Coupling mechanisms for damped vortex motion in superfluids

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We investigate different dissipative dynamics for a vortex line immersed into a superfluid where density fluctuations have been excited. For this sake, we consider various linear coupling models where the vortex interacts with the quasiparticles of the normal fluid through its coordinate or momentum. We can unambiguously show that one and only one combination of these variables leads to damped evolution in agreement with the phenomenological descriptions. [S0163-1829(97)07434-1]

I. INTRODUCTION

Quantum dissipation has been a vivid branch of research for several decades[1,2] in view of both the fundamental theoretical aspects of the subject and the vast application fields, that include quantum optics, physical chemistry, nuclear physics, and condensed-matter physics. The best known models resort to a system-plus-reservoir description, where the irreversible approach to thermal equilibrium of a quantum object is examined using projection and/or reduction techniques. The key ingredient to starting such a study is the Hamiltonian, which consists of three terms, respectively, corresponding to the free system, to the reservoir, and to their mutual interaction. Among the most popular choices for the latter, those being linear in operators representing the coordinate or the momentum of the quantum particle are preferred for applications in the frame of standard nonequilibrium statistical mechanics. In particular, those interaction models known as the rotating wave approximation (RWA) and the full coupling (FC) model, have been favored by many authors.[3–6]

In spite of the fact that the RWA has strong foundations, especially concerning applications to quantum optics, some of its drawbacks have been pointed out by several authors. In particular, being a velocity-dependent interaction, it breaks the equivalence between velocity and momentum mediated by the inertia parameter.[7] This brings some undesirable consequences, mainly the fact that one may not recover the classical limit of a semiclassical evolution. The FC model is, in contrast, well behaved in the classical limit of the semiclassical description of quantum dissipative motion.[7] However, it is well known that such interaction mechanisms induce potential terms which localize the Brownian particle, which ought to be artificially removed adding a counterterm in the original, unperturbed Hamiltonian.[1,2]

We have investigated these aspects of the RWA and the FC model focusing upon quantum harmonic motion[7] and spin relaxation.[8–10] However, the realm of physical systems which can be mapped onto a simple quantal Brownian motion model is much wider; recently, we have shown that a single vortex moving in a superfluid containing quanta of density fluctuations can be regarded as a quantum Brownian particle interacting with a reservoir.[11] The description of the subsequent damped motion presents a substantial agreement with the phenomenological approaches. The most important aspect of the model is the interaction mechanism adopted, which involves the vortex velocity in a linear approximation.

In this work we investigate to a deeper extent the different dissipative dynamics that coupling models of the FC form assign to point vortices immersed into a superfluid which contains some excitations. In particular, we show that FC Hamiltonians expressed in terms of either the coordinate or the momentum of the vortex give rise to localization phenomena in phase space. In order to suppress phase-space localization, we search the kind of mixed couplings that involve linear combinations of coordinate and momentum in the spirit of Ref. 11. We find that there is a unique interaction mechanism that leads to damped evolution in agreement with the phenomenological descriptions. This mechanism is precisely the one investigated in Ref. 11, where it appeared as a natural choice in view of the fact that the free motion of a point vortex in a fluid is electromagnetic, in other words, the driving force acts on the vortex velocity similarly to the Lorentz force on a charged particle.

This paper is organized as follows. In Sec. II we review the description of the motion of a free vortex filament with cylindrical symmetry in a superfluid at zero temperature and set our notation. In Sec. III, different choices for the FC interaction Hamiltonian are discussed and it is shown that if the coupling involves only the coordinate or the momentum of the point particle, its subsequent motion is not physically acceptable. The solution is presented in Sec. IV, where we show that the only Hamiltonian leading to realistic evolution of the vortex involves both the coordinate and the momentum of the particle linearly combined as in the velocity of the free motion. Finally, some concluding remarks are presented in Sec. V.

