Instability and stratification of a two-component Bose-Einstein condensate in a trapped ultracold gas

Eugene P. Bashkin and Alexei V. Vagov
Department of Physics, The University of Western Australia, Nedlands, Perth, Western Australia 6907, Australia
(Received 13 January 1997; revised manuscript received 24 March 1997)

The thermodynamic stability of a trapped Bose gas with a two-component condensate, e.g., a gas of atoms in two distinct hyperfine states or a binary gaseous mixture, is considered on the basis of modified Gross-Pitaevskii equations. Under certain conditions the system becomes unstable with respect to stratification, i.e., to spatial separation between the two condensates. One condensate assumes a donutlike spatial distribution, while the other retains a centered distribution. Possible applications of the theory to recent experimental data on two overlapping condensates in ^{87}Rb are discussed. [S0163-1829(97)05630-0]

The recent observation of Bose-Einstein condensation (BEC) and related phenomena in a system of magnetically trapped ultracold atoms[1–3] rekindled interest in a subject whose basic theoretical framework was developed by Bogolyubov[4] in the late 1940's.

In this paper we will study quite unusual properties of a two-component Bose-Einstein condensate localized in a potential trap. The behavior of a mixture of two Bose superfluids does not reduce at all to a trivial doubling of equations for a one-component system but exhibits a number of qualitatively new features. A general theoretical description of a two-component Bose quantum superfluid was given in several articles (see Refs. 5,6 and references therein). A detailed theory of macroscopic properties of superfluid $H_{\downarrow}$ was derived in 1989,[7] at time when most experimental efforts throughout the world were aimed at achieving BEC in spin-polarized atomic hydrogen $H_{\downarrow}$. The description was based on the fact that under normal conditions gaseous $H_{\downarrow}$ is a mixture of atoms in two distinct hyperfine states (the so-called $|a\rangle$ and $|b\rangle$ states). Inasmuch as the process of longitudinal nuclear magnetic relaxation related to the relativistic dipole interaction is very slow, the system in question can be considered as a two-component mixture of Bose particles with different orientation of nuclear spin, in which the concentrations of both species are conserved over a period less than the corresponding relaxation time. Such a state implies equilibrium momentum and energy distributions whereas the occupation numbers for the individual hyperfine states are definitely out of equilibrium. Under these conditions the system behaves thermodynamically as a binary fluid consisting of two different kinds of particles. Of course interactions within each component as well as interaction between the components play an extremely important role when considering macroscopic properties of the system. A similar approach was applied earlier to ^{3}He systems where atoms with nuclear spin up and down were considered as two different species provided the observation time scale was short enough in comparison to the longitudinal spin-relaxation time. The thermodynamics of such a system in ''partial'' equilibrium appeared to be quite different than usual (for review and references see Refs. 8,9). This was directly proved in experiments on the velocity of sound in a degenerate Fermi gas of ^{3}He atoms carried out by Greywall and Paalanen.[10] The phenomenon of spin diffusion in ^{3}He fluids, which has reliably been observed and measured for more than 30 years (see Ref. 8 and references therein), is based on the fact that ^{3}He atoms with spin up and down form two different and distinguishable components. The self-diffusion between these two components is what one normally calls ''spin diffusion'' in ^{3}He. If transitions between ''up'' and ''down'' states played a crucial role there would be no spin diffusion at all, by definition.

Indeed, the arguments described above fully apply to a supercooled atomic vapor of alkali atoms that can certainly be in different hyperfine states. Despite the fact that the nuclear spin-flip transition rate in a low-density alkali metal vapor is probably somewhat higher than in the case of gaseous atomic hydrogen, the longitudinal relaxation time related to weak processes of relativistic origin is still much longer than the BEC equilibration time determined by direct elastic collisions between the atoms. This circumstance allows us to treat such a system as a two-component dilute solution in which the number of particles in each species remains unchanged for the duration of the experiment. Recently the JILA group reported a successful experiment on the creation of two strongly overlapping rarefied clouds of ^{87}Rb atoms in the different hyperfine states, $|F=1,m=-1\rangle$ and $|F=2,m=2\rangle$, under conditions where both components underwent BEC phase transition.[11]

Another natural objective for experimental study is a real mix of two different gases, i.e., a real binary (or, generally speaking, multicomponent) solution. Experimentally BEC has been achieved so far in three alkali metal vapors, namely, in Rb, Li, and Na (see references above). In theory, at least, this means we already have three combinations that may be studied in experiment.

