

# Superconducting fluctuations in the thermal conductivity of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ and $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$ materials

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The thermal conductivity  $\kappa$  of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystals is analyzed near the critical temperature ( $T_c = 79.5$  and  $83.8$  K, respectively) in order to extract the superconducting fluctuation contribution  $\kappa_{\text{fl}}$ . The fluctuationless background is calculated in a formal way, taking into account both an electronic and a phonon contribution above and below the critical temperature. The fluctuation contribution to the electronic thermal conductivity is then extracted. A crossover from a critical to a Gaussian fluctuation regime ( $|T_G - T_c| \approx 2.2$  K) is observed on the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample both above and below  $T_c$ , followed by a crossover from a three-dimensional (3D) to a 2D Gaussian behavior ( $|T_{\text{VL}} - T_c| \approx 3.7$  K) as the sample temperature is further moved away from  $T_c$ . On the other hand, a 3D behavior only is observed on the  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  material in the normal and superconducting states. In both systems, the critical exponents are found to be those theoretically predicted. Moreover, the crossover temperatures are consistent with those expected from the differently anisotropic structures of these compounds. From the crossover temperatures and the thermal conductivity fluctuation contribution amplitudes, quite realistic values for the Ginzburg-Landau parameter  $k_{\text{GL}}$ , the interlayer coupling energy  $J_c$ , and the electronic mean free path  $l_e$  near  $T_c$  are found in those materials. [S0163-1829(97)03425-5]

## I. INTRODUCTION

The thermal conductivity  $\kappa$  of high- $T_c$  superconductors (HTS's) remains one of the most interesting transport property of these materials. Most works on  $\kappa$  are devoted to the understanding of the peak structure observed in the  $a$ - $b$  plane component below  $T_c$  in various high- $T_c$  materials.<sup>1</sup> For some authors, this peak is essentially due to a phonon contribution  $\kappa_{\text{ph}}$ .<sup>1-3</sup> Others explain this feature by considering an alternative interpretation based on an electronic scattering model.<sup>4-9</sup> As concerns the fluctuation contribution, much experimental work has been devoted to the electrical resistivity  $\rho$  (Refs. 10-13) and thermopower  $S$ .<sup>14-18</sup> Only a couple of papers<sup>19,20</sup> report the fluctuation contribution to the thermal conductivity of high- $T_c$  cuprates.

In the present work, precise experimental results on the thermal conductivity of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and magnetically textured  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystals are reported and analyzed near  $T_c$  in order to extract the contribution of fluctuations. The normal-state background is first calculated in a formal way, considering an electronic and a phonon contribution. The fluctuation contribution to the thermal conductivity is then extracted and analyzed above  $T_c$  along the lines of the theoretical model of Varlamov and Livanov (VL).<sup>21</sup>

Based on previous calculations of the thermal conductivity of HTS's below  $T_c$ ,<sup>6,7</sup> the background can be extracted below  $T_c$  as well. The fluctuation contribution behavior to

the thermal conductivity  $\kappa$  below  $T_c$  is thus also reported. It is shown that the analysis leads to the observation of both Gaussian and truly critical fluctuation regimes on both sides of the transition temperature. Furthermore, interesting physical parameters can be derived, like the Ginzburg-Landau parameter  $k_{\text{GL}}$ , the interlayer coupling energy between the superconducting layers  $J_c$ , and the electronic mean free path  $l_e$  near  $T_c$ .

The theoretical results of Varlamov and Livanov are briefly recalled in Sec. II. The normal-state background is calculated in Sec. III. The fluctuation contribution to the thermal conductivity of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystals is then extracted and analyzed within the VL theory. Conclusions are finally drawn in Sec. IV.

## II. THEORETICAL MODEL

Varlamov and Livanov<sup>21</sup> have calculated the effect of superconducting fluctuations on the electronic contribution  $\kappa_{e,\text{fl}}$  to the thermal conductivity of layered superconductors in the Gaussian fluctuation range. They have considered a Lawrence-Doniach energy spectrum<sup>22</sup>

$$\mathcal{E}(\mathbf{k}) - \varepsilon_F = \frac{\hbar^2}{2m_{ab}^*} (k_x^2 + k_y^2) + J_c \cos(k_z d), \quad (1)$$

where  $m_{ab}^*$  is the electron effective mass and  $J_c$  the coupling energy between the superconducting layers separated by a distance  $d$ . The expression of  $J_c$  is given by<sup>22</sup>

$$J_c = \frac{2\hbar}{\gamma d} \sqrt{\frac{\varepsilon_F}{m_{ab}^*}}, \quad (2)$$

where  $\gamma = \sqrt{m_c^*/m_{ab}^*}$  is the anisotropy parameter.

