Weak-localization, Aslamazov-Larkin, and Maki-Thompson superconducting fluctuation effects in disordered $Zr_{1-x}Rh_x$ films above T_c

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The transverse magnetoresistance of disordered $Zr_{1-x}Rh_x$ thin films has been measured above the superconducting transition temperature T_c as a function of temperature in a magnetic field up to 60 kG. The investigated films are disordered enough to indicate quantum corrections due to localization and electron-electron interaction effects. The field and temperature dependence of the observed magnetoresistance is interpreted in terms of weak-localization, Aslamazov-Larkin, and Maki-Thompson superconducting fluctuations effects. From the comparison of the experimental results with theoretical calculations, the electron-electron attraction strength, $\beta(T/T_c)$, is derived and is in good agreement with Larkin's theory. The total phase-breaking rate τ_φ^{-1} has been estimated and ascribed to electron-phonon, electron-electron, electron-fluctuation, and spin-flip scattering mechanisms. [S0163-1829(97)00934-X]

I. INTRODUCTION

The disordered thin films of superconductors show above their T_c several quantum corrections to the conductivity. These corrections are based on (i) weak localization, (ii) the Coulomb contribution in the particle-hole channel (diffusion channel), (iii) the Coulomb contribution in the particle-particle channel (Cooper channel), and (iv) the Aslamazov-Larkin and Maki-Thompson fluctuations. The contribution of these corrections to the conductivity of two-dimensional disordered superconductors in the fluctuation region has been intensively studied in several theoretical and experimental investigations.[1–14]

Generally, the superconducting fluctuations cause a broadening of the resistance transition curve above the critical temperature T_c in highly disordered thin films. The films, which have high normal resistance and exhibit superconducting fluctuations for temperatures close to T_c, are dominated by the Aslamazov-Larkin direct process[15,16] Abrahams *et al.*[17] and later Redi,[18] using a phenomenological approach and a microscopical calculation respectively, extended the Aslamazov-Larkin theory to include the effects of applied magnetic field. In the two-dimensional (2D) limit the transversal magnetoresistance due to Aslamazov-Larkin fluctuations is given by the Redi expression:

$$\left[\frac{\Delta R_\square(H,T)}{R_\square^2(0,T)}\right]^{\mathrm{AL}} = -\frac{e^2}{2\pi^2\hbar}\frac{\pi^2}{8\ln\left(\frac{T}{T_c}\right)} \times\left\{8\left(\frac{H^*}{H}\right)^2\left[\psi\left(\frac{1}{2}+\frac{H^*}{H}\right) - \psi\left(1+\frac{H^*}{H}\right)+\frac{H}{2H^*}\right]-1\right\}, \quad (1)$$

where $\psi(x)$ is the di-gamma function and H^* is the characteristic field which is defined by the relation

$$H^* = (2k_BT/\pi eD)\ln(T/T_c). \quad (2)$$

In relation (2) D is the electron diffusion constant. The Aslamazov-Larkin term makes a significant contribution to the magnetoresistance only very close to T_c.

In contradiction to the Aslamazov-Larkin direct process, Maki[19] and later Thompson,[20] in order to explain the conductivity of clean superconductors, have suggested an indirect fluctuation process, which originates from the interaction of superconducting fluctuations with normal-state quasiparticles. The two-dimensional magnetoresistance due to the Maki-Thompson process is calculated by Larkin[1] and is given by

$$\left[\frac{\Delta R_\square(H,T)}{R_\square^2(0,T)}\right]^{\mathrm{MT}} = \frac{e^2}{2\pi^2\hbar}\beta_L(T/T_c) \times\left[\psi\left(\frac{1}{2}+\frac{H_\varphi}{H}\right)+\ln\left(\frac{H}{H_\varphi}\right)\right], \quad (3)$$

where $\beta_L(T/T_c)$ is the Larkin electron-electron interaction strength parameter and is tabulated by Larkin.[1] The superconducting fluctuation contribution to conductivity corresponding to the Maki-Thompson process[19,20] has the same magnetic-field dependence as the weak localization contribution,[21] but it has the opposite sign and differs by the coefficient $\beta_L(T/T_c)$. For temperature in the immediate vicinity of T_c, $\beta_L(T/T_c)$ is approximated by the expression $\beta_L(T/T_c)\sim\pi^2/[4\ln(T/T_c)]$. The characteristic field H_φ is related to the electron phase-breaking rate τ_φ^{-1} by the relation $H_\varphi=\hbar/4eD\tau_\varphi$. The phase-breaking rate τ_φ^{-1} is given by