The physical nature of the phenomena we are interested in is very simple. In a one-component system the criterion of thermodynamic stability reduces to the requirement that the compressibility (or the velocity of sound) should not be negative. In a multicomponent fluid the corresponding criterion is much more complex and is equivalent to a number of inequalities for various thermodynamic derivatives.[12] One can easily convince oneself that a rarefied mixture of two

Bose gases, each of which is thermodynamically stable, can turn out to be unstable with respect to stratification into pure species. Put simply, two such components will always try to separate spatially and stay far away from each other. In a trap, however, such a ''falling apart'' and full separation are impossible simply because both species are confined to the same potential well. The spatial density distribution of both components should now be found by minimizing the free energy of the fluid in the trap. Inasmuch as the instability and related structural transformation occur at temperatures well below the BEC critical point, one ends up with a quite peculiar spatial distribution of the two coexisting condensate fractions which can be directly detected in the trap. As will be seen below the onset of instability depends essentially on the magnitudes of the s-wave scattering lengths, a_{ij}, $i,j=1,2$ for elastic two-body collisions between all particles in the two-component gaseous mixture (i.e., the a_{ij} describe interactions both within and between the components). Regretfully experimental data of sufficient accuracy on the magnitudes of the quantities a_{ij} (particularly of a_{12}) are unavailable at present and a concrete prediction of what mixture should be taken as most suitable for experimental study cannot be made. However, it is worth emphasizing that in the case of two hyperfine components of the same gas even a tiny difference in the s-wave scattering lengths due to relativistic effects may lead to instability and related structural transformation.

We will start with the Hamiltonian of a two-component Bose gas in an external potential of the magnetic trap which has the form:

$$\begin{aligned} H=&\frac{\hbar^2}{2m}\sum_{i=1,2}\int \nabla\Psi_i^+(r)\nabla\Psi_i(r)d^3r \\ &+\frac{1}{2}\sum_{i,j=1,2}\int \Psi_i^+(r)\Psi_j^+(r')U_{ij}(r-r') \\ &\times\Psi_j(r')\Psi_i(r)d^3rd^3r' \\ &+\sum_{i=1,2}\int V_i(r)\Psi_i^+(r)\Psi_i(r)d^3r, \end{aligned} \tag{1}$$

where $\Psi_{1,2}$ are the standard Bose field operators[13] for the first and second components, 1 and 2, respectively, and $V_i, i=1,2$ denotes the external potentials for those components which, in principle, could differ from each other. One can easily see that the Hamiltonian (1) conserves the number of particles in each of the two components.

A two-component system described by the Hamiltonian (1) has two different transition temperatures, T_{c1} and T_{c2},[7] that are determined by the concentrations of the components. If the temperature T is higher than both critical temperatures then we have a normal mixture of two Bose gases and there is no BEC at all. At $T_{c1}<T<T_{c2}$ the system manifests itself as a quantum solution of the normal $|1\rangle$ component in the superfluid background of the $|2\rangle$ component. When lowering the temperature, $T<T_{c1}<T_{c2}$, we finally enter the range where two interacting Bose-Einstein condensates coexist in the fluid. In this paper we will focus on the latter case with two coexisting condensates as this is most amenable to experiment. The theoretical description of a two-component BE condensate is based on the generalized Gross-Pitaevskii equations (GPE):[14,7]

$$i\hbar\dot{\Phi}_i=\left(-\frac{\hbar^2}{2m}\Delta-\mu_i+u_{ii}|\Phi_i|^2+u_{ij}|\Phi_j|^2+V_i\right)\Phi_i, \tag{2}$$

where $i\neq j$, Φ_i is the macroscopic wave function of the ith condensate, μ_i is the chemical potential. The quantities u_{ij} are the coupling constants that in the case of a dilute system can be expressed in terms of the corresponding scattering lengths, $u_{ij}=4\pi\hbar^2a_{ij}/m$. We consider here the case of same atoms in different hyperfine states so that $m_1=m_2=m$ where m is the mass of the atom. Symmetry of the scattering matrix implies $u_{12}=u_{21}$.