Using the linear response theory and the temperature-dependent Matsubara diagrammatic technique, Varlamov and Livanov<sup>21</sup> obtained the following expression for  $\kappa_{e,n}$  in the clean limit (i.e., neglecting the effect of impurities on the fluctuation-induced propagator):

$$\frac{\kappa_{e,n}}{\kappa_{e,n}(T_c)} = \frac{9\pi^5}{128[7\zeta(3)]^2} \frac{\hbar}{\varepsilon_F \tau_e(T_c)} \times \left[ \frac{T-T_c}{T_c} \left( \frac{T-T_c}{T_c} + \delta^2 \right) \right]^{-1/2}, \quad (3)$$

where  $\kappa_{e,n}(T_c)$  is the normal-state *electronic* contribution to the thermal conductivity extrapolated at  $T_c$ ,  $\zeta(3) = 1.202$  the Riemann zeta function,  $\varepsilon_F$  the Fermi energy, and  $\tau_e$  the electronic relaxation time. The so-called dimensionality parameter  $\delta$  (Ref. 21) is related to the interlayer coupling energy  $J_c$  by the expression

$$\delta = \left( \frac{7\zeta(3)J_c^2}{8\pi^2(k_B T_c)^2} \right)^{1/2}. \quad (4)$$

In the limiting case of strictly two or three dimensions, the fluctuation contribution to the thermal conductivity reads [cf. Eq. (3)]

$$\frac{\kappa_{e,n}}{\kappa_{e,n}(T_c)} = A \begin{cases} (1/\delta)\varepsilon^{-1/2} & \text{if } \delta^2 \gg \varepsilon \quad (3D), \\ \varepsilon^{-1} & \text{if } \delta^2 \ll \varepsilon \quad (2D), \end{cases} \quad (5)$$

where  $A = (9\pi^5\hbar)/\{128[7\zeta(3)]^2\varepsilon_F\tau_e(T_c)\}$  and  $\varepsilon = (T - T_c)/T_c$ .

The Varlamov-Livanov calculations thus predict a crossover from a two-dimensional (2D) to a 3D Gaussian fluctuation behavior at the  $T_{VL}$  temperature given by

$$T_{VL} = T_c + T_c \delta^2. \quad (6)$$

The experimental derivation of  $T_{VL}$  should thus allow us to estimate the interlayer coupling energy  $J_c$  in the material from Eqs. (4) and (6). Besides, the electronic relaxation time  $\tau_e$  at  $T_c$  should be obtained from the amplitude  $A$  of the fluctuation contribution to the thermal conductivity; cfr. Eq. (3). We point out that Eqs. (3)–(6) have been obtained by considering a simple parabolic energy spectrum and an isotropic *s*-wave energy gap parameter. Since the band structure of high- $T_c$  superconductors is more complex and the gap parameter most probably of *d*-wave type,<sup>23</sup> the values of  $J_c$  and  $\tau_e$  obtained from the above equations should be considered as crude estimates. We recall here that Tewordt *et al.*<sup>24</sup> have calculated the fluctuation contribution to the electrical conductivity  $\sigma$  of quasi-two-dimensional superconductors within a Hubbard model, considering various pairing symmetries for the energy gap. These authors found that the

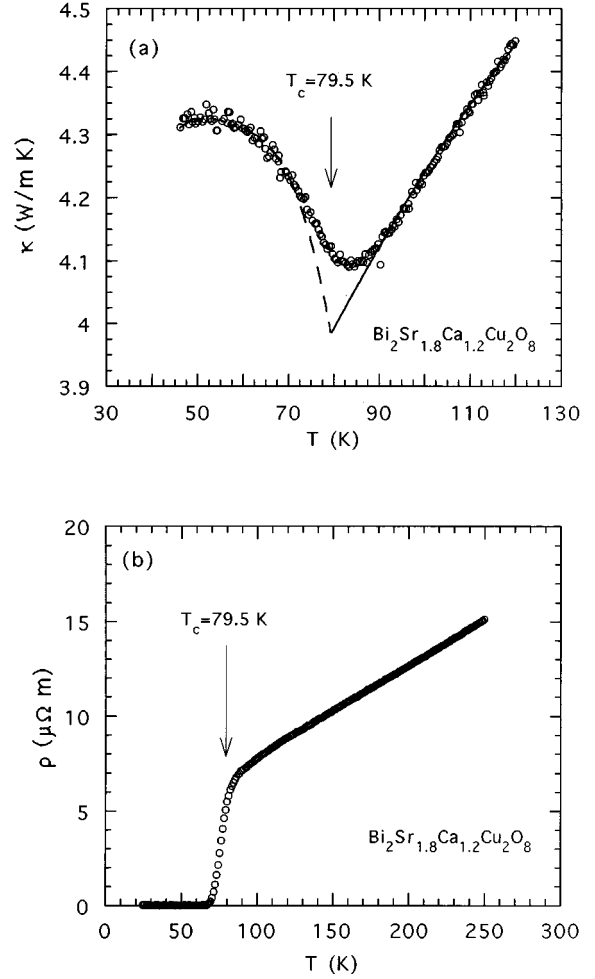


FIG. 1. (a) Thermal conductivity  $\kappa$  and (b) electrical resistivity  $\rho$  of a  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal vs temperature  $T$ . The solid and dashed lines are the fluctuationless thermal conductivity backgrounds in the normal state and superconducting state, respectively, theoretically derived as explained in the text.

dimensionality parameter  $\delta$  defined in Eq. (4) as well as the relaxation time  $\tau_e$  depend on the band structure, its filling, and the pairing symmetry.