II. CYCLOTRON VORTEX MOTION

The cyclotron motion of a cylindrical vortex parallel to the z axis in a superfluid at zero temperature is provoked by the Magnus force, which provides the lift upon a cylinder that moves with velocity $\mathbf{v}-\mathbf{v}_s$ relative to the fluid and exhibits circulation around the z axis, $\mathbf{v}_s$ being the superfluid velocity and $q_v=\pm 1$ is the sign of the vorticity according to the right-handed convention. The structure of this force is

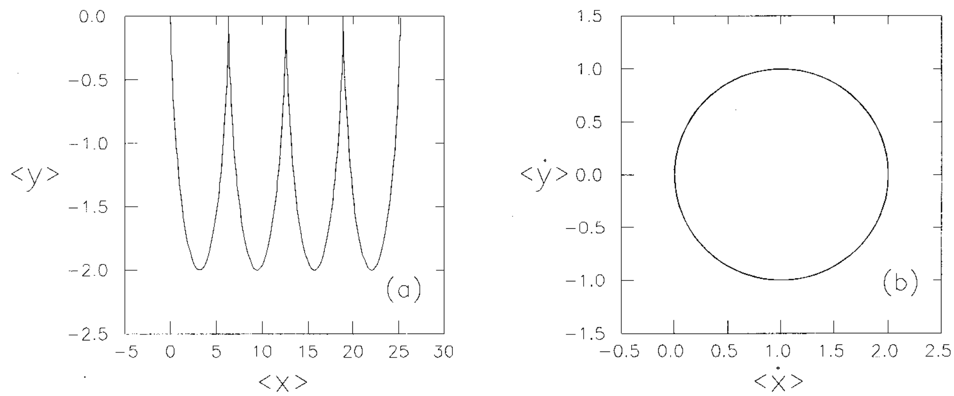

FIG. 1. Trajectories of mean values of Heisenberg operators in the coordinate (a) and velocity (b) planes for the cyclotron motion in Eq. (2.8). The coordinates in (a) and the velocities in (b) are, respectively, given in units of v_s/Ω and v_s. We have assumed a vortex initially at rest at the origin of the coordinate plane. Note that the center of the circle in (b) corresponds to the superfluid velocity.

$$\mathbf{F}_M = q_v h \rho_s l \hat{\mathbf{z}} \mathrm{x} (\mathbf{v} - \mathbf{v_s}). \tag{2.1}$$

In what follows we shall assume a uniform superfluid velocity pointing along the x axis. This motion has been described in Ref. 12 and corresponds to the Hamiltonian dynamics generated by

$$H = \frac{1}{2M}[\mathbf{p} - q_v \mathbf{A}(\mathbf{r})]^2 + M\Omega v_s y \tag{2.2}$$

with the vector potential

$$\mathbf{A}(\mathbf{r}) = \frac{h\rho_s l}{2}(y, -x), \tag{2.3}$$

which gives rise to the component of the lift proportional to the vortex velocity. The component of this force depending upon the superfluid velocity is minus the gradient of the scalar potential $M\Omega v_s y$. Here M is the dynamical mass of a vortex element of length l and ρ_s is the number density of the superfluid, which at zero temperature coincides with the total density per unit mass ρ/m, m being the mass of a single atom. Furthermore,

$$\Omega = \frac{q_v h \rho_s l}{M} \tag{2.4}$$

is the unperturbed cyclotron frequency, where h is Planck's constant.

In terms of the complex position and momentum operators

$$q = x + iy; \quad p = p_x + i p_y, \tag{2.5}$$

the Heisenberg equations of motion are

$$\dot{q} = \frac{p}{M} + i\frac{\Omega}{2} q, \tag{2.6}$$

$$\dot{p} = i\frac{\Omega}{2} p - \frac{M\Omega^2}{4} q - iM\Omega v_s, \tag{2.7}$$

or, equivalently

$$\ddot{q} = i\Omega(\dot{q} - v_s), \tag{2.8}$$

where the right-hand side (rhs) displays the complex form of the classical Magnus force.

Integration of Eqs. (2.6) and (2.7) gives

$$q(t) = f_+(t) q_0 - i\frac{2}{M\Omega} f_-(t) p_0 + v_s\left[t + i\frac{2}{\Omega} f_-(t)\right], \tag{2.9}$$

$$p(t) = i\frac{M\Omega}{2} f_-(t) q_0 + f_+(t) p_0 - i\frac{\Omega}{2} M v_s\left[t - i\frac{2}{\Omega} f_-(t)\right], \tag{2.10}$$

where q_0, p_0 are the initial position and momentum and

$$f_\pm(t) = \frac{e^{i\Omega t} \pm 1}{2}. \tag{2.11}$$

The above equations describe the simple cyclotron motion whose trajectories for the mean values of the Heisenberg operators in the coordinate and velocity planes are depicted in Figs. 1(a) and 1(b).