$$\tau_\varphi^{-1} = \tau_{\mathrm{in}}^{-1}(T) + 2\tau_s^{-1}, \quad (4)$$

where $\tau_{\rm in}^{-1}(T)$ is the dephasing rate due to inelastic scattering and τ_s^{-1} is the temperature-independent spin-flip scattering rate. A feature of Larkin's expression of the magnetoresistance Eq. (3) is that it does not saturate at high fields. Lopes dos Santos and Abrahams[6] have shown that at temperatures close to T_c [$\ln(T/T_c) \ll 1$] and for fields $H \ll k_B T/4eD$, Eq. (3) should be replaced by the approximate form

$$\left(\frac{\Delta R_\square}{R_\square^2}\right)^{\rm MT} = \frac{e^2}{2\pi^2\hbar}\beta_{LS,A}(T/T_c,\delta)\left[\psi\left(\frac{1}{2}+\frac{H_\varphi}{H}\right) - \psi\left(\frac{1}{2}+\frac{H^*}{H}\right)+\ln\left(\frac{H^*}{H_\varphi}\right)\right]. \quad (5)$$

The $\beta_{LS,A}(T/T_c,\delta)$ parameter differs from Larkin's tabulated values only very near to T_c. In the limit $\ln(T/T_c) \ll 1$, the $\beta_{LS,A}(T/T_c,\delta)$ becomes

$$\beta_{LS,A}(T/T_c,\delta) = (\pi^2/4)[1/\ln(T/T_c) - \delta], \quad (6)$$

where $\delta = \pi\hbar/8k_BT\tau_\varphi$ is the Maki-Thompson pair-breaking parameter. The characteristic field H^* is defined by Eq. (2) and is related with δ via the expression $H^* = (H_\varphi/\delta)\ln(T/T_c)$. It must be mentioned that the Lopes dos Santos-Abrahams equation (5) underestimates the magnetoresistance at $T \gg T_c$ and the Larkin equation (3) should be used. In order to analyze the experimental data of the magnetoresistance one must include the Aslamazov-Larkin and Maki-Thompson fluctuations together with the localization effects.[21]

At temperatures close to T_c, where $\ln(T/T_c) \ll 1$, one must use the Aslamazov-Larkin fluctuations [Eq. (1)] and the Lopes dos Santos and Abrahams expression [Eq. (5)] for the Maki-Thompson fluctuations together with the weak-localization term. The localization term contains only the phase-breaking field H_φ, because it is assumed that the spin-orbit coupling is very strong and the digamma function which contains the spin-orbit characteristic field $H_{\rm so}$ can be omitted. The parameters necessary to define the theoretical curve are the characteristic fields H^* and H_φ as well as the prefactor $\beta_{LS,A}(T/T_c\delta)$.

At temperatures far above T_c, where $\ln(T/T_c) \gg 1$, one can neglect the Aslamazov-Larkin term and must use the Larkin expression [Eq. (3)] for the Maki-Thompson fluctuations together with the weak-localization term, which contains only the phase-breaking field H_φ. The characteristic field H_φ and the prefactor $\beta_L(T/T_c)$ can be defined as adjustable parameters.

Experimental conductivity studies of the three-dimensional amorphous superconductor $Zr_{75}Rh_{25}$ were carried out by Johnson, Tsuei, and Chaudhari[22] and Gumbatov *et al.*[23] By analysis of their data in zero field, Johnson, Tsuei, and Chaudhari[22] have found that the Aslamazov-Larkin[15,16] prediction for the conductivity gives good agreement with the experimental results near T_c both in magnitude and temperature dependence. They have attributed the absence of the Maki-Thompson[19,20] term in contributions to the conductivity from pair-breaking effects due to thermal phonons.[24] On the other hand, Gumbatov *et al.*[23] studying the same three-dimensional metallic glass system have analyzed their data mainly in terms of the contribution from the weak-localization and the electron-electron interaction in the diffusion and Cooper channels. The above authors have attributed the discrepancy between the calculations and the experimental data to the Maki-Thompson fluctuations, which they have not taken into account in the analysis.

