of a lattice point, except that the integral (30) is restricted to the first BZ while the integral in $U_{\rm self}$ extends over the entire $\mathbf{k}$ space.

To $F_{\rm elast}$ (30) one has to add the energy $-U_{\rm self}-2U_B$ required to remove one lattice point. Here U_B is the binding energy of each lattice point,

$$\begin{aligned}U_B&=\frac{1}{2}\sum_{\mu\neq 0}V_3(\mathbf{R}_\mu)\\&=\frac{1}{2}\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})\left(\sum_\mu e^{i\mathbf{k}\mathbf{R}_\mu}-1\right)\\&=\frac{n_3}{2}\sum_{\mathbf{K}}\tilde{V}_3(\mathbf{K})-\frac{1}{2}\int\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k}),\end{aligned} \tag{31}$$

where Eqs. (3) and (12) were used. The last integral in Eq. (31) is over all space and equals $-U_{\rm self}$. In order to eliminate the self-energy from the total energy of a vacancy, we consider now a Frenkel vacancy, which means that during creation of the vacancy we keep the total number N of lattice points and the total volume N/n constant.[2] This is achieved by building the removed lattice point back into the ideal lattice, which is possible at the surface or if a large number of such vacancies is considered. The addition of a lattice point increases the lattice density $n=n_3$ or $n=n_2$ and requires an energy $U_{\rm self}+(\partial/\partial n)(nU_B)=U_{\rm self}+U_B+n\partial U_B/\partial n$. The total energy of the vacancy is thus within linear continuum theory $F_{\rm vac}=-U_{\rm self}-2U_B+U_{\rm self}+U_B+n\partial U_B/\partial n+F_{\rm elast}$, yielding

$$F_{\rm vac}=n\partial U_B/\partial n-U_B+F_{\rm elast} \tag{32}$$

with $F_{\rm elast}$ and U_B given by Eqs. (30) and (31) for 3D lattices. The corresponding expressions for 2D lattices are obvious from the above derivation. The energy of a Frenkel interstitial (a lattice point removed from the surface and squeezed into the lattice) within this linear continuum approximation is given by $F_{\rm int}=-F_{\rm vac}$ (32).

To discuss the vacancy energy (32) we first note that the binding energy U_B (31) in soft lattices may be approximated by

$$U_B\approx\frac{n_3}{2}\tilde{V}_3(0)-\frac{1}{2}\int_{\rm BZ}\frac{d^3k}{8\pi^3}V_3(\mathbf{k}), \tag{33}$$

since the sum over $\mathbf{K}\neq 0$ and the integral over $\mathbf{k}$ outside the first BZ approximately cancel. The integral in Eq. (33) equals $F_{\rm elast}$ (30); these two terms thus cancel in $F_{\rm vac}$ (32). Since $\tilde{V}_3(0)$ does not depend on n, two more terms $\pm(n_3/2)\tilde{V}_3(0)$ cancel in $F_{\rm vac}$. The only remaining term is thus

$$F_{\rm vac}\approx-\frac{n}{2}\frac{\partial}{\partial n}\int_{\rm BZ}\frac{d^3k}{8\pi^3}V_3(\mathbf{k}). \tag{34}$$

The derivative $\partial/\partial n$ $(n=n_3,n_2)$ can be taken explicitly noting that $V(\mathbf{k})$ depends only on $k=|\mathbf{k}|$ and that the radius of the first BZ (i.e., the radius of a sphere or circle of same volume or area as the first BZ) is $k_{\rm BZ}^3=6\pi^2n_3$ in isotropic 3D lattices and $k_{\rm BZ}^2=q_{\rm BZ}^2=4\pi n_2$ in 2D. The results for 3D and 2D isotropic lattices look the same,

$$F_{\rm vac}\approx\frac{n}{2}\tilde{V}(k_{\rm BZ})\sim V(a). \tag{35}$$

This residual term, comparable to the nearest neighbor interaction, is of the same order as other terms that were disregarded in this continuum approximation, which assumes just that $\tilde{V}(k_{\rm BZ})\ll\tilde{V}(0)$. Therefore, our result is that within continuum approximation the energy of a vacancy (and of an interstitial) *vanishes* or is of the order of the (disregarded) nearest neighbor interaction.

This finding might have been expected by handwaving arguments, but from the above derivation one notes that *the vanishing energy of a point defect results from the compensation of several large terms*. Omitting or approximating one of these terms, e.g., the elastic energy, may yield artificial

final results of both signs. The correct energy of point defects should, therefore, be calculated either by evaluating Eqs. (30)–(32) numerically or by direct computation from Eq. (1), minimizing the energy with respect to all lattice positions $\mathbf{r}_\mu$, as was done for the 2D vortex lattice in Refs. 2,22.