From the mathematical point of view the GPE (2) are nonlinear coupled Schrödinger equations. Similar equations are frequently used in optics[15] to describe the so-called ''couplers,'' nonlinear devices where light beams interact with each other. Solutions to the coupled nonlinear Schrödinger equations are known to be very sensitive to the magnitudes of the coupling constants, u_{ij}. As was demonstrated in Ref. 16 for one-dimensional coupled Schrödinger equations, at certain threshold values of u_{ij} solutions of quite different structure and symmetry can emerge. This critical phenomenon is usually referred to as a bifurcation and the threshold magnitudes of the parameters at which it occurs are called the bifurcation points.

The GPE (2) applied to an infinite two-condensate fluid in the absence of an external potential are characterized by a critical surface given by $u_{12}^2=u_{11}u_{22}$. This can easily be demonstrated when considering the spectrum of small excitations about the constant solution to the GPE equations:

$$\mu_i=u_{ii}\rho_i+u_{ij}\rho_j, \quad \rho_i=|\Phi_i|^2. \tag{3}$$

Substituting $\Phi_i=\rho_i^{1/2}+\phi_i$ into Eq. (2) we obtain after linearization:

$$\begin{aligned} i\hbar\dot{\phi}_i=&-\left(\frac{\hbar^2}{2m}\Delta+\mu_i\right)\phi_i+u_{ii}\rho_i(2\phi_i+\overline{\phi_i})+u_{ij}\rho_j\phi_i \\ &+u_{ij}\sqrt{\rho_i\rho_j}(\phi_j+\overline{\phi_j}), \end{aligned} \tag{4}$$

where the bar denotes complex conjugation. The solution to this equation can conveniently be sought in the traditional form $\phi_i=A_i\exp(ikx-i\omega t)+\overline{B_i}\exp(-ikx+i\omega t)$ with constants A_i and B_i to be determined. Substituting this into Eq. (4) one can easily obtain two branches of the excitation spectrum:

$$\begin{aligned} \omega_\pm(k)=&\frac{k}{\sqrt{2m}}\Bigg[\frac{k^2\hbar^2}{2m}+(u_{11}\rho_1+u_{22}\rho_2) \\ &\times\left(1\pm\sqrt{1+4\frac{u_{12}^2-u_{11}u_{22}}{(u_{11}\rho_1+u_{22}\rho_2)^2}\rho_1\rho_2}\right)\Bigg]^{1/2}. \end{aligned} \tag{5}$$

The lower branch of the spectrum, ω_-, becomes imaginary when $u_{12}^2>u_{11}u_{22}$, and the homogeneous solution (3) is no longer stable. It should be pointed out that the opposite inequality, $u_{12}^2<u_{11}u_{22}$, is the standard condition of the ther-

modynamic stability of the two-component system.[12] Thus $u_{12}^2 = u_{11}u_{22}$ is a critical point for the GPE (2) in the absence of an external potential.

When deriving the stability criterion we actually restrict ourselves to considering the low-temperature limit $T \ll T_{c1}, T_{c2}$. In other words, it has implicitly been assumed that the system becomes unstable and stratifies only well below the BEC transition whereas above the transition temperature the fluid is thermodynamically stable. The question might arise whether this assumption leads to a loss of generality of the developed theory. Firstly, one can easily convince oneself that a rarefied two-component fluid is always stable at high enough temperatures. At $T \gg T_{c1}, T_{c2}$ when all particles obey Maxwell-Boltzmann statistics, the statement is obvious as the main contribution to the stability criterion comes from the terms pertinent to an ideal gas. Virial corrections related to the interaction, as well as the quantum-statistical corrections, are always small in comparison to the ideal-gas terms at such high temperatures. Using the results of Ref. 7 one can also verify that the statement remains valid at intermediate temperatures, $T \leqslant T_{ci}$ or $T \geqslant T_{ci}, i = 1,2$. Thus the mixture can lose its stability only at very low temperatures when the temperature-related contribution to the energy becomes smaller than the terms related to the interaction between particles. As can be seen from Ref. 7, if the system is exactly at the point of instability, $u_{12}^2 = u_{11}u_{22}$, (or close to it) at $T = 0$ even a small phonon correction will make the mixture stable at higher temperatures. Secondly, even if the mixture was unstable above the BEC transition this would not render the results obtained in this paper meaningless. The stratified structure found below corresponds to a minimum of the free energy. Therefore such a phase definitely could exist (maybe as a metastable one) no matter what might happen to the system at higher temperatures.