III. DISSIPATIVE DYNAMICS

If the superfluid contains elementary excitations, these can behave as a reservoir to which the vortex may couple. This interaction provides a dissipation mechanism that damps the cyclotron motion. If we denote by σ the density operator of the vortex, we can derive, in the weak coupling–non-Markovian limit,[9,10] a generalized master equation (GME) with time-dependent coefficients; this is achieved combining the standard reduction-projection procedure of nonequilibrium statistical mechanics[13] with the time convolutionless method developed by Chaturvedi and Shibata.[14] For an interaction of the form

$$H_{\mathrm{int}} = \sum_i \lambda_i S_i B_i, \tag{3.1}$$

where S_i and B_i are vortex and reservoir operators, respectively, the GME reads

$$\dot{\sigma}(t)+\frac{i}{\hbar}[H,\sigma]$$

$$=-\frac{1}{\hbar^2}\sum_{i,j}\lambda_i\lambda_j\int_0^t d\tau\{[S_i,[S_j(-\tau),\sigma]]\phi_{ij}(\tau)$$

$$+i[S_i,[S_j(-\tau),\sigma]_+]\psi_{ij}(\tau)\}, \tag{3.2}$$

where $[a,b]_+$ denotes an anticommutator. The operators S_i appearing in this expression will be either the coordinate or the momentum of the free cyclotron motion displayed in Eqs. (2.9) and (2.10). In spite of its integrodifferential equation structure, the GME is a differential one, since the unknown σ under the integral sign is taken at time t; the time-dependent functions $\phi_{ij}(t)$ and $\psi_{ij}(t)$ are the real and imaginary parts, respectively, of the correlation between heat bath operators[6]

$$\langle B_i(\tau)B_j\rangle=\phi_{ij}(\tau)+i\psi_{ij}(\tau). \tag{3.3}$$

We shall derive equations of motion for the expectation values of the position and momentum components of the vortex. Taking into account very general relations involving operators a, b, and c, namely

$$\mathrm{Tr}(a[b,[c,\sigma]])=\mathrm{Tr}([[a,b],c]\sigma)\equiv\langle[[a,b],c]\rangle, \tag{3.4}$$

$$\mathrm{Tr}(a[b,[c,\sigma]_+])=\mathrm{Tr}([[a,b],c]_+\sigma)\equiv\langle[[a,b],c]_+\rangle, \tag{3.5}$$

we readily obtain for an observable O

$$\langle\dot{O}\rangle+\frac{i}{\hbar}\langle[O,H]\rangle$$

$$=-\frac{1}{\hbar^2}\sum_{i,j}\lambda_i\lambda_j\int_0^t d\tau\{\langle[[O,S_i],S_j(-\tau)]\rangle\phi_{ij}(\tau)$$

$$+i\langle[[O,S_i],S_j(-\tau)]_+\rangle\psi_{ij}(\tau)\}. \tag{3.6}$$

Since the general coupling (3.1) is linear in q and/or p, the motion equations for the expectation values are of the form

$$\langle\dot{q}\rangle=a_{qq}\langle q\rangle+a_{qp}\langle p\rangle+c_q,$$

$$\langle\dot{p}\rangle=a_{pq}\langle q\rangle+a_{pp}\langle p\rangle+c_p. \tag{3.7}$$

In general, the coefficients in Eq. (3.7) are time dependent and involve time integrals of the correlation function of the reservoir. The Markovian regime suppresses these time dependences, since in that case all upper integration limits become infinite. In such a situation, similarly to the free cyclotron motion, one can merge Eqs. (3.7) into a single force equation, namely

$$\langle\ddot{q}\rangle=i\Omega(1+\gamma)\langle\dot{q}\rangle+\omega^2\langle q\rangle+a, \tag{3.8}$$

where

$$i\Omega(1+\gamma)=a_{qq}+a_{pp} \tag{3.9}$$

is a complex friction constant,

$$\omega^2=a_{qp}a_{pq}-a_{qq}a_{pp} \tag{3.10}$$

is the determinant of the linear system at the rhs of Eq. (3.7) and

$$a=a_{qp}c_p-a_{pp}c_q, \tag{3.11}$$

is a reference acceleration.

