Comment on "Intrinsic resistance fluctuations in mesoscopic superconducting wires"

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Experiments on superconducting nanostructures have discovered an interesting resistance anomaly: at temperatures just above superconducting transition the sample resistance exceeds its normal-state value. The recent model proposed to explain this effect [Moshchalkov *et al.*, Phys. Rev. B **49**, R15 412 (1994)] is strongly connected to the small sample size. We believe, however, that this anomaly is due to an extra resistance of the normal-superconducting interface in combination with some geometrical factors. In this Comment we present a simple model to explain experimental results. [S0163-1829(97)02525-3]

We discuss here resistance anomalies observed on superconducting transition curves of narrow aluminum strips. It is a surprising behavior when $R(T)$ curves show a pronounced increase of the sample resistance at the top of superconducting transition.[1–4] The recent model proposed to explain this effect is based on a concept of phase slip centers.[1] However, the approach seems to be wrong. The main assumption of the model is that normal and superconducting currents cannot coexist. It is in obvious contradiction to most of the theoretical papers on this subject (see, for example, Ref. 5 and references therein). The co-existence of currents is one of the main features of phase slip centers and it has nothing to do with spatial distribution of the modulus of the superconducting order parameter as it was supposed in Ref. 1. We do not discuss the model proposed in Ref. 1 in more detail as well as other models[2–4] because, in our opinion, these models have no relation to the experimental effect they have to explain. Instead we present a simple qualitative model, which is based on well-known phenomena. Our model has virtually nothing new, but it can explain rather well all features observed experimentally. We want to mention, also, recent papers[6,7] where a similar resistance anomaly was observed on aluminum strips that were about 50 times wider than in Refs. 1–4. These experiments clearly show that the resistance anomaly is not an intrinsic property of mesoscopic samples.

We strongly believe that the origin of the resistance anomaly observed in Refs. 1–4 is the same as in Refs. 6 and 7. This origin is the normal-superconducting (NS) boundary that enters into the space between potential probes. At temperatures close to superconducting critical temperature T_c the boundary has an electric resistance. The resistance of the NS interface was discovered experimentally in the intermediate state of type-1 superconductors more than 25 years ago.[8] Numerous theoretical studies of this problem have been published[9–17] as well as experimental papers where the interface resistance has been investigated in detail.[10–11,14,16,18–20] The boundary resistance manifests itself in two phenomena: the quasipartical charge imbalance, which gives an exponential decay of the electric field into superconductor (see Ref. 21 for details), and a step of the electric potential Φ just on the boundary. The last term is essential to provide the continuity of the electric current near the interface. The charge imbalance cannot give resistance values higher than in the normal state (it is clearly seen in Fig. 14 of Ref. 21). In this case one could think that only the step of Φ can be responsible for the resistance anomaly. However, the analysis below shows that the actual experimental situation is more complicated and one has to take into account the shape of the NS boundary when it crosses the contact region. It will be shown that in some cases both parts of the boundary resistance contribute to the resistance anomaly.

Electrical resistance of the NS boundary

We consider the "clean limit" when the mean free path of electrons l is much greater than all other characteristics distances because in this way one can show very clearly the origin of the interface resistance. At the end of this section we point out corrections to this simplified picture.

Let us consider a superconducting half-space ($x<0$) in contact with a half-space made of a normal metal with a weak electric current across the boundary plain.[22] At $T \ll T_c$ the energy of current carriers in the normal metal is less than the superconducting energy gap Δ and they cannot penetrate into the superconductor. In this case electrons must be reflected as holes and vice versa (Andreev reflection) and the boundary has no resistance.[23] However, the situation is different close to T_c where the reflection coefficient W is less or even much less than 1 and in this case the NS boundary has an electric resistance.

To calculate this resistance let us consider, first, electric conductivity in the normal metal far from the boundary. In the electric field all electrons have an additional drift velocity V_{dr} along the field. The drift velocity is due to acceleration of electrons by the electric field and can be written as $V_{dr}=eEl/p_F$, where e is the electron charge, E is an electric field, and p_F is a Fermi momentum. The current density $j=en_eV_{dr}$ (n_e is density of electrons).

