Effects of disorder on two strongly correlated coupled chains

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We study the effects of disorder on a system of two coupled chains of strongly correlated fermions (the ladder system), using a renormalization-group technique. The stability of the phases of the pure system has been investigated as a function of interactions both for fermions with spin and spinless fermions. For spinless fermions the repulsive side is strongly localized whereas the system with attractive interactions is *stable* with respect to disorder, at variance with the single-chain case. For fermions with spins, the repulsive side is also localized, and in particular the d-wave superconducting phase found for the pure system is totally destroyed by an arbitrarily small amount of disorder. On the other hand, the attractive side is again remarkably stable with respect to localization. We have also computed the charge stiffness, the localization length, and the temperature dependence of the conductivity for the various phases. In the parameter range where d-wave superconductivity would occur for the pure system the conductivity is found to *decrease* monotonically with temperature, even at high temperature, and we discuss this surprising result. For a model with one-site repulsion and nearest-neighbor attraction, the most stable phase is an orbital antiferromagnet. Although this phase has no divergent superconducting fluctuation it can have a divergent conductivity at low temperature. Finally, to make a comparison of our results with experimental ladder systems, we treated the interladder coupling in a mean-field approximation. We argue based on our results that the superconductivity observed in some of these compounds cannot be a simple stabilization of the d-wave phase found for a pure single ladder. The application of our results to systems such as quantum wires is also discussed. In particular, the corrections to conductance in a two-channel quantum wire have been obtained as a function of system length, temperature, and interactions. [S0163-1829(97)08735-3]

I. INTRODUCTION

Strongly interacting systems constitute one of the most challenging problems of condensed-matter physics. In one dimension a fairly complete solution of the interacting problem can be obtained, and it is well known that one-dimensional systems are some of the simplest realizations of non-Fermi liquids, and have generic properties known as Luttinger liquids.[1–4] Prompted by a variety of experimental situations ranging from organic conductors to high-T_c superconductors, there has been, in the recent years, a growing interest in systems of coupled interacting electron chains. Unfortunately, despite a good understanding of purely one-dimensional systems, the effects of interchain hopping, allowing us to go from one to higher (two or three) dimensions are much less known. Whether non-Fermi liquid properties can be retained even in the presence of finite hopping or not is still a highly controversial issue.[5,6]

Many studies have therefore focused on systems of few coupled chains[7] (two coupled chains being the so-called ladder systems), for which much more controlled analytical[8–15] or numerical[16–20] techniques can be applied allowing for a deeper understanding of their physical properties. For commensurate filling, i.e., one electron per site, the system becomes equivalent to coupled spin chains, since the charge degrees of freedom are frozen by a Mott transition. Important differences between ladders with an even and odd number of legs were expected, in a way reminiscent of the Haldane conjecture between one-dimensional systems with integer and half integer spins. In particular, ladders with an even number of legs were predicted to have a spin gap. Good experimental realizations of such coupled spin chains like $Sr_{n-1}Cu_{n+1}O_{2n}$ (Refs. 21 and 22) and $VO_2P_2O_7$ (Refs. 23 and 24) compounds have confirmed such behavior. Due to the presence of such a spin gap an even more spectacular effect is expected upon doping. Opposite the single chains, which exhibit either a spin density wave or charge density wave ground state for repulsive interactions, the ladder system is believed to have a superconducting ground state involving pairing across the chains. That superconducting state has similarities with d-wave paring that has been advocated in some two-dimensional models of strongly correlated electrons for high-T_c superconductors[25–31] such as the existence of a spin gap and a sign change of the superconducting order parameter when one moves on the ''Fermi surface.'' In the strong coupling limit, i.e., the t-J model, the d-wave phase can also be viewed as a resonating-valence-band state.[32]

However all the studies of ladder systems have been, up to now, restricted to pure systems. Unfortunately (or maybe fortunately) it is well known, that for one-dimensional systems disorder has extremely strong effects. For a noninteracting system, it is well known that all states get localized in the presence of an infinitesimal random potential.[33,34] Interactions can modify this picture, but for a one-chain system delocalization occurs only for strongly attractive interactions. In particular even normal s-wave superconducting phases are destroyed by nonmagnetic impurities except for exceedingly attractive interactions (see, e.g., Ref. 35 and references therein), and no Anderson's theorem exists even for weakly coupled one-dimensional systems.[36]

In order to compare the theoretical predictions of d-wave superconductivity in doped ladder systems with experiments,

it is of prime importance to understand the effects of disorder on the phase diagram of the pure ladder system. One of the important questions is of course the stability of the newly found d-wave superconducting phase since there are no obvious reasons why it would survive the introduction of a small amount of disorder. Such a study is also relevant to the physics of quantum wires with few channels.[37–41] In quantum wire systems, the situation is however complicated by the occurrence of long range Coulomb forces[41] that can induce a one-dimensional analog of the Wigner crystal,[42,43] which can drastically modify the response of the system to disorder.[45] However, the presence of charges in the grids of the quantum wire systems can be cleverly used to screen completely the long range interactions, and have an experimental realization of a Luttinger liquid.[40] By changing the gate voltage it is possible to have more than one band at the Fermi level, in a controlled way. The quantum wire is thus a possible realization of a two (or more) leg ladder. Interband tunneling plays the role of interchain hopping. They provide ideal systems in which to check for the effect of disorder.[39,40]

Besides the exciting possibility of testing the ability of Luttinger liquid models to describe accurately the now available quasi-one-dimensional experimental systems, investigation of disorder effects in ladders presents in its own right a great theoretical interest. Indeed, the two-chain problem is the simplest one to study the effects of interchain hopping onto the Anderson localization in the presence of interactions, giving some clues to this difficult topic in more than one dimension. In particular, one would be interested in obtaining boundaries between localized and delocalized phases and the dependence on localization lengths on disorder. Another question of particular interest is the effect of interactions on physical quantities controlled by disorder such as the conductivity for a macroscopic system, or for a mesoscopic one the persistent currents. In particular, for a one-chain system, it was shown that for a system with spin degrees of freedom persistent currents were enhanced[45] by repulsive interactions, at variance on what happened for a spinless system. It is therefore important to check whether this striking result still holds in a more two-dimensional system.

In this paper, we consider the effects of a weak random potential scattering on systems of coupled fermionic chains both with spin and spinless using bosonization and renormalization-group (RG) techniques. A short account of some of the results of this paper were presented in Ref. 46. Besides giving the phase boundaries, the RG also provides us with expressions of the localization lengths, temperature dependence of conductivity, and dependence of persistent currents with system size. The plan of the paper is as follows.

In Sec. II, we discuss the spinless fermions two legs ladder problem. We first recall the phase diagram of the pure system,[14] then consider the effects of disorder. This allows for a detailed comparison of the transport properties of the two-chain system with the ones of the one-chain spinless fermion system and the ones of one chain of fermion with spin. We show that contrarily to naive expectations, the ladder spinless fermions system is very different from the one-chain system with spin, and that the effect of interactions on persistent currents is even more violent on a two-chain spinless fermions system than it was for a one-chain system.

In Sec. III, we discuss the technically more involved case of fermions with spins. Following the same methodology, we first recall the phase diagram of the nonrandom two-chain system and then consider the effects of a weak random potential on the phase diagram. As for the spinless problem, we give a detailed discussion of the transport properties in the disordered phases. We compare these results with the ones already known for one chain, and show that the reduction of persistent current by attractive interactions that is observed in one chain of fermions with spin should be almost absent in the two-chain system. The d-wave superconductivity of the fermionic two-chain system is a feature that is not preserved in the presence of a very small amount of disorder. On the other hand, for some values of the parameter, an orbital antiferromagnetic phase exists. This phase has an infinite conductivity *even in the presence of disorder* although it has exponentially decaying superconducting correlations.

In order to compare our results with experiments on doped ladder systems, it is necessary to treat interchain coupling which stabilizes superconductivity at finite temperature in real systems, and may also reduce the sensitivity of the system to disorder. Thus in Sec. IV we examine the mean-field theory for the d-wave superconductor in an array of coupled disordered ladders. We give a criterion for persistence of superconductivity in the presence of disorder and show that d-wave superconductivity remains unstable except in very pure samples or in the presence of a very strong Josephson coupling between ladders.

In Sec. V, we summarize the implications for experimental systems such as doped $Sr_xCu_{1-x}O$ chains and quantum wire with two channels. We claim that in recently synthesized doped ladder systems, the physics of the superconducting phase is more likely to be of two-dimensional origin rather than just a stabilization of a ladder d-wave superconductivity. Conclusions can be found in Sec. VI. Finally, most of the technicalities can be found in the Appendixes.

II. SPINLESS FERMIONS

A. Pure system

Let us consider first two chains of spinless fermions coupled by an interchain hopping $t_\perp$. For simplicity we first consider only nearest-neighbor interactions, and interchain interactions. The effect of more complicated interactions will be detailed below. The Hamiltonian for the pure system reads

$$H=-t\sum_{i,p} c^\dagger_{i,p}c_{i+1,p}+\text{H.c.}+V\sum_i n_{i,p}n_{i+1,p}$$
$$+t_\perp\sum_i c^\dagger_{i,1}c_{i,-1}+\text{H.c.}+U\sum_i n_{i,1}n_{i,-1}, \qquad (1)$$

where $p=-1,1$ is the chain index and i is the site index. To treat the interactions, it is convenient to rewrite the Hamiltonian in term of boson operators.[4,3,47] To do so, we linearize the fermions dispersion relation around k_F, introduce right (R) and left movers (L) for each chain, and take the continuum limit $c_{n,r,p}\to\sqrt{\alpha}\psi_{r,p}(n\alpha)$ with $r=L,R$, $p=\pm1$ the chain index and α the lattice spacing. We use the bonding

$\psi_o=[(\psi_1+\psi_{-1})/\sqrt{2}]$ and antibonding $\psi_\pi=[(\psi_1-\psi-1)/\sqrt{2}]$ bands base and introduce the densities $\rho_{r,o,\pi}(x)=:\psi^\dagger_{r,o,\pi}(x)\psi_{r,o,\pi}(x):$. We then define the canonically conjugate fields $\phi_{\rho,\|}$ and $\Pi_{\rho,\|}$ via

$$\partial_x\phi_{\rho,\|}=-\frac{\pi}{\sqrt{2}}(\rho_{L,o}+\rho_{R,o}\pm\rho_{L,\pi}\pm\rho_{R,\pi}), \tag{2}$$

$$\Pi_{\rho,\|}=\frac{1}{\sqrt{2}}(\rho_{R,o}\pm\rho_{R,\pi}-\rho_{L,o}\mp\rho_{L,\pi}), \tag{3}$$

and the field $\theta_{\rho,\|}(x)=\int_{-\infty}^{x}\Pi_{\rho,\|}(x')dx'$. More details on the bosonization technique can be found in Appendix A. In term of these fields the Hamiltonian becomes[14]

$$H=H_\rho+H_\|,\quad H_\rho=\int\frac{dx}{2\pi}\left[u_\rho K_\rho(\pi\Pi_\rho)^2+\frac{u_\rho}{K_\rho}(\partial_x\phi_\rho)^2\right],$$

$$H_\|=\int\frac{dx}{2\pi}\left[u_\| K_\|(\pi\Pi_\|)^2+\frac{u_\|}{K_\|}(\partial_x\phi_\|)^2\right]+\int dx\ t_\perp\frac{\sqrt{2}}{\pi}\partial_x\phi_\|$$
$$+\int dx\left[\frac{2g_\perp}{(2\pi\alpha)^2}\cos(\sqrt{8}\phi_\|)+\frac{2g_f}{(2\pi\alpha)^2}\cos(\sqrt{8}\theta_\|)\right]. \tag{4}$$

For the microscopic Hamiltonian (1), one finds

$$K_\|=1+\frac{Ua}{2\pi v_F},$$

$$u_\|=v_F\left(1-\frac{Ua}{2\pi v_F}+\frac{Va}{\pi v_F}[1-\cos(2k_Fa)]\right),$$

$$g_f=-Va[1-\cos(2k_Fa)],$$

$$g_\perp=Ua-Va[1-\cos(2k_Fa)],$$

$$u_\rho=v_F\left(1+\frac{Ua}{2\pi v_F}+\frac{Va}{\pi v_F}[1-\cos(2k_Fa)]\right),$$

$$K_\rho=1-\frac{Ua}{2\pi v_F}-\frac{Va}{\pi v_F}[1-\cos(2k_Fa)], \tag{5}$$

with $v_F=2ta\ \sin(2k_Fa)$. Therefore for the pure t-V model, one has $K_\rho<1$ (respectively, $K_\rho>1$) and $g_f<0$ (respectively, $g_f>0$) for repulsive (respectively, attractive) interactions and $K_\|=1$ for all t,V. In fact (4) describes the most general two-chain spinless system. More complicated interactions (i.e., longer range and interchain interactions) lead only to a change in the parameters K, u, and g. By adding interchain interactions such as U in formula (5) or longer range interactions one can in particular access the other regimes $K_\rho>1$ and $g_f<0$ or $K_\rho<1$ and $g_f>0$. The physics of the system is readily seen on Eq. (4). The $t_\perp$ term suppresses $\cos(\sqrt{8}\phi_\|)$. Depending on the value of $K_\|$, the $\theta_\|$ can either remain massless or develop a gap. We concentrate here on the case where $\theta_\|$ develops a gap and acquires a nonzero expectation value determined by minimizing the ground state energy (see Appendix A). This situation always occurs for the t-V model.[14] By mapping (4) on a problem of one chain of fermions with spin and spin-anisotropic interactions in a magnetic field,[48] one can obtain the complete phase diagram for the pure case.[14] Since, due to the one-dimensional nature of the problem, no true ordered state exists, one has to find the most divergent instability. As for one chain, two main types of instabilities are possible: particle-hole (density, current, etc.) instabilities or particle-particle (i.e., superconducting) ones. The operators with the most divergent susceptibilities are in a boson form,

TABLE I. The four sectors of the pure two-chain spinless fermions model, as a function of K_ρ and g_f. The average value of the massive field $\langle\theta_\|\rangle$ are indicated together with the phase with the most divergent susceptibility.

	I	II	III	IV
g_f	+	+	−	−
K_ρ	<1	>1	>1	<1
$\langle\theta_\|\rangle=$	$\frac{\pi}{\sqrt{8}}$	$\frac{\pi}{\sqrt{8}}$	0	0
phase	OAF	SC^s	SC^d	CDW^π

$$O_{\mathrm{CDW}^\pi}=\psi^\dagger_{R,1}(x)\psi_{L,1}(x)-\psi^\dagger_{R,-1}\psi_{L,-1}(x)$$
$$\sim e^{i\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\|),$$

$$O_{\mathrm{SC}^s}=\psi_{L,o}(x)\psi_{R,\pi}+\psi_{L,\pi}\psi_{R,o}\sim e^{i\sqrt{2}\theta_\rho}\sin(\sqrt{2}\theta_\|),$$

$$O_{\mathrm{OAF}}=i[\psi^\dagger_{R,1}(x)\psi_{L,-1}(x)-\psi^\dagger_{R,-1}(x)\psi_{L,1}(x)]$$
$$\sim e^{i\sqrt{2}\phi_\rho}\sin(\sqrt{2}\theta_\|),$$

$$O_{\mathrm{SC}^d}=\psi_{L,o}\psi_{R,\pi}-\psi_{L,\pi}\psi_{R,o}\sim e^{i\sqrt{2}\theta_\rho}\cos(\sqrt{2}\theta_\|).$$

They describe, respectively, out of phase charge density waves, an orbital antiferromagnetic phase and chain symmetric ''s'' and antisymmetric ''d'' type superconductivity. The out of phase charge density has a $2k_F$ modulation of the density along the chain and a change of sign across the chains. In the orbital antiferromagnet currents go from one chain to the other with wave vector $2k_F$, giving currents circulating around plaquettes of length π/k_F. The superconducting phases are the standard ones, given on the original model by

$$O_{\mathrm{SC}^d}(n)=c_{n,1}c_{n,2},$$

$$O_{\mathrm{SC}^s}(n)=c_{n+1,1}c_{n,1}-c_{n+1,2}c_{n,2}. \tag{6}$$

The most stable phase depends on the parameters K and g. The various cases are given in Table I, and the phase diagram shown in Fig. 1. In Ref. 14 the bosonized forms of O_{SC^s} and O_{SC^d} are exchanged due to the neglect of anticommuting operators (see Appendix A), so that the two superconducting phases have been erroneously exchanged.