In order to appreciate the effect of the reservoir constituted by the excitations of the superfluid, at this point it will be instructive to compare the expectation value of the Magnus force equation (2.8) and the force equation (3.8). In fact, we observe that the reservoir induces both a dissipative and a conservative coupling, respectively, measured by the parameters $\mathrm{Im}(\gamma)$ and ω^2. We also realize that the strength and direction of the original Magnus force [rhs of Eq. (2.8)] will be modified through the parameters $\mathrm{Re}(\gamma)$ and a. In other words, in addition to a damping force, an harmonic restoring force plus a gravitylike component appear. This means that, in general, the motion takes place around the minimum of the induced conservative potential, which provides then a fixed point to the dynamics, namely

$$\langle q\rangle_F=-\frac{a}{\omega^2}. \tag{3.12}$$

However, it is important to remark that such a fixed point disappears whenever the harmonic force in Eq. (3.8) vanishes, i.e., when the determinant of the linear system at the rhs of Eq. (3.7) is identically zero.

A. Coordinate-dependent coupling

We first consider a coupling model that describes the damping mechanism by means of an interaction Hamiltonian of the form

$$H_{\mathrm{int}}=\lambda_r\mathbf{B}\cdot\mathbf{r} \tag{3.13}$$

with $\mathbf{r}$ the position vector of the point vortex and $\mathbf{B}$ a vector function of operators that create density fluctuations in the liquid. In the frame of the current description, this function is related to the Hermitian part of the Feynman-Cohen operator $\hat{O}_{\mathbf{k}}$ that creates a phonon or a roton with momentum k (Ref. 15)

$$\hat{O}_{\mathbf{k}}^\dagger=\rho_{\mathbf{k}}^\dagger-\frac{1}{N}\sum_{\mathbf{k}'\neq\mathbf{k}}\frac{\mathbf{k}'\cdot\mathbf{k}}{k'^2}\rho_{\mathbf{k}'}^\dagger\rho_{\mathbf{k}-\mathbf{k}'}^\dagger, \tag{3.14}$$

where N is the number of atoms in the liquid and $k=|\mathbf{k}|$. Notice that if, for example, the Hermitian operator $\mathbf{B}$ is chosen as proportional to

$$B_k=\frac{\hat{O}_{\mathbf{k}}^\dagger+\hat{O}_{\mathbf{k}}}{\sqrt{2}}, \tag{3.15}$$

the function

$$\langle B_k(\tau)B_k\rangle=\phi_{\mathbf{k}}(\tau)+i\psi_{\mathbf{k}}(\tau) \tag{3.16}$$

is related to the Fourier transform of the dynamical structure factor $S(\mathbf{k},\omega)$ of the superfluid, which for the case of liquid-helium isotopes is experimentally known for a wide range of transferred momenta and energy.[16,17]

From Eq. (3.6) we readily get

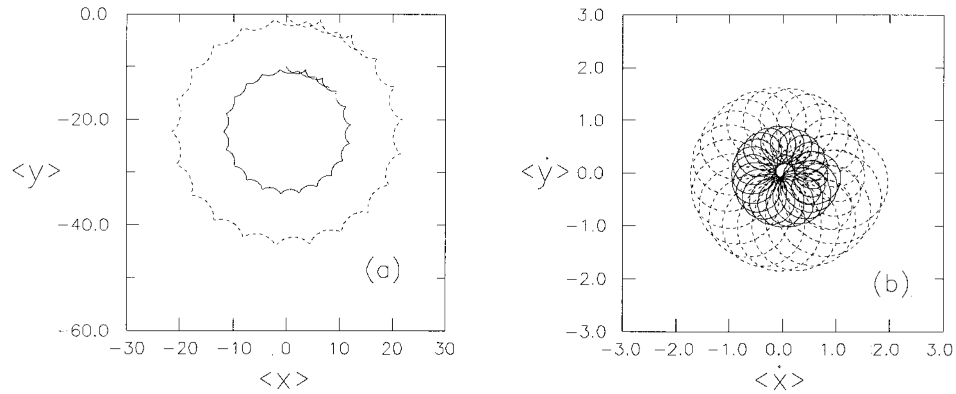

FIG. 2. Same as Fig. 1 for trajectories arising from Eq. (3.28) for a friction coefficient Im $(\gamma)=0.0045$. The dashed line trajectories correspond to the initial conditions of Fig. 1, while the continuous line ones assume the same initial velocity but an initial position $\langle q_0\rangle=(0,-10)$.