In the next section, we present experimental results on the thermal conductivity of a  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_{8+y}$  and a textured  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystal and the subsequent parameter values as deduced from the above formulas.

### III. RESULTS AND DISCUSSION

#### A. $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$

The  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal was prepared using the method described in Refs. 20 and 25. The thermal conductivity of this material was measured using a steady-state longitudinal method presented elsewhere.<sup>26</sup> The temperature dependence of the total thermal conductivity (i.e., both including the electron and phonon contributions)  $\kappa$  of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal ( $T_c = 79.5$  K) is shown in Fig. 1(a). The critical temperature of this sample was obtained from the inflection point of the electrical resistivity  $\rho$  as shown in Fig. 1(b). From Fig. 1(a), one can see that  $\kappa$

TABLE I. Values of the free parameters used to calculate the electronic contribution to the thermal conductivity of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  polycrystals.

Sample	$J_c$ (meV)	$\lambda_{tr}$	$N$
$\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$	4.1	0.28	0.27
$\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$	38	0.54	0.19

decreases quasilinearly with decreasing temperature in the normal state and markedly presents an upturn between 90 and 70 K. The thermal conductivity then increases in the superconducting state and reaches a maximum at about 51 K. The smooth upturn between 90 and 70 K can be attributed to the contribution of superconducting fluctuations as shown below.

### 1. Fluctuation contribution above $T_c$

The solid line in Fig. 1(a) represents the normal-state thermal conductivity background  $\kappa_n(T)$  which includes an electronic  $\kappa_{e,n}$  and a phonon  $\kappa_{ph,n}$  contribution, i.e.,

$$\kappa_n(T) = \kappa_{e,n}(T) + \kappa_{ph,n}(T). \quad (7)$$

The electronic contribution to the thermal conductivity  $\kappa_{e,n}$  can be calculated using a variational method<sup>27</sup> with the Lawrence-Doniach spectrum, Eq. (1), for the case of such high- $T_c$  cuprates; cf. Refs. 6–8. The free parameters of the model are the electron-phonon transport coupling constant  $\lambda_{tr}$ , the interlayer coupling energy  $J_c$  between the superconducting layers, and the electron-point defect fraction in the sample,  $N$ . The temperature dependence of the overall thermal conductivity of a  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  single crystal has been previously analyzed with this model, in particular below  $T_c$ .<sup>7</sup> The values found for  $\lambda_{tr}$  and  $J_c$  in this material from data below  $T_c$  and at very low temperature<sup>28</sup> are recalled in Table I. These values should stand for the overall temperature range and have thus been used here to calculate the electronic contribution to the thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal. Besides, in order to estimate the fraction of point defects,  $N$ , in this latter sample, we use the Wiedemann-Franz law<sup>27</sup>

$$\kappa_{e,n}(T) = L_0 T / \rho(T), \quad (8)$$

where  $L_0 = 2.44 \times 10^{-8} \text{ (V/K)}^2$  is the Lorentz number and  $\rho(T)$  the temperature-dependent electrical resistivity. This law should be valid near the Debye temperature,<sup>27</sup> i.e., near 250–300 K. Since the normal-state electrical resistivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample shown in Fig. 1(b) is linear with temperature,  $\kappa_{e,n}$  should be constant in the high-temperature range ( $T \geq 250$  K) according to Eq. (8). The normal-state electronic contribution value to the thermal conductivity can thus be estimated at  $T = 250$  K, i.e., to be 0.4 W/m K. This value corresponds to  $N = 0.27$  from our model.<sup>6</sup>

Having  $\lambda_{tr}$ ,  $J_c$ , and  $N$  determined as described above, the variational method can be used to calculate  $\kappa_{e,n}$  over the whole temperature range.<sup>6–8</sup> The calculated temperature dependence of the electronic thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample in *normal state* is shown by a solid line in Fig. 2 (the superconducting-state contribution

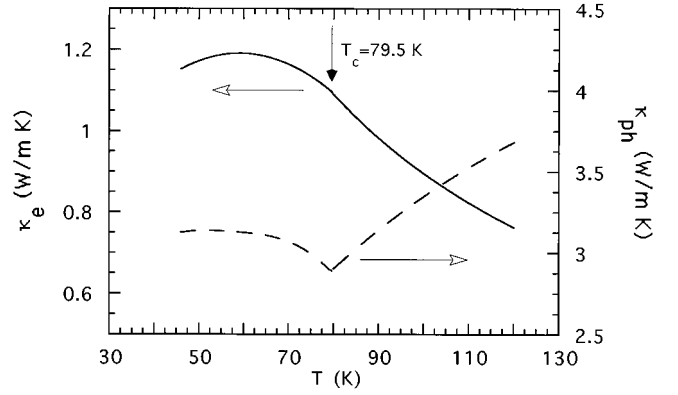


FIG. 2. Electronic  $\kappa_e$  and phonon  $\kappa_{ph}$  contributions to the thermal conductivity of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  as a function of temperature. The solid line is the electronic contribution, and the dashed line represents the phonon contribution (see text).