In the present work, we report in a series of $Zr_{1-x}Rh_x$ thin films with thicknesses varying from 120 to 1200 Å, a detailed study on the inelastic-scattering mechanism and the superconducting parameter $\beta(T/T_c)$, where for $T \gg T_c$ $\beta(T/T_c)$ means the Larkin's parameter $\beta_L(T/T_c)$ and for T close to T_c $\beta(T/T_c)$ is the Lopes dos Santos and Abrahams parameter $\beta_{LS,A}(T/T_c,\delta)$, and provide a quantitative evaluation based on recent quantum transport theories.

II. EXPERIMENTAL PROCEDURE

The $Zr_{1-x}Rh_x$ master alloy was prepared from pure zirconium (99.99%) and rhodium (99.95%) by HF-levitation melting under an ultrapure helium gas at atmospheric pressure. The $Zr_{1-x}Rh_x$ thin films are deposited in an ultrahigh vacuum chamber ($<10^{-8}$ mbar) by electron-beam evaporation on Si wafers coated with about 2000 Å thick Si_3N_4 film held at room temperature. The Si_3N_4 films, prepared by chemical vapor deposition, were used as substrates to avoid interface oxidation between $Zr_{1-x}Rh_x$ films and the substrate. In order to obtain large deposition rates a graphite boat was used as a crucible, which was placed on the *e*-gun. The composition of the film was determined by Rutherford-backscattering spectroscopy. The film thickness was determined using a crystal thickness monitor, which was calibrated with a DEKTAK II profilometer. The sample dimensions 20×2 mm^2 were defined with a stainless-steel mask held in close contact with the substrate. The four-terminal resistance measurements were all carried out at different fixed temperature by varying the magnetic field. The dc voltage was measured with a HP 3457 A digital voltmeter, giving 61/2 steady digits of accuracy. The measuring dc currents were kept below 100 μA to minimize Joule heating and to ensure that the currents would not destroy superconductivity. The low-temperature measurements, between 1.6 and 20 K, were carried out in a standard stainless-steel He^4 cryostat. The transverse magnetoresistance was measured in a superconducting solenoid which produces a magnetic field up to 60 kG.

III. RESULTS

Figure 1 shows a representative zero magnetic-field resistance as a function of the temperature T between 1.6 and 4.5 K for $Zr_{1-x}Rh_x$ thin films. The broad resistive transition is due to the presence of superconducting fluctuations. The solid curve of Fig. 1 is a least-squares fit using the theoretical expression for two-dimensional conductivity of a superconductor derived by Aslamazov and Larkin[15,16] and Maki-Thompson.[19,20] The full expression for the conductance is given by

$$R_\square^{-1} = R_{\square n}^{-1} + R_0/\varepsilon(T) + 2R_0\ \ln[\varepsilon(T)/\delta]/[\varepsilon(T)-\delta], \quad (7)$$