B. Effects of disorder

Now we consider the effect of the disorder on Eqs. (1)–(4). We introduce a random on-site potential $\epsilon_{i,p}$ uncorrelated from site to site and from chain to chain:

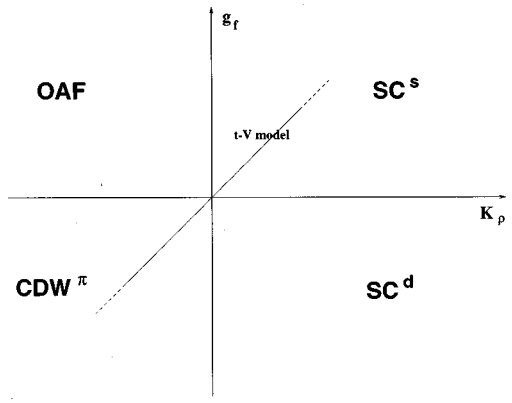

FIG. 1. The phase diagram of a generic spinless ladder in terms of g_f and K_ρ. $K_\rho>1$ means attraction in the symmetric charge sector and $K_\rho<1$ repulsion. The line depicts the phase spanned by the pure t-V ladder, leading to a CDW$^\pi$ phase for $V>0$ and a superconducting SC^s phase for $V<0$.

$$H_{\text{random}}=\sum_{\substack{i\\ p=\pm 1}} \epsilon_{i,p} c^\dagger_{i,p} c_{i,p}, \tag{7}$$

with $\overline{\epsilon_{i,p}\epsilon_{j,p'}}=D\delta_{i,j}\delta_{p,p'}$. In the continuum limit and using the bonding antibonding basis the disorder becomes

$$H_{\text{random}}=\int dx\{\epsilon_s(x)[\psi_0^\dagger(x)\psi_0(x)+\psi_\pi^\dagger(x)\psi_\pi(x)] + \epsilon_a(x)[\psi_0^\dagger(x)\psi_\pi(x)+\psi_\pi^\dagger(x)\psi_0(x)]\}, \tag{8}$$

with $\epsilon_{s,a}=(\epsilon_1\pm\epsilon_{-1})/2$ and $\overline{\epsilon_\alpha(x)\epsilon_\beta(x')}=(Da/2)\,\delta(x-x')\delta_{\alpha,\beta}$. Using the expression of fermion operators defined in Appendix A and Eq. (2) one obtains for the disorder term

$$\begin{aligned} H_{\text{random}}=&\int dx\Big[\eta_s(x)\frac{\sqrt{2}}{\pi}\partial_x\phi_\rho(x)+\frac{\xi_s(x)}{\pi\alpha}e^{\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\phi_\parallel) \\ &+\frac{\xi_s^*(x)}{\pi\alpha}e^{-\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\phi_\parallel)\Big] \\ &+\int dx\Big[\frac{\eta_a(x)}{\pi\alpha}\cos(\sqrt{2}\phi_\parallel)\cos(\sqrt{2}\theta_\parallel) \\ &+\frac{\xi_a(x)}{\pi\alpha}e^{\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\parallel) \\ &+\frac{\xi_s^*(x)}{\pi\alpha}e^{-\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\parallel)\Big], \end{aligned} \tag{9}$$

where the disorder has been split in a $q\sim 0$ component ($\eta_{s,a}$) and a $q\sim 2k_F$ one ($\xi_{s,a}$). As for one chain the η and ξ are uncorrelated and

$$\overline{\eta_{s,a}(x)\,\eta_{s,a}(x')}=D_{s,a}a\,\delta(x-x'), \tag{10}$$

$$\overline{\xi_{s,a}(x)\xi_{s,a}(x')}=0, \tag{11}$$

$$\overline{\xi_{s,a}(x)\xi^*_{s,a}(x')}=D_{s,a}a\,\delta(x-x'). \tag{12}$$

The $q\sim 0$ (forward scattering) part of the disorder does not affect the conductivity and cannot lead to localization,[34] but could, in principle, modify the phase diagram and in particular destroy the gaps of the pure phase. As for one chain, one can eliminate the $\eta_s\partial_x\phi_\rho$ by a transformation $\phi_\rho\to\phi_\rho+(\sqrt{8}K_\rho/u_\rho)\int dx\ \eta_s(x)$. The only effect of this term is therefore to give an additional exponential decay in the density-density correlation functions.

Due to the presence of a gap in $\theta_\parallel$ (see Table I), the $\eta_a(x)\cos(\sqrt{2}\phi_\parallel)\cos(\sqrt{2}\theta_\parallel)$ term is always suppressed at lowest order. It could however generate relevant terms at higher order. However higher order terms are either identical to backscattering terms already present in the Hamiltonian, or adds random contributions to $g_\perp\cos\sqrt{8}\phi_\parallel$ and $g_f\cos\sqrt{8}\theta_\parallel$. At small disorder these contributions are negligiable, and one can completely disregard the forward scattering. We can therefore keep only for the coupling to disorder H_s+H_a

$$H_s=\int\frac{dx}{\pi\alpha}\xi_s(x)e^{\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\phi_\parallel)+\text{H.c.}, \tag{13}$$

$$H_a=\int\frac{dx}{\pi\alpha}\xi_a(x)e^{\imath\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\parallel)+\text{H.c.} \tag{14}$$

In H_s the symmetric part of the disorder couples to the in-phase charge density wave order parameter $O_{\text{CDW}^0}=(e^{\imath\sqrt{2}\phi_\rho}/\pi\alpha)\cos(\sqrt{2}\phi_\parallel)$, whereas the antisymmetric part involves O_{CDW^π}. Due to the gap in $\theta_\parallel$, $\phi_\parallel$ has huge quantum fluctuations, and consequently the symmetric part of the disorder D_s is always less relevant than the antisymmetric one D_a. We can therefore focus on the latter and forget about the former. The effect of Eq. (14) again depends on the values of g_f and K.

1. $g_f<0$

For $g_f<0$ (i.e., $V>0$ for the t-V model) we can replace $\cos(\sqrt{2}\theta_\parallel)$ by its (nonzero) mean value and the coupling to disorder Eq. (14) reduces to $C\int dx\ \xi_a(x)e^{i\sqrt{2}\phi_\rho(x)}+\text{H.c.}$, where C is a constant. The effect of such a term can be determined, as for a single chain,[35] by using a RG procedure. Upon varying a cutoff α, similar to a lattice spacing in the original lattice problem, one find the following renormalization for the disorder:

$$\frac{dK_\rho}{dl}=-C_2D_a, \tag{15}$$

$$\frac{dD_a}{dl}=D_a(3-K_\rho), \tag{16}$$

where $l=\ln(\alpha)$ and C_2 a constant. Equation (16) implies a localization-delocalization transition at $K_\rho=3$. For $K_\rho>3$ the disorder is irrelevant and the corresponding phase in the pure system is stable. For $K_\rho<3$ disorder grows. Although the system flows to a strong coupling fixed point, it is natural[35] to interpret this phase as localized by disorder, since the disorder will pin the massless field ϕ_ρ. As a consequence, the d-wave superconducting phase is unstable in the presence of disorder except for huge attractive interactions. In the case of the t-V model at $V>0$, we have $K_\rho<1$ and therefore the CDW$^\pi$ is always pinned by the disorder.

Similarly to the one-chain problem the localization length can be computed using the RG. For very weak disorder and far from the transition one can neglect the renormalization of the exponent K_ρ induced by D_a. Using that approximation, we obtain

$$D_a(l)=e^{(3-K_\rho)l}D_a(0). \tag{17}$$

For $D_a(l)\sim v_F^2/\alpha$ that scheme breaks down and we have a strongly disordered system. For such a system the localization length, i.e., the scale of variation of the phase ϕ_ρ is of the order of the (renormalized) lattice spacing α^*. This occurs for $e^{l^*}\sim(v_F^2/D_a(0)\alpha)^{[1/(3-K_\rho)]}$. Therefore

$$L_{2\,\mathrm{ch.}}=\alpha(0)\left(\frac{v_F^2}{D\alpha}\right)^{[1/(3-K_\rho)]}. \tag{18}$$

Let us recall that for a noninteracting system, the localization length is of the order of the mean free path, i.e., $L_{\mathrm{loc.}}\sim(v_F^2/D)$.

Using the renormalization equation it is also possible[36] to obtain the temperature dependence of the conductivity for temperatures above the the pinning temperature $u/L_{2\,\mathrm{ch}}$. Below the pinning temperature, the conductivity is expected to decrease as $\exp-(T_{\mathrm{pin}}/T)^\mu$, by analogy with noninteracting electrons. A derivation of the temperature dependence of conductivity has been given in Ref. 35. Another method to derive the temperature (or frequency) dependence of the conductivity is given in Appendix B. If one neglects the renormalization of the exponents the conductivity behaves as

$$\sigma(T)\sim T^{2-K_\rho}. \tag{19}$$

Therefore, for $K_\rho<2$, the conductivity decreases, and there is *no remnant* of any superconducting behavior effect well above the temperature at which the system is effectively pinned $T_{\mathrm{pin}}\sim u/L_{2\,\mathrm{ch.}}$. Thus the existence of d-wave superconductivity in the pure system affects the transport properties of the disordered system only for quite a large attraction. Analogous effects will occur for fermions with spins as will be discussed in Sec. III.

2. $g_f>0$

For $g_f>0$ (i.e., attractive interactions for a t-V model), $\langle\theta_\parallel\rangle=(\pi/\sqrt{8})$ and in a first approximation the coupling (14) vanishes. Obviously, this approximation is too crude and one must integrate the fluctuations of $\theta_\parallel$ around its mean value to get the effective coupling. This is done in Appendix C and gives the following effective action for ϕ_ρ:

$$S_\rho=\int dx\ d\tau\left[\frac{(\nabla\phi_\rho)^2}{2\pi K_\rho}+[\xi(x)e^{i\sqrt{8}\phi_\rho(x,\tau)}+\mathrm{H.c.}]\right], \tag{20}$$

with $\overline{\xi(x)\xi^*(x')}=D\delta(x-x')$ and $D\sim D_a^2$.

The renormalization of the disorder is given by an equation similar to Eq. (16):

$$\frac{dD}{dl}=(3-4K_\rho)D(l). \tag{21}$$

The disorder is now relevant only for $K_\rho<3/4$, leading to three different phases for $g_f>0$: a random orbital antiferromagnet for $K_\rho<3/4$, an ordered orbital antiferromagnet for $3/4<K_\rho<1$, and an s-wave superconducting phase for $K_\rho>1$. For the t-V model, $K_\rho>1$, and the s-wave superconducting phase is therefore *stable* with respect to weak disorder, which differs from the single-chain problem. For the latter the delocalization only occured for *extremely* attractive interactions, i.e., $K_\rho>3/2$. For the two-chains problem the localization-delocalization transition arises in the immediate vicinity of the noninteracting point. Contrarily to the case of repulsive interactions, interchain hopping now strongly reduces the localization effects.

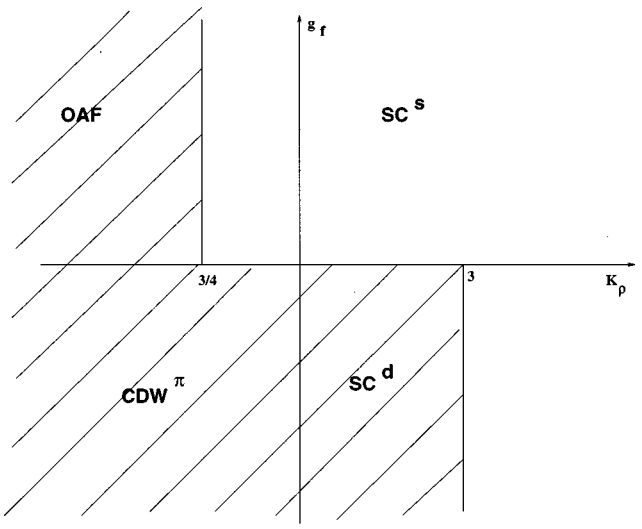

FIG. 2. The phase diagram of the disordered two-chain t-V model in terms of g_f and K_ρ. For a single chain the system is localized for $K_\rho<3/2$. Ladder effects therefore *delocalize* for attractive interactions and *enhance* localization for repulsive ones.

The localization length in the random orbital antiferromagnet is now given by

$$\frac{L_{2\,\mathrm{ch.}}}{\alpha}=(1/D)^{[1/(3-4K_\rho)]}=\left(\frac{v_F^2}{D_a\alpha}\right)^{[2/(3-4K_\rho)]}. \tag{22}$$

The conductivity behaves both in the OAF and the s-wave phase as

$$\sigma(T)\sim T^{2-4K_\rho} \tag{23}$$

diverges as $T\to0$, since the ground state is superconducting. It is to be noted that although the OAF has no superconducting order parameter, its conductivity can also be divergent for $K_\rho>3/4$ *even* in the presence of disorder. An expanded discussion of orbital antiferromagnet phases can be found in Sec. III C 1 and Appendix D. The resulting phase diagram is summarized in Fig. 2, together with the single-chain phase diagram.

C. Physical consequences

The ladder system shows drastically different sensitivity to disorder depending on the sign of g_f: at $g_f<0$ localization effects are much stronger than at $g_f>0$. This is obvious both on the phase diagram shown on Fig. 2, and in the expressions (18) and (22) for the localization length. For the case of a pure t-V model, $g_f>0$ $K_\rho>1$ when $V<0$ (attractive interactions) and as can be seen from Fig. 2 the system is

delocalized. Although our calculation does not allow us to come arbitrarily close to the $V=0$ point for finite disorder, since the disorder has to be smaller than the gaps of the pure system, we see that if we have a very small disorder, the insulator superconductor transition does occur in the vicinity of the noninteracting point. This is remarkable and in marked contrast with the single-chain system where the delocalization transition occurs for $K=3/2$, i.e., very strongly attractive interactions even for arbitrarily weak disorder. One could naively think that this effect is simply a manifestation of the delocalization effect seen for noninteracting electrons when one increases the number of channel (or the number of chains). The mechanism is more subtle however, and is in fact controlled by the interactions. Contrarily to the noninteracting case where the localization length is simply proportional to the number of chains, we have here a *complete* delocalization of the attractive region, and the localization length becomes infinite.