The larger the range of the potential $V(\mathbf{r})$ is, the smaller are the defect energies, and the larger will be the numerical effort. This effort can be reduced and the accuracy improved by using an Ewald summation method.[23] Computations are also required to account for the nonlinear elastic relaxation and when the detailed position dependence of the defect energy is of interest. But both the nonlinearity and this position dependence become weaker when the range of the lattice interaction is larger. One can show that for a logarithmic 2D interaction potential $V_2(r) \propto \ln r$ the expansion of the elastic energy of a lattice point surrounded by a stiff lattice of sixfold rotational symmetry and displaced by u starts with u^6, and for fourfold symmetry with u^4. This means the potential well is very flat and has vanishing curvature in its minimum. Therefore, the position-dependent energy term of an interstitial is not so large, as long as the interstitial does not come too close to a neighboring lattice point.

V. INTERACTION OF POINT DEFECTS

In this section it will be shown that the interaction between point defects also vanishes within linear continuum theory. The self energy of these defects is of the order of the nearest neighbor interaction $V(a)$, which is small compared to the binding energy U_B (31), and their interaction energy is much smaller than the unscreened interaction. When the interaction between lattice points is of long range, the interaction of lattice defects is ideally screened by the relaxation of the lattice. I show this screening for the example of a vacancy-interstitial pair, but the results apply also to pairs of two vacancies or two interstitials, and obviously to any number of such point defects.

The energy of a vacancy-interstitial pair is composed of the two self energies and the interaction energy $F_{\rm int}$ between these two point defects. In its turn $F_{\rm int}=F^0_{\rm int}+F^{\rm elast}_{\rm int}$ is composed of the unscreened interaction and the energy of the elastic relaxation. For a vacany at $\mathbf{r}=0$ and an interstitial at $\mathbf{r}=\mathbf{r}_p$ (pair distance r_p) one has with Eqs. (1) and (3) for a 3D lattice

$$F^0_{\rm int}=-V_3(\mathbf{r}_p)=-\int \frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})\cos\mathbf{k}\mathbf{r}_p\,, \tag{36}$$

where the integral is over all space. The forces exerted on the undisplaced lattice points by the defect pair are

$$\begin{aligned}\mathbf{f}_\mu&=\nabla V_3(\mathbf{R}_\mu)-\nabla V_3(\mathbf{R}_\mu-\mathbf{r}_p)\\&=\int \frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})i\mathbf{k}e^{i\mathbf{k}\mathbf{R}_\mu}(1-e^{-i\mathbf{k}\mathbf{r}_p}).\end{aligned} \tag{37}$$

Comparing this with Eq. (21), one sees that $\tilde{V}_3(\mathbf{k})$ is replaced by $\tilde{V}_3(\mathbf{k})(1-e^{-i\mathbf{k}\mathbf{r}_p})$. One can thus use Eqs. (22)–(25) to obtain for 3D isotropic lattices

$$\tilde{\mathbf{f}}(\mathbf{k})\approx i\mathbf{k}\tilde{V}_3(\mathbf{k})(1-e^{-i\mathbf{k}\mathbf{r}_p}), \tag{38}$$

$$\tilde{\mathbf{u}}(\mathbf{k})=\frac{i\mathbf{k}}{k^2 n_3}(1-e^{-i\mathbf{k}\mathbf{r}_p}), \tag{39}$$

$$\mathbf{u}_\mu=\frac{-1}{4\pi n_3}\left(\frac{\mathbf{R}_\mu}{R^3_\mu}-\frac{\mathbf{R}_\mu-\mathbf{r}_p}{|\mathbf{R}_\mu-\mathbf{r}_p|^3}\right). \tag{40}$$

The corresponding expressions for 2D lattices are obvious from Eqs. (25) and (26). The elastic displacements $\mathbf{u}_\mu$ (40) caused by the two defects superimpose linearly, as it should be within linear elasticity theory. Therefore, the screening of the magnetic field in Sec. IV B applies also when several defects are present.