below  $T_c$  is discussed in the next section). One can see that  $\kappa_{e,n}$  behaves like  $T^\beta$  with  $\beta = -0.88$  at high temperature.<sup>6</sup>

The *normal-state* phonon contribution to the thermal conductivity  $\kappa_{ph,n}(T)$  is then obtained by subtracting  $\kappa_{e,n}$  from the high-temperature data ( $T > 90$  K). This former contribution can then be fitted to the expression given by Tewordt and Wölkhausen,<sup>2</sup> i.e.,

$$\kappa_{ph,n} = \frac{k_B}{2\pi^2 v_s} \left( \frac{k_B}{\hbar} \right)^3 T^3 \int_0^{T_D/T} dx \frac{x^4 e^x}{(e^x - 1)^2} \tau_{ph}(T, x), \quad (9)$$

where  $v_s$  is the sound velocity in the material,  $x = \hbar\omega/(k_B T)$  the reduced phonon frequency, and  $\tau_{ph}$  the phonon transport relaxation time, which is given by<sup>2,29</sup>

$$\tau_{ph}(T, x) = [D_p T^4 x^4 + ETx + UT]^{-1}. \quad (10)$$

The first term in Eq. (10) represents the scattering of phonons by point defects, and the coefficient  $D_p$  is related to the fraction of point defects,  $N$ , by<sup>29</sup>

$$D_p = (k_B^4 V N) / (4\pi \hbar^4 v_s^3), \quad (11)$$

where  $V$  is the unit cell volume. The second term in Eq. (10) results from the scattering of phonons by electrons. The parameter  $E$  is expressed as a function of the transport electron-phonon coupling constant  $\lambda_{tr}$  through the formula<sup>29</sup>

$$E = (\pi \lambda_{tr} k_B^3 T_D^2) / (5 \sqrt{2m^*} \hbar v_s \epsilon_F^{3/2}). \quad (12)$$

The last term in Eq. (10) corresponds to the phonon-phonon umklapp scattering process, and the coefficient  $U$  is related to the Gruneisen constant  $\gamma_G$  by<sup>29</sup>

$$U = (k_B \gamma_G^2) / (M v_s a), \quad (13)$$

where  $M$  is the unit cell mass and  $a$  the average interatomic distance.

The dashed line shown in Fig. 2 results from a fit to the data using Eqs. (9)–(13), leaving  $T_D$  and  $\gamma_G$  as free parameters, and fixing the following realistic values of the other physical parameters appearing in these latter equations:  $\lambda_{tr} = 0.28$  and  $N = 0.27$  (see Table I),  $\epsilon_F = 0.08$  eV,  $m^* = 8m_0$ ,  $V = 905.3 \text{ \AA}^3$ ,  $M = 5.9 \times 10^{-27}$  g,  $a = 5.4 \text{ \AA}$ ,<sup>30</sup> and

TABLE II. Values of the Debye temperature  $T_D$  and the Grun-eisen constant  $\gamma_G$  of  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  determined from the fit of Eqs. (9)–(13) to the thermal conductivity data.

Sample	$T_D$ (K)		$\gamma_G$	
	(this work)	$T_D$ (K)	(this work)	$\gamma_G$
$\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$	288	250 <sup>a</sup>	0.79	1.06 <sup>b</sup>
$\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$	394	356 <sup>c</sup>	2.35	2.7 <sup>b</sup>

<sup>a</sup>Reference 32.

<sup>b</sup>Reference 33.

<sup>c</sup>Reference 34.

$v_s = 3000$  m/s.<sup>31</sup> The values found for  $T_D$  and  $\gamma_G$  are shown in Table II. They lie in the same range of magnitudes as the values found in the literature.<sup>32–34</sup>

Summing the normal electronic and phonon contributions to the thermal conductivity as shown in Fig. 2, we obtain the total normal-state background contribution  $\kappa_n = \kappa_{e,n} + \kappa_{\text{ph},n}$  as shown by a solid line in Fig. 1(a). The fluctuation contribution  $\kappa_{e,\text{fl}}$  results from subtracting  $\kappa_n$  from the total  $\kappa$  data, i.e.,  $\kappa_{e,\text{fl}} = \kappa - \kappa_n$ .

In order to search for the values of the critical exponents, the normalized fluctuation contribution  $\kappa_{e,\text{fl}}/\kappa_{e,n}(T_c)$  to the thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample for  $T > T_c$  is presented in Fig. 3 as a function of  $\varepsilon = (T - T_c)/T_c$  on a log-log plot. From Fig. 3, one observes that  $\kappa_{e,\text{fl}}$  behaves like  $(T - T_c)^{-1/3}$  for  $0.56 \text{ K} < T - T_c < 2.15 \text{ K}$ . This is characteristic of the 3D XY model for *critical fluctuations*.<sup>35–37</sup> A crossover to a 3D Gaussian behavior  $\kappa_{e,\text{fl}}/\kappa_{e,n}(T_c) = 0.014\varepsilon^{-1/2}$  occurs at the Ginzburg temperature  $T_G = 81.65 \text{ K}$  ( $\varepsilon_G = 0.027$ ). The Ginzburg temperature is related to physical constants through<sup>36</sup>

$$|T_G - T_c| = 1.07 \times 10^{-9} \frac{k_{\text{GL}}^4 T_c^3}{B_{c2}(0)}, \quad (14)$$

where  $k_{\text{GL}}$  is the Ginzburg-Landau parameter and  $B_{c2}(0)$  the upper critical field (in gauss). Fixing  $B_{c2}(0) = 100 \text{ T}$ ,<sup>30</sup> one