$$\langle\dot{q}\rangle=\frac{\langle p\rangle}{M}+i\frac{\Omega}{2}\langle q\rangle, \tag{3.17}$$

$$\langle\dot{p}\rangle=i\frac{\Omega}{2}\langle p\rangle-\frac{M\Omega^2}{4}\langle q\rangle-iM\Omega v_s -\frac{2\lambda_r^2}{\hbar}\int_0^t d\tau\langle q(-\tau)\rangle\psi(\tau). \tag{3.18}$$

Inserting Eq. (2.9) in Eq. (3.18) we obtain the coefficients corresponding to Eq. (3.7) as

$$a_{qq}=i\frac{\Omega}{2}, \tag{3.19}$$

$$a_{qp}=\frac{1}{M}, \tag{3.20}$$

$$c_q=0, \tag{3.21}$$

$$a_{pq}=-\frac{M\Omega^2}{4}\left(1-\frac{4\lambda_r^2}{M\Omega^2}\gamma_+\right), \tag{3.22}$$

$$a_{pp}=i\frac{\Omega}{2}\left(1-\frac{4\lambda_r^2}{M\Omega^2}\gamma_-\right), \tag{3.23}$$

$$c_p=-iM\Omega v_s\left[1-i\frac{\lambda_r^2}{M\Omega}\left(\gamma_0-i\frac{2}{\Omega}\gamma_-\right)\right], \tag{3.24}$$

where the involved time-dependent coefficients read

$$\gamma_\pm(t)=-\frac{2}{\hbar}\int_0^t d\tau f_\pm(-\tau)\psi(\tau), \tag{3.25}$$

$$\gamma_0(t)=-\frac{2}{\hbar}\int_0^t d\tau\tau\psi(\tau) \tag{3.26}$$

with $f_\pm(\tau)$ given by Eq. (2.11).

Specifying now the Markovian limit where any time dependence originated in a time integral is suppressed, we obtain the harmonic restoring strength

$$\omega^2=\frac{\lambda_r^2}{M}(\gamma_+-\gamma_-) \tag{3.27}$$

and the second-order equation for the acceleration is

$$\langle\ddot{q}\rangle=i\Omega\left(1-\frac{2\lambda_r^2}{M\Omega^2}\gamma_-\right)(\langle\dot{q}\rangle-v_s) +\frac{\lambda_r^2}{M}(\gamma_+-\gamma_-)\langle q\rangle-v_s\frac{\lambda_r^2}{M}\gamma_0. \tag{3.28}$$

In order to illustrate with some numerical results, we have considered a Gaussian reservoir.[18] In this model the imaginary part of the correlation function (3.3) is given by the exponential decay law $\psi(\tau)=-Aue^{-u\tau}$, where u is the inverse of the reservoir relaxation time and A is a positive constant depending on the specific form of the **B** operators. Then all parameters of the force equation (3.28) are determined by a unique dimensionless friction coefficient Im(γ) [cf. Eq. (3.8)]:

$$\mathrm{Im}(\gamma)=\mathrm{Im}\left(-\frac{2\lambda_r^2}{M\Omega^2}\gamma_-\right)\simeq\frac{2A\lambda_r^2}{M\hbar\Omega u}, \tag{3.29}$$

in addition to the dimensional parameters Ω and v_s.

Therefore Eq. (3.28) can be straightforwardly integrated and some of the trajectories are depicted in Fig. 2, where we set $u=10\Omega$ in order to enforce the Markovian hypothesis. One expects that in a superfluid with velocity v_s, the only initial condition which determines the dynamics at later times is the vortex velocity. However, Fig. 2 shows that for a given initial velocity, different choices of the initial vortex position give rise to significant departures between the corresponding velocities at later times. This reflects another undesired consequence of the presence of the fixed point Eq. (3.12).