The elastic energy of the defect pair is obtained by inserting $\tilde{\mathbf{u}}(\mathbf{k})$ (39) into $-F_{\rm elast}$ (13), cf. the derivation of Eq. (30). Noting that $(1-e^{-i\mathbf{k}\mathbf{r}_p})(1-e^{i\mathbf{k}\mathbf{r}_p})=2-2\cos\mathbf{k}\mathbf{r}_p$ we find for 3D isotropic lattices

$$F_{\rm elast}=-\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})(1-\cos\mathbf{k}\mathbf{r}_p). \tag{41}$$

To get the interaction part of Eq. (41) one has to subtract from it twice the elastic energy (30) of one point defect, equal to the limit $r_p\to\infty$ of Eq. (41), yielding

$$F^{\rm elast}_{\rm int}=\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})\cos\mathbf{k}\mathbf{r}_p\,. \tag{42}$$

This expression is almost identical to $-F^0_{\rm int}$ (36), except that in Eq. (42) the integral is over the first BZ. These two terms therefore almost cancel each other. The total interaction energy of the defect pair is thus

$$F_{\rm int}=F^0_{\rm int}+F^{\rm elast}_{\rm int}=\int_{\rm k>k_{BZ}}\frac{d^3k}{8\pi^3}\tilde{V}_3(\mathbf{k})\cos\mathbf{k}\mathbf{r}_p\,. \tag{43}$$

This integral is very small since $\tilde{V}(k)\ll\tilde{V}(0)$ for $\mathbf{k}$ outside the first BZ when the lattice interaction $V(\mathbf{r})$ is smooth and of long range. For $\mathbf{r}_p=a$ the $F_{\rm int}$ (43) is thus much smaller than the nearest neighbor interaction $V(a)$, which equals the same integral (42) but taken over all $\mathbf{k}$ space. For $r_p\gg a$ the integral (43) becomes even smaller since the integrand oscillates rapidly.

Our result is thus that the interaction of isotropic point defects *vanishes* within the linear continuum approximation. For lattices with short-range interaction this result might have been expected since the strain around a point defect is a pure shear strain, which does not interact with an isotropic defect.[24,25] Our finding is, however, more general, stating that in 3D and 2D isotropic lattices with long-range interaction $V(r)$ the unscreened *direct* interaction $\pm V(r_p)$ between two point defects is exactly compensated by the linear elastic *indirect* interaction between the defects.

This result means that in *nonlocal* lattices there is in fact an elastic interaction between point defects, of same range and size as the direct interaction, thus compensating the latter. This feature is particularly important for lattices with very long-range interaction, e.g., the lattices of vortex lines and pancake vortices,[10] which will be considered in more detail in Sec. VI.

As opposed to these soft lattices, in atomic lattices with nearest neighbor interaction, where the usual local elasticity theory is a good approximation, a small short-range interaction of point defects may come from the *nonlinear* elastic deformation of the lattice near the defect, and a long-range elastic interaction $\propto 1/r^3$ may arise from the anisotropy of specific point defects and of the anisotropic elasticity of the material. The interation between point defects in various metals is calculated, e.g., in Ref. 26.

VI. APPLICATION TO LAYERED SUPERCONDUCTORS

A. Forces and displacements

We now apply the results derived in Secs. IV and V for isotropic 3D and 2D lattices to the anisotropic lattice of pancake vortices in layered high-T_c superconductors. The two differences to the isotropic 3D lattice are that the forces $\mathbf{f}_\mu$ and displacements $\mathbf{u}_\mu$ in layered superconductors have no z component and that the elastic matrix has the uniaxial form (15). For example, the forces exerted on the pancakes by an antipancake-pancake vortex pair, corresponding to a vacancy-interstitial pair, positioned at $\mathbf{r}=0$ and $\mathbf{r}=\mathbf{r}_p=(x_p,0,z_p)$ (i.e., with in-plane distance x_p and interlayer distance $z_p=ms$, m integer) are obtained by putting $\tilde{\mathbf{f}}_z=0$ in Eq. (38),

$$\tilde{\mathbf{f}}(\mathbf{k})\approx i\mathbf{q}\tilde{V}_3(\mathbf{k})(1-e^{-i\mathbf{k}\mathbf{r}_p}). \tag{44}$$

Here $\mathbf{q}=(q_x,q_y)=(k_x,k_y,0)$ and $\tilde{V}_3(\mathbf{k})$ is the 3D pancake interaction, Eq. (4). The elastic matrix (15) of the pancake lattice has the inverse

$$\Phi^{-1}_{\alpha\beta}(\mathbf{k})=\frac{(c_{66}-c_{11})k_\alpha k_\beta+\delta_{\alpha\beta}(c_{11}q^2+c_{44}k_z^2)}{(c_{11}q^2+c_{44}k_z^2)(c_{66}q^2+c_{44}k_z^2)} \tag{45}$$

with the uniaxial compression modulus $c_{11}(\mathbf{k})$ (16) and the tilt moduls $c_{44}(\mathbf{k})$ (18); the shear modulus $c_{\mathbf{66}}$ will drop out from the point-defect results. The products $\mathbf{q}\Phi$ and $\mathbf{q}\Phi^{-1}$ required below are thus for the highly anisotropic pancake lattice

$$q_\alpha\Phi_{\alpha\beta}(\mathbf{k})=q_\beta[q^2c_{11}(\mathbf{k})+k_z^2c_{44}(\mathbf{k})], \tag{46}$$

$$q_\alpha\Phi^{-1}_{\alpha\beta}(\mathbf{k})=q_\beta[q^2c_{11}(\mathbf{k})+k_z^2c_{44}(\mathbf{k})]^{-1}. \tag{47}$$