The situation near the boundary is different. At temperatures close to T_c, where $W \ll 1$, all electrons, which are moving towards the boundary freely penetrate into the superconductor and disappear from our consideration: their drift velocity will be lost somewhere inside the superconductor due to scattering. About the same number of electrons is coming into the normal metal from the superconductor. However, if there is no electric field in the superconductor one would have for these electrons $V_{\mathrm{dr}}=0$. Therefore, only half of the electrons near the boundary have the drift velocity and the current density would be twice smaller than that in the normal metal far from the boundary. Since it is not possible, one has to expect some additional electric field in the boundary region. The solution is the formation of a step of electric potential Φ on the NS boundary.[9,10] The amplitude of this step Φ_{NS} can be easily calculated in the one-dimensional case. The continuity of the electric current gives for $W=0$

$$\Phi_{\mathrm{N-S}}=El.$$

If $W \neq 0$ one has to take into account that the reflection takes place at $x<0$ and reflected quasiparticles pass the potential step twice while quasiparticles coming from the superconductor pass it only once. In this case

$$\Phi_{\mathrm{N-S}}=El\,\frac{1-W}{1+W}. \tag{1}$$

The boundary resistance per unit area

$$\rho_{\mathrm{NS}}=\frac{\Phi_{\mathrm{NS}}}{j}=\frac{\rho_F}{e^2 n_e}\,\frac{1-W}{1+W}, \quad \text{where } \frac{p_F}{e^2 n_e}\approx 10^{-11}\ \Omega\ \mathrm{cm}^2.$$

In the clean limit ρ_{NS} is independent of the mean free path and it is about the same in different metals.

In the three-dimensional case one has to use the values averaged over the Fermi surface and it makes the procedure rather complicated. At the same time it does not change the picture qualitatively. That is why we do not consider this case here. Another reason is that the continuity of the current in the normal metal, discussed above, gives only a part of the interface resistance. Another part is due to transformation of the normal current into supercurrent at $x<0$. This transformation produces the quasiparticle charge imbalance and it gives an exponential decay of the electric field into the superconductor.[10–15] However, the charge imbalance alone cannot provide the continuity of the electric current and the potential step on the NS boundary must exist as well. The shorter is l, the smaller is a part of the total interface resistance, which corresponds to the potential step. The important point is that the absolute value of the potential step also decreases with decreasing l and Eq. (1) gives the upper limit for Φ_{NS} that can be realized only in very pure samples. At the same time a total value of the interface resistance increases due to enhancement of the quasiparticle charge imbalance.

Superconducting transition in narrow strips

The experiments we discuss here have been made on narrow strips made of aluminum film.[24] Samples of this kind are not perfectly uniform and the superconducting critical temperature T_c should be slightly different in different parts of the strip. Direct evidences of this nonuniformity can be found in Ref. 2 where transitions of different parts of the same sample were studied. In this case superconducting transition (in decreasing temperature) should start in one or a few places with highest T_c, and then at lower temperatures superconductivity will spread throughout the sample. For a small part of the strip between potential probes there will be a gradual movement of the NS boundary from one probe to another. Since the boundary has a resistance, its entering into the space between probes increases the voltage. For an idealized sample with infinitely narrow potential probes there is a jumpwise increase of the resistance (Fig. 1). However, in the experiments the probe width was about the same as the width of the strip and one should expect a smooth resistance maximum.

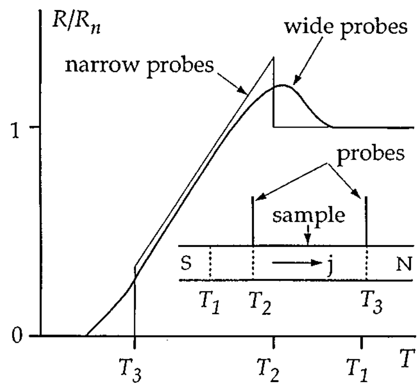

FIG. 1. Schematic dependence of normalized resistance on temperature for infinitely narrow and for wide potential probes. The inset shows positions of the NS boundary for three different temperatures (dashed lines). The superconducting domain grows from the left to the right with decreasing temperature.