B. Momentum-dependent coupling

A similar behavior takes place if one assumes a coupling that depends upon the vortex momentum, i.e., if the interaction Hamiltonian is

$$H_{\mathrm{int}}=\lambda_p\mathbf{B}_p\cdot\mathbf{p}. \tag{3.30}$$

In this case, the corresponding equations of motion for the expectation values of complex position and momentum are

$$\langle\dot{q}\rangle=\frac{\langle p\rangle}{M}+i\frac{\Omega}{2}\langle q\rangle+\frac{2\lambda_p}{\hbar}\int_0^t d\tau\langle p(-\tau)\rangle\psi(\tau), \tag{3.31}$$

$$\langle\dot{p}\rangle = i\frac{\Omega}{2}\langle p\rangle - \frac{M\Omega^2}{4}\langle q\rangle - iM\Omega v_s. \tag{3.32}$$

Using Eq. (2.10) we obtain the coefficients

$$a_{qq} = i\frac{\Omega}{2}(1-\lambda_p^2 M\gamma_-), \tag{3.33}$$

$$a_{qp} = \frac{1}{M}(1-\lambda_p^2 M\gamma_+), \tag{3.34}$$

$$c_q = M\lambda_p^2 v_s\left(\gamma_- - i\frac{\Omega}{2}\gamma_0\right), \tag{3.35}$$

$$a_{pq} = -\frac{M\Omega^2}{4}, \tag{3.36}$$

$$a_{pp} = i\frac{\Omega}{2}, \tag{3.37}$$

$$c_p = -iM\Omega v_s. \tag{3.38}$$

In this case

$$\omega^2 = \frac{\lambda_p^2 M\Omega^2}{4}(\gamma_+ - \gamma_-) \tag{3.39}$$

and the corresponding acceleration in the Markovian regime is

$$\langle\ddot{q}\rangle = i\Omega\left(1-\frac{\lambda_p^2 M}{2}\gamma_-\right)(\langle\dot{q}\rangle - v_s) + \lambda_p^2\frac{M\Omega^2}{4}(\gamma_+ - \gamma_-)\langle q\rangle - \lambda_p^2\frac{M\Omega^2}{4}v_s\left[\gamma_0 - i\frac{4}{\Omega}(\gamma_+ - \gamma_-)\right]. \tag{3.40}$$

It should be noticed that similarly to what happens for the coordinate-dependent coupling [cf. Eq. (3.28)], the trajectories will develop around a fixed point, representing a nonrealistic dynamics. Moreover, if we set $\mu_p \equiv \lambda_p$ and $\mu_r \equiv 2\lambda_r/(M\Omega)$ Eqs. (3.28) and (3.40) can be written as

$$\langle\ddot{q}\rangle = i\Omega\left(1-\mu_j^2\frac{M}{2}\gamma_-\right)(\langle\dot{q}\rangle - v_s) + \mu_j^2\frac{M\Omega^2}{4}(\gamma_+ - \gamma_-)\langle q\rangle - \mu_j^2\frac{M\Omega^2}{4}v_s\gamma_0 - i\delta_{jp}M\Omega v_s\mu_p^2(\gamma_+ - \gamma_-) \tag{3.41}$$

for $j = r,p$. Then assuming $\mu_r = \mu_p$ the fixed point lies at

$$\langle q\rangle_F^p = \langle q\rangle_F^r - i\frac{4}{\Omega}v_s, \tag{3.42}$$

where the superscripts r,p indicate the type of coupling Hamiltonian under consideration.

IV. COMBINED COUPLING MODEL

In view of the results presented in the previous section, it appears natural to inquire whether a suitable linear combination of the previous coupling Hamiltonians could suppress the fixed point intrinsic to the coupling mechanisms there discussed. For this sake, we now investigate the phase-space dynamics that the GME associates to the interaction Hamiltonian

$$H_{\rm int} = \lambda_r \mathbf{r}\cdot\mathbf{A} + \lambda_p \mathbf{p}\cdot\mathbf{B}. \tag{4.1}$$