For comparison we note that for a dense flux-line lattice in *isotropic* superconductors one has $c_{11}(\mathbf{k})\approx c_{44}(\mathbf{k})\approx(B^2/\mu_0)(1+k^2\lambda^2)$, thus

$$q_\alpha\Phi_{\alpha\beta}(\mathbf{k})=q_\beta k^2c_{11}(k),\quad q_\alpha\Phi^{-1}_{\alpha\beta}(\mathbf{k})=\frac{q_\beta}{k^2c_{11}(k)}. \tag{48}$$

Formulas (46)–(48) are required to calculate the elastic response of vortex lattices to forces exerted by structural defects or by pins.

The displacements caused by an additional antipancake-pancake pair are obtained from Eqs. (23), (44), and (47),

$$\tilde{\mathbf{u}}(\mathbf{k})=\frac{i\mathbf{q}\tilde{V}_3(\mathbf{k})(1-e^{-i\mathbf{k}\mathbf{r}_p})n_3}{q^2c_{11}(\mathbf{k})+k_z^2c_{44}(k_z)}, \tag{49}$$

with $n_3=B/(\Phi_0 s)$. Inserting here $c_{11}(\mathbf{k})$ (16) we obtain

$$\tilde{\mathbf{u}}(\mathbf{k})=\frac{i\mathbf{q}(1-e^{-i\mathbf{k}\mathbf{r}_p})}{q^2n_3}\left(1+\frac{k_z^2c_{44}(k_z)}{q^2c_{11}(\mathbf{k})}\right)^{-1}. \tag{50}$$

Using c_{11} (17) and c_{44} (18) we see that

$$g(\mathbf{k})\equiv\frac{k_z^2c_{44}(k_z)}{q^2c_{11}(\mathbf{k})}=\frac{\ln(1+k_z^2/q_{\rm BZ}^2)}{2q_{\rm BZ}^2\lambda^2}\,\frac{1+k^2\lambda^2}{k^2\lambda^2} \tag{51}$$

is a small correction, $g\ll 1$, when $q_{\rm BZ}^2=4\pi B/\Phi_0\gg\lambda^{-2}$, corresponding to $a\ll\pi\lambda$ or $B\gg B_{c1}/\ln\kappa$, and when $k^2\lambda^2$ is not too small.

To visualize the difference to isotropic lattices we consider the displacements $\mathbf{u}_\mu$ caused by a vacancy (antipancake) positioned at $\mathbf{r}=0$, given by the first term in Eq. (50) (the unity). The corresponding result for a vacancy at $\mathbf{r}=\mathbf{r}_p$ is then obtained by replacing $\mathbf{R}_\mu$ by $\mathbf{R}_\mu-\mathbf{r}_p$, and for an interstitial (additional pancake) by changing the sign. At not too large $|z_\mu|<\lambda$ in the back transform (11) for $\mathbf{u}_\mu$ mainly large $|k_z|\gg\lambda^{-1}$ contribute to the integral. One may thus replace the factor $1+1/k^2\lambda^2$ in $g(\mathbf{k})$ (51) by unity. The integrals over k_z and $\mathbf{q}$ in the back transform (11) then separate and one obtains the displacement at $\mathbf{R}_\mu=\mathbf{R}_{\mu 2}+z_\mu\hat{\mathbf{z}}$,

$$\mathbf{u}_\mu=\frac{-\mathbf{R}_{\mu 2}}{2\pi R_{\mu 2}^2 n_2}\,\frac{s}{\pi}\int_0^{\pi/s}dk_z\frac{\cos(k_z z_\mu)}{1+g(k_z)}, \tag{52}$$

where $\mathbf{R}_{\mu 2}$ are the 2D ideal lattice vectors, $n_2=B/\Phi_0$, and $g(k_z)\approx\ln(1+k_z/q_{\rm BZ})/(q_{\rm BZ}^2\lambda^2)$. For the displacements in the same layer as the vacancy ($z_\mu=0$), the general formula (52) reproduces the 2D universal result (26) with an approximate correction factor $1-g(k_z=\pi/s)\approx 1-(a^2/\pi^2\lambda^2)\ln(a/s)\approx 1$.