For the repulsive case $V>0$ (i.e., $g_f<0$ $K_\rho<1$) the opposite effect occurs and the ladder system is *more* localized than the corresponding one-chain system. Indeed for one chain the localization length is given by[35,49]

$$\frac{L_{1\,\mathrm{ch.}}}{\alpha}\sim\left(\frac{v_F^2}{D\alpha}\right)^{[1/(3-2K)]} \tag{24}$$

and is therefore longer than the one of the ladder system shown in Eq. (18). For very large repulsion ($K\to 0$) these two lengths give back the standard Fukuyama-Lee pinning length of classical charge density waves.[50] For finite repulsion the localization length of the ladder system is much shorter than the one of the corresponding one-dimensional system with the same K. Close to the noninteracting point $K\sim 1$, the localization length of the open chain is just the mean free path $L_{1\,\mathrm{ch.}}\sim v_F^2/D$, whereas the ladder one is $L_{2\,\mathrm{ch.}}\sim\alpha\sqrt{v_F^2/D\alpha}$.

This peculiar behavior of the spinless ladder system is due to the gapping of some charge modes that is different depending on whether the interaction is attractive or repulsive. For the repulsive side $2k_F$ charge fluctuations are still there and the gap just reduces some of the quantum fluctuation and hence reinforces the effects of disorder, whereas for the attractive side the gap kills the dominant charge fluctuation coupled to disorder and helps to delocalize. The sensitivity to disorder is therefore *not* directly related to the presence or absence of the superconducting fluctuations in the pure system, but more on how the *density* fluctuations behave. The smoother the density fluctuations are, the less localized the system is. These effects will be even more transparent for the system with spins as will be examined in detail in Sec. III. As a consequence the transport properties *cannot* simply be guessed by looking at the phase diagram of the pure system. They even can be opposite to what our intuition based on higher dimensional system could suggest: the more ''superconducting'' the system is the better the transport is (see, e.g., Sec. III C).

D. Persistent currents in the ladder system

In addition to the temperature dependence of the conductivity, one can compute the charge stiffness of the system[51–53] D, which measures the strength of the Drude peak in a macroscopic system $\sigma(\omega)=\mathcal{D}\delta(\omega)+\sigma_{\mathrm{reg}}$. The stiffness $\mathcal{D}$ can be related to the change of the energy of the ground state of the system in the presence of an external flux by

$$\mathcal{D}=\frac{L}{2}\left.\frac{d^2E_0}{d\phi^2}\right|_{\phi=0}, \tag{25}$$

E_0 being the ground state energy of a ring in a field. ϕ denotes the boundary angle $\phi=2\pi f/f_0$, where f is the flux threading the ring and $f_0=hc/e$ is the flux quantum. This quantity is directly related to the persistent currents for a mesoscopic system.[54–58] For a mesoscopic system, the persistent current measures the response to a finite flux by

$$J=L\left.\frac{dE_0}{d\phi}\right|_{\phi}. \tag{26}$$

Therefore the stiffness $\mathcal{D}$ provides a measure of the persistent currents for a small (or close to a multiple of 2π) flux since $J=2\mathcal{D}\phi$. Although the complete calculation of the persistent currents at finite flux is also possible for a one-dimensional interacting system, the calculation is more complicated in the presence of disorder, and the stiffness carries enough information for our present purposes.

The effects of interactions on persistent currents is an extremely difficult question to answer in two or three dimensions. Perturbative calculations suggest that interactions could enhance persistent currents.[59–62] For a single spinless chain the persistent currents were found to *decrease* with more repulsive interactions.[63–65] This effect can naturally be explained using a renormalization-group technique, and it was shown that such behavior is peculiar to the spinless problem and that for a single chain of electrons with spins persistent currents should be enhanced by repulsive interactions.[45,64] For the ladder system it is therefore very interesting to see if the same effects occur and, in particular, to check again for the differences between the spinless system and the system with spins. In particular, one could imagine that the chain index acts in a similar way than a spin index for a single chain. As we will see this idea is far too naive. We examine the spinless system in this Section and the system with spins will be investigated in Sec. III.

For the ladder system, the conductivity stiffness[51] is obtained using Eq. (A13) as $\mathcal{D}=2u_\rho K_\rho$. The factor of two compared to the single chain expression (A13) is due to the fact that there are twice as many degrees of freedom in the two chain system. In the following, we consider a finite system, the size L of which is smaller than the localization length.

From the renormalization-group equation for u_ρ, K_ρ,[35] one can obtain[45] the renormalization-group equation for $\mathcal{D}$,

$$\frac{d\mathcal{D}}{dl}=-D(l). \tag{27}$$

The conductivity stiffness of a disordered system of size L, $\mathcal{D}(L)$ is then obtained by stopping the RG equation at $\alpha(l)=L$ and taking $\mathcal{D}(L)=\mathcal{D}(l)$. In the case $g_f<0$, we have seen that $D(l)=D(0)e^{(3-K_\rho)l}$, at least when $\alpha(l)\ll L_{2\,\mathrm{ch.}}$. Putting that approximation for $D(l)$ in Eq. (27) gives us

$$\mathcal{D}(L)=\mathcal{D}(0)-\mathcal{C}D(0)\left[\left(\frac{L}{\alpha(0)}\right)^{3-K_\rho}-1\right]. \quad (28)$$

Using the expression for $L_{2\text{ch.}}$, Eq. (28) simplifies for a length smaller than the localization length into

$$\mathcal{D}_{g_f<0}(L)=\mathcal{D}(0)-\mathcal{C}\left[\left(\frac{L}{L_{2\text{ch.}(g_f<0)}}\right)^{3-K_\rho}-1\right], \quad (29)$$

$$\mathcal{D}_{g_f>0}(L)=\mathcal{D}(0)-\mathcal{C}'\left[\left(\frac{L}{L_{2\text{ch.}(g_f>0)}}\right)^{3-4K_\rho}-1\right]. \quad (30)$$

Thus for $g_f>0$ the reduction of the stiffness is less important than for $g_f<0$.

Therefore, the length dependence of the conductivity stiffness (and the persistent currents) is extremely sensitive to the attractive or repulsive character of the interactions for the t-V model or any model with intrachain-only interactions. By comparison with the one-chain case,[45,63] we see that the effects of the interactions on the conductivity stiffness are qualitatively the same (i.e., repulsive interactions help in reducing the conductivity stiffness, while attractive interactions reduce the decrease of conductivity stiffness by disorder), but they are much stronger for two chains than for one chain. In fact, for a t-V model, the reduction of conductivity stiffness would be *finite* for attractive interactions, even in an infinite system since then the disorder is completely irrelevant.

It is noteworthy that the chain index does *not* act in a similar way as a spin degree of freedom for which there would be an *increase* of the persistent currents showing again the important difference between a system with and without spin.[45] The physical reasons for this difference are examined in more details in the next section.

E. Spinless ladder vs one chain with spin

Naively, one could think that going from one-chain to two-chain amounts to having one internal degree of freedom that is equivalent to the spin, and thus that the results for the system with spin will apply straightforwardly to the ladder system. However, from what we have seen precedingly, this is definitely not the case. In fact, we have properties for the spinless ladder that are just the contrary of the ones of the fermions with spin. Attractive interactions delocalize in the spinless fermions case, whereas they increase localization in the case of fermions with spin. Persistent currents are enhanced for more attractive interactions in the spinless ladder whereas repulsive interactions would enhance the persistent currents[45] in a spin system. The reason for that is that the spinless ladder has no SU(2) symmetry (except for $V=0$) contrarily to one chain with spin. The minimum of the ground state energy of the spinless ladder corresponds to states that break the SU(2) symmetry because $t_\perp$ plays the role of a magnetic field.[14,48] Thus such phases cannot be obtained in an isotropic system of fermions with spin.

For attractive interactions, the only way for the symmetric fermions with spin system to preserve SU(2) symmetry is to form singlet phases such as $2k_F$ charge density waves or singlet superconducting state. Coupling the charge density wave fluctuations with a random potential implies strong localization effects. On the other hand, the spinless ladder simply form pairs along the chains and can avoid to form $2k_F$ fluctuations. Translated in the spin language, such a phase would be an anisotropic triplet superconductor with a spin gap, and would be forbidden by symmetry. In the same way, for repulsive interactions, preserving SU(2) symmetry prevents the formation of a gap, whereas a gap formation is possible for the spinless ladder giving an out of phase charge density wave. In the spin language, this corresponds to an anisotropic spin density wave.

Adding random potentials to the spinless ladder results in a rather artificial model of fermions in a random potential and a random field parallel to the z axis. Because of the anisotropy, the system is more sensitive to the random field parallel to the z axis than to the random potential. Thus, for repulsive interactions, the anisotropic system has a very strong coupling to disorder, whereas for repulsive interactions, it is only weakly coupled. On the other hand, the isotropic system is only feeling a random potential. When interactions are attractive, there is a spin gap and CDW fluctuations that can couple to disorder, making the system more localized. When interactions are repulsive, on the other hand, there is no spin gap thus reducing the coupling of the CDW fluctuations with disorder.

We conclude that for interacting systems, contrarily to their noninteracting counterparts, not only the number of available internal degrees of freedom but also the internal symmetries determine the response to random perturbations. Losing some symmetries allows for a larger variety of ground states, and thus to very different responses to weak perturbations.

III. FERMIONS WITH SPIN

A. Pure system

The pure case has been analyzed in great detail both analytically[8–13] and numerically.[16,17,19,20] A very interesting feature of that model is the existence of a ''d-wave'' superconducting phase for purely repulsive interactions and the existence of a spin gap. The Hamiltonian is in the extended Hubbard case:

$$H=-t\sum_{i,\sigma,p} c^\dagger_{i+1,\sigma,p}c_{i,\sigma,p}+\text{H.c.}-t_\perp\sum_{i,\sigma,p} c^\dagger_{i,\sigma,p}c_{i,\sigma,-p}$$
$$+U\sum_{i,p} n_{i,\uparrow,p}n_{i,\downarrow,p}+V\sum_{i,p} n_{i,p}n_{i+1,p}, \quad (31)$$

where $p=\pm 1$ is the chain index and $\sigma=\uparrow,\downarrow$ labels the spin. In order to treat this Hamiltonian using bosonization one has to separate the bonding o and antibonding π bands as was done for spinless fermions. Then, within each band, one can apply the standard bosonization formulas for fermions with spins. As a consequence, the system is described by four fields $\phi^\pi_\rho,\phi^\pi_\sigma\phi^o_\rho,\phi^o_\sigma$ instead of 2 in the spinning case. For the pure case we follow closely the derivation of Ref. 11. It is convenient in the following to replace the fields $\phi^{o,\pi}_\nu$ $(\nu=\rho,\sigma)$ by linear combinations: $\phi_{\nu\pm}=1/\sqrt{2}\,(\phi_{\nu,o}\pm\phi_{\nu,\pi})$. The low energy physics depends on the signs of two constants g_1,g_2. Physically, g_2 repre-

TABLE II. The four sectors of the pure two-chain Hubbard model, as a function of K_ρ and g_1. The average value of the field developing a gap are indicated together with the phase with the most divergent susceptibility.

	I	II	III	IV
g_1	+	+	−	−
$K_{\rho+}$	<1	>1	>1	<1
$\langle\theta_{\rho-}\rangle$	0	0	0	0
$\langle\phi_{\sigma+}\rangle$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	0	0
σ^-	$\langle\phi_{\sigma-}\rangle=\frac{\pi}{2}$	$\langle\theta_{\sigma-}\rangle=0$	$\langle\phi_{\sigma-}\rangle=0$	$\langle\theta_{\sigma-}\rangle=\frac{\pi}{2}$
phase	SC^d	OAF	SC^s	CDW^π

sents the forward scattering interaction, while g_1 represents the backward scattering interactions. The Hamiltonian consists of a free part,

$$H=\sum_{\substack{\nu=\rho,\sigma\\ r=\pm}}\int\frac{dx}{2\pi}\left[u_{\nu r}K_{\nu r}(\pi\Pi_{\nu r})^2+\frac{u_{\nu r}}{K_{\nu r}}(\partial_x\phi_{\nu r})^2\right], \tag{32}$$

and two sine-Gordon like parts, one associated with interband processes induced by intrachain forward scattering,

$$H_{\text{int},2}=\frac{g_2}{2(\pi\alpha)^2}\int dx\ \cos2\theta_{\rho-}(\cos2\phi_{\sigma-}+\cos2\theta_{\sigma-}), \tag{33}$$

and the other associated with the intrachain backward scattering,

$$H_{\text{int},1}=\frac{2g_1^*}{(2\pi\alpha)^2}\int dx[\cos2\phi_{\sigma+}(\cos2\theta_{\rho-}+\cos2\phi_{\sigma-}+\cos2\theta_{\sigma-})-\cos2\theta_{\rho-}\ \cos2\theta_{\sigma-}]. \tag{34}$$

In all cases, only one of the four bosonic fields ($\phi_{\rho+}$) is gapless[11] and all physical quantities depend on a parameter $K_{\rho+}$ of the symmetric charge mode, analogous to the K_ρ of the spinless problem. In terms of $g_1,g_2,K_{\rho+}$ is given by

$$K_{\rho+}=\left(\frac{2\pi v_F+(g_1-2g_2)}{2\pi v_F-(g_1-2g_2)}\right)^{1/2}. \tag{35}$$

That expression is valid for the generic g-ological model. For the extended Hubbard model, we can go further as g_1,g_2 can be expressed in terms of U,V,k_F as

$$g_1=Ua+2Va\ \cos(2k_Fa),$$
$$g_1-2g_2=-[Ua+2Va(2-\cos(2k_Fa)], \tag{36}$$

where a is the lattice spacing. The mean values of the three other fields are determined by minimizing the energy of the ground state. Depending on the interactions one can distinguish four sectors that are summarized in Table II.