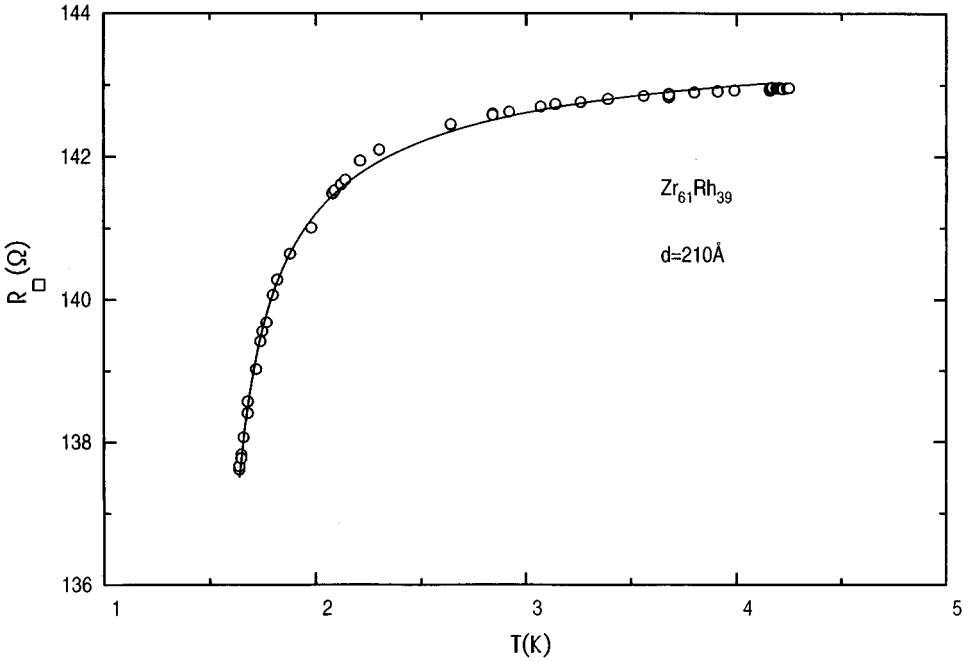

FIG. 1. The temperature dependence of resistance of the 210 Å thick film in zero field.

where R_n is the normal resistance, $R_0=e^2/16\hbar=6.58\times10^{-4}\Omega/\square$, $\varepsilon(T)=(T/T_c-1)$, $\delta=\pi\hbar/8k_BT\tau_\varphi$ is the pair-breaking parameter, and T_c is the BCS mean-field transition temperature. The calculated T_c and δ value are presented in Table I.

In Fig. 2 is plotted the transverse upper critical field H_{c2} of a $Zr_{87}Rh_{13}$ thin films with a thickness of 800 Å as a function of the temperature. From the temperature dependence of H_{c2} the electron diffusion coefficient D can be extracted using de Gennes's expression for dirty superconductors[25]

$$D=-\frac{4k_B}{\pi e(dH_{c2}/dT)_{T_c}}=-\frac{1.098\times10^{-4}}{(dH_{c2}/dT)_{T_c}}(\mathrm{m^2/s}). \quad (8)$$

The calculated values of D are listed in Table I. On the other hand, if one uses the Einstein relation for the diffusion constant $D=1/2e^2\varrho N(\varepsilon_F)$, where ϱ is the measured resistivity of thin films at 4.2 K and $N(\varepsilon_F)=2.64\times10^{42}$ states/J mole or 5.22×10^{46} states/J m^3 the density of states at Fermi energy of amorphous Zr_3Rh alloy,[26] obtained from measurements of the low-temperature electronic specific heat, one would take the values of D, which are listed also in Table I.

In Fig. 3 is shown the superconducting transition temperature T_c as a function of the sheet resistance $R_\square$ for the present investigated films. This figure shows that T_c decreases as $R_\square$ increases, a behavior observed also in other systems.[27,28] Figure 4 shows indicatively the transversal magnetoresistance of a 120 Å $Zr_{61}Rh_{39}$ thin film as a function of the magnetic field in a semilogarithmic diagram at different temperatures above T_c. In the entire magnetic-field region the magnetoresistance is positive at all measuring temperatures and also increases with decreasing T. The symbols represent the experimental results.

IV. DISCUSSION

The decrease of the superconducting transition temperature T_c with increasing the sheet resistance $R_\square$ shown in Fig. 3, can be attributed to the localization effects. According to

TABLE I. Value of relevant parameters for $Zr_{1-x}Rh_x$ thin films.

Thickness (Å)	120	185	210	215	355	800	1200
$R_\square$ (4.2 K)($\Omega/\square$)	246	106	143	105	81	13.3	9.3
ϱ_0 (4.2 K)($\mu\Omega$ cm)	295	196	300	225	288	106	111
T_c (K)	1.30	2.76	1.50	2.41	2.85	3.39	3.4
δ [from Eq. (7)]	0.2	0.1	0.3	0.2	0.1	0.1	0.2
$D(\times10^{-5})$ (m^2/s)		1.51			2.09	2.72	2.46
$D=\frac{1}{2e^2\varrho_0N(\varepsilon_F)}(\times10^{-4})$(m^2/s)	1.26	1.91	1.24	1.66	1.25	3.52	3.36
λ	0.46	0.55	0.47	0.53	0.56	0.59	0.59
$L_T=(\hbar D/k_BT)^{1/2}$ (Å)	139	170	138	159	138	232	226
Composition							
(at. %) Zr	61	75	61	75	75	87	87
(at. %) Rh	39	25	39	25	25	13	13