The displacements in other layers ($z_\mu\neq 0$) are much smaller. If the small correction $g(k_z)\ll 1$ is disregarded, the integral over k_z yields $\sin(\pi z_\mu/s)/(\pi z_\mu/s)=\delta_{m0}$ since $z_\mu=ms$ with m integer. This means the displacements in the layers $m\neq 0$ *vanish*. This approximation is the better the higher the induction B is, since the perturbation $g(k_z)$ contains a factor

$$\gamma=\frac{1}{q_{\rm BZ}^2\lambda^2}=\frac{\sqrt{3}a^2}{8\pi\lambda^2}\approx\frac{B_{c2}}{B\ln\kappa}\ll 1. \tag{53}$$

Accounting for this perturbation we get for $|z_\mu|<\lambda$ the pancake displacements caused by a vacancy sitting at the origin, with $\delta_{m0}=1$ ($m=0$) and $\delta_{m0}=0$ ($m\neq 0$),

$$\mathbf{u}_\mu=\frac{-\mathbf{R}_{\mu 2}}{2\pi R_{\mu 2}^2 n_2}\left(\delta_{m0}-\frac{\gamma}{2}\alpha_m\right). \tag{54}$$

Here the constants α_m (m integer) are

$$\alpha_m=\frac{s}{\pi}\int_0^{\pi/s}\ln\left(1+\frac{k_z^2}{q_{\rm BZ}^2}\right)\cos(k_z ms)\,dk_z$$

$$=\int_0^1\ln(1+\beta u^2)\cos(m\pi u)\,du$$

$$\approx\delta_{m0}(\ln\beta-2)-\frac{1-\delta_{m0}}{|m|} \tag{55}$$

with $\beta=(\pi/s)^2/q_{\rm BZ}^2\approx a^2/s^2\gg 1$. Thus,

$$\mathbf{u}_\mu\approx\frac{-\mathbf{R}_{\mu 2}}{2\pi R_{\mu 2}^2 n_2}\left[\left(1-\gamma\ln\frac{a}{es}\right)\delta_{m0}-\gamma\frac{1-\delta_{m0}}{2|m|}\right] \tag{56}$$

with $e=2.718$. Note that the displacements in the layer $m=0$ and in the other layers have *opposite sign*. The displacements in the mth layer decrease thus only slowly, approximately as $1/|m|=s/|z|$ when $|z|<\lambda$, but the prefactor γ is typically small.

B. Magnetic field

The magnetic field caused by a pancake vortex at $\mathbf{r}_\mu=0$ in layered superconductors with $s\ll\lambda$ is[3,6,15]

$$\mathbf{B}_p(\mathbf{r})=\int\frac{d^3k}{8\pi^3}\tilde{\mathbf{B}}_p(\mathbf{k})e^{i\mathbf{kr}},$$

$$\tilde{\mathbf{B}}_p(\mathbf{k})=\Phi_0 s\frac{\hat{\mathbf{z}}-\mathbf{q}k_z/q^2}{1+k^2\lambda^2}. \tag{57}$$

Explicitly one has, with $\vec{\rho}=(x,y)$ and $\mathbf{r}=(x,y,z)$,[3]

$$\mathbf{B}_p(\mathbf{r})=\frac{s\Phi_0}{4\pi\lambda^2}\left[\frac{e^{-r/\lambda}}{r}\hat{\mathbf{z}}+\frac{z}{\rho}\left(\frac{e^{-|z|/\lambda}}{|z|}-\frac{e^{-r/\lambda}}{r}\right)\hat{\rho}\right]. \tag{58}$$

The magnetic field of a 3D pancake lattice is the linear superposition of the fields $\mathbf{B}_p(\mathbf{r}-\mathbf{r}_\mu)$. The perturbation $\mathbf{B}_1(\mathbf{r})$ caused by pancake displacements $\mathbf{u}_\mu=\mathbf{r}_\mu-R_\mu$ is derived in analogy to Eq. (29),

$$\mathbf{B}_1(\mathbf{r})=-n_3\int\frac{d^3k}{8\pi^3}\tilde{\mathbf{B}}_p(\mathbf{k})i\mathbf{q}\tilde{\mathbf{u}}(\mathbf{k})e^{i\mathbf{kr}}. \tag{59}$$

Inserting here the displacement field caused by a vacancy or antipancake positioned at the origin, $\tilde{\mathbf{u}}(\mathbf{k})=i\mathbf{q}/\{q^2n_3[1+g(\mathbf{k})]\}$, cf. Eq. (50), one obtains

$$\mathbf{B}_1(\mathbf{r})=\int\frac{d^3k}{8\pi^3}\frac{\tilde{\mathbf{B}}_p(\mathbf{k})}{1+g(\mathbf{k})}e^{i\mathbf{kr}}. \tag{60}$$