As for the spinless case one has to consider the various operators with divergent susceptibilities,

$$O_{\text{CDW}^\pi}(n)=\sum_{p,\sigma}pc^\dagger_{n,\sigma,p}c_{n,\sigma,p}, \tag{37}$$

$$O_{\text{OAF}}(n)=\sum_{p,\sigma}pc^\dagger_{n,\sigma,p}c_{n,\sigma,-p}, \tag{38}$$

$$O_{\text{SC}^s}(n)=\sum_p c_{n,\sigma,p}c_{n,-\sigma,p}, \tag{39}$$

$$O_{\text{SC}^d}(n)=\sum_p c_{n,\sigma,p}c_{n,-\sigma,-p}. \tag{40}$$

When taking the continuum limit these expressions become

$$O_{\text{CDW}^\pi}=\sum_\sigma(\psi^\dagger_{L1\sigma}\psi_{R1\sigma}-\psi^\dagger_{L-1\sigma}\psi_{R-1\sigma}), \tag{41}$$

$$O_{\text{OAF}}=\imath\sum_\sigma(\psi^\dagger_{L1\sigma}\psi_{R-1\sigma}-\psi^\dagger_{L-1\sigma}\psi_{R1\sigma}), \tag{42}$$

$$O_{\text{SC}^s}=\sum_\sigma(\psi_{L0\sigma}\psi_{R0,-\sigma}+\psi_{L\pi\sigma}\psi_{R\pi,-\sigma}), \tag{43}$$

$$O_{\text{SC}^d}=\sum_\sigma(\psi_{L0\sigma}\psi_{R0,-\sigma}-\psi_{L\pi\sigma}\psi_{R\pi,-\sigma}), \tag{44}$$

where for the SC operators, one has to retain the $q\sim0$ component, while for the OAF and CDW^π the $q\sim2k_F$ component gives the dominant contribution. To get the correct bosonized expression one has to pay extra care to the anticommuting U operators[66] and one obtains

$$O_{\text{CDW}^\pi}=\frac{2}{\pi\alpha}\ e^{\imath\phi_{\rho+}}\cos\phi_{\sigma+}\sin\theta_{\sigma-}, \tag{45}$$

$$O_{\text{OAF}}=\frac{2\imath}{\pi\alpha}\ e^{\imath\phi_{\rho+}}\sin\ \phi_{\sigma+}\cos\theta_{\sigma-}, \tag{46}$$

$$O_{\text{SC}^s}=\frac{2}{\pi\alpha}\ e^{-\imath\theta_{\rho-}}\cos\phi_{\sigma+}\cos\phi_{\sigma-}, \tag{47}$$

$$O_{\text{SC}^d}=\frac{2}{\pi\alpha}\ e^{-\imath\theta\rho+}\sin\phi_{\sigma+}\sin\phi_{\sigma-}. \tag{48}$$

From the bosonized form of these operators (simplified by the fact that $\langle\theta_{\rho-}\rangle=0$) everywhere and the expressions given in Table II one can deduce that sector I is a SC^d phase, sector II an OAF phase, sector III a SC^s phase, and sector IV a CDW^π phase. The phase diagram of the pure system is summarized in Fig. 3. Note that for the pure Hubbard model, which corresponds to $V=0$ in Eq. (36), one can only have the SC^d phase (for $U>0$) or the SC^s phase (for $U<0$). The other phases could be obtained for a more general model such as the extended Hubbard model. We will come back to that point later.

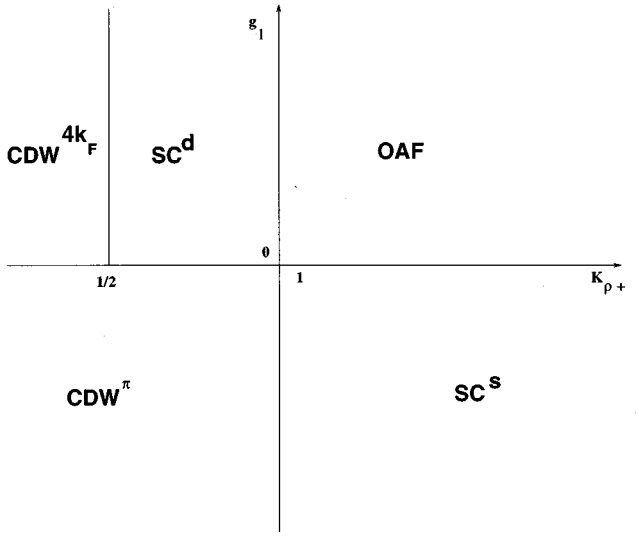

FIG. 3. The phase diagram of the pure two-chain Hubbard model in terms of g_1 and $K_{\rho+}$. $K_{\rho+}>1$ and $g_1<0$ corresponds to purely attractive interactions. $K_{\rho+}<1$ and $g_1>1$ corresponds to purely attractive interactions. For a Hubbard model, this leads to a SC^d phase for $U>0$ and a SC^s phase for $U<0$.

B. Effects of disorder

Let us now add a weak random on-site potential:

$$H_{\text{random potential}}=\sum_{i,\sigma,p}\epsilon_{i,p}n_{i,\sigma,p}, \tag{49}$$

with $n_{i,p}=c^\dagger_{i,\uparrow}c_{i,\uparrow}+c^\dagger_{i,\downarrow}c_{i,\downarrow}$ and $\overline{\epsilon_{i,p}\epsilon_{j,p'}}=D\delta_{i,j}\delta_{p,p'}$. We go through the same steps as in the spinless fermions section. We get to the continuum limit, introduce the bonding and antibonding band, and bosonize the resulting coupling to disorder. Let us first consider the $q\sim 0$ part of the coupling to disorder. For the symmetric part of the disorder this coupling is of the form

$$H_{s,q\sim 0}=\int \eta_s(x)\partial_x\phi_\rho(x)dx. \tag{50}$$

It is clear that this part of the disorder can be eliminated by the transformation

$$\phi_\rho(x)\rightarrow\phi_\rho(x)+\int^x(\pi K_{\rho+}/u_{\rho+})\,\eta_s(x')dx'.$$

For the $q\sim 0$ part of the antisymmetric random potential, we obtain

$$H_{a,q\sim 0}=\int dx\;\eta_a(x)\sum_\sigma[\psi^\dagger_{R,0,\sigma}\psi_{R,\pi,\sigma}+\psi^\dagger_{L,0,\sigma}\psi_{L,\pi,\sigma}+\text{H.c.}]. \tag{51}$$

The bosonized form of that operator is the following:

$$H_{a,q\sim 0}=\int dx\,\frac{\eta_a(x)}{\pi\alpha}[e^{i(\phi_{\rho-}+\theta_{\rho-})}\cos(\phi_{\sigma-}+\theta_{\sigma-})+e^{i(-\phi_{\rho-}+\theta_{\rho-})}\cos(\phi_{\sigma-}-\theta_{\sigma-})+\text{H.c.}]. \tag{52}$$

From that equation, we see that the $q\sim 0$ part of the antisymmetric disorder is not coupled to the gapless charge symmetric mode. Moreover, it always contains one term that has exponentially decaying correlations. Therefore, it cannot break any gap by an effect *à la* Imry Ma and cannot generate any relevant term by a massive mode integration. It will thus be possible to drop it safely in the following. Then, we have to consider the $2k_F$ part of the disorder. We have for the $2k_F$ coupling to disorder two terms:

$$H_a=\int \xi_a(x)O_{\text{CDW}^\pi}(x)+\xi^*_a(x)O^\dagger_{\text{CDW}^\pi}(x)dx, \tag{53}$$

$$H_s=\int \xi_s(x)O_{\text{CDW}^o}(x)+\xi^*_s(x)O^\dagger_{\text{CDW}^o}(x)dx. \tag{54}$$

Where $\overline{\xi_n(x)\xi_{n'}(x')^*}=D_n\delta_{n,n'}\delta(x-x')(n,n'=a,s)$, the ξ_n are random Gaussian distributed potentials. The operators O_{CDW^o} represent the in-phase charge density wave, and O_{CDW^π} the out of phase one.

As before we assume that the disorder is weak enough not to destroy the gaps in the system. We have already argued that the $q\sim 0$ is irrelevant to our problem. Concerning the $2k_F$ part, we only retain the massless mode. The situation is quite similar to the one of an XXZ spin chain in a random magnetic field. A XXZ spin chain is a Hubbard chain at half filling, and thus has a charge gap. The random magnetic field couples to the spin density that contains (frozen) charge degrees of freedom. However, the random magnetic field only affects the spin degrees of freedom and does not break the charge gap. By analogy, we expect that even when the random potential gets relevant it will not break the spin gap or the gap in the antisymmetric charge mode. Since the gaps are stable, we can obtain simplified forms for the couplings by replacing the fields by their mean values as we did in the spinless fermions problem.

1. SC^d sector

We want to analyze the effect of the weak random potential introduced through Eqs. (53)–(54). Making use of the full expressions of $O_{\text{CDW}^{o,\pi}}$ and replacing the gapped fields by their mean values (see sector I of Table II), we obtain the following simplified forms:

$$O_{\text{CDW}^o}\sim e^{i\phi_{\rho+}}\sin(\phi_{\rho-}), \tag{55}$$

$$O_{\text{CDW}^\pi}\sim e^{i\phi_{\rho+}}\sin(\theta_{\sigma-})\cos\phi_{\sigma+}. \tag{56}$$

These two operators have exponentially decaying correlation functions and no direct coupling with disorder would exist if one just took into account the mean values of the fields $\phi_{\rho,-}$ and $\theta_{\sigma,-}$. As in the spinless case one should integrate over fluctuations to get the effective coupling

$$S^{\text{disorder}}_{\rho+}=\int \xi_{\text{eff.}}(x)e^{i2\phi_{\rho+}(x,\tau)}dx\;d\tau+\text{H.c.} \tag{57}$$

Equation (57) can be viewed as the coupling of the fermions with the $2(k_{Fo}\pm k_{F\pi})$ Fourier component of the disordered potential, i.e., to a $4k_F$ charge density wave. The origin for such a $4k_F$ charge density wave can be understood in simple terms: at half filling, the strong on site repulsion puts one fermion per site, meaning that there are no $2k_F$ CDW fluctuations. However, the fermion density is maximum on the

lattice site and minimum in between giving the $4k_F$ charge density wave fluctuations. In addition due to the spin gap occurring in a ladder with an even number of legs there are no $2k_F$ fluctuations in the spin density as well. As we move away from half filling, the spin gap will survive as well as the absence of $2k_F$ fluctuations. Therefore, a random potential can only couple to the $4k_F$ component of the fermion density even away from half filling. This is to be contrasted to the case of a single chain where the dominant coupling occurs through the $2k_F$ charge fluctuation. One thus expects the disorder effects to be weaker in the ladder system. One can also recover directly the $4k_F$ CDW by looking at higher Fourier components of the density in the bosonization formulas. The physics of the metal insulator transition can be interpreted here as the pinning-depinning transition of this $4k_F$ charge density wave.

Due to the presence of the gaps, the problem has in fact been formally reduced to a problem of one chain of spinless fermions with disorder. Using the results from the one chain problem we find that the localization-delocalization occurs at $K_{\rho+}=3/2$. Since purely repulsive interaction imply $K_{\rho+}<1$ the d-wave phase is therefore unstable to arbitrarily weak disorder. The symmetric (54) and the antisymmetric (53) part of the disorder contribute equally to destroy the d-wave superconductivity, in contrast with the spinless case where the antisymmetric part was the most relevant. The localization length of the two-chain system with spin and purely repulsive interactions can be obtained by a similar method to that for the spinless case and is

$$\frac{L_{2\,\mathrm{ch.}}}{\alpha}\sim\left(\frac{v_F^2}{D\alpha}\right)^{2/(3-2K_{\rho+})}, \tag{58}$$

and therefore longer than the corresponding one for one chain with repulsive interactions[49,35]

$$\frac{L_{1\,\mathrm{ch.}}}{\alpha}\sim\left(\frac{v_F^2}{D\alpha}\right)^{1/(2-K_{\rho+})}. \tag{59}$$

As for the spinless case Eq. (58) is applicable if one is far enough from the noninteracting point so that disorder does not destroy the gaps created by the interactions. In that case one sees from Eq. (58) that there is a considerable delocalization in the ladder. Indeed for weakly repulsive interactions $K_{\rho+}\sim 1$, the localization length becomes much longer than the mean free path l, since $L_{2\,\mathrm{ch.}}\sim\alpha(l/\alpha)^2$, instead of $L\sim l$ for a single chain. However the more repulsive the interactions become, the more the system localizes (one recovers $L_{2\,\mathrm{ch.}}\sim l$ for $K=1/2$).

The temperature dependence of the conductivity can be obtained above the pinning temperature $T_{\mathrm{pin.}}=(u_{\rho+}/L_{2\mathrm{ch.}})$ (see Appendix B). One gets

$$\sigma(T)\propto T^{2-2K_{\rho+}}. \tag{60}$$

For $K_{\rho+}<1$, the conductivity *decreases* as $T\to 0$ even for temperatures much higher than $T_{\mathrm{pin.}}$. There is no remnant of the ''superconducting'' behavior of the pure system in the whole SC^d sector ($K_{\rho+}<1$).

2. SC^s sector

For sector III, the O operators take a different simplified form, due to the different gaps in the system,

$$O_{\mathrm{CDW}^o}\sim e^{\imath\phi_{\rho+}}\cos(\phi_{\rho-}), \tag{61}$$

$$O_{\mathrm{CDW}^\pi}\sim e^{\imath\phi_{\rho+}}\sin(\theta_{\sigma-}). \tag{62}$$

By substituting in Eqs. (53) and (54) and integrating over fluctuations we end with an action of the form (58). This time, $K_{\rho+}>1$, so the localization-delocalization transition can be reached at $K_{\rho+}=3/2$. This transition arises for a much weaker attraction than in the one dimensional case[35] where $K_\rho=3$. This critical value of K can be realized for a simple Hubbard model [the maximum K for the Hubbard model is $K=2$ (Refs. 67 and 45)], whereas the one chain Hubbard model is always localized even for very negative U.[46] In addition the localization length is increased,

$$\frac{L_{2\,\mathrm{ch.}}}{\alpha}=\left(\frac{v_F^2}{D\alpha}\right)^{[2/(3-2K_{\rho+})]}, \tag{63}$$

whereas in the one-chain case

$$\frac{L_{1\,\mathrm{ch.}}}{\alpha}=\left(\frac{v_F^2}{D\alpha}\right)^{[1/(3-K_\rho)]}. \tag{64}$$

Note that here the localization length has the same dependence in disorder on the attractive (63) and the repulsive (58) side, whereas for a single chain the localization length is *reduced* on the attractive side due to the formation of a spin gap [compare Eq. (64) and Eq. (59)]. For the ladder this comes from the fact that both in the attractive and repulsive sector, three of the modes are always gapped.

The conductivity above the pinning temperature behaves as

$$\sigma(T)\sim T^{2-2K_{\rho+}}, \tag{65}$$

with again the same exponent as in the d-wave sector (60). However, since now $K_{\rho+}>1$ the conductivity now *decreases* with decreasing T. There will thus be for $1<K_{\rho+}<3/2$ a maximum in the conductivity for $T\sim T_{\mathrm{pin.}}$, and the resistivity will go to zero for high values of $K_{\rho+}$. This maximum can be seen as a remnant of the superconducting behavior of the pure system. For $K_{\rho+}>3/2$, the system has infinite conductivity for $T\to 0$.

3. CDW$^\pi$ sector

Let us now consider Sec. IV. In that sector, one has strong fluctuations towards a CDW$^\pi$ phase. Such a phase is the analog of the CDW$^\pi$ that existed in the spinless fermion problem. We see that the coupling to disorder reduces to (see Table II)

$$\int dx\ \xi_a(x)e^{\imath\phi_{\rho+}}+\mathrm{H.c.} \tag{66}$$

As in the spinless fermion case that antisymmetric charge density wave only couples to the antisymmetric disorder. The RG equation for disorder is

$$\frac{dD_a}{dl}=\left(3-\frac{K_{\rho+}}{2}\right)D_a(l). \tag{67}$$

The antisymmetric disorder is thus relevant for $K_{\rho+}<6$. Since the CDW$^{\pi}$ phase only exists at $K_{\rho+}<1$ the CDW$^{\pi}$ is always very strongly pinned by disorder. Using Eq. (67) we obtain for the localization length in that phase

$$\frac{L_{\text{loc.,CDW}^{\pi}}}{\alpha}\sim\left(\frac{v_F^2}{D_a\alpha}\right)^{[2/(6-K_{\rho}+)]}. \tag{68}$$

In the classical limit $K_{\rho+}\to 0$ one recovers again the standard result[50] for the pinning of a classical CDW.