The boundary resistance is strongly dependent on the probability of the Andreev reflection. To find the reflection coefficient one has to consider $\Delta(x)$ near $x=0$. In this case spatial variations of Δ are due to variations of T_c along the sample. Using the standard expression for $\Delta(T)$ close to T_c, it can be written as

$$\Delta(x)=\Delta_{T=0}\left\{1-\left[\frac{T}{T_c(x)}\right]^4\right\}^{1/2}.$$

Figure 2 shows $\Delta(x)$ for $dT_c/dx=\mathrm{const}$ at different values of the applied magnetic field. Parameters for this figure have been chosen to be of the order of experimental values: dT_c/dx can be estimated from the width of the superconducting transition assuming that the sample nonuniformity is a main reason for the transition broadening.[25] We used the data of Ref. 2 to find dT_c/dx and the value of $\xi(T=0)$ is taken from Ref. 1. One can see that normal excitations have to overcome some distance L_S inside the superconductor before reflection will happen and there is a finite probability to be scatter. In the case of scattering the reflection process cannot be considered as the Andreev reflection. Thus, the scattering inside the superconductor effectively decreases the probability of the Andreev reflection. L_S can be found from a simple relation: $\Delta(L_S)=\varepsilon$, where ε is an energy of a normal

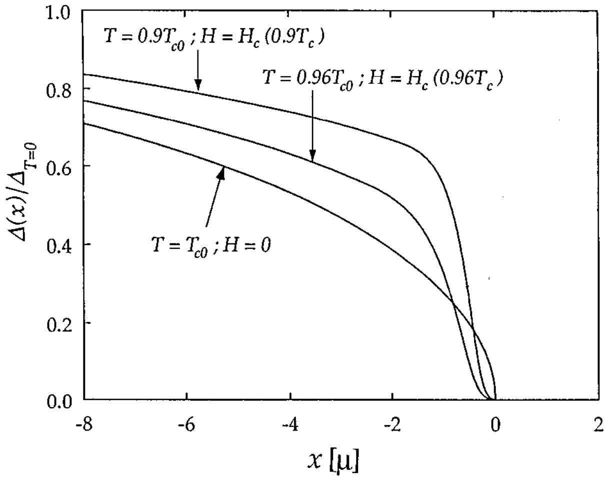

FIG. 2. $\Delta(x)$ for the boundary created by the gradient of T_c with and without the external magnetic field. $T_{c0} \equiv T_c(x=0)$; $\Delta_{T=0}$ is an equilibrium value of Δ at $T=0$. $dT_c/dx = 0.02T_{c0}\mu^{-1}$; $\xi(T=0) = 0.13\mu$.

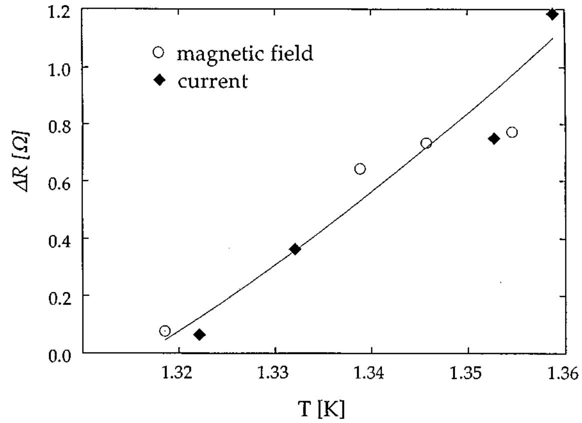

FIG. 4. Amplitude of the resistance anomaly ΔR as a function of the position of the resistance maximum for various values of the current and the magnetic field according to Ref. 4. $\Delta R = R_{\max} - R_n$ where $R_{\max}$ is the maximum value of the sample resistance and R_n is the normal-state resistance.

excitation above the Fermi level. L_S depends on ε and also on the magnetic field because the magnetic field changes the $\Delta(x)$ dependence (see Fig. 2).

As has been already pointed out the charge imbalance gives an exponential decay of the electric field into the superconducting region and in the case of a flat boundary perpendicular to the current it cannot produce resistance values higher than in the normal state. On the other hand, in experiments l is short and in this case the potential step is far too small to explain the magnitude of the experimentally observed resistance anomaly. However, the boundary normal to the current is not necessarily the case in experiments. Deviation of the boundary from the perpendicular direction can make the resistance anomaly significantly greater as well as significantly weaker depending on the sign of this deviation. To illustrate this influence a few different boundaries are shown in Fig. 3.[26] The preferable way for electric current is along the superconductor and it makes the current density nonuniform across the sample. Boundaries A and A' produce the higher current density near the potential probe and the