Expanding this with respect to the small perturbation $g(\mathbf{k})$ (51) one obtains

$$\mathbf{B}_1(\mathbf{r})=\mathbf{B}_p()-\int\frac{d^3k}{8\pi^3}\tilde{\mathbf{B}}_p(\mathbf{k})g(\mathbf{k})e^{i\mathbf{kr}}. \tag{61}$$

This means that the relaxation of the 3D pancake lattice around a pancake vacancy restores the field of the removed pancake almost exactly. But while the screening in Sec. IV B was shown to be perfect for a vacancy in the 2D lattice of stiff vortex lines, and generally in isotropic 2D or 3D lattices, the compensation of the pancake-vacancy field $-\mathbf{B}_p(\mathbf{r})$ by the field of the lattice relaxation (59) is not perfect, since the pancake lattice is anisotropic and the pancakes can move only within the layers. The small residual field around the vacancy or antipancake is given by the second term in Eq. (61). Evaluating this one obtains for the screened magnetic field of a vacancy

$$\mathbf{B}_{\rm vac}(\mathbf{r})=\mathbf{B}_1-\mathbf{B}_p\approx-\int\frac{d^3k}{8\pi^3}\tilde{\mathbf{B}}_p(\mathbf{k})g(\mathbf{k})e^{i\mathbf{kr}}$$

$$=-\frac{\Phi_0 s}{q_{\rm BZ}^2\lambda^4}\int\frac{d^3k}{8\pi^3}\frac{\hat{\mathbf{z}}-\mathbf{q}k_z/q^2}{2k^2}\ln\left(1+\frac{k_z^2}{q_{\rm BZ}^2}\right)e^{i\mathbf{kr}}. \tag{62}$$

For the far field at $|z|\gg a$ one has $k_z^2\ll q_{\rm BZ}^2$ and thus may expand the logarithm in Eq. (62) and replace k_z^2 by $-\partial^2/\partial z^2$. The z component of $\mathbf{B}_{\rm vac}$ (62) may now be obtained using the formula

$$\int\frac{d^3k}{2\pi^2}\frac{e^{i\mathbf{kr}}}{k^2}=\frac{1}{r}. \tag{63}$$

The in-plane component then follows from $\mathrm{div}\mathbf{B}=0$. This yields $\mathbf{B}_{\rm vac}\propto(\partial^2/\partial z^2)\,(\hat{\mathbf{z}}/r-\hat{\rho}z/\rho^2 r)$ and finally

$$\mathbf{B}_{\rm vac}(\mathbf{r})\approx\gamma^2\frac{\Phi_0 s}{8\pi}\frac{3z\mathbf{r}-r^2\hat{\mathbf{z}}}{r^5}$$

$$=\gamma^2\frac{\Phi_0 s}{8\pi}\frac{3z\rho\hat{\rho}+(2z^2-\rho^2)\hat{\mathbf{z}}}{r^5} \tag{64}$$

with $\gamma=\lambda^{-2}q_{\rm BZ}^{-2}$ from Eq. (53).

Looking at the z component of the screened magnetic field (64) of a vacancy, we notice that this decreases as $1/r^3$ whereas the unscreened pancake (or vacancy) field, Eq. (58), decreases exponentially. Due to the large factors $q_{\rm BZ}^2\lambda^2\gg 1$ and $q_{\rm BZ}^2 r^2\gg 1$ in the denominator of Eq. (64) (far field means $r\gg a$) the amplitude of this long-range screened vacancy field is small. The word screening here refers to the magnetic field reduction caused by the relaxation of the pancake vortices close to the vacancy. But since the displacement field (56) is of longer range than the z component of the magnetic field (58) of the unscreened vacancy, *the screened magnetic field at large distances exceeds the unscreened field*.

Two compensations have occurred in Eqs. (62) and (64). First, the two factors $1+k^2\lambda^2$ in the pancake field (57) and in the compression modulus $c_{11}(\mathbf{k})$ (17) [which enters the small correction $g(\mathbf{k})$ (51) and thus Eq. (62)] have compensated. As a consequence, the far field $B_{\rm vac}(\mathbf{r})$ decreases not exponentially but algebraically. Such a compensation does not occur in the displacement field, Eq. (52).