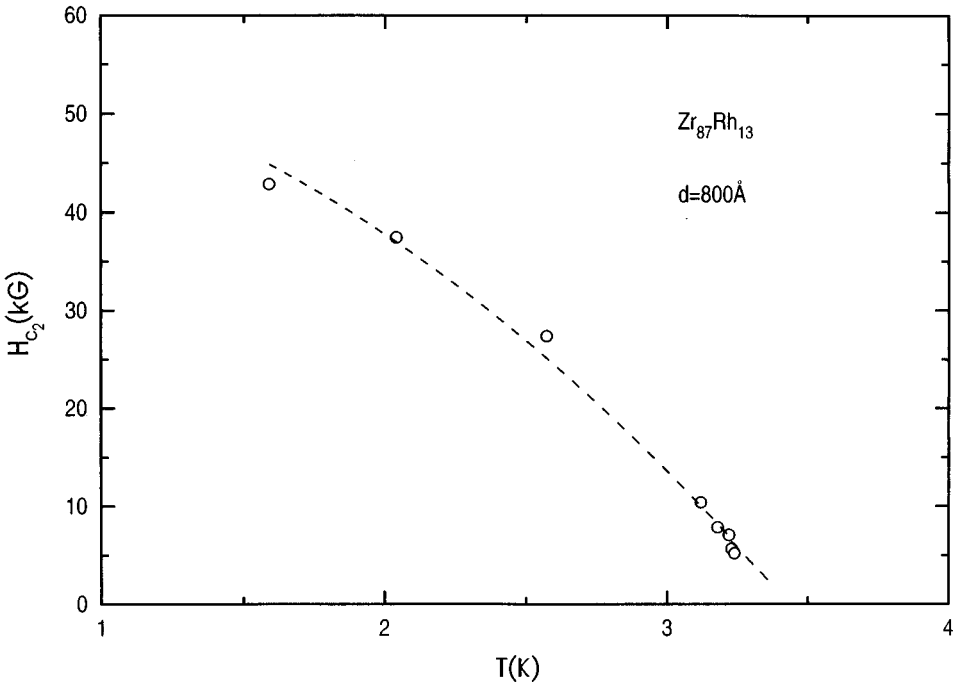

FIG. 2. The temperature dependence of the upper critical field H_{c2} of a 800 Å thick film.

Maekawa and Fukuyama[2] and Tagaki and Kuroda[3] the quantum interference effects in 2D disordered metals influence the superconducting transition temperature T_c.

The interplay between the interaction and disorder leads to the enhancement of the Coulomb repulsive interaction and to the depressing of the electronic density of states $N(0)$. Both effects contribute to the depression of T_c, however, the influence of the $N(0)$ change in dirty 2D superconducting films is negligibly small as compared with that of the increased Coulomb interaction.[29] Specifically, Maekawa and Fukuyama,[2] using perturbation theory, have found that the presence of disorder lowers the temperature T_c of the superconducting transition of films in the following way:

$$\ln\left[\frac{T_c}{T_{c0}}\right]=-\left(\frac{e^2}{2\pi^2\hbar}\right)R_{\square}g_1N(0)\left\{\frac{1}{2}\left[\ln\left(5.4\frac{\xi_0 T_{c0}}{lT_c}\right)\right]^2+\frac{1}{3}\left[\ln\left(5.4\frac{\xi_0 T_{c0}}{lT_c}\right)\right]^3\right\}, \tag{9}$$

where T_c and T_{c0} are the critical temperatures of the film with and without impurity scattering, $R_{\square}$ is the sheet resistance, g_l is the electron-electron repulsion coupling constant, $N(0)$ is the density of states, $\xi_0=0.18\hbar v_F/k_BT_{c0}$ is the