From now on we assume that the two-dimensional vector operators **A** and **B** of the reservoir satisfy

$$\langle A_i(t)A_j\rangle = \langle B_i(t)B_j\rangle = 0 \quad \text{for } i\neq j \tag{4.2}$$

and that $\mathbf{B} = R(\varphi)\mathbf{A}$, where $R(\varphi)$ is the matrix associated with a rotation in an angle φ. This hypothesis implies that the physical operator which represents the elementary excitation is unique, and that its different manifestations in interaction Hamiltonians involving either the coordinate or the momentum of the vortex cannot differ but in the orientation of the vector in the direction perpendicular to the vortex filament. Accordingly [cf. Eq. (3.3)], we have

$$\psi_{A_iB_i}(\tau) = \psi_{B_iA_i}(\tau) = \psi(\tau)\cos\varphi, \tag{4.3}$$

$$\psi_{A_xB_y}(\tau) = \psi_{B_yA_x}(\tau) = \psi(\tau)\sin\varphi, \tag{4.4}$$

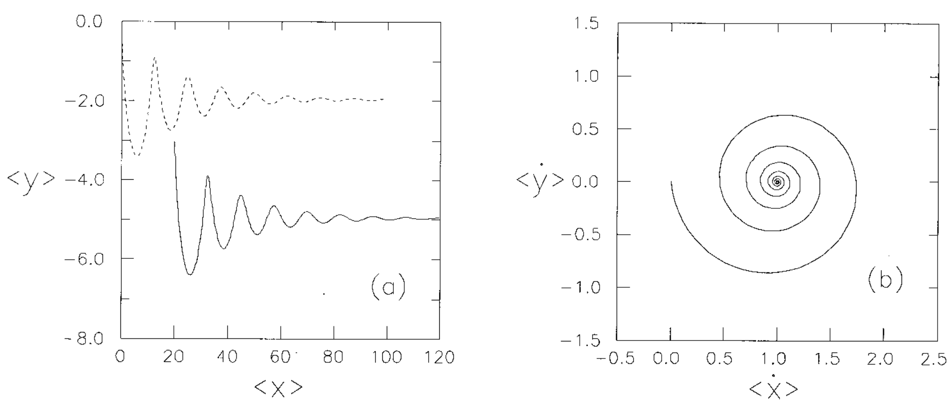

FIG. 3. Same as Fig. 1 for the trajectories arising from Eq. (4.18) for a friction coefficient $\mathrm{Im}(\gamma)=0.05$. The dashed line in (a) corresponds to the initial conditions of Fig. 1 while the continuous line assumes the same initial velocity but an initial position $\langle q_0\rangle = (20,-3)$. Both cases yield in (b) the same trajectory on the velocity plane, i.e., a spiral curve converging to the superfluid velocity.

$$\psi_{A_yB_x}(\tau)=\psi_{B_xA_y}(\tau)=-\psi(\tau)\sin\varphi, \qquad (4.5)$$

where

$$\psi(\tau)\equiv\psi_{A_xA_x}(\tau)=\psi_{B_xB_x}(\tau). \qquad (4.6)$$

The evolution of the mean values of the complex position and momentum are given by

$$\langle\dot{q}\rangle=\frac{\langle p\rangle}{M}+i\frac{\Omega}{2}\langle q\rangle+\frac{2\lambda_p}{\hbar}\int_0^t d\tau\{\lambda_r\langle q(-\tau)\rangle e^{i\varphi}+\lambda_p\langle p(-\tau)\rangle\}\psi(\tau), \qquad (4.7)$$

$$\langle\dot{p}\rangle=i\frac{\Omega}{2}\langle p\rangle-\frac{M\Omega^2}{4}\langle q\rangle-iM\Omega v_s-\frac{2\lambda_r}{\hbar}\int_0^t d\tau\{\lambda_r\langle q(-\tau)\rangle+\lambda_p\langle p(-\tau)\rangle e^{-i\varphi}\}\psi(\tau), \qquad (4.8)$$

and the corresponding coefficients are

$$a_{qq}=i\frac{\Omega}{2}(1+iM\mu_r\mu_p e^{i\varphi}\gamma_+-M\mu_p^2\gamma_-), \qquad (4.9)$$

$$a_{qp}=\frac{1}{M}(1+iM\mu_r\mu_p e^{i\varphi}\gamma_--M\mu_p^2\gamma_+), \qquad (4.10)$$

$$c_q=Mv_s\left[\mu_r\mu_p\ \frac{\Omega}{2}e^{i\varphi}\left(\gamma_0-i\frac{2}{\Omega}\gamma_-\right)+\mu_p^2\left(\gamma_--i\frac{\Omega}{2}\gamma_0\right)\right], \qquad (4.11)$$