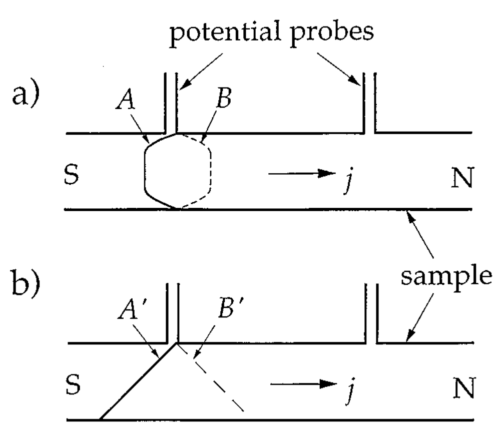

FIG. 3. Possible shapes of the NS boundary. Only symmetrical boundaries are shown. However, one can expect more complicated shapes especially in the contact region.

resistance anomaly should be more pronounced. For boundaries B and B' the current density near the probe is smaller and the anomaly is to be smaller as well. In the latter case the boundary crosses the potential probe when the sample resistance is already less than its normal-state value and the anomaly can be unobservable on $R(T)$ curves. It is difficult to imagine that T_c distribution can be reproducible and, therefore, the resistance anomaly will be different for different samples. This simple conclusion is in agreement with experimental results.[1–4]

We can consider also the influence of the magnetic field and the current density on the effect. An increase of the magnetic field or of the current density should move all of the transition curve to lower temperatures. In this case, in addition to space variations of T_c, we have also depression of the superconductivity by the magnetic field (or current) and it makes $\Delta(x)$ steeper near $x=0$ (Fig. 2). Thus, the reflection coefficient W must be enlarged and the resistance anomaly should be correspondingly decreased. The stronger is a shift of the transition curve to low temperatures the higher is W.[27] Thus, the temperature shift of the resistance maximum can be used as a parameter, which defines the amplitude of the effect independently whether this shift is caused by the magnetic field, by the current, or by their combination. To check this we have plotted the amplitude of the resistance maximum versus its position for different magnetic fields and different currents from Ref. 4 (Fig. 4). Both types of symbols are lying along the same curve in complete agreement with the consideration above.

The quantitative analysis is difficult due to an uncertain experimental situation. The reflection coefficient depends greatly on $\Delta(x)$, however, this dependence is not known and the actual experimental situation can be sufficiently different from $dT_c/dx = \text{const}$ used in Fig. 2. The shape of the boundary when it crosses the contact region is also unpredictable. That is why we have considered only the amplitude of the resistance anomaly. One could see how many parameters define this amplitude and how few of them are known. In this case we do not see any sense in discussing the particular form of $R(T)$ curves. It should be noted that our consideration does not depend on the sample size and, therefore, it is applicable to experiments[6,7] as well.

In conclusion we want to say that the effects discussed above cannot be avoided in experiments. They can explain rather well all features observed experimentally. However, uncertainty of the experimental situation does not permit quantitative analysis of experimental results. It should be noted also that most of theoretical results have been obtained for the superconducting half-space and they cannot be applied to strips directly. At the same time, developing the theory to describe thin and narrow strips is not justified unless the experimental situation is sufficiently improved.

ACKNOWLEDGMENTS

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[22] We assume that the superconducting and normal parts are made of the same metal. Experimentally the NS boundary can be created by an external magnetic field or by a temperature gradient.
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[24] In some experiments small loops were studied. For our consideration a particular shape of the sample is not important.
[25] This estimation gives dT_c/dx along the sample. At the same time variations of T_c in a perpendicular direction should be stronger because films are usually thinner near the edges, and the T_c of aluminum films is strongly dependent on the film thickness.
[26] The boundary conditions for the superconducting order parameter in the case when the boundary is created by the gradient of T_c (or by the temperature gradient in uniform samples): $\Psi_{x=0}=0$ and $\Psi'_{x=0}=\text{const}$. There are no special limitations on the boundary shape and the size of superconducting domains. For the boundary created by the magnetic field $\Psi'_{x=0}=0$ and it makes the formation of small superconducting domains impossible. The difference is clearly seen in Fig. 2.
[27] The magnetic field or the current, if they are strong enough, change also the boundary conditions for Ψ. It is another way in which the magnetic field and the current can affect the resistance anomaly.