The conductivity of the CDW$^{\pi}$ above the pinning temperature behaves as

$$\sigma(T)\sim T^{2-K_{\rho+}/2}, \tag{69}$$

showing since $K_{\rho+}<1$ a very rapid decrease in the conductivity as $T\to 0$. This behavior is a consequence of the very strong pinning of the CDW$^{\pi}$. This feature of the antisymmetric CDW is similar to the one occurring for the spinless ladder.

4. OAF sector

In the case of the orbital antiferromagnet, the coupling to disorder is made of two terms: One term comes from O_{CDW^o} the other one from $O_{\text{CDW}^{\pi}}$. According to the preceding sections, these terms contain, respectively, $\cos\phi_{\rho-}$ and $\cos\phi_{\sigma+}\sin\theta_{\sigma-}$ and (see Table II) therefore have exponentially decaying fluctuations. In order to get nontrivial results, the massive modes have to be integrated out as in the preceding sections. This again leads to an action of the form (57) and the disorder in the OAF phase is relevant for $K_{\rho+}<3/2$. The OAF phase is therefore *as delocalized* as the superconducting SC^s phase, although the pure system does not exhibit any obvious superconducting order parameter. The localization length in the disordered OAF is

$$\frac{L_{\text{loc.}}}{\alpha}=\left(\frac{v_F^2}{D\alpha}\right)^{[2/(3-2K_{\rho}+)]}. \tag{70}$$

For $K_{\rho+}>3/2$ we have a metallic phase.

The disorder leads to a conductivity of the form

$$\sigma(T)\propto T^{2-2K_{\rho+}}. \tag{71}$$

The conductivity in the OAF is therefore identical, as far as the temperature dependence is concerned, to the one in the SC^s. It will exhibit in the localized phase $1<K_{\rho+}<3/2$ the same maximum in the conductivity for $T\sim T_{\text{pin.}}$. Once again one sees that the transport properties can hardly be guessed from the phase diagram of the pure system. The OAF is thus also an excellent candidate for a ''superconducting'' behavior.

Using Eq. (36), it is possible to get some hints on the parameter regime of the extended Hubbard model in which the OAF could be achieved. One is in the OAF sector if $g_1>0$ and $K_{\rho+}>1$. In the extended Hubbard language it means

$$2V[2-\cos(2k_Fa)]<-U<2V\cos(2k_Fa). \tag{72}$$

Let us assume a local repulsion $U>0$ and that one is close to half filling $\cos(2k_Fa)\sim -1$. In that case one reaches the OAF for moderate nearest-neighbor attraction $V<-U/6$. Such a situation is likely enough to be realized, especially if additional attractive mechanisms such as phonons are taken into account.

5. Differences with the spinless ladder

The spinless ladder and the ladder with spin show some marked physical differences. Some of them are due to the fact that interchain hopping has a different impact on fermions with spin and spinless fermions. In a system of spinless fermions, energy can be gained from hopping only if one site of the rung is occupied and the other one is empty due to the Pauli principle. This induces an enhancement of density fluctuations. On the other hand, a system with spin can gain energy from interchain hopping by having the two sites of the rung occupied by fermions of opposite spins. This leads to spin gap formation and a *smoothening* of density fluctuations. This effect is enhanced in the presence of a purely repulsive interaction as it tends to smooth the density fluctuations in a system with spins, whereas it enhances them in a spinless system.[45] This has already important consequences in the pure case. In particular, the positions of the SC^d and OAF phases are different (see Figs. 1 and 3) as the d wave in the spinless system needs some amount of attraction whereas it is achieved from completely repulsive interactions in the ladder with spin. In the presence of disorder, the d-wave phase of the spinless system can be stabilized by sufficiently attractive interactions, whereas in the system with spin it is *always* unstable (see Figs. 2 and 5), being replaced by an s-wave superconducting phase for attractive interactions. Also, in the presence of disorder, the system with spin due to the smoothening of the density shows delocalization compared to the one-chain case *both* for the attractive and the repulsive side. On the other hand, for the spinless system the reinforcement of the density fluctuations enhances localization on the repulsive side. The attractive side on the other hand is totally delocalized.

In both cases the s-wave phase (occuring for attractive interactions) is very strongly stabilized by the interchain hopping. This can be understood by a picture of tightly bound pairs that behave in both cases as hard core bosons. In that case, the statistics do not influence qualitatively the transport properties anymore. Similarly both systems tend to form charge density waves that are extremely well pinned by disorder (usually much more easily than their one-chain counterpart). In the case of fermions with spin, this requires some mixing of attractive and repulsive interactions so that a pair of fermions of opposite spins are formed in the chains. These pairs then have hard core bosons interactions so that the situation becomes an analog to the spinless fermions case. This explains the enhancement of pinning for the antisymmetric charge density wave phase. However, in the system with spin with purely repulsive interactions there is *no* CDW$^{\pi}$ in contrast with the spinless system. Both systems also present an OAF phase that is revealed to be quite stable in the presence of a small disorder. For the spinless ladder the OAF is even stable close to the noninteracting point. Finally, an interesting similarity between the system of fermions with spin and the system of spinless fermions is that

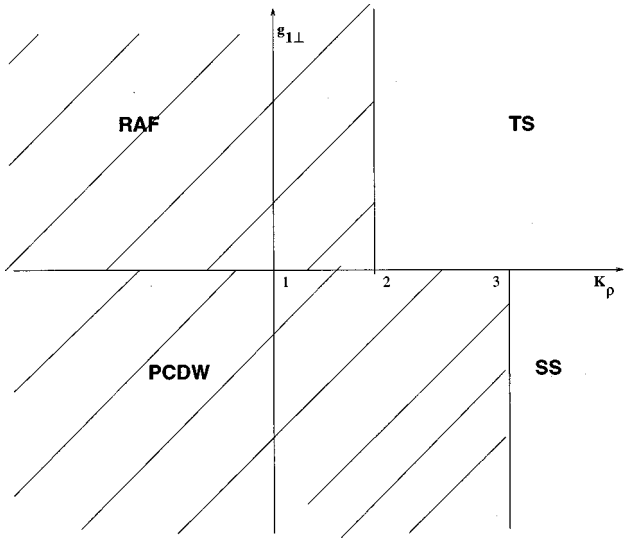

FIG. 4. The phase diagram of the disordered one-chain Hubbard model in terms of $g_{1\perp}$ and K_ρ. Delocalization occurs for $K_\rho>3$ for $g_{1\perp}<0$ and for $K_\rho>2$ for $g_{1\perp}>0$.

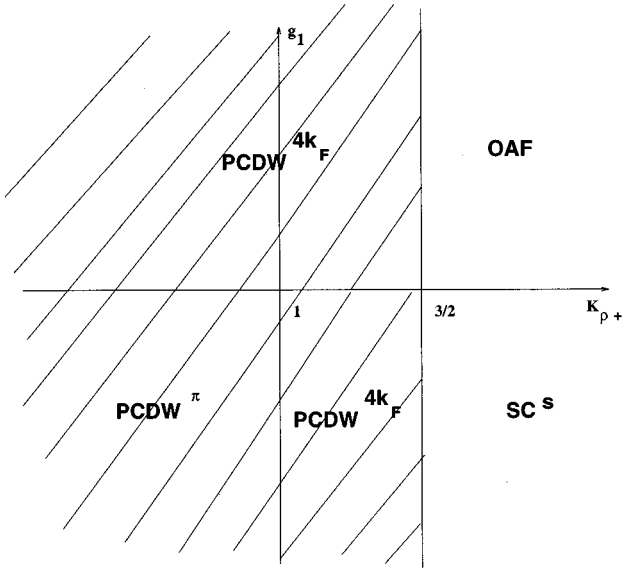

FIG. 5. The phase diagram of the disordered two-chain Hubbard model in terms of g_1 and $K_{\rho+}$. The SC^d phase is completely eaten by the PCDW^{4k_F} phase, whereas the OAF and the SC^s persist if there is enough attraction. Delocalization occurs for $K>3/2$, i.e., for less attractive interactions than in the one-chain case.

pinning on *two* different CDW phases are possible depending on the interactions: either the antisymmetric $2k_F$ CDW or a $4k_F$ CDW. In these two localized phases the behavior of the conductivity at high frequency or high temperature and of the localization length at small disorder are very different (the difference appears in the exponents) the $4k_F$ being much less well pinned than the $2k_F$. This is to be contrasted to the one-chain case where only one pinned charge density wave phase is realized. Therefore, we may expect to see, for weak disorder, a crossover between two different pinned charge density wave phases in the two-chain system when varying the strength of the interactions. Such a crossover needs a more detailed study. Unfortunately it cannot be tackled by the RG since it occurs deep in the localized regime. One interesting question is whether such a transition still occurs for strong disorder.

C. Transport properties

The ladder with spin shows therefore in the presence of disorder transport properties drastically different from the one that one could naively expect form the pure phase diagram. In particular, the *d*-wave phase disappears and does not exhibit any remarkable conductivity. Let us look in more detail at the transport properties and compare them to what happens in a single chain[35] for the various sectors.

1. Conductivity

As was mentioned in Sec. III B 2, the ladder *s*-wave phase is much more stable to disorder than its one-chain counterpart (see Figs. 4 and 5). This effect manifests itself in the location of the superconducting-localized transition, and in the localization length. As for the spinless case, this effect is entirely controlled by the interactions and going from one to two chains affects the *power law dependence* of the localization length with disorder. It is thus much stronger than the increase of localization length occuring for a noninteracting system (proportional to the number of channels). In the presence of interactions the behavior of the localization length cannot be guessed by analogies with the noninteracting system. The case of spinless fermions where repulsive interactions make the two-chain system more localized than the one-chain system is an excellent counterexample.

The resistivity [see Eq. (65)] is also dropping much faster than for one chain for which $\sigma_{1\text{ch.}}(T)\sim T^{2-K_\rho}$. The ladder is thus a much better conductor than a single chain both because of the scale of localization and because of the better temperature dependence. In addition, even in the localized phase the conductivity will increase for all values of $K_{\rho+}$ for which the *s*-wave phase exists in the pure system, until one reaches the localization temperature T_{pin}. This behavior is qualitatively sketched on Fig. 6. The SC^s phase shows therefore all the ''good'' characteristics of a ''superconducting'' phase, and in that respect is much more normal than its one-chain counterpart.

For repulsive interactions a different physical situation occurs. The system is still less localized than the one-chain

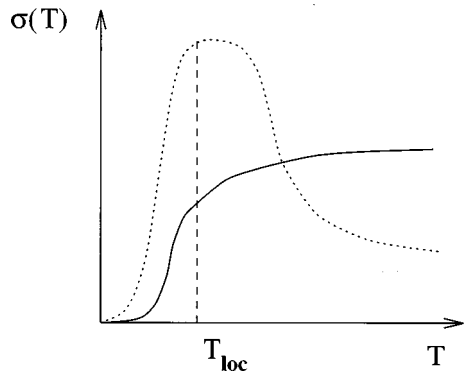

FIG. 6. Behavior of the conductivities of the *s*-wave (dotted line) and *d*-wave (solid line) superconductor as a function of temperature. For $T\gg T_{\text{loc.}}$, $\sigma(T)\propto T^{2-2K_{\rho^+}}$. For the *d* wave, there is no maximum in the conductivity and therefore no remnant of superconductivity in the localized phase.

counterpart. The transition occurs for a smaller value of $K_{\rho+}=3/2$ (versus $K_\rho=2$ for a single chain), and the localization length is larger than for one chain [see Eq. (58)]. Contrarily to the single chain where the pinned phase is a random antiferromagnet, here the presence of the spin gap forces the localized phase to become a pinned $4k_F$ CDW. However the SC^d phase is completely wiped out by the disorder, and what is more surprising, no trace of this ''superconducting'' phase can be found in the high temperature ($T>T_{\rm pin}$) of the conductivity [see Eq. (65)]. In particular, $\sigma(T)$ decreases monotonically even at high temperature in stark contrast with the SC^s phase as shown on Fig. 6. This again illustrates the fact that the transport properties are not linked to the behavior of the superconducting order parameter but to the *density* fluctuations. For a single chain since the density exponent and the superconducting one are related by $K_{\rm density}\sim 1/K_{\rm supra}$ when superconducting fluctuation increases density fluctuations necessarily decrease and the system becomes a better conductor. Or course this is also true in the presence of a true superconducting order in higher dimensional systems. For the single chain the fact that superconducting fluctuations do not necessarily imply better transport also appears from the fact that the attractive Hubbard model is *more* localized than the repulsive one:[45] when the interactions go from repulsive to attractive a spin gap opens and the density fluctuations are suddenly lowered making the system more easy to pin. A similar effect occurs in the d-wave phase of the ladder, in a more dramatic way: the d-wave phase does not look superconducting at all since it leaves enough room for enough $4k_F$ charge fluctuations. Note that the more repulsive the interactions are the worse the conductivity, in a similar way to the single chain where the phase is a spin density wave. The interchain hopping has thus two effects : on the one hand it leads to the appearance of the spin gap that wipes the SDW and replaces it by the SC^d wave and on the other hand it freezes the density fluctuations (in particular, the transverse charge modes). Those gaps suppress $2k_F$ CDW fluctuations, and localization happens only through coupling to $4k_F$ CDW fluctuations. Since the mechanism for localization is the same for all signs of the interactions, the transport properties are only weakly dependent of the sign of the interactions. This charge freezing is the dominant effect on transport. The two effects are essentially unrelated.

The most remarkable phase is the OAF which is an illustration of the above. This phase has a localization length and a $\sigma(T)$ *as good as* a genuine SC^s wave phase, and yet has no genuine superconducting order parameter. In fact the absence of order parameter is here also due to the spin gap since for a single chain the corresponding phase is a triplet superconducting phase. However the fact that density fluctuations are already very small in this phase remains (and is helped by the freezing of transverse charge fluctuations), giving the remarkable transport properties of this phase. This remarkable property is not an artifact of the potential scattering and persists even if coupling to different form of disorder is included. In particular, the superconductinglike transport properties of the OAF also exist in the presence of a random hopping along the chains and a random interchain hopping amplitude (see Appendix D). Note that this phase has analogies of the so-called flux phase,[68–71] the size of the plaquette is fixed here by the interparticle distance, and of course this phase could not be reached for a pure Hubbard model (at the opposite of what was claimed in a higher dimension). Whether for such a phase, a sort of Meissner effect also exists is of course a very interesting question. The connection between the one-dimensional antiferromagnet and its $2d$ or $3d$ counterparts clearly deserves further investigation. In particular, in two dimensions a phase offering some similarities with the one-dimensional OAF, has been proposed for the high-T_c superconductors.[72]

2. Persistent currents

In a similar way to the spinless case one can compute the charge stiffness. For the ladder with attractive interactions one has for $K_{\rho+},K_\rho<3/2$,

$$\mathcal{D}(L)=\mathcal{D}(0)-\left(\frac{L}{L_{\rm loc.,1ch.}}\right)^{3-K_\rho}, \tag{73}$$

whereas for a two-chain one, it is

$$\mathcal{D}(L)=\mathcal{D}(0)-\left(\frac{L}{L_{\rm loc.,1ch.}}\right)^{3-2K_{\rho+}}. \tag{74}$$

These formulas are valid for $\alpha\ll L\ll L_{\rm loc.}$. It is easy to see that they lead to a smaller reduction of the conductivity stiffness in the two-chain case, in agreement with the fact that $L_{2\,\rm ch.}>L_{1\,\rm ch}$. For repulsive interactions, it is of the form

$$\mathcal{D}(L)=\mathcal{D}(0)-\left(\frac{L}{L_{\rm loc.,1ch.}}\right)^{3-K_\rho}. \tag{75}$$

Whereas for the two-chain case, it is of the form

$$\mathcal{D}(L)=\mathcal{D}(0)-\left(\frac{L}{L_{\rm loc.,1ch.}}\right)^{3-2K_{\rho+}}, \tag{76}$$

and the two-chain system has a smaller reduction of conductivity stiffness than the one-chain system. So up to prefactors the reduction in stiffness in the ladder system with spins is identical for repulsive and attractive interactions and the reduction of conductivity stiffness also shows no abrupt change as one goes from attractive to repulsive interactions. By contrast, in the one chain case, attractive interactions induce a spin gap and localization arises from coupling of a single massless mode to $2k_F$ disorder. This gap closes for repulsive interactions and localization arises from the coupling of two massless modes with the $2k_F$ random potential. This causes the abrupt change in transport properties and charge stiffness[35,45] when one goes from attractive to repulsive interactions. This is related to the fact that the localization lengths for attractive and repulsive interactions have the same dependence on disorder, in marked contrast both with the spinless problem and the single chain with spins. The effect of increase of persistent current by repulsive interactions occurring in the single chain[45] is thus either absent or strongly reduced (not an exponent effect any more) in the ladder. It would of course be interesting to investigate ladder with more than two legs to see if this effect reappears and check for a possible difference of behavior between odd and even legs ladders.