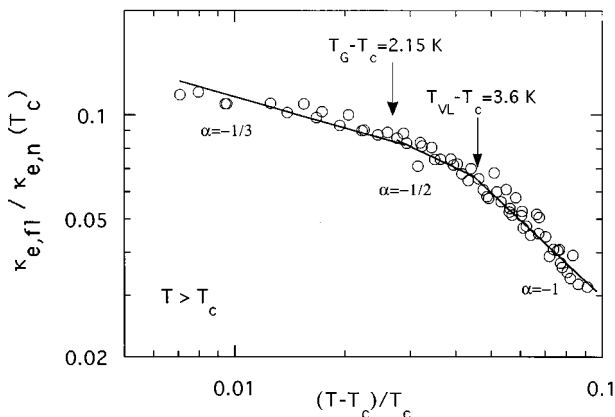


FIG. 3. Normalized fluctuation contribution  $\kappa_{e,\text{fl}}/\kappa_{e,n}(T_c)$  to the electronic thermal conductivity of a  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal vs reduced temperature  $(T - T_c)/T_c$  for  $T > T_c$ .

estimates  $k_{\text{GL}} = 250$ , in good agreement with values found in the literature,<sup>30,38,39</sup> i.e.,  $k_{\text{GL}} \in [100, 300]$ . As the temperature is further increased, a crossover to a 2D behavior  $\kappa_{e,\text{fl}}/\kappa_{e,n}(T_c) = 0.0029\varepsilon^{-1}$  occurs when  $T_{\text{VL}} - T_c = 3.6 \text{ K}$ , in good agreement with the theoretical predictions recalled in Sec. II. From Eqs. (4) and (6), the interlayer coupling energy  $J_c$  in the sample is estimated to be  $4.5 \text{ meV}$ , in quite good agreement with values found by other studies.<sup>7,11</sup>

We can also estimate the order of magnitude of the electron relaxation time  $\tau_e$  at  $T_c$  from the amplitude of the fluctuation contribution: cf. Eq. (3). Fixing  $\varepsilon_F = 0.08 \text{ eV}$ ,<sup>30</sup> we obtain  $\tau_e(T_c) = 9.07 \times 10^{-13} \text{ s}$ , which corresponds to an electronic mean free path  $l_e = v_F \tau_e = 538 \text{ \AA}$ . This latter value is a little bit large as compared to values found in the literature, i.e.,  $l_e(T_c) \approx 100\text{--}200 \text{ \AA}$ .<sup>40</sup> This discrepancy can be attributed to the fact that the Varlamov-Livanov theory does not take into account the effect of impurities, an effect which should contribute to a renormalization of the electron Green's functions; see also Ref. 24. As explained in Ref. 21, this should lead to a somewhat different amplitude in Eq. (3); i.e., the amplitude should be multiplied by  $(\hbar \tau_e^{-1}/k_B T_c)$ . Considering this substitution in Eq. (3), we obtain a more realistic value for the electronic mean free path  $l_e(T_c) = 168 \text{ \AA}$ . Besides, this result confirms the validity of the clean limit in high- $T_c$  superconductors;<sup>41</sup> i.e., the electronic mean free path is much greater than the zero-temperature coherence length  $\xi(0) \approx 10 \text{ \AA}$ .

## 2. Fluctuation contribution below $T_c$

Up to now, it seems that there has been no report on the superconducting fluctuations below  $T_c$  for the case of transport properties in HTS's. That temperature region is usually more difficult to analyze in HTS's, like in magnetic materials,<sup>42–44</sup> because of the finite value of the order parameter. Moreover, since the electrical resistivity and the thermoelectric power of HTS's vanish over a very small temperature range below  $T_c$ , it is very hard indeed to obtain reliable fluctuation contributions in that region. Since the thermal conductivity has a finite value in the superconducting state, one can expect that the fluctuation contribution could be observed below  $T_c$  by subtracting the appropriate background arising from superconductivity effects.

Like in the previous section, we have calculated the (fluctuationless) background below  $T_c$  by assuming a linear superposition of an electronic  $\kappa_{e,s}$  and a phonon contribution  $\kappa_{\text{ph},s}$ . The former contribution is again obtained by using an electronic model described in Refs. 6 and 7, considering a *d*-wave superconducting energy gap parameter, which has been shown to be reliable in the case of the transport properties.<sup>28,45,46</sup> The resulting electronic contribution is shown in Fig. 2 by a solid line. The physical parameters are (self-consistently) fixed to the same values as those given in the previous section. Moreover, the zero-temperature superconducting energy gap parameter  $\Delta(0)$  is fixed to be  $20 \text{ meV}$ .<sup>30</sup> One can see from Fig. 2 that  $\kappa_{e,s}$  reaches a peak near  $60 \text{ K}$  and presents a slight slope break at  $T_c$ .