A further issue which may lead to confusion is the following. Ideal gases do not separate. Therefore, the instability in question is related to nonideality of the gas mixture. For that reason the general stability condition for a two-component solution[12] should, in principle, involve the densities of both condensates. The above criterion, however, involves only interaction constants u_{ij} which seems to contradict the general thermodynamic statement. In actual fact, there is no contradiction here. One can demonstrate that in the case of a rarefied gas mixture the instability criterion at $T \to 0$ would involve only the interaction constants. At low densities the free energy F of the Bogolyubov gas reduces to a quadratic form in N_1 and N_2 (similar to the conventional virial expansion). To have a minimum this form should be positive definite, i.e., $\det\| \partial^2 F / \partial N_i \partial N_k \| \geqslant 0$. Inasmuch as F contains only quadratic terms in N_i the corresponding stability criterion involves only density-independent constants in the order of approximation used. Of course, if one was to take into account higher-order corrections in density (that are responsible, e.g., for the interaction-dependent term in the BEC-transition temperature in the Bogolyubov gas), a density-dependent contribution to the instability condition would come into effect. In the approximation used applying the general criterion of thermodynamic stability from Ref. 12 expressed in terms of N_1 and N_2, rather than through the pressure and concentration, yields the above result.

It should be emphasized that most rarefied binary gases are stable. However, there is no law of nature to say that a low-density mixture always has to be stable. There is a number of dilute two-component systems in which instabilities and related phase transformations can exhibit themselves even at high temperatures (see Ref. 9 and references therein). From that point of view this paper may be seen as revealing another example of such an instability at very low temperatures.

Now we will turn to the experimental situation where two condensates are placed in a harmonic external potential of a magnetic trap. For the sake of simplicity we will consider a symmetric trap and assume that the same external potential acts on both components:

$$V_1(r) = V_2(r) = V(r) = \frac{1}{2} m \Omega^2 r^2. \tag{6}$$

The existence of the critical point where the solutions change their character, can most easily be seen in the case where the number density of one of the components (condensates) is small, $N_2 \ll N_1$. Under these circumstances the terms proportional to $|\Phi_2|^2$ in Eqs. (2) can be neglected, which immediately yields

$$i\hbar \dot{\Phi}_i = \left(-\frac{\hbar^2}{2m} \Delta - \mu_i + u_{i1} |\Phi_1|^2 + V(r) \right) \Phi_i . \tag{7}$$

The first equation, $i = 1$, in Eq. (7) is nonlinear and has to be solved numerically or by means of some approximation. It was shown in Ref. 17 that if $8\pi a_{11} N_1 / l \gg 1$, where $l = \sqrt{\hbar / m\Omega}$ is the characteristic length of the harmonic potential, a Thomas-Fermi-like approximation is of good accuracy:

$$|\Phi_1(r)|^2 = \frac{1}{u_{11}} \begin{cases} \mu_1 - V(r), & |r| < r_0 \\ 0, & |r| > r_0 . \end{cases} \tag{8}$$

This solution is quite accurate provided r is not too close to the turning point, $r_0 = \sqrt{2\mu_1 / m\Omega}$. Substituting Eq. (8) into the second equation, $i = 2$, in Eq. (7) we obtain

$$i\hbar \dot{\Phi}_2 = \left(-\frac{\hbar^2}{2m} \Delta - \mu_2 + U(r) \right) \Phi_2 , \tag{9}$$

where

$$U(r) = \begin{cases} \mu_1 \dfrac{u_{21}}{u_{11}} + \left(1 - \dfrac{u_{12}}{u_{11}} \right) V(r), & |r| < r_0 \\ V(r), & |r| > r_0 . \end{cases} \tag{10}$$