IV. COUPLED LADDERS

A. Mean-field treatment

In the preceding sections, we have been considering isolated bichains. To describe realistic compounds, such as $Sr_xCu_{1-x}O$, and have a finite temperature phase transition, interchain coupling should be taken into account. A realistic coupling is of course single particle hopping between the ladders. However in ladders, due to the existence of single particle gaps (spin and antisymmetric charge mode) for the ladder, single particle hopping is irrelevant, provided that the interladder hopping is much smaller than the gaps of the system. One has therefore to consider only the particle-hole (or particle-particle) coupling generated by the single particle hopping.[73] Such couplings can lead to an ordered phase at a finite temperature. As is very reasonable on physical grounds such interchain couplings stabilize the dominant one-dimensional instability. We focus here on the existence of a stable d-wave superconducting phase. This allows us to keep only the particle-particle (or Josephson) coupling between the ladders. The Hamiltonian for the coupled ladders system is

$$H=\sum_n \Bigg[H_{\text{disordered 2 chain system},n} + \frac{J}{2}\int dx[O^{\dagger}_{\text{SC},n}(x)O_{\text{SC},n+1}(x) + O^{\dagger}_{\text{SC},n+1}(x)O_{\text{SC},n}(x)]\Bigg], \tag{77}$$

where $O_{\text{SC},n}$ is the operator for (d-wave or s-wave) superconductivity for the nth ladder and J is the strength of the Josephson coupling. On can simplify further the Hamiltonian (77) by keeping only massless modes in the ladder. Doing so we assume that the spin gap and the interchain gap of the two chain system are much larger than the disorder and much larger than the Josephson coupling. However, we make no assumption on the relative magnitude of the Josephson coupling and the strength of the random potential. The resulting Hamiltonian is, both for the case where the dominant instability is an s-wave or d-wave superconductivity,

$$H=\sum_n \Bigg(\int \frac{dx}{2\pi}\left[u_{\rho+}K_{\rho+}(\pi\Pi_{\rho+,n})^2+\frac{u_{\rho+}}{K_{\rho+}}(\partial_x\phi_{\rho+,n})^2\right] + \int \frac{dx}{\pi\alpha}[\xi_{\text{eff.},n}(x)e^{i2\phi_{\rho+,n}}+\text{H.c.}] + J\int dx\,\cos(\theta_{\rho+,n}-\theta_{\rho+,n+1})\Bigg). \tag{78}$$

To solve Eq. (78) we treat the Josephson coupling in a mean field assuming the existence of a finite superconducting order parameter $\langle\cos(\theta_{\rho+})\rangle$. By making the replacement $\cos(\theta_{\rho+,n}-\theta_{\rho+,n+1})\rightarrow\langle\cos(\theta_{\rho+})\rangle\cos(\theta_{\rho+,n})$, the Hamiltonian (78) becomes the one of an isolated ladder system in an external field, the value of which is determined by a self-consistency condition. The Hamiltonian is then

$$H_{\text{MF}}=\int \frac{dx}{2\pi}\left[u_{\rho+}K_{\rho+}(\pi\Pi_{\rho+})^2+\frac{u_{\rho+}}{K_{\rho+}}(\partial_x\phi_{\rho+})^2\right] + \int \frac{dx}{\pi\alpha}[\xi_{\text{eff.}}(x)e^{i2\phi_{\rho+}}+\text{H.c.}] - \frac{W}{(2\pi\alpha)^2}\int dx\,\cos(\theta_{\rho+}), \tag{79}$$

with the self-consistency condition $W=J\langle\cos(\theta_{\rho+})\rangle$.

The equation determining T_c is

$$\frac{1}{J}=\frac{1}{(2\pi\alpha)^2}\int dx\int_0^{\beta_c} d\tau\langle T_\tau\cos\theta_{\rho+}(x,\tau)\cos\,\theta(0,0)\rangle_{H_0}, \tag{80}$$

with $\beta_c=(1/T_c)$ and H_0 is H_{MF} for $W=0$. To solve Eq. (80), one has to compute the finite temperature superconducting response function of a ladder in the presence of disorder. There are presently no methods to do this exactly, but one can get an accurate solution for T_c by making some simplifying approximations.[36] First, one notices that a finite temperature induces a cutoff length $l(T)=u_{\rho+}/T$ beyond which all correlation functions decay exponentially to zero. We make thus the approximation that beyond $l(T)$ all correlation functions are truly zero and below $l(T)$ they are equal to the $T=0$ correlation functions. This allows us to use the RG equations introduced in Sec. III. If we denote by χ the superconducting response function, when we change the running cutoff $\alpha(l)\rightarrow\alpha(l)e^{dl}$, we have $\chi\rightarrow\chi\exp\{-[dl/2K(l)]\}$. Thus to compute correlation functions at length scale R it is sufficient to integrate the RG equation from the cutoff up to R and follow the renormalization of the response function. Making use of these two approximations, the equation giving T_c simplifies into

$$\frac{1}{J}=\int_\alpha^{u_{\rho+}/T}\frac{RdR}{2\pi\alpha^2}\exp\left(-\int_0^{\ln(R/\alpha)}\frac{dl}{2K(l)}\right). \tag{81}$$

The values of $K(l)$ are obtained by numerically solving the RG equations:

$$\frac{dK}{dl}=-D(l)K(l)^2, \tag{82}$$

$$\frac{dD}{dl}=[3-2K(l)]D(l), \tag{83}$$

the values of T_c for $K=0.5,1.2$ and $J=0.1$ as a function of D are shown on Figs. 7 and 8, respectively. We note that for $K=1.2$ we have an s-wave superconducting phase and for $K=0.5$, a d-wave phase. This can be expected since the interchain coupling stabilizes the dominant one dimensional fluctuation (see Fig. 5). We see that (see Fig. 8) as in the case of the single chain mean-field theory[36] of superconductivity we have an initial linear decrease of the critical temperature with disorder strength. This is to be contrasted with the standard mean-field theory of the s-wave superconductor in three dimensions being based on a diffusion approximation that does not include Anderson localization effects and gives T_c independent of the disorder. This is the well known Anderson theorem. The linear decrease of T_c with the strength of

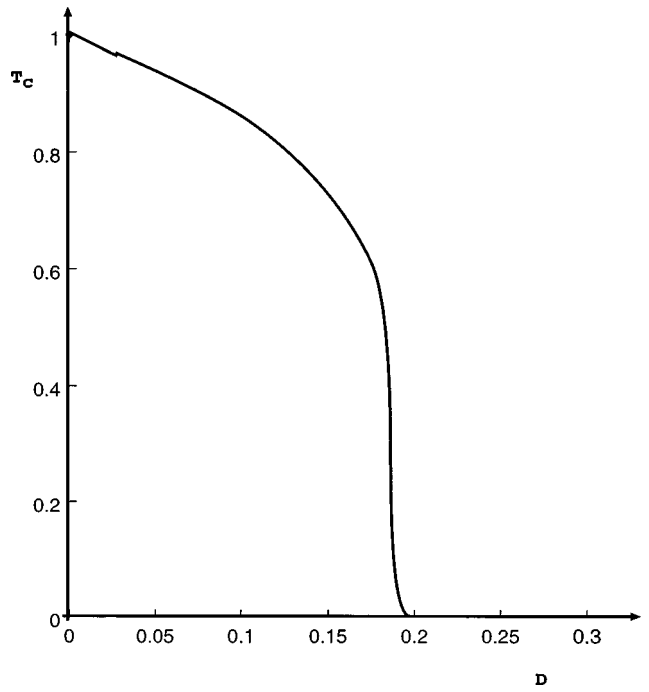

FIG. 7. T_c as a function of disorder for the d-wave phase ($K_\rho = 0.5$). T_c drops quickly to zero for $D \simeq 0.2$.

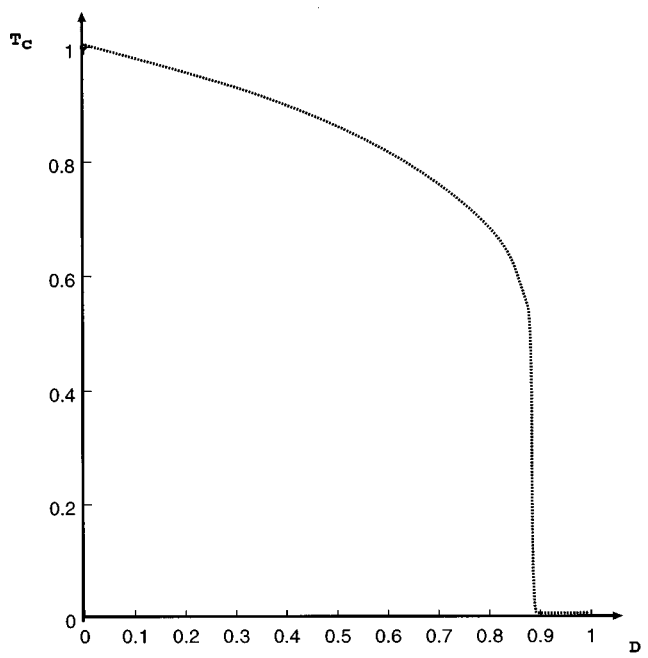

FIG. 8. T_c as a function of disorder for the s-wave phase ($K_\rho = 1.2$). T_c drops to zero for $D \simeq 0.9$. Note the initial linear decay of T_c that shows that Anderson theorem does not hold in coupled chain system due to strong localization effects and absence of a diffusive regime.

disorder for s-wave superconductivity in our chain mean field is due to localization effects. This peculiar situation is due to the absence of a diffusive regime in one-dimensional disordered systems, which implies that their response functions are always affected by localization effects.

For a d-wave superconductor one expects in mean-field theory a linear decrease of T_c as a function of D (see, e.g., Ref. 74). For the ladder system however the decrease of T_c is mainly due to the localization effects, similarly to the s-wave superconductor case. Although it indeed starts linearly for small disorder (see Fig. 7), localization effects manifest themselves by the the sudden drop to $T_c = 0$ at a critical disorder strength (see Figs. 8, 7). For identical Josephson coupling between the bichains, the critical disorder strength is smaller for the d-wave superconductor than for the s-wave one.

B. Simplified treatment

Although the mean-field theory allows an accurate description of the effects of disorder on T_c the critical value of disorder above which superconductivity is destroyed can also be obtained by a very simple physical argument. Let $T_c^{(\mathrm{pure})}(J)$ be the temperature at which the superconducting transition would occur in the array of ladders if there were no impurities. Just above $T_c^{(\mathrm{pure})}$, the thermal length is $(u_{\rho+}/T_c^{(\mathrm{pure})})$ and beyond that length all phase coherence is lost. Clearly, if the thermal length is smaller than the localization length in a single chain containing impurities $\xi_{\mathrm{loc.}}$, phase coherence is lost before coherent backscattering can build Anderson localization. The system will escape localization due to the building of the (mean-field) superconductivity. Thus, if

$$\frac{u_{\rho+}}{T_c^{(\mathrm{pure})}} < \xi_{\mathrm{loc}} \tag{84}$$

Anderson localization will not suppress the superconducting transition. Equation (84) gives a simplified criterion for the stability of superconductivity. For fixed Josephson coupling J, Eq. (80) leads to $T_c^{(\mathrm{pure})} \sim J[2K_{\rho+}/(2K_{\rho+}-1)]$. Thus, the higher $K_{\rho+}$ the higher $T_c^{(\mathrm{pure})}$. From the preceding section, the localization length both in the s-wave and d-wave superconducting phase is $\xi_{\mathrm{loc.}} \sim (1/D)[2/(3-2K_{\rho+})]$. Increasing $K_{\rho+}$ also reduces $\xi_{\mathrm{loc.}}$. Thus, the two effects reinforce each other, and make the SC^s phase that exists for $K_{\rho+} > 1$ more stable against Anderson localization than the SC^d phase that exists only at $K_{\rho+} < 1$.

V. EXPERIMENTAL CONSEQUENCES

The theoretical results obtained in the preceding sections have important consequences for experimental systems that are believed to be well approximated by coupled chains systems, namely, the doped ladder systems which present a superconducting transition and the two band quantum wire. In the former case, one would like to know if the superconducting transition is related to the divergence of superconducting fluctuations in the strictly one-dimensional system that results at the mean-field level in a finite T superconducting transition or if the physics of the transition is a two- or three-dimensional one. We believe that the resilience of superconductivity to disorder is a stringent test of effective dimensionality. In the case of quantum wires, we discuss the experimental consequences of our results for the conductivity and charge stiffness in the interacting system. Measurements of the conductance would allow us to check the above

theories for the ladders and provide a measurement of the Luttinger liquid parameter in the charge sector, providing some insight on the strength of interactions in these systems.

A. Superconductivity of doped ladder systems

Our study has various experimental consequences for the observation of superconductivity in Ladder systems. First, if the superconductivity is to come from purely repulsive interactions (i.e., to be of the d-wave type), it should be extremely sensitive to disorder as we showed in Sec. IV A. In fact, any randomness would induce a conductivity that never increases as temperature decreases (see Fig. 6), so that superconductivity would be impossible to probe except in extremely pure samples. Such sensitivity with respect to disorder is certainly consistent with the difficulty in observing any type of superconductivity in the ladder systems $Sr_{n-1}Cu_{n+1}O_{2n}$.[21,22] However superconductivity seems indeed to be observed in $Sr_{0.4}Ca_{13.6}Cu_{24}O_{41.36}$ (Ref. 75) under pressure (~ 3 GPa). Whether such superconductivity is of the d-wave type is of course still open. Various experimental facts, however seem to indicate that if it is the case, it is unlikely that such a superconducting phase could be described by weakly coupled ladder systems. Indeed one could use the criterion (84) to estimate the localization length. Taking a reasonable value of 10^6 ms^{-1} for the Fermi velocity, one obtains from the observed $T_c \sim 10$ K, a minimal localization length of $\xi \sim 10\,000$ Å. Using Eq. (58), this leads to extremely long mean-free paths ($l = \xi$ for $K = 1/2$) when one is in the d-wave sector. So unless the chains are extremely pure, a fact not likely to be true in such doped materials, one expects based on one-dimensional physics alone that the superconductivity should be totally suppressed. If the presence of superconductivity is due to an extremely pure system (which is doubtful) then, introducing more disorder in the system (for instance, by irradiation) should induce a dramatic decrease of the critical temperature.