The phonon contribution can also be calculated by using Eqs. (9)–(13). However, in the superconducting state, the second term in Eq. (10) has to be multiplied by the function

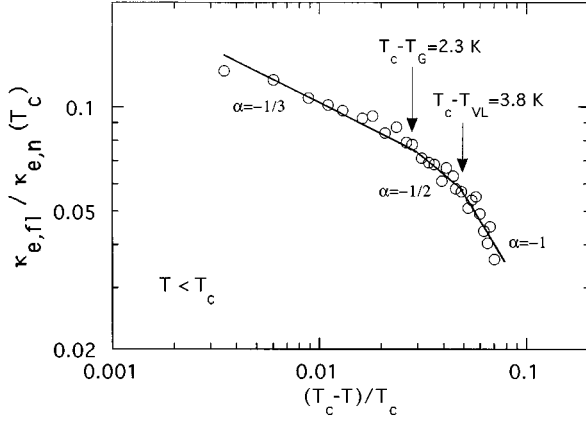


FIG. 4. Normalized fluctuation contribution  $\kappa_{e,fl}/\kappa_{e,n}(T_c)$  to the electronic thermal conductivity of a  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal vs reduced temperature  $(T_c - T)/T_c$  for  $T > T_c$ .

$g(x, y) = \tau_{e,ph}^n / \tau_{e,ph}^s$ , where  $x = \hbar\omega/k_B T$  and  $y = \Delta(T)/k_B T$ , in order to take into account the gap parameter dependence through the phonon mean free path. The expression of this latter function is in fact explicitly given in Refs. 2 and 47. The phonon contribution to the thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  polycrystal is shown in Fig. 2 by the dashed line when fixing the parameters to the values given in Sec. III A 1. One can observe that  $\kappa_{ph,s}$  also presents a well-marked slope break at  $T_c$  and a broad maximum near 55 K.

The total superconducting contribution  $\kappa_s = \kappa_{e,s} + \kappa_{ph,s}$  to the thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample is shown in Fig. 1(a) by a dashed line. We point out that the thermal conductivity enhancement observed below  $T_c$  in Fig. 1(a) is found to result from both an electronic and a phonon contribution.

The normalized fluctuation contribution  $\kappa_{e,fl}/\kappa_{e,n}(T_c)$  to the thermal conductivity of the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample for  $T < T_c$  is presented in Fig. 4 vs  $\varepsilon = (T - T_c)/T_c$  on a log-log plot. Like in the normal-state case, one can see that  $\kappa_{e,fl}$  behaves like  $(T - T_c)^{-1/3}$  for  $0.32 \text{ K} < T - T_c < 2.3 \text{ K}$ , followed by a crossover to a 3D Gaussian fluctuation behavior  $\kappa_{e,fl}/\kappa_{e,n}(T_c) = 0.013\varepsilon^{-1/2}$  at  $T_c - T_{GL} = 2.3 \text{ K}$ . This leads to a Ginzburg-Landau parameter  $k_{GL} = 256$ . Finally, a crossover to a 2D behavior is also observed for  $\kappa_{e,fl}/\kappa_{e,n}(T_c) = 0.0028/\varepsilon^{-1}$  at  $T_c - T_{VL} = 3.8 \text{ K}$ , corresponding to the interlayer coupling energy  $J_c = 4.6 \text{ meV}$ .

### B. $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$

The  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  sample was prepared by using a magnetic-field melt-texturing technique in order to obtain high-quality strongly coupled superconducting grains.<sup>48,49</sup> We have melt-textured a 80/20% weight Dy-123/Dy-211 composite in the presence of a 0.6-T magnetic field. The synthesis of the 123 and 211 precursors starts, respectively, from a corresponding stoichiometric mixing of  $\text{Dy}_2\text{O}_3$ ,  $\text{BaCO}_3$ , and  $\text{CuO}$  pretreated at  $920^\circ\text{C}$  for 48 h, including intermediate grindings. The Dy-123 and Dy-211 powders are then mixed together in the appropriate ratio, compacted into a strip or a pellet, and transferred into an alumina crucible

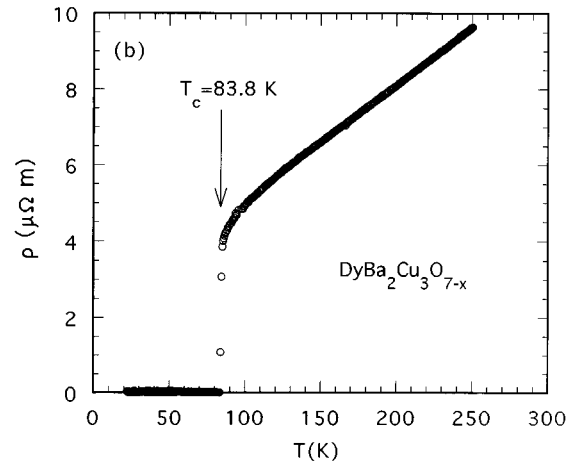
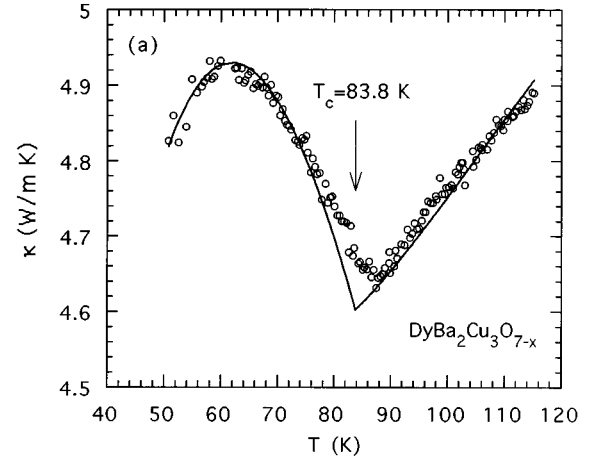