If $u_{11} > u_{12}$ the potential $U(r)$ reaches its minimum at $r = 0$. In the opposite case where $u_{11} < u_{12}$, the minimum on the curve $U(r)$ occurs at r_0. This means that the maximum of the stationary solution of the linear equation (9) with lowest energy (the peak of the spatial distribution of the condensate fraction) will be located at the points $r = 0$ and $r = r_0$, respectively. For that reason the second BE condensate is not centered in the middle of the magnetic trap any longer but is displaced towards the peripheral part of the potential well forming a donutlike structure provided $u_{11} < u_{12}$. The exact numerical solution for Eq. (8) confirms this result.

In the more relevant case where both condensates have a substantial number of particles, a linear approximation does not work any more, and one has to seek a solution to the original system of coupled equations (2). As before, we can try a Thomas-Fermi-like approximation as a starting point, which in this case leads to the following equations:

$$u_{ii}|\Phi_i|^2+u_{ij}|\Phi_j|^2=\begin{cases}\mu_i-V(r) & |r|<r_i\\ 0 & |r|>r_i,\end{cases} \tag{11}$$

where $r_i=\sqrt{2\mu_i/m\Omega^2}$. After simple algebra we obtain

$$|\Phi_i|^2=\left\{\left(\frac{15m^{3/2}\Omega^3}{16\sqrt{2}\pi}\right)^{2/5}[u_{jj}(u_{ii}N_i+u_{ij}N_j)^{2/5}-u_{ij}(u_{jj}N_j+u_{ij}N_i)^{2/5}]+V(r)(u_{ij}-u_{ii})\right\}(u_{ii}u_{jj}-u_{ij}^2)^{-1}, \tag{12}$$

where we made use of the conditions

$$N_i=4\pi\int|\Phi_i|^2d^3r, \quad i=1,2. \tag{13}$$

The result is less satisfactory than in the previous case. The reason is that the solution (12) can become negative near the turning points r_i in Eq. (11) as well as in between them. We can still attempt to study the instabilities, however, if we use the fact that the solution (12) is correct in the vicinity of $r=0$. Therefore if one of the condensate densities, $|\Phi_1(0)|^2$ or $|\Phi_2(0)|^2$, is negative one can conclude that there is no stable solution with both densities having their maxima at $r=0$. This conclusion leads to the condition

$$\frac{u_{jj}(u_{ii}N_i+u_{ij}N_j)^{2/5}-u_{ij}(u_{jj}N_j+u_{ij}N_i)^{2/5}}{u_{ii}u_{jj}-u_{ij}^2}\geqslant 0. \tag{14}$$

As can be seen from Eq. (14), $u_{11}u_{22}=u_{12}^2$ is a singular point similar to the one in a uniform bulk two-component fluid. However, a system confined to a trap exhibits another critical point, at which the condition of thermodynamic stability is not violated. To demonstrate this we will consider the two following cases: (a) $N_1>N_2, u_{11}>u_{22}$; (b) $N_1>N_2, u_{11}<u_{22}$.

Without loss of generality we can set $u_{22}=1$. Let quantities u,v, and x be defined as $u_{11}=u, u_{12}=v\sqrt{u}, N_2/N_1=x$. Case (a) then corresponds to $u>1, x<1$ and case (b) is equivalent to $u<1, x<1$. The condition $u_{12}^2<u_{11}u_{22}$ reduces simply to $v<1$. With this notation Eq. (14) reads

$$(u+xv\sqrt{u})^{2/5}>v\sqrt{u}(x+v\sqrt{u})^{2/5},$$
$$u(x+v\sqrt{u})^{2/5}>v\sqrt{u}(u+xv\sqrt{u})^{2/5}. \tag{15}$$

It is not difficult to solve these equations numerically and demonstrate that in both cases, $u<1$ and $u>1$, one of these conditions is broken while the other is not. The same conclusion can also be derived analytically. One can easily convince oneself that the equations obtained from Eq. (15) by equating the left-hand and the right-hand sides [i.e., by considering the equalities rather than the inequalities in Eq. (15)], have a unique real solution $v=1$ when $u=1$. Solving a linearized version of Eq. (15) by setting $v=1+\delta v$ and $u=1+\delta u$ yields

$$\delta v<-\frac{3}{2}\frac{x+1}{3x+7}|\delta u|<0, \tag{16}$$

which proves the above statement.