Besides the extreme sensitivity of T_c to disorder other arguments are again a simple stabilization of one-dimensional physics in the experimental compound: even if one could be below the critical disorder strength determined by Eq. (84) and Fig. 7, the physics above T_c should be dominated by the one-dimensional (ladder) effects. In this regime the resistivity goes *up* with decreasing temperature as described in Sec. III C. The observed resistivity showing a monotonic *decrease* of the resistivity (roughly with a T^2 or T law) is again incompatible with the one-dimensional description. If one is in the purely repulsive sector, the most likely explanation of the main experimental features is that under pressure the interchain hopping between the ladders become strong enough so that the system does not retain its one-dimensional feature, but is more accurately described by two-dimensional physics. Such an interpretation is also compatible with the fact that the system at ambient pressure is insulating. In that case, the coupled bichains treatment becomes extremely questionable, and it is probably better to start from a two-dimensional description, for which disorder effects are probably weaker, but for which the nature of the superconducting phase has yet to be completely elucidated.

Another interesting, but probably more farfetched, possibility could be that the system is in fact in the orbital antiferromagnetic sector. In such a sector the effects of disorder are much more reduced, and even very large localization lengths can lead to reasonable mean-free paths [ξ/a is at worst $(l/a)^2$, and diverges for $K = 3/2$, see Eq. (70)]. The resistivity decreases with temperature according to Eq. (71). Here the difficulty lies more in getting the interactions corresponding to this phase, since one needs local repulsion and a sizeable nearest-neighbor attraction. In any case, careful measurements of the temperature dependence of the resistivity above T_c could help to decide if such OAF effects are present. Of course here again, one cannot exclude that the physics is two dimensional to start with, but at least now the one-dimensional starting point is more consistent with the dominant experimental features.

B. Application to quantum wires

Progress in nanostructure technologies have allowed for measurements of the transport properties of low dimensional electronic systems. In particular, in recent experiments on quantum wires,[37–40] the *conductance* of a quasi-one-dimensional electron gas has been measured at very low temperatures. For the pure system, or extremely weak disorder one finds quantized values of the conductance[41] in good agreement with the theoretical predictions[76,77] at fractions of h/e^2 as a function of width of the quantum wire (i.e., of the number of subbands at the Fermi level). The relation between the number of channels and resistance has been verified.[40] Impurities on the other hand induce backward scattering that is known to cause Anderson localization in a sufficiently long system. In small enough system, it leads to a reduction of conductance as the length of the system is increased or the temperature is lowered. Deviations of conductance from e^2/h as a function of temperature have indeed been obtained in experiments[78] as well as deviations as a function of the length of the wire[40] and can be related to the Luttinger liquid exponent. The correction to the conductance due to impurities[79–81] is of the form

$$G(T) = \frac{e^2}{h} - gT^{-\nu}, \tag{85}$$

where $\nu = 1 - K_\rho$ is the conductivity exponent.[82] The derivation is similar to the derivation of the T dependence of conductivity in Appendix B. For finite size systems T can be replaced by the lower cutoff v_F/L in Eq. (85). This formula only holds at high enough temperatures or for systems of length L shorter than the localization length for which the corrections term is small.

If two channels are present in the wire, the system becomes then equivalent to a ladder system. Two bands present at the Fermi level are the equivalent of the bonding and antibonding bands of the ladder system. One then expects that the whole physics derived in Sec. III C should apply to these wires. In particular, since one expects reasonably repulsive interactions one should be in the SC^d of the CDW^{4k_F} phase. Going from a single chain to the ladder should have observable consequences on the transport properties. First since the localization length increases drastically in the ladder system one would expect the conductance corrections due to disorder to be much weaker for two channels. This of course assumes that the typical interactions do not vary too

TABLE III. The conductivities and localization lengths in the spinless fermions case. The phases are the ones of the nondisordered system that are turned into localized ones upon introduction of a small disorder.

Phase	$L_{\mathrm{loc.}}$	$\sigma(T)$
PCDW^{4k_F}	$\left(\frac{1}{D_a}\right)^{[2/(3-4K_{\rho+})]}$	$T^{2-4K_{\rho+}}$
PCDW$^{\pi}$	$\left(\frac{1}{D_a}\right)^{[1/(3-K_{\rho+})]}$	$T^{2-4K_{\rho+}}$

TABLE IV. The conductivities and localization lengths in the fermions with spin case. The phases are the ones of the nondisordered system that are turned into localized ones upon introduction of a small disorder.

Phase	$L_{\mathrm{loc.}}$	$\sigma(T)$
OAF, SC^{d}SC^s	$\left(\frac{1}{D_a}\right)^{[2/(3-2K_{\rho+})]}$	$T^{2-2K_{\rho+}}$
CDW$^{\pi}$	$\left(\frac{1}{D_a}\right)^{[2/(6-K_{\rho+})]}$	$T^{2-(K_{\rho+}/2)}$

much when going from one channel to two channel, a fact which is not certain. Second, by doing the expansion of the corrections to conductance for the ladder and using the conductivity exponent (Sec. III C), one would obtain $\nu=2-2K_{\rho+}$. A fit of the temperature dependence of the conductance in Ref. 41, could allow to extract the Luttinger liquid exponents for the ladder system, as well as check the above predictions.[83]

VI. CONCLUSION

In this paper we have examined the effects of disorder on a two-legs ladder system, using RG techniques. We have computed the effects of disorder on the phase diagram as well as the localization length. Disorder has drastic effects on the phase diagram. For spinless fermions, it leads to an extremely strong localization of the charge density wave phase that exists for repulsive interactions. Such localization is even stronger than for a single chain. On the other hand, for the ladder system there is a remarkable stability of the s-wave superconducting phase (for attractive interactions), compared to the single-chain case. The insulator-superconductor transition occurs in the vicinity of the noninteracting point for a pure t-V model whereas in the one-chain system it occurs for strongly attractive interactions.

For fermions with spin, the repulsive part of the phase diagram is also strongly localized by disorder. In particular, the d-wave superconducting phase found for ladder systems is completely suppressed by an arbitrarily small amount of disorder. We emphasize that this is not *only* a pair breaking effect but a much stronger Anderson localization effect. On the other hand, the s-wave superconducting phase occuring for attractive interactions is again much more stable to disorder than its one-chain counterpart.

Besides obtaining the phase diagram, we have also investigated the transport properties of the t-V and Hubbard two-chain systems. The RG enabled us to compute the localization length and the charge stiffness as a function of disorder (see Tables III, IV) and the temperature and frequency dependence of the conductivity. Various remarkable fact emerged. First, the behaviors of the spinless ladder and the ladder with spins are very different. In particular, the spinless ladder shows the same tendency as the single spinless chain, namely, that attractive interactions decrease localization whereas repulsive interactions enhance it. In the two-chain case, that effect is even stronger. For attractive interactions there is *no localization*, whereas for repulsive ones the system is much more localized than its one-chain counterpart (the *exponent* in the dependence of localization length with disorder is changed). On the other hand, for the Hubbard ladder there is no such effect: up to a prefactor, the localization length with attractive interaction is the same as for repulsive ones. As a consequence, the corrections to conductivity stiffness are the same for attractive and for repulsive interactions.

The temperature dependence of the conductivity follows a power law of the form $\sigma(T)\sim T^{2-2K_{\rho+}}$, for temperatures above the localization scale T_{loc}, where K_{ρ}. For the repulsive side, where for the pure system one would have the d-wave superconducting phase, $K_{\rho+}<1$, and thus the conductivity *decreases* as a function of the temperature even well above T_{loc}. The transport thus shows *no remnant* of the superconducting behavior one could have naively expected when looking at the pure system. This remarkable fact illustrates that transport is in fact controlled by the density fluctuations of the system and *not* by the existence of slowly decreasing superconducting correlation functions. The ladder system provides evidence of a phase that is genuinely a d-wave superconductor as far as phase diagram is concerned, but would from the transport point of view be closer to an insulator. Of course such an interesting behavior would clearly deserve more studying. In particular, it would be interesting to know how the correlation between the density fluctuations and the superconducting one evolves as the number of chain is increased, and how the crossover to the three-dimensional situation occurs. Such a study goes of course far beyond the goals of the present paper.

We have applied our results to two types of experimental systems. First, our results should be relevant for quantum wires with two channels. Here the prediction for the exponent in the conductivity can be directly checked by measuring the temperature dependence of the conductance of the system. Note that the conductivity/conductance exponent $2-2K_{\rho+}$ for the ladder systems is different from the one for a single channel (or a single chain) $1-K$. Due to the increase of the localization length when going from one channel to two channels one would also expect overall smaller corrections to the conductance for a given strength of the disorder, and roughly constant interactions. Investigation of systems with more than two chains would be useful in order to get a better understanding of the role of internal symmetries and gaps on the transport properties of quasi-one-dimensional systems. This is of course also useful in connection with experiments on quantum wires. In particular, we expect that the behavior of systems with an even number of legs is dominated by gap formation whereas the behavior of

systems with an odd number of legs should be closer to the one of a one-chain system.

The other experimental system on which our results could be applied are of course the coupled bichains where superconductivity has recently been obtained[75] under pressure. To compare with this system of coupled ladders, it was necessary to treat the coupling between different ladders, which we did using a mean-field approximation. The results indicate that even in the presence of coupling between ladders the d-wave phase is still much too sensitive to disorder to be the one experimentally observed. In addition, the observed temperature dependence of the conductivity would be incompatible with the one computed here should these systems be dominated by one-dimensional (ladder) physics. These observations and the fact that the conductivity occurs under very large pressure tends to indicate that the mechanism for superconductivity in these systems is very likely to be of a two- or three-dimensional nature and not just the mere stabilization of the ladder superconducting phase. On the other hand, the system without pressure has a resistivity that could be more compatible with the localization effects described here. Of course one interesting question would be whether one can get a one (ladder) to higher dimensional crossover as the pressure is applied. This of course could only be decided by a more quantitative comparison with experiments as well as further theoretical and experimental work. Adding additional impurities, for example by irradiation, could allow us to distinguish if the system is in a one-dimensional regime, since one expects much more drastic localization effects in that case.

Finally, a ladder system with spins exhibits an extremely interesting orbital antiferromagnetic phase. Although such a phase cannot occur in a pure Hubbard system it can in principle be stabilized if some nearest-neighbor attraction is added. Although such a phase has *no* superconducting order parameter, it has perfect conductivity in the presence of a random potential. Moreover that perfect conductivity is also robust in the presence of random hopping both along the chains and perpendicular to the chains. As far as transport is concerned this phase is therefore a one-dimensional ''superconductor.'' Nevertheless, it has only subdominant (in the spinless case) or exponentially decaying (in the case of fermions with spin) superconducting correlations, again an illustration that looking at the superconducting fluctuations is *not* a good criterion to determine the transport properties. Due to the peculiar nature of this phase it would be interesting to check whether it survives in ladder systems with more than two legs. More generally it also deserves further investigation in dimensions higher than one, both in relation of flux phases of two-dimensional systems and other orbital phases proposed for the normal state of cuprate superconductors.[72]

The study of the disorder effects could also be extended in various directions. In particular, a more detailed description of the physics inside the localized phase would be suitable. However such a description is beyond the reach of the simple RG calculation. Going to strong but diluted disorder is also a challenging problem. In particular, understanding the crossover from the results of our paper to the limit where disorder suppresses gaps altogether in the system remains yet to be done.

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APPENDIX A: BOSONIZATION TECHNIQUE

In this section, we will give a short review of the bosonization technique in order to fix the notations. We give the expressions for a single chain of spinless fermions. For more species of fermions, one can bosonize each specie individually, and the corresponding expressions are given in the text.

1. Representation of fermion operators in terms of boson ones

Noninteracting one-dimensional spinless fermions on a lattice are described by the kinetic energy,

$$H=-t\sum_{n=1}^{N}(c_{n+1}^{\dagger}c_n+c_n^{\dagger}c_{n+1})=\sum_k \epsilon(k)c_k^{\dagger}c_k, \quad \text{(A1)}$$

where $\epsilon(k)=-2t\cos k$ and $c_n=(1/\sqrt{N})\Sigma_k e^{\imath k r_n}c_k$.

To obtain the asymptotic (low energy, long wavelength) properties of the system one can linearize the spectrum near the Fermi ''surface'' ($\pm k_F$) and take the continuum limit by introducing $\psi(x)=(c_n/\sqrt{a})$ with a the lattice spacing and $x=na$. With our definition the ψ's have the commutation relations of continuum fermion operators. We define the R (respectively L) (right and left movers) fermions as fermions with momentum close to $+k_F$ (respectively, $-k_F$) as

$$\psi_R(x)=\frac{1}{\sqrt{Na}}\sum_{|k|<\Lambda} e^{\imath kx}c_{k_F+k}, \quad \text{(A2)}$$

and similarly for $\psi_L(x)$ with $k_F\rightarrow -k_F$. Λ is a cutoff needed not to double count fermion states, and imposed by the linearization of the dispersion relation. All asymptotic properties can be expressed in term of $\psi_{R,L}$. In particular, the full fermion operator becomes $\psi(x)=e^{\imath k_F x}\psi_R(x)+e^{-\imath k_F x}\psi_L(x)$. The Hamiltonian (A1) becomes

$$H=-iv_F(\psi_R^{\dagger}\partial_x\psi_R-\psi_L^{\dagger}\partial_x\psi_L), \quad \text{(A3)}$$

with $v_F=2ta\sin(k_F a)$.

