Envelope of Weiss oscillations and the role of disorder in surface superlattices

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We study the magnitude of the Weiss magnetoresistance commensurability oscillations in surface superlattices, in samples with a long sequence of such features. The high index oscillations are suppressed faster than the conventional exponential term—apparently by a term of the form $\exp\{-(B_0/B)^3\}$. We discuss the role of small-angle scattering in the suppression of these oscillations and suggest a heuristic derivation of this envelope function. [S0163-1829(97)02235-2]

It is well established that when a surface superlattice (SSL), namely, a periodically modulated two-dimensional electron gas (2DEG), is placed in a magnetic field B, the resistance oscillates with field.[1–6] These magnetoresistance commensurability oscillations (MCO's), also known as Weiss oscillations, are periodic in $1/B$ and reflect the recurring commensurability between a, the underlying period of the SSL, and the cyclotron orbit diameter $2R_c$, as the field is varied. Here $R_c=\hbar k_F/eB$ with k_F the Fermi wave vector.

An explicit form for the magnetoresistance was derived by Beenakker[3] using a classical Boltzmann equation; in the case of a weak sinusoidal SSL potential $V(x)=V_0\sin(2\pi x/a)$, and in the limits of low field and weak scattering, it reduces to

$$\frac{\Delta\rho_{xx}}{\rho}=A\times B\cos^2\left(\frac{2\pi R_c}{a}-\frac{\pi}{4}\right), \tag{1}$$

with

$$A=\frac{(2V_0\tau)^2}{(m\nu_F)^3}\,\frac{e^3}{a}, \tag{1a}$$

where $\rho=m/ne^2\tau$ is the zero-field resistivity, ν_F the Fermi velocity, and τ the momentum relaxation time. The classical physics behind this effect was explained by Beenakker as resulting from a lateral drift of the cyclotron orbit center under the influence of the SSL potential. An essentially identical result has been derived from a quantum-mechanical picture as well, where the effect is understood in terms of Landau bands broadened by the SSL potential.[1,2,4,5]

Aside from correctly predicting the position of the MCO peaks, Eq. (1) suggests their amplitude to be linear in B and quadratic in the strength of the periodic potential, V_0. In fact many workers have used Eq. (1) to evaluate V_0. However, the linearity in B is quite far from experimental reality; this is true especially for the higher index peaks, namely at lower B, where the peaks are found to be substantially suppressed with respect to the above prediction. This in itself may come as little surprise, since Eq. (1) is obtained under the assumption that the cyclotron frequency ω_c is $\gg\tau^{-1}$, which is not entirely justified under typical experimental conditions. Indeed this discrepancy has been noted in several works,[7–9] but a systematic study of the amplitude has not yet been put forward.

In this paper we analyze the amplitude of MCO's in samples which, through the combination of geometric and material properties, show a particularly long sequence of such peaks. The high index peaks appear to be exponentially suppressed, though as many as twenty peaks can be found. We propose an analysis of the suppression of the MCO's in terms of a finite scattering rate. Interestingly, this suppression cannot be described by a simple exponential law, as we will soon show. We discuss several approaches to understanding this suppression; however, these leave important questions open, and a satisfactory quantitative theory remains to be developed.

The samples consisted of a 2DEG with density $n=3.6\times10^{11}$ cm^{-2} and mobility $\mu=6\times10^5$ cm^2/V s, located some 70 nm below the surface, patterned into conventional Hall bars with evaporated Ohmic contacts. Gratings of the 100-nm period were formed on the Hall bars by electron beam lithography and shallow, low-energy plasma etching. After etching an additional ''blanket'' gate was deposited over the entire grating. The purpose of this gate is threefold: controlling the 2DEG density, protecting from continual oxidation, and smoothening the surface potential. Details of sample preparation and characterization have been described elsewhere.[10] Measurements were carried out in a helium-flow cryostat at temperatures ranging from 1.5 to 30 K, using standard ac lock-in techniques.

In Fig. 1 we show the magnetoresistance of a 100-nm SSL. The oscillations between 0.1 and 0.5 T are the MCO's described above. Note the remarkable number of oscillations, which we attribute to the combination of the short period and the sample quality. The amplitude of the MCO's increases with field, and one can readily see that the increase is faster than the linear dependence of Eq. (1). At higher fields we see the onset of Shubnikov–de Haas (SdH) oscillations as well.