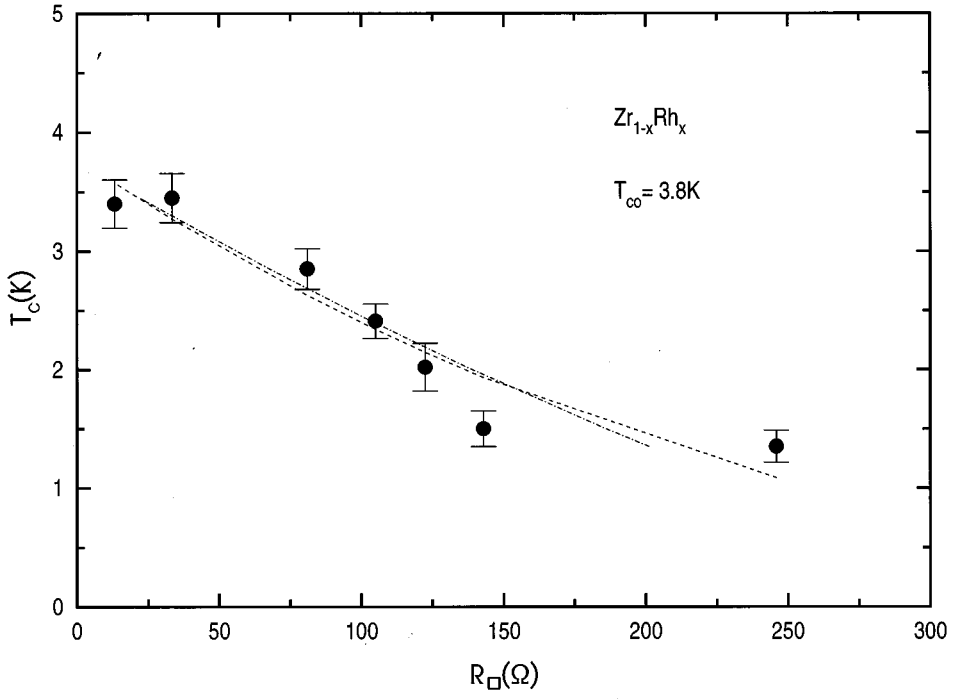

FIG. 3. Variation of the superconducting T_c as a function of $R_{\square}$ at 4.2 K. The dash-pointed and dashed lines are theoretical fits with Eqs. (9) and (10), respectively.

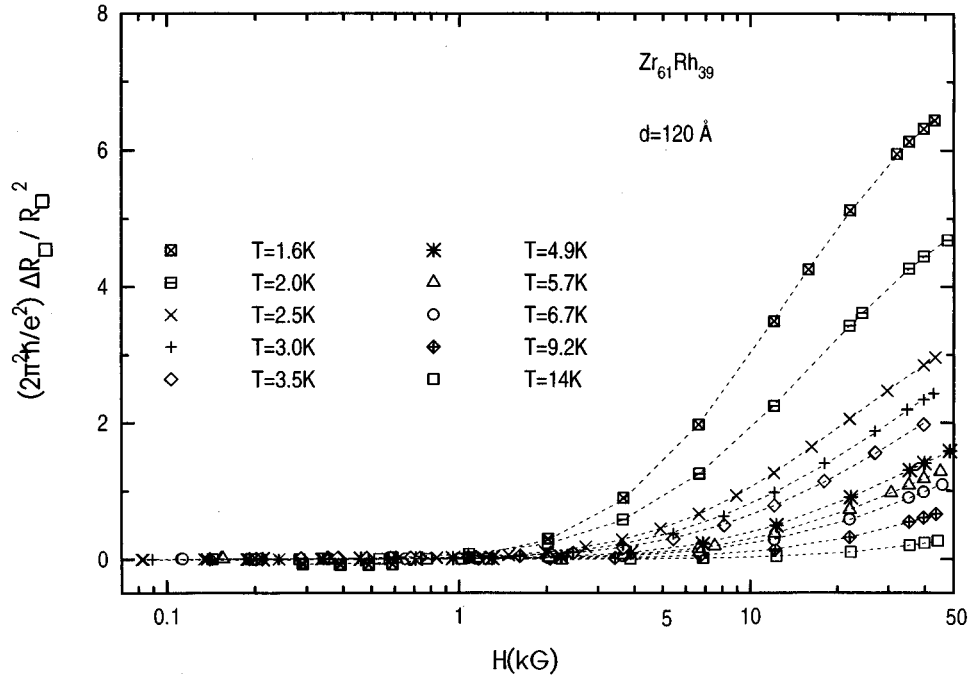

FIG. 4. Transverse magnetoresistance of a 120 Å thick film at different fixed temperatures above T_c.

zero-temperature coherence length corresponding to T_{c0}, l is the elastic mean free path and $(e^2/2\pi^2\hbar)=1.235\times10^{-5}$ Ω^{-1}. The quadratic term of Eq. (9) is due to the corrections to the pair of states, while the cubic term is due to the enhancement of the Coulomb repulsion between the electrons due to impurities and it is much stronger than the quadratic term.