Second, the magnetic field of the displaced pancakes, which determines the long-range algebraic tail of $B_{\rm vac}(\mathbf{r})$, Eq. (64), does not depend on the layer spacing s, apart from the trivial prefactor $s\Phi_0$ originating from the unscreened pancake field, Eq. (58). This is so since the factor s in the displacement field $\mathbf{u}_\mu\propto s/|z_\mu|$ (56) is compensated by a factor $1/s$ originating from the density of the layers. Therefore, in the limit $s\to 0$ the pancake displacements (53) caused at a constant distance $|z_\mu|\neq 0$ by removing a pancake *vanish*, but the additional magnetic field caused by these displacements stays finite.

C. Interaction energy

The self-energy of an added or removed pancake is largely compensated by the relaxation of the surrounding

dense pancake lattice as described in Sec. IV. The exact value depends on the displacements of the nearest neighbors and thus requires computation. The screened long-range interaction between pancakes, however, can be calculated analytically. Due to the *anisotropy* of layered superconductors, this screened interaction does not vanish in continuum approximation, while the screened interaction in *isotropic* materials vanishes, Sec. V.

Consider a removed pancake (a vacancy) at the origin $\mathbf{r}=0$ and an added pancake (an interstitial) at some position $\mathbf{r}_p=(\rho_p,z_m)$ with $z_m=ms$. The unscreened interaction $F_{\rm int}^0(\mathbf{r}_p)$ of this antipancake-pancake pair is given by Eq. (36) with $\tilde V_3(\mathbf{k})$ from Eq. (4) inserted. The elastic energy of the pancake displacements caused by the defect pair is given by Eqs. (41) and (42), but now with $\tilde V_3(\mathbf{k})$ replaced by $\tilde V_3(\mathbf{k})/[1+g(\mathbf{k})]$ with $g(\mathbf{k})$ from Eq. (51). The factor $(1+g)^{-1}$ occurs in $F_{\rm elast}$, Eq. (13), since the $\tilde{\mathbf{u}}(\mathbf{k})$ (50) contribute two factors $(1+g)^{-1}$, one of which is compensated by $q_\alpha\Phi_{\alpha\beta}q_\beta=q^4c_{11}(\mathbf{k})[1+g(\mathbf{k})]$, cf. Eq. (46). The screened pair interaction is thus

$$F_{\rm int}=-\int_\infty\frac{d^3k}{8\pi^3}\tilde V_3(\mathbf{k})\cos\mathbf{k}\mathbf{r}_p+\int_{\rm BZ}\frac{d^3k}{8\pi^3}\tilde V_3(\mathbf{k})\frac{\cos\mathbf{k}\mathbf{r}_p}{1+g(\mathbf{k})}. \tag{65}$$

Since we are interested in the interaction at large intraplane distances ρ_p, only small values of $q\ll q_{\rm BZ}$ enter in Eq. (65), and the different integration areas in the two integrals over $\mathbf{q}$ are irrelevant; the k_z integration in Eq. (65) is over $-\pi/s\le k_z\le\pi/s$. Expanding with respect to the small perturbation $g(\mathbf{k})\ll1$, we obtain thus

$$\begin{aligned}F_{\rm int}&=-\int\frac{d^3k}{8\pi^3}\tilde V_3(\mathbf{k})g(\mathbf{k})\cos\mathbf{k}\mathbf{r}_p\\&=-\frac{\Phi_0^2s^2}{2\mu_0q_{\rm BZ}^2\lambda^4}\int\frac{d^3k}{8\pi^3}\frac{e^{i\mathbf{k}\mathbf{r}_p}}{q^2}\ln\left(1+\frac{k_z^2}{q_{\rm BZ}^2}\right)\\&=-\frac{\Phi_0^2s}{4\pi\mu_0q_{\rm BZ}^2\lambda^4}\ln\frac{R}{\rho_p}\alpha_m\end{aligned} \tag{66}$$

with α_m from Eq. (55). Note that the separation into two factors depending only on ρ_p and z_m is exact in Eq. (66) since the factors $k^2/(1+k^2\lambda^2)$ in $\tilde V_3(\mathbf{k})$ (4) and $(1+k^2\lambda^2)/k^2$ in $g(\mathbf{k})$ exactly cancel. In contrast, with the displacement field $\mathbf{u}_\mu$, Eq. (52), one had to assume $|z|<\lambda$ to obtain $(1+k^2\lambda^2)/k^2\lambda^2\approx1$ and arrive at the factors α_m, Eq. (55). Equation (66) therefore describes the pair interaction at arbitrarily large distances $r_p\gg a$. An inner cutoff in the q integration was chosen such that the interaction vanishes when the pair separation reaches the specimen size R. If $|z_m|$ is not too large one may use for α_m the approximation given by the last line of Eq. (55). The screened intraplane interaction between an antipancake and a pancake is then,