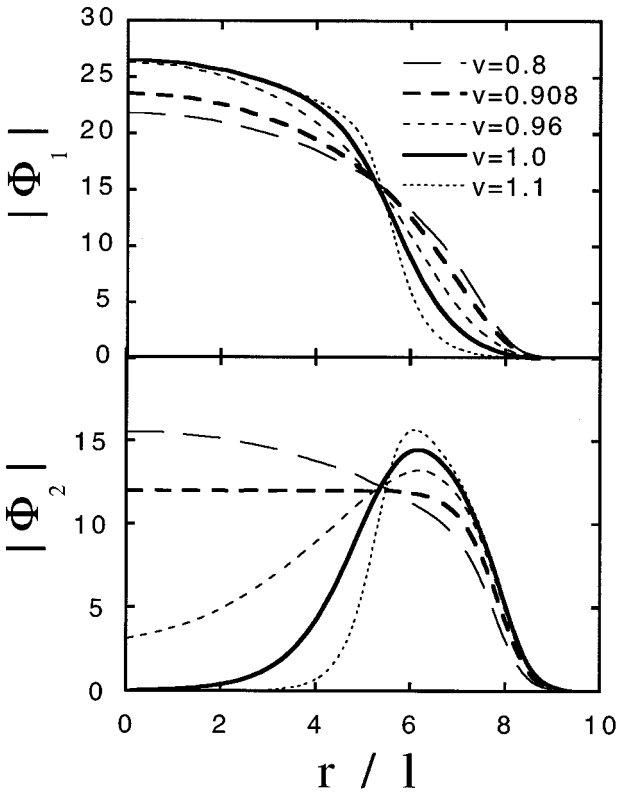

FIG. 1. Macroscopic wave functions, case (a).

To study what the actual solutions of the GPE (2) are, we performed an exact numerical minimization of the Gross-Pitaevskii free-energy functional:

$$F=\int d^3r\left(\frac{\hbar^2}{2m}\sum_{i=1,2}|\nabla\Phi_i|^2+\frac{1}{2}\sum_{i,j=1,2}u_{ij}|\Phi_i|^2|\Phi_j|^2+V(r)\sum_{i=1,2}|\Phi_i|^2\right), \tag{17}$$

provided conditions (13) are fulfilled. To obtain the ''ground'' state we sought spherically symmetric solutions. Parameters entering the functional in Eq. (17) were chosen close to a typical experiment with Rb,[1] $\Omega/2\pi=220$ Hz, $a_{22}=100a_0$ with a_0 the Bohr radii. Also we set $N_1=10^6$ and $N_2=0.710^6$. In case (a) we chose $2u_{11}=u_{22}$, while in case (b) we set $u_{11}=2u_{22}$. The results of the minimization are shown in Figs. 1 and 2, respectively.

As is shown in Fig. 1, the structure of the solutions changes when u_{12} crosses the ''line'' $u_{12}\approx 0.91\sqrt{u_{11}u_{22}}$. When this happens the peak of the second condensate fraction, Φ_2, is shifted from the center of the trap $r=0$ towards $r=r_0$, although the number density at the center remains finite, $\Phi_2(0)\neq 0$. If u_{12} crosses the second critical ''line,''