It is neither new nor surprising that Eq. (1) overestimates the MCO's amplitude, since this expression was derived ignoring the finite mean free path of electrons. It has been suggested before[7] that in order to account correctly for the MCO peak height one must include a factor $\exp(-\pi/\omega_c\tau)$ in Eq. (1), namely,

$$\frac{\Delta\rho_{xx}}{\rho}=A\times\exp\left(\frac{-\pi}{\omega_c\tau}\right)\times B\cos^2\left(\frac{2\pi R_c}{a}-\frac{\pi}{4}\right), \tag{2}$$

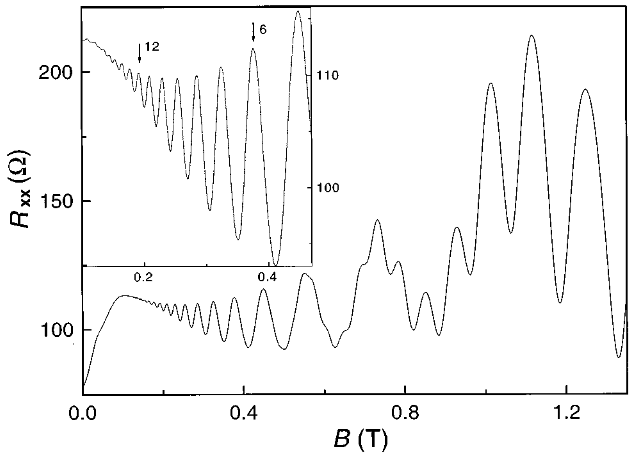

FIG. 1. Magnetoresistance of a 100-nm period SSL at 1.5 K, showing a long sequence of Weiss oscillations in the range of 0.1–0.5 T. Inset: an expanded view of the low-field oscillations, where one can observe that their amplitude grows faster than linearly in B.

which accounts for the fact that due to scattering only a fraction of the electrons actually complete a cyclotron orbit. Furthermore, it was implied that the scattering time that should go into this factor is the single-particle lifetime, τ_s, rather than the transport time, τ_t. In general the latter exceeds the former in a high mobility 2DEG, by up to two orders of magnitude. Indeed, inserting τ_t into Eq. (2) would result in a negligible effect on the MCO, where as with τ_s it is at least in the right ballpark.

However, Fig. 2 shows that a simple exponential factor cannot account for the deviations from Eq. (1). Here, by plotting the amplitude of the MCO's—after dividing by B—on a semilog scale vs $1/B$, we see a substantial deviation from a straight line, which clearly rules out this possibility. This is the case despite the interesting observation that the

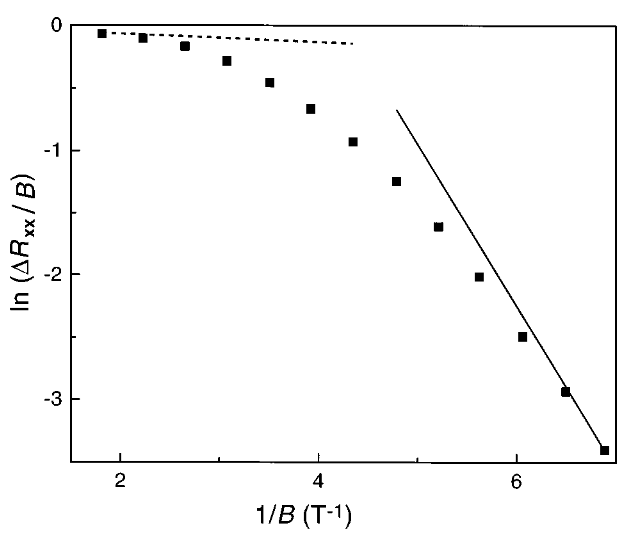

FIG. 2. Amplitude of the Weiss MCO, divided by B and plotted on a semilog scale vs $1/B$. Rather than the expected exponential suppression, $\exp(-\pi/\omega_c\tau)$, which would have led to a straight line, we find that the data falls on a strongly rounded curve. The solid (dashed) line corresponds to the slope on an exponential term with τ_s (τ_t) in the exponent, as explained in the text.

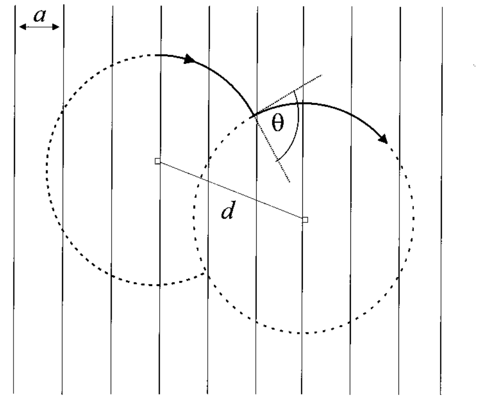

FIG. 3. Elastic scattering of an electron in cyclotron orbit can be pictured as a sudden displacement, $d=2R_c\sin(\theta/2)$, of the center of the orbit. Since the drift velocity in the SSL depends on the x position of the orbit center as well as on its radius, such scattering leads to a sudden change in the drift velocity, $d\sim a$ is a criterion for the importance of scattering. Thus for larger R_c (i.e., smaller B) there is greater sensitivity to small-angle scattering.

asymptotic slope in Fig. 2 at the higher indices ($B\rightarrow 0$), shown by the solid line, corresponds quite closely to $\exp(\pi/\omega_c\tau_s)$, using the value of τ_s determined from SdH. At high fields, however, the slope is much weaker and arguably approaches an exponent with τ_t replacing τ_s (dashed line). We turn first to explain why Eq. (2) in its simple form is unlikely to describe the entire range of MCO's, and later proceed to suggest alternative ways of quantifying the effect of scattering on the amplitude of MCO's.