Due to the separation into two branch of fermions and the linearization of the spectrum, the Fourier components of the fermion density operators

$$\rho_{R,L}(q)=\sum_k \psi_{R,L,k+q}^{\dagger}\psi_{R,k} \quad \text{(A4)}$$

have boson commutation relations,[2,4,3]

$$[\rho_R(q),\rho_R(-q')]=-\frac{L}{2\pi}q\delta_{q,q'},$$

$$[\rho_L(q),\rho_L(-q')]=\frac{L}{2\pi}q\delta_{q,q'},$$

$$[\rho_L(q),\rho_R(-q')]=0. \quad \text{(A5)}$$

This allows us to rewrite (A3) as

$$H=\pi v_F \int dx[\rho_R(x)^2+\rho_L(x)^2], \tag{A6}$$

with $\rho_s(x)=\psi_s^\dagger(x)\psi_s(x)$ for $s=L,R$. Instead of using the density operators themselves it is more convenient to introduce

$$\Pi(x)=\rho_R-\rho_L,$$

$$\frac{-1}{\pi}\partial_x\phi=(\rho_R+\rho_L). \tag{A7}$$

Physically, Π is a momentum density while $\partial_x\phi$ is proportional to the deviation of the fermion density from its average value. The commutation relations for the ρ's imply that $[\phi(x),\Pi(x')]=\imath\delta(x-x')$. Also, the Hamiltonian rewritten in terms of Π and gf is

$$H=\int dx\frac{v_F}{2\pi}[(\pi\Pi)^2+(\partial_x\phi)^2], \tag{A8}$$

which is just the continuum limit of the Hamiltonian of a one-dimensional harmonic chain. Note that the following procedure could have been applied to a more complicated lattice Hamiltonian than Eq. (A1). All that is needed is that the Fermi surface reduces to two points. The effectiveness of bosonization stems from the fact that it is possible to express the fermions operators in terms of $\Pi(x)$ and $\phi(x)$. If one introduces $\theta(x)=\pi\int_{-\infty}^{x}\Pi(x')dx'$, one has the following relations:

$$\psi_R(x)=\frac{1}{\sqrt{2\pi\alpha}}e^{\imath[\theta(x)-\phi(x)]}U_R,$$

$$\psi_L(x)=\frac{1}{\sqrt{2\pi\alpha}}e^{\imath[\theta(x)+\phi(x)]}U_L, \tag{A9}$$

α being a cutoff, the presence of which is imposed by the cutoff needed in the linearization of the dispersion relations. The U_R and U_L are anticommuting operators introduced by Haldane that annihilate one fermion at the Fermi level. These operators also anticommute with their Hermitian conjugates. It can be verified explicitly that those relations reproduce correctly the commutators of fermion operators. These U operators give, in general, corrections vanishing in the thermodynamic limit and can be safely dropped. On the other hand, if there are different species of fermions (such as up and down spin fermions or band degeneracies), one must bosonize separately each fermion specie using the formulas for spinless fermions. It is needed to introduce $U_{L,n},U_{R,n}$ operators and their complex conjugates (n indexing the internal degrees of freedom such as spin) to enforce proper fermions anticommutation relations. In order to make that bookkeeping less tedious,[84] one can introduce η operators such that

$$\eta_\alpha\eta_\beta+\eta_\beta\eta_\alpha=2\delta_{\alpha,\beta},$$

$$\eta_\alpha^\dagger=\eta_\alpha. \tag{A10}$$

Where $\alpha=(L,n)$ or $\alpha=(R,n)$ these operators can therefore introduce minus signs in the various bosonized expressions.

2. Handling the interactions with bosonization

Let us consider spinless fermions. Interactions can then be handled straightforwardly: If one adds a density coupling of the form $\int dx\ U\rho(x)^2$, the density can be decomposed in a slowly varying part $\rho_R(x)+\rho_L(x)$ and a $2k_F$ part $e^{2\imath k_F x}\psi_L^\dagger(x)\psi_R(x)+\text{H.c.}$.

In the Hamiltonian, one retains only the slowly varying terms (the other term gives a zero value when integrated over x). The $2k_F$ always disappear, while the $4k_F$ can persist in a half filled lattice system.[47] As a consequence, at a noncommensurate filling, the Hamiltonian reduces to

$$H=\int \frac{dx}{2\pi}\left[uK(\pi\Pi)^2+\frac{u}{K}(\partial_x\phi)^2\right], \tag{A11}$$

with $uK=v_F$ as a consequence of Galilean invariance. If one makes the rescaling $\phi\to\phi/\sqrt{K}$ and $\Pi\to\Pi\sqrt{K}$, one has the same Hamiltonian as in Eq. (A8) with the correct commutation relation for ϕ and Π. If one computes the physical correlation functions at 0 K such as the $2k_F$ part of the fermion Green's function $G(x-x',t-t')=-\imath\langle T\psi_R(x,t)\psi_L^\dagger(x',t')\rangle$, it is easily seen that K controls their power law decay while u controls the propagation of excitations. u and K are also related to physical quantities such as the charge stiffness[51] and the compressibility. More specifically, defining the compressibility by $\chi=-1/L(\partial P/\partial L)_T$, $P=-(\partial F/\partial L)_T$ and taking, $T\to 0$ K, we have

$$\chi=\frac{\pi K}{uk_F^2}. \tag{A12}$$

The charge stiffness is defined by $\mathcal{D}=(L/2)[d^2E(\varphi)/d^2\varphi^2]_{\varphi=0}$, φ being a flux threading the system. From that definition, one obtains

$$\mathcal{D}=uK. \tag{A13}$$

The case of fermions with internal degrees of freedom is usually more complicated, because some of the interactions cannot be reduced to $(\partial_x\phi)^2$ terms, the most well known example being the backscattering of two fermions with opposite spins.[3,4] Usually, one finds sine-Gordon Hamiltonians of the form

$$H_{\text{SG}}=\int \frac{dx}{2\pi}\left[uK(\pi\Pi)^2+\frac{u}{K}(\partial_x\phi)^2\right]+\Delta\int dx\ \cos(\beta\phi). \tag{A14}$$

These Hamiltonians can be studied using RG techniques.[4,85] The flow equations for K and Δ are of the Kosterlitz-Thouless form.[86,87] Δ has scaling dimension $2-\beta^2K/4$. Therefore a small Δ is relevant for $K<8/\beta^2$. From the RG equation for Δ one sees that there are two regimes: one small K or large enough Δ regime, where Δ is relevant and a large K, small enough Δ regime where Δ is irrelevant. When Δ is irrelevant, the correlation functions keep their power law character up to logarithmic corrections.[85] On the other hand, if Δ is relevant, ϕ will acquire a nonzero expectation value

that minimizes the ground state energy and a gap will be generated. It can then be shown[4] that there $\langle f(\phi)\rangle \sim f(\langle\phi\rangle)$ and that $\langle T_\tau e^{\imath\alpha\theta(x,\tau)} e^{-\imath\alpha\theta(0,0)}\rangle \sim \exp(\{[-\sqrt{x^2+(u\tau)^2}]/\xi\})$, where ξ is a correlation length. These results are used extensively in the paper.

APPENDIX B: MEMORY FUNCTION CALCULATION OF ac AND dc CONDUCTIVITY

For the sake of clarity, we will explain the technique in the example of one chain of spinless fermions (technically this is the simplest case), and then explain how the calculation can be extended to more complicated cases. First let us describe the memory function approximation.[88] The conductivity is given by linear response theory as

$$\sigma(\omega) = -\imath \frac{\chi(0)-\chi(\omega)}{\omega}, \tag{B1}$$

where $\chi(\omega)$ is the current-current response function.[88,89] The memory function $M(\omega)$ is defined by

$$\sigma(\omega) = \frac{-\imath\chi(0)}{\omega + M(\omega)}. \tag{B2}$$

This gives the exact formula:[88]

$$M(\omega) = \frac{\omega\chi(\omega)}{\chi(0)-\chi(\omega)}, \tag{B3}$$

an expansion[88] at high frequency and the small impurity concentration gives

$$M(\omega) = \frac{(\langle\langle F;F\rangle\rangle_\omega - \langle\langle F;F\rangle\rangle_{\omega=0})/\omega}{-\chi(0)}, \tag{B4}$$

where $\langle\langle ;\rangle\rangle$ is a retarded correlator evaluated for the pure system and $F=[J,H]$, J being the total current. To use that formalism in the framework of bosonization, we first need an expression for the current.[89] This can be obtained from the definition of the fermion density $\rho(x)=\rho_0-(\partial_x\phi/\pi)$, and the current conservation equation: $\partial_t\rho+\partial_x j=0$. One obtains $j(x)=(\partial_t\phi/\pi)$. Using the Heisenberg equation of motion for ϕ and noting that the total current $J=\int dx\ j(x)$, one finds $J=uK\int \Pi(x)dx$. The coupling to disorder being

$$H_{\rm imp} = \int dx \frac{\xi(x)}{2\pi\alpha} e^{\imath 2\phi(x)} + \text{H.c.} \tag{B5}$$

We get $F\propto\int[\xi(x)/2\pi\alpha]e^{\imath 2\phi(x)} - [\xi^*(x)/2\pi\alpha]e^{-\imath 2\phi(x)}$.

This gives

$$\overline{\langle T_\tau F(\tau)F(0)\rangle} \propto \int dx\ D\delta(x)\left(\frac{1}{x^2+(u\tau)^2}\right)^K \propto \tau^{-2K}. \tag{B6}$$

Therefore, $\langle\langle F;F\rangle\rangle_\omega \propto \int d\tau\ e^{\imath\omega\tau}\tau^{-2K} \propto \omega^{2K-1}$. This gives $M(\omega)\propto\omega^{2K-2}$ and for $K>3/2\sigma(\omega)\propto\omega^{2-2K}$. The formula we have obtained is valid only at high frequency. We can get from it a high-temperature formula by using the dimensional equivalence of temperature and frequency (e.g., $\hbar\omega\sim k_B T$). To generalize the calculation to a more complicated case, we must first note that in the formula for the current, ϕ will be replaced by ϕ_ρ in the case of two chains of spinless fermions and $\phi_{\rho+}$ in the case of two chains of fermions with spin. The coupling to disorder being some $\int dx\ \xi(x)e^{\imath n\phi}+\text{H.c.}$, n depending on the problem at hand, we see that in the general case we will just have to make the replacement $2K\to(n^2/2)K$ in the formulas giving $\sigma(\omega),\sigma(T)$.

APPENDIX C: EFFECTIVE RANDOM POTENTIAL IN THE PRESENCE OF GAPS

In that section, we will give a derivation of the RG equation for D_a at $g_f>0$. We start with the method of[36] computing perturbatively the correlation function: $\langle T_\tau e^{\imath\sqrt{2}\phi_\rho(x_1,\tau_1)} e^{-\imath\sqrt{2}\phi_\rho(x_2,\tau_2)}\rangle$. In second order in the random potential, since $\langle T_\tau \sin(\sqrt{2}\phi_\parallel)(x,\tau)\sin(\sqrt{2}\phi_\parallel)(0,0)\rangle \sim e^{-r/l}$, there is no singular contribution. Therefore, we must go to fourth order. We will drop the combinatorics since we are only interested in the renormalization of D. The fourth order term is of the following form:

$$\begin{aligned} D_a^2 \int & \frac{dx_1 d\tau_1 dx_2 d\tau_2 dx_3 d\tau_3 dx_4 d\tau_4}{(\pi\alpha)^4} [\delta(x_1-x_4)\delta(x_2-x_3)+\delta(x_1-x_2)\delta(x_3-x_4)] \\ & \times\langle T_\tau e^{\imath\sqrt{2}[\phi_\rho(x,\tau)+\phi_\rho(x_1,\tau_1)+\phi_\rho(x_3,\tau_3)-\phi_\rho(x_2,\tau_2)-\phi_\rho(x_4,\tau_4)-\phi_\rho(0,0)]}\rangle \\ & \times\langle T_\tau \sin(\sqrt{2}\phi_\parallel)(x_1,\tau_1)\sin(\sqrt{2}\phi_\parallel)(x_2,\tau_2)\sin(\sqrt{2}\phi_\parallel)(x_3,\tau_3)\sin(\sqrt{2}\phi_\parallel)(x_4,\tau_4)\rangle. \end{aligned} \tag{C1}$$

The $\phi_\parallel$ will be exponentially small except when $|r_1-r_3|\ll l$ and $|r_2-r_4|\ll l$ or $|r_1-r_2|\ll l$ and $|r_3-r_4|\ll l$ (the other cases are equivalent to these two ones up to a relabeling of dummy integration variables). It is easily seen that the second case is in fact trivial. Therefore, the only interesting contribution comes from the first term. This term reduces to the simple form

$$\begin{aligned} & D_a^2 l^2 C \int dx_1 d\tau_1 dx_2 d\tau_2 \delta(x_1-x_2) \\ & \times\langle T_\tau e^{\imath\sqrt{2}[\phi_\rho(x,\tau)+2\phi_\rho(x_1,\tau_1)-2\phi_\rho(x_2,\tau_2)-\phi_\rho(0,0)]}\rangle, \end{aligned} \tag{C2}$$

where C is a constant that depends on the regularization of the continuum model. It can be seen that the term that we

obtain can be generated by the following effective coupling:

$$H_{\text{effective}}=\int dx\,\xi_{\text{eff.}}(x)e^{\imath\sqrt{8}\phi_\rho}+\text{H.c.}, \tag{C3}$$

with $\overline{\xi_{\text{eff.}}(x)\xi_{\text{eff.}}(x')}=D\delta(x-x')$ and $D\propto CD_a^2$. It is clear that for couplings of the form $e^{\imath\phi_\rho}\cos(\theta_\parallel)$, the same argumentation will be equally valid. Note that using a self-consistent harmonic approximation (SCHA) approximation gives different results; This is due to the fact that normal ordering in SCHA is done without taking the presence of the gaps into account. Therefore standard scaling, irrespective of the presence of the gaps, always holds when one uses SCHA.

APPENDIX D: THE ORBITAL ANTIFERROMAGNET IN THE PRESENCE OF RANDOM INTRACHAIN HOPPING AND RANDOM INTERCHAIN HOPPING

We consider the following two types of random hopping: a random hopping along the chains

$$H_{\text{intrachain}}=\sum_{i,\sigma}\,[\,\delta t_i^1(c_{i+1,\sigma,1}^\dagger c_{i,\sigma,1}+c_{i,\sigma,1}^\dagger c_{i+1,\sigma,1}) + \delta t_i^2(c_{i+1,\sigma,2}^\dagger c_{i,\sigma,2}+c_{i,\sigma,2}^\dagger c_{i+1,\sigma,2})], \tag{D1}$$

and a random interchain hopping *amplitude*,

$$H_{\text{interchain}}=\sum_{i,\sigma}\,\delta t_{\perp,i}(c_{i,1}^\dagger c_{i,2}+c_{i,2}^\dagger c_{i,1}), \tag{D2}$$

where $\delta t_\perp$ is *real*. Bosonization of (D1) leads to an expression *identical* to the one that obtains by bosonizing a random on-site potential. It is then evident that the transport properties of the orbital antiferromagnet are the same in the presence of a random potential or random hopping along the chains. Bosonization of equation (D2) gives the following expression:

$$H_{\text{interchain}}=\int\,\frac{2dx}{\pi\alpha}t_\perp^{2k_F}(x)e^{-\imath\phi_{\rho+}} \times[\imath\,\sin\,\phi_{\rho+}\cos\phi_{\sigma-}\,\cos\phi_{\sigma+} + \cos\phi_{\rho-}\,\sin\phi_{\sigma-}\,\sin\phi_{\sigma+}]+\text{H.c.} \tag{D3}$$

It is not difficult to see that such a term has exponentially decaying correlations since $\theta_{\rho-}$ develops a gap. Integration of the massive ρ mode leads to a coupling that is identical to the coupling to a random potential. Therefore a random amplitude of the hopping term also does not affect the transport properties of the OAF more severely than a random potential and thus the ''superconducting'' transport properties of the OAF are not an artifact of restricting to random potentials. All physically admissible random perturbations of the two-chain system lead to the same limit for localization delocalization ($K_{\rho+}=3/2$), the same behavior for conductivity as a function of frequency and temperature, and the same dependence of localization length as a function of disorder.

On the other hand, if the random hopping term has a random *phase*, there is a direct coupling to the OAF order parameter and then the OAF phase is suppressed. Such terms are allowed, for instance, in a tight binding picture only if the phases on the atoms of the two-chain system cannot be made real. This could be achieved with a random magnetic flux in each plaquette of the two-chain system.

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