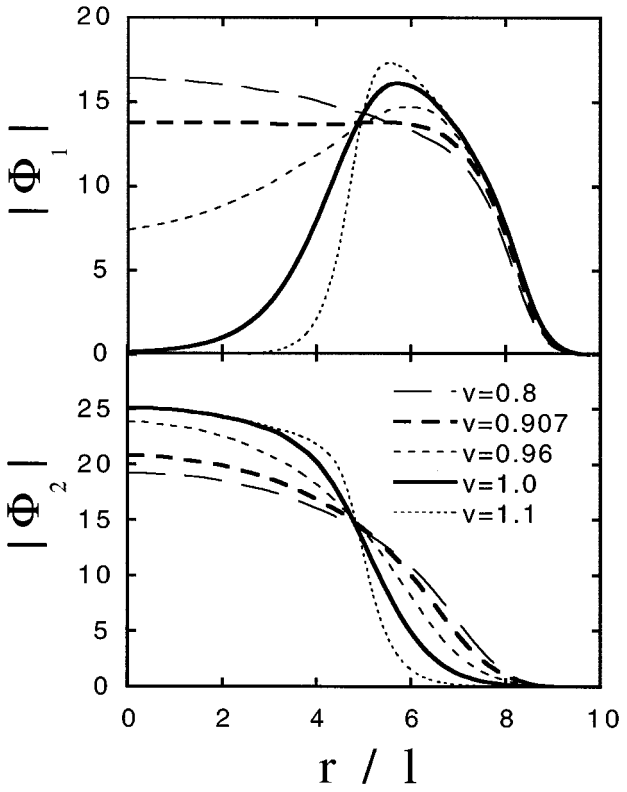

FIG. 2. Macroscopic wave functions, case (b).

$u_{12}^2 = u_{11}u_{22}$, the condensate fraction of the second component at the center, $\Phi_2(0)$, vanishes.

The results for case (b) shown in Fig. 2 are similar to those displayed in Fig. 1 except that here the component Φ_1, which has a bigger number density, is pushed out of the center of the trap. Thus one can conclude that as a result of instability the condensate with a weaker self-interaction will occupy the ''lowest'' places in the external potential ''landscape.'' The condensate pushed away from the center of the trap will give rise to a donutlike structure where the condensate fraction at the center of the potential well is significantly depleted or even zero. This is similar to the stratification of immiscible liquids with different mass densities in a gravitational field.

In the experiment[11] cited above two BE condensates were kept separated due to the difference in external potentials, V_1 and V_2. However, the density profile of one condensate was noticeably altered in the presence of the other one. The authors of Ref. 11 attributed this to the fact that the two condensates effectively repel each other or, in other words, that the scattering length for the collisions between atoms in the two different states is positive. The result of our calculations shows that such a change in the density profile may in fact indicate that the scattering length in question falls into the instability range. Unfortunately the separation of the condensates in the experiment[11] was too large to state with certainty whether the instability observed in Ref. 11 manifests itself in the two-condensate system we investigate here. To the best of our knowledge no reliable theoretical calculation of the scattering length for elastic collisions of Rb atoms in differing hyperfine states is available. For that reason one cannot exclude an explanation of the experimental data in Ref. 11 in terms of the phenomenon discussed in this paper, although the whole question of whether the stability criterion was broken under the experimental conditions still remains open. In order to answer this question it would be most helpful if an experiment could achieve a much smaller separation between the two interacting condensates (small in comparison to the width of the spatial condensate distribution). If the stability condition is broken we predict a structural transformation in both condensate clouds, in particular, the formation of a centered peak in the density distribution of one condensate and of a donutlike distribution for the other.

Let us point out that in the case of a mixture of two different hyperfine states such a structural transformation implies a strongly inhomogeneous and quite unusual distribution of nuclear magnetization, kind of a ''domain wall,'' across the trap. Of course, in this case the inhomogeneous distribution of the nuclear magnetization as well as the donutlike structure cannot exist eternally, since the state under consideration does not correspond to full thermodynamic equilibrium (see the introduction). In the long run the nuclear spin-flip transitions will bring the system to full equilibrium in which the fluid will possess one BE condensate only, so that Nozieres' arguments[18] about the absence of degeneracy in Bose-Einstein condensates hold. However, the corresponding equilibration time (related to the weak magnetic interaction) is usually much greater than the typical observation time in experiments which justifies the approach used in this paper. In the case of a binary mixture of atoms of different kind the state in question is in full thermodynamic equilibrium and therefore can live forever.