Since MCO's are essentially a classical effect,[3,6] it is intuitively clear that scattering by infinitesimally small angles should have little influence on this phenomenon. The question is, at what angles an effect does set in, and how much weight to assign to different angles. One can readily see that the larger the cyclotron radius R_c is, the greater the sensitivity to small-angle scattering. This is because scattering of an electron in cyclotron orbit by an angle θ is equivalent to shifting the position of the orbit guiding center by $d=2R_c\sin(\theta/2)$, as depicted schematically in Fig. 3. Now, since the drift velocity of an orbit whose center coordinate is at x is given by $\nu_d(x)=\nu_0\cos(2\pi x/a)$, where $\nu_0=2\pi V_0/aB$, it follows that the change in ν_d can be neglected to the extent that $d\ll a$, namely, we should only be concerned with θ comparable to, or exceeding, $2\arcsin(a/2R_c)$. Thus we have an angular scale which ultimately results from the existence of two length scales in the problem, namely, the SSL period and the cyclotron diameter.

The simplest way to quantify this argument is to define an ''effective'' scattering time $\tau(B)$ for inserting into Eq. (2), which only counts scattering events whose angle exceeds $\theta(B)\equiv\eta\times\arcsin(a/2R_c)$ with η a numerical factor of order unity:

$$\frac{1}{\tau(B)}=\int_{\theta(B)}^{\pi}\sigma(\theta)d\theta. \tag{3}$$

In the limit of small field we get $\tau(B\rightarrow 0)\approx\tau_s$, since

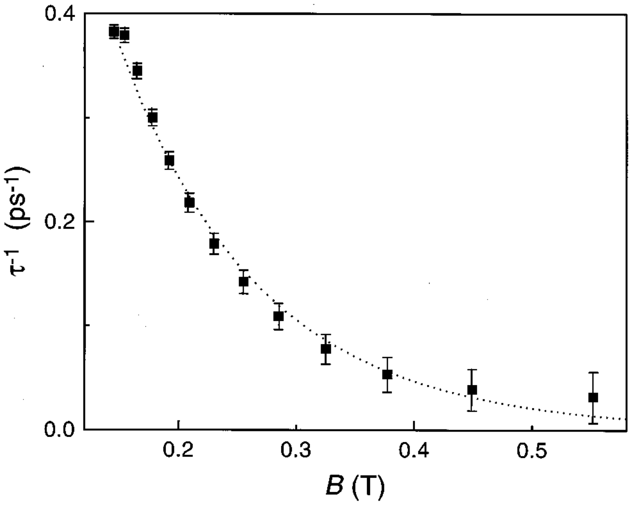

FIG. 4. The effective scattering time vs magnetic field: The symbols correspond to the experimental data, where an ad hoc parameter $\tau(B)$ is obtained from our MCO data via Eq. (2), including an estimated error. The dashed line is the result of Eq. (3), using the theoretical elastic cross section for the 2DEG and the single fitting parameter $\eta=0.6$, as explained in the text.

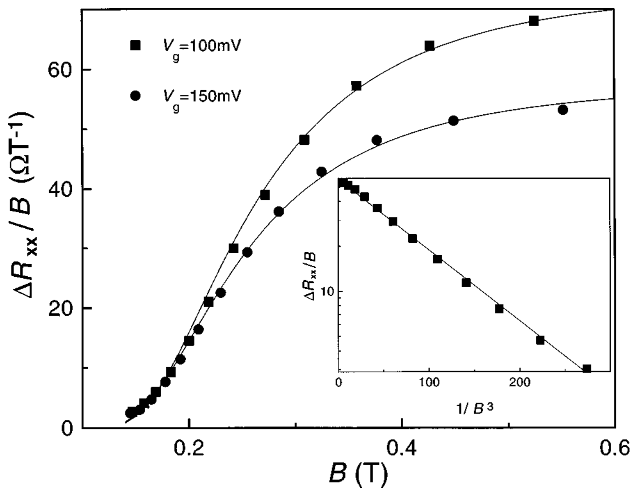

FIG. 5. Comparison between the experimental data and the $\exp(-B^3)$ model discussed in the text, shown as the amplitude of the MCO divided by B. The symbols correspond to data points for two different gate voltages, hence somewhat different ν_F and V_0. Both are fitted to Eq. (6) with the dimensionless fitting parameter $\gamma=0.06$. Inset: the same data on a semilog plot vs $1/B^3$, showing good qualitative agreement with Eq. (6).