FIG. 5. (a) Thermal conductivity  $\kappa$  and (b) electrical resistivity  $\rho$  of a magnetically textured  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  polycrystal vs temperature  $T$ . The solid line represents the fluctuationless thermal conductivity background as explained in the text.

which was vertically inserted in a specially built furnace placed between the polar heads of the magnet. The magnetic field has been applied during the whole process. The thermal cycle starts at room temperature with heating to  $1035^\circ\text{C}$  at a rate of  $150^\circ\text{C/h}$ . A slow decrease of  $2^\circ\text{C/h}$  over several hours to  $980^\circ\text{C}$  is followed by a cooling process at  $50^\circ\text{C/h}$  to room temperature under oxygen atmosphere. The sample was then annealed at  $400^\circ\text{C}$  during 1 day. The sample dimensions are  $\sim 15 \times 10 \times 3 \text{ mm}^3$ .

The experimental results on the thermal conductivity  $\kappa$  of such a textured  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystal are shown in Fig. 5(a). The critical temperature  $T_c = 83.8 \text{ K}$  was estimated from the inflection point of the electrical resistivity data shown in Fig. 5(b). Notice that  $T_c$  is not so close to 90 K as in “ordinary” Y-Ba-Cu-O materials. It is known indeed that such rare-earth-doped 123 materials are not so easily oxygenated.

The solid line in Fig. 5(a) is the fluctuationless thermal conductivity background calculated in the same way as explained in Sec. III A, i.e., superposing an electronic and a phonon contribution with the appropriate parameters. It is observed that the data are more scattered than those for

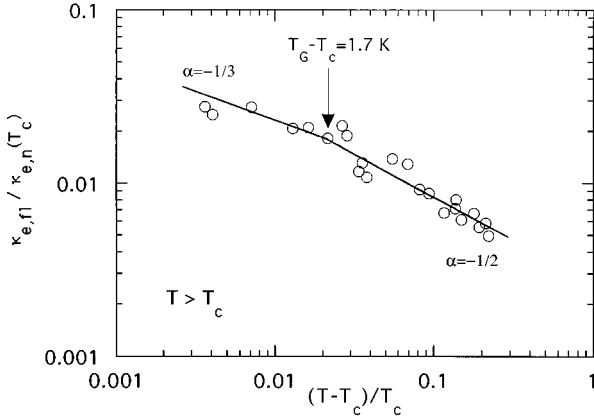


FIG. 6. Normalized fluctuation contribution  $\kappa_{e,fl}/\kappa_{e,n}(T_c)$  to the electronic thermal conductivity of a  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  polycrystal vs temperature reduced  $(T-T_c)/T_c$  for  $T > T_c$ .

$\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  presented in Fig. 1(a), resulting in a less precise fit in the normal state as shown in Fig. 5(a). This is likely due to thermal instabilities in heterogeneous grain boundaries. The following values of the physical parameters were used:  $J_c = 38$  meV,  $\lambda_{tr} = 0.54$ ,  $N = 0.19$  (see Table I),  $\Delta(0) = 20$  meV,  $\varepsilon_F = 0.1$  eV,  $m^* = 5m_0$ ,  $V = 173.2 \text{ \AA}^3$ ,  $M = 1.1 \times 10^{-27}$  g,  $a = 3.8 \text{ \AA}$ ,<sup>30</sup> and  $\nu_S = 5000$  m/s.<sup>31</sup> The derived values from the fit for  $T_D$  and  $\gamma_G$  as shown in Table II are also comparable to those found in the literature.

The normalized fluctuation contribution  $\kappa_{e,fl}/\kappa_{e,n}(T_c)$  to the electronic thermal conductivity of the  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  sample is shown in Figs. 6 and 7, respectively, for  $T > T_c$  and  $T < T_c$  vs  $|T-T_c|/T_c$ . The behavior seems to be only three dimensional in the  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  compound, i.e.,  $\kappa_{e,fl}/\kappa_{e,n}(T_c) = 0.003\varepsilon^{-1/2}$  in the temperature ranges  $0.3 \text{ K} < T-T_c < 21.8 \text{ K}$  for  $T > T_c$  and  $0.4 \text{ K} < T-T_c < 7.5 \text{ K}$  for  $T < T_c$ , respectively, with no evidence of a crossover to a two-dimensional behavior. This result seems consistent with the *a priori* supposedly high value of the interlayer coupling energy in the least anisotropic  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  material, i.e.,  $J_c \approx 40$  meV (Ref. 7) (see also Table I). As a matter of fact, from Eqs. (4) and (6), such a  $J_c$  value leads to a crossover