In all calculations presented above the presence of an external potential of the magnetic trap was essential for the effect. The trapping potentials are often highly inhomogeneous which results in significant density gradients even above the BEC transition. In the latter case, $T > T_{ci}, i = 1,2$, the density distributions of both components can be found directly from the conditions $\mu_i + U =$ const. At low enough temperatures almost all atoms belong to the BE condensate, and their spatial distribution is given by the Gross-Pitaevskii equations with an external potential. All properties including the instability criterion can be inferred from these equations. The exact solutions for the density distributions (condensate fractions) obtained by directly minimizing the Gross-Pitaevskii free energy, are shown in Figs. 1 and 2. Note that a perturbative approach was not used here when numerically calculating these density profiles. In analytical calculations the Thomas-Fermi-like approximation was used, in which density gradients were taken into account explicitly. As can be seen from the manuscript, the role of the trap does not reduce to only a confining force; the trap causes significant qualitative changes as well. As was demonstrated above, under certain conditions (even in the case where the bulk mixture in the absence of any external potential would definitely be stable) the same mixture in the trap becomes unstable and stratifies. Instead of there being a single point of thermodynamic instability in a two-component bulk fluid, one may end up with a number of extra critical points for a mixture in a trap, and the whole picture becomes much more complicated. This is probably the most important outcome of this paper.

In the case of a bulk fluid in the absence of any external field the instability itself can definitely occur but it might result in a quite different structural transformation. One of the possible structures might be an intermittent chain of clusters with a higher and lower condensate fraction density which is qualitatively similar to the case of two unmixable liquids (like water and petrol) in the absence of gravity. However, this case, being relevant to no experimental geometry, is not attended to here.

This work was supported in part by the Australian Research Council under Grant No. A69600107.

[1] M. H. Anderson, J. R. Ensher, M. R. Mattews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] C. C. Bradley, C. A. Sackett, J. J. Toleett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[3] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] N. N. Bogolyubov, Izv. Akad. Nauk SSSR, Ser. Fiz. **11**, 77 (1947).
[5] V. P. Mineev, Zh. Éksp. Teor. Fiz. **67**, 263 (1974) [Sov. Phys. JETP **40**, 132 (1974)].
[6] Yu. A. Nepomnyashchii, Zh. Éksp. Teor. Fiz. **70**, 1070 (1976) [Sov. Phys. JETP **43**, 559 (1976)].
[7] E. P. Bashkin and L. A. Kuchuk, Zh. Éksp. Teor. Fiz. **96**, 1714 (1989) [Sov. Phys. JETP **69(5)**, 968 (1989)].
[8] E. P. Bashkin and A. E. Meyerovich, Adv. Phys. **30**, 1 (1981).
[9] E. P. Bashkin, Usp. Fiz. Nauk **148**, 443 (1986) [Sov. Phys. Usp. **29(3)**, 238 (1986)].
[10] D. S. Greywall and M. A. Paalanen, Phys. Rev. Lett. **46**, 1292 (1981).
[11] C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wiemann, Phys. Rev. Lett. **78**, 586 (1997).
[12] L. D. Landau and E. M. Lifshits, *Statistical Physics* (Pergamon, New York, 1977), Pt. 1.
[13] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzialoshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice Hall, Princeton, NJ, 1963).
[14] E. M. Lifshits and L. P. Pitaevskii, *Statistical Physics* (Pergamon, New York, 1980), Pt. 2.
[15] N. Akhmediev and A. Ankiewicz, Phys. Rev. Lett. **70**, 2395 (1993); JEEE Australia **15**, 277 (1995).
[16] V. M. Eleonskii, V. G. Korolev, N. E. Kulagin, and L. P. Shil'nikov, Zh. Éksp. Teor. Fiz. **99**, 1113 (1991) [Sov. Phys. JETP **72**, 619 (1991)].
[17] F. Dalfovo and S. Stringari, Phys. Rev. A **53**, 2477 (1996).
[18] P. Nozieres, in *Bose Einstein Condensation*, edited by A. Griffin, D. W. Snoke, and S. Stringari (Cambridge University Press, Cambridge, 1995).