$$a_{pq}=-\frac{M\Omega^2}{4}(1-iM\mu_r\mu_p e^{-i\varphi}\gamma_--M\mu_r^2\gamma_+), \qquad (4.12)$$

$$a_{pp}=i\frac{\Omega}{2}(1-iM\mu_r\mu_p e^{-i\varphi}\gamma_+-M\mu_r^2\gamma_-), \qquad (4.13)$$

$$c_p=Mv_s\Omega\left[-i+M\mu_r^2\frac{\Omega}{4}\left(-\gamma_0+i\gamma_-\frac{2}{\Omega}\right)+\frac{M}{2}\mu_p\mu_r e^{-i\varphi}\left(i\frac{\Omega}{2}\gamma_0-\gamma_-\right)\right], \qquad (4.14)$$

with μ_r and μ_p defined below Eq. (3.40). The fixed point is suppressed if the determinant of the system vanishes. This condition yields

$$\frac{\mu_r}{\mu_p}=\sin\varphi\pm\sqrt{\sin^2\varphi-1}, \qquad (4.15)$$

and since μ_r and μ_p must be real quantities, this implies that

$$\varphi=\pm\frac{\pi}{2}, \quad \mu_r=\pm\mu_p. \qquad (4.16)$$

One can realize that the above relationship requires

$$H_{\mathrm{int}}=\pm\lambda_p\left\{\left(p_x-\frac{M\Omega}{2}y\right)B_x+\left(p_y+\frac{M\Omega}{2}x\right)B_y\right\}, \qquad (4.17)$$

and this in turn, indicates that the only interaction mechanism which does not localize the vortex in phase space, involves the particular combination of coordinate and momentum giving the free velocity of a point vortex moving under the Magnus force. The coupling Hamiltonian thus obtained corresponds to the case considered in Ref. 11. The second-order equation for the acceleration now reads

$$\langle\ddot{q}\rangle=i\Omega[1-M\mu_r^2(\gamma_++\gamma_-)](\langle\dot{q}\rangle-v_s), \qquad (4.18)$$

where one can identify the friction coefficient $\mathrm{Im}(\gamma)$ of Eq. (3.8) as

$$\mathrm{Im}(\gamma)=\mathrm{Im}[-M\mu_r^2(\gamma_++\gamma_-)], \qquad (4.19)$$

which may be approximated to $\mathrm{Im}(\gamma)\simeq 8A\lambda_r^2/M\hbar\Omega u$, under the same assumptions leading to the rhs of Eq. (3.29). Integration of Eq. (4.18) yields the trajectories depicted in Fig. 3 where it is shown that the shape of the orbits does not depend on the initial vortex position, as expected.

V. CONCLUDING REMARKS

The damped motion of a vortex in a superfluid was investigated under different linear couplings in the vortex observables. We have shown that a coupling proportional to the free velocity reproduces the basic features of a realistic damped motion. It is worth noting that this Hamiltonian coupling is equivalent to a Lagrangian coupling proportional to the true vortex velocity. Moreover, we have shown that any other linear combination of coordinate and momentum must be discarded, since it produces nonrealistic trajectories around a fixed point.

Note that in contrast to superfluids, the interaction Hamiltonian for vortices in superconductors may be proportional to the vortex coordinate[19] since in such a case, the pinning potential gives rise to localization. This is equivalent to taking $\lambda_p=0$ in our equations.

It is interesting to discuss some features regarding the simplest dissipative dynamics without localization, i.e., that undergone by a free Brownian particle. In such a case, the coupling to the heat bath is usually modeled by a translationally invariant form involving only coordinates.[1] However, an interaction linear in the momentum, which in this case is proportional to the free velocity, has been shown to be useful in situations where both the system and the reservoir variables are functions of the same set of degrees of freedom.[20] This is precisely the case if the Brownian particle represents a collective excitation interacting with intrinsic ones, and one can then show that the dynamics do not contain any fixed point.[21] In fact, the limit $\Omega\to 0$ in our equations should indeed correspond to such dynamics, with $\lambda_r=0$, i.e., a coupling proportional to the momentum or free velocity is the condition for nonappearance of fixed points [Eq. (4.15)]. Finally, we have shown that in the presence of ''external fields'' with $\Omega\neq 0$, realistic dynamics take place only when both λ_r and λ_p are different from zero and related according to $\lambda_p=\pm 2\lambda_r/M\Omega$.

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