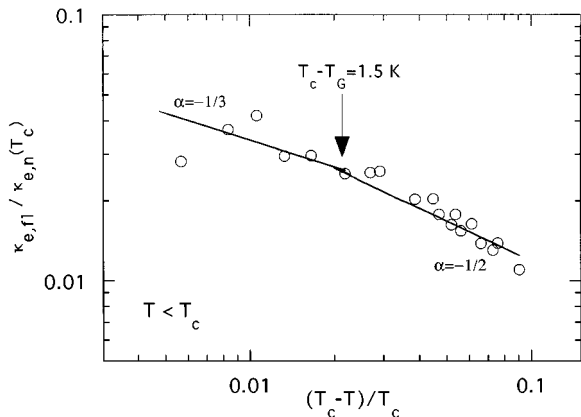


FIG. 7. Normalized fluctuation contribution  $\kappa_{e,fl}/\kappa_{e,n}(T_c)$  to the electronic thermal conductivity of a  $\text{DyBa}_2\text{Cu}_3\text{O}_{7-x}$  polycrystal vs temperature  $(T_c-T)/T_c$  for  $T < T_c$ .

temperature  $T_{VL} = 270$  K. It should be pointed out that the 3D XY critical behavior and true critical region seem observable quite close to  $T_c$  at  $|T_G - T_c| \approx 1.6$  K. However, due to the data scattering which are much more important than in the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample, it is more difficult to distinguish between a  $-1/2$  and a  $-1/3$  exponent on the experimental results close to  $T_c$ , but the trend seems to be realistic.

From the amplitude of the fluctuation contribution, the electronic relaxation time is found to be  $\tau_e(T_c) = 2.07 \times 10^{-13}$  s, corresponding to a mean free path  $l_e = 174 \text{ \AA}$ , in reasonable agreement with the value  $l_e \approx 90 \text{ \AA}$  recently estimated from the analysis of the thermal Hall conductivity in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  by Krishana *et al.*<sup>50</sup>

#### IV. CONCLUSIONS

We have discussed in this paper the effect of superconducting fluctuations on the thermal conductivity of a  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  and a magnetically textured  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  polycrystal. The (fluctuationless) thermal conductivity background has been calculated in a formal theoretical way, taking into account both electronic and phonon contributions through standard formulas and using physical parameters leading to the correct orders of magnitude. The fluctuation contribution to the electronic thermal conductivity of these materials has then been extracted both *above* and *below* the critical temperature. When analyzed within the theoretical model of Varlamov and Livanov,<sup>21</sup>  $\kappa_{e,fl}$  presents a well-marked crossover at  $\varepsilon_{VL} \approx 0.046$  from the 2D to 3D Gaussian fluctuation regime as the critical temperature is approached in the  $\text{Bi}_2\text{Sr}_{1.8}\text{Ca}_{1.2}\text{Cu}_2\text{O}_8$  sample. This is indeed expected due to the low value of the interlayer coupling energy  $J_c \approx 4$  meV in this highly anisotropic material. Besides, when  $T_c$  is further approached, a crossover from the Gaussian to the critical fluctuation region is observed at the Ginzburg temperature interval  $|T_G - T_c| \approx 2.2$  K, leading to a reasonable value of the Ginzburg-Landau parameter  $k_{GL} \approx 250$ . It should be stressed that it is the first time, to our knowledge, that the true critical behavior is observed from thermal conductivity measurements, in particular below  $T_c$ . Besides, this critical behavior is consistent with the 3D XY model,<sup>37</sup> in agreement with previous predictions by Lobb.<sup>36</sup>

On the other hand,  $\kappa_{e,fl}$  is found to have a three-dimensional behavior in the  $\text{DyBa}_2\text{Cu}_2\text{O}_{7-x}$  sample in the whole observed Gaussian fluctuation temperature range, i.e., for  $0.31 \text{ K} < T-T_c < 21.79 \text{ K}$  above  $T_c$  and  $0.47 \text{ K} < T_c - T < 7.54 \text{ K}$  below  $T_c$ , respectively. This indicates a high value of  $J_c$  in this less-anisotropic compound. Though the truly critical exponent is more difficult to distinguish from the Gaussian one due to greater data scattering in the  $\kappa_{e,fl}$  of this sample, the Ginzburg temperature is estimated to be  $|T_G - T_c| \approx 1.6$  K.

Finally, the values obtained for the electronic mean free path (i.e., between about 500 and 150  $\text{ \AA}$ ) appear to be a little bit larger than expected. This could be due to the fact that the Varlamov-Livanov theory<sup>21</sup> is based on an isotropic *s*-wave gap parameter and a simple parabolic energy spectrum, and does not account for the effect of impurity scattering: see Ref. 24. An extension to the Varlamov-Livanov calculations for the fluctuation contribution to the thermal conductivity

with more realistic band structures and anisotropic  $d$ -wave pairing should be thus useful.

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