$$F_{\rm int}(z_m=0)=-\gamma\frac{\Phi_0^2s}{2\pi\mu_0\lambda^2}\ln\frac{R}{\rho_p}\ln\frac{a}{es}, \tag{67}$$

and the screened interplane interaction is

$$F_{\rm int}(z_m\ne0)=-\gamma\frac{\Phi_0^2s}{4\pi\mu_0\lambda^2}\ln\frac{R}{\rho_p}\frac{1}{|m|} \tag{68}$$

with $\gamma\approx(a/\pi\lambda)^2\ll1$, Eq. (52). Note that this screened antipancake-pancake interaction is *attractive* for all layer indices m since all the logarithmic factors are positive. In contrast, the unscreened pair interaction, Sec. II, is attractive for $m=0$ but repulsive for $m\ne0$.

Obviously, the screened interaction between two pancakes or between two antipancakes is also given by Eqs. (66)–(68) but has opposite sign. The ratio between the screened intraplane and interplane interaction across $|m|$ layers in all these cases is

$$\frac{F_{\rm int}(z_m=0)}{F_{\rm int}(z_m\ne0)}=\frac{\alpha_{m=0}}{\alpha_{m\ne0}}=2|m|\ln\frac{a}{es}. \tag{69}$$

The ratio between the screened and unscreened interaction of pancakes or antipancakes, $S=F_{\rm int}^{\rm scr}/F_{\rm int}^{\rm unscr}$, follows from Eqs. (6), (7), (67), and (68),

$$S(\rho\gg\xi,z_m=0)=\gamma\ln\frac{a}{es}, \tag{70}$$

$$S(\rho\gg\lambda,z_m\ne0)=-\gamma\frac{\lambda}{|z_m|}\exp\frac{|z_m|}{\lambda}. \tag{71}$$

Since $\gamma\approx a^2/(\pi\lambda)^2\ll1$, the intraplane interaction is always reduced by screening, while the interplane interaction changes sign and at $z_m\gg\lambda$ even exceeds in magnitude the exponentially decreasing unscreened interaction.

Though the exact value of the screened self-energies is not known, it appears natural to assume that inclusion of the self-energies of the antipancake-pancake pair changes the factor $\ln(R/\rho_p)$ in Eq. (67) to $\ln(\xi/\rho_p)$, yielding the total screened energy of a pair in the same layer,

$$F_{\rm pair}(z_m=0)=\gamma\frac{\Phi_0^2s}{2\pi\mu_0\lambda^2}\ln\frac{\rho_p}{\xi}\ln\frac{a}{es}, \tag{72}$$

in analogy to Eq. (8). The reduction by screening is again $F_{\rm pair}^{\rm scr}/F_{\rm pair}^{\rm unscr}=\gamma\ln(a/es)\ll1$.

VII. SUMMARY

In 2D and 3D lattices with isotropic long-range interaction between the lattice points or lines, the energy of point defects is strongly reduced by the relaxation of the surrounding lattice. Within a consistent continuum approximation, the screened self- and interaction energies of vacancies and interstitials even vanish, i.e., the direct interaction is compensated by the indirect elastic interaction. Since this ideal screening results from an exact compensation of large energy terms, all these terms should *not* be approximated in order to avoid spurious results. The lattice relaxation around point defects within continuum approximation is such that, after introduction of the defect, the number of lattice points in any given area or volume stays constant, even if this area or volume contains the point defect, cf. Eq. (26).

In anisotropic lattices, the screening of point defects is not complete, even within continuum approximation. For the 3D lattice of point vortices (pancakes) in layered superconduct-

ors, the continuum approximation is excellent since the interaction of the pancake vortices is much longer ranged than the typical spacing of pancakes. The elastic displacements and magnetic field around a screened vacancy are given in Secs. VI A and B, and the screened interaction between pancakes or antipancakes in Sec. VI C. The screened magnetic field and interaction of pancakes contain a small prefactor $\gamma \approx (a/\pi\lambda)^2 \ll 1$, which means strong screening of the near field. However, at large distances, the relaxation of the pancake lattice *increases* the magnetic field and the interaction energy: These have long-ranging tails which decrease more slowly than the unscreened pancake field or interaction and which originate from the algebraically decreasing elastic displacements. In addition, the screening changes the sign of the interaction of pancakes located in different layers. The screening and modification of the pancake interaction by the static relaxation of the surrounding pancake lattice possibly should be considered in future theories of pinning and thermal depinning, and in the prediction of phase diagrams of layered superconductors in a magnetic field.

ACKNOWLEDGMENTS

Stimulating discussions with Roman Mints are acknowledged. This work was supported by the German–Israeli Foundation for Research and Development, Grant No. I-300-101.07/93.

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