$$\frac{1}{\tau_s}=\int_0^{\pi}\sigma(\theta)d\theta. \tag{3a}$$

Here $\sigma(\theta)$ is the elastic-scattering cross section for the 2DEG.[11] The experimental data can be fitted quite well using Eqs. (2) *and* (3): This is seen in Fig. 4, where the symbols correspond to the value of $\tau(B)$ extracted *ad hoc* from the MCO data at each point using Eq. (2), and the dashed curve results from a numerical calculation using Eq. (3) and Ref. 11. The fit yields $\eta\approx 0.6$. This simple model could be improved by replacing the cutoff integral of Eq. (3) by a smooth weight function, which could be derived on more detailed physical grounds; still, the *ad hoc* nature of the exponential term is artificial and highly simplified.

In order to get a more satisfactory explanation we tried a different, though still quite straightforward, approach to quantifying the effect of a high rate of small-angle scattering events, while still building on Beenakker's original analysis. The idea is to consider small-angle events—which can occur many times during a single cyclotron period—as an effect which partially averages $\nu_d(x)$ among nearby orbits, thus essentially diminishing its values.

As already shown, elastic-scattering shifts the position of the guiding center, hence modifies $\nu_d(x)$. These events occur at an average rate given by $1/\tau_s$, and over a cyclotron orbit time of $2\pi/\omega_c$ we have diffusion of the guiding center over a characteristic distance δ_0. We can estimate the latter in terms of a random walk of $N=2\pi/\omega_c\tau_s$ steps of size d, namely,

$$\langle(\delta_0)^2\rangle=(2\pi/\omega_c\tau_s)\times\langle[2R_c\sin(\theta/2)]^2\rangle=(2\pi/\omega_c\tau_t)R_c{}^2 \tag{4}$$

using $\tau_s/\tau_t\equiv\langle 1-\cos\theta\rangle=\langle 2\sin^2(\theta/2)\rangle$, where another factor $\frac{1}{2}$ accounts for taking only the x component of d. The actual diffusion spreading of the guiding center will be given by $\delta=\gamma\delta_0$, with γ a yet unknown numerical prefactor. Hence $\nu_d(x)=\nu_d(0)\times\cos(2\pi x/a)$ is now replaced by $\overline{\nu_d}(x)$, namely, its value averaged over the vicinity of x by a convolution with a Gaussian function $\propto\exp\{-(x'-x)^2/2\delta^2\}$. The convolution of a cosine with a Gaussian can be carried out analytically and readily yields

$$\overline{\nu_d}(x)=\nu_d(x)\exp(-2\pi^2\delta^2/a^2), \tag{5}$$

which in turn can be replaced in Beenakker's derivation of Eq. (1) to obtain

$$\frac{\Delta\rho_{xx}}{\rho}=A\exp\left[-4\pi^3\gamma^2\left(\frac{R_c}{a}\right)^2\frac{1}{\omega_c\tau_t}\right]\times B\cos^2\left(\frac{2\pi R_c}{a}=\frac{\pi}{4}\right). \tag{6}$$

Two comments are in place. The first is that only the transport time, τ_t, and not τ_s, comes into this expression, a highly desirable feature given that this is a classically explained phenomenon. Second, this result implies that the MCO peak height envelope scales like $B\times\exp\{-(B_0/B)^3\}$, where B_0 contains, among other things, the unknown factor γ.

In Fig. 5 we plot the experimental MCO amplitude for two gate voltages, along with a fit to Eq. (6). The inset shows the data on a semilog plot vs B^{-3}, resulting in a straight line, which provides encouraging support to the suggested model. The value of γ derived from the slope is quite small, $\gamma\approx 0.06$. The smallness of γ underlines the limitations of the method of estimating δ_0.

The exact choice of $2\pi/\omega_c$ as the diffusion time, and the use of a single Gaussian to describe this process, are clearly of limited validity at best. Given this weakness of the model it is remarkable that the derived $\exp\{-(B_0/B)^3\}$ agrees so well with experiment. Theoretical studies by several workers are currently under way, focused on deriving a more detailed and rigorous account for our observations, so hopefully a more substantive theory will be available in the near feature.

In conclusion, we have studied the amplitude of the magnetoresistance (Weiss) oscillations in an SSL, in an attempt to reach quantitative understanding of their suppression at low magnetic fields. This suppression is not described by a conventional exponent, and its peculiar nature can be generally understood in terms of the angular dispersion of elastic scattering in a 2DEG. In fact, due to the interplay between the cyclotron resonance and the underlying SSL period, the sensitivity to small-angle scattering increases at lower fields, and this issue is the key to understanding the unusual envelope of the oscillations.

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