The dashed-pointed curve in Fig. 3 represents the fit of the experimental data using Eq. (9), and is obtained taking $T_{c0}=3.8$ K, $\xi_0/l=1470$, and $g_1N(0)$ as the fitting parameter. The best-fit value for $g_1N(0)$ is 1.1 and compares well the value of $g_1N(0)=1$ of the screened Coulomb interaction in the limit of long wavelength.[30] The ratio ξ_0/l is calculated from the zero-temperature coherence length $\xi_0=4400$ Å, using $v_F=1.2\times10^6$ m/s [corresponding to $\varepsilon_F=4$ eV (Ref. 31)] and $T_{c0}=3.8$ K, and from the mean value of $l=3$ Å. The value of l is estimated from the electrical resistivity ϱ_0 ($\mu\Omega$ cm) at helium temperature using the expression: l(Å) $=92(\mathbf{r}_s/a_0)^2/\varrho_0$ ($\mu\Omega$ cm),[32] where a_0 is the Bohr radius and $\mathbf{r}_s$ is defined as $\mathbf{r}_s=0.75(r_s)_{\mathrm{Zr}}+0.25(r_s)_{\mathrm{Rh}}$ [$r_s=(3/4\pi n)^{1/3}$, and n is the number of electrons per cm^3].

Finkel'stein[33] has shown that Eq. (9) is valid at $R_\square<0.5$ kΩ. For systems with sheet resistance $R_\square>0.5$ kΩ the transition temperature $T_c(R_\square)$ can no longer be determined by simply using the first correction in $(e^2/2\pi^2\hbar)R_\square$ and the renormalization group equations should be used. Finkel'stein[33] has obtained for the dependence $T_c(R_\square)$ the expression:

$$\frac{T_c}{T_{c0}}=\exp(-1/\gamma)\left[\left(1+\frac{(t/2)^{1/2}}{\gamma-t/4}\right)\Big/\left(1-\frac{(t/2)^{1/2}}{\gamma-t/4}\right)\right]^{1/\sqrt{2t}}, \tag{10}$$

where $\gamma=1/\ln(k_BT_{c0}\tau_{\mathrm{tr}}/\hbar)$, $\gamma<0$, $t=(e^2/2\pi^2\hbar)R_\square$, τ_{tr} is the transport relaxation time, T_c is the transition temperature of the film, and T_{c0} is the bulk value of the transition temperature. γ is used as a fitting parameter. The dashed curve in Fig. 3 is a fit to Eq. (10) with $\gamma^{-1}=-12.5$ for $T_{c0}=3.8$ K. From the value of γ^{-1} the transport relaxation time is calculated to the 7.5×10^{-18} s. Using for the logarithmic term of Eq. (9) $\ln[5.4(\xi_0T_{c0}/lT_c)]$ the equivalent expression $\ln(\hbar/k_BT_{c0}\tau_0)$ and taking $g_1N(0)=1.1$, the best fit to the experimental data of Fig. 3 gives a relaxation time $\tau_0=2.4\times10^{-16}$ s. This relaxation time is about a factor of 30 larger than the transport relaxation time extracted from the Finkel'stein's Eq. (10). On the other hand, using $g_1N(0)=0.5$ in Eq. (9) [the value assumed by Finkel'stein[33] in Eq. (10)], the fitting procedure gives a relaxation time $\tau_0=10.5\times10^{-18}$ s. This value is near to the transport relaxation time τ_{tr} extracted from Eq. (10). Consequently the discrepancy in the relaxation time between Eqs. (9) and (10) is only due to the different values of the parameter $g_1N(0)$.

We can compare our experimental results with the theory, assuming that the measured magnetoresistance arises from weak-localization, Aslamazov-Larkin, and Maki-Thompson superconducting fluctuations suppression, using the procedure mentioned above. The Maki-Thompson superconducting fluctuations can be represented by the Lopes dos Santos and Abrahams expression [Eq. (5)] for temperatures near T_c and for the case that the magnetoresistance saturates, and by the Larkin expression [Eq. (3)] for temperatures far above T_c. From the temperature dependence of the magnetoresistance data of Fig. 4, one can obtain values for the $\beta(T/T_c)$, the characteristic field H^* and therefore the diffusion constant D, the characteristics field H_φ and therefore the phase-breaking times τ_φ and the phase-breaking parameter δ. The dashed curves in Fig. 4 are the best fits. This procedure has been applied to all $Zr_{1-x}Rh_x$ investigated thin films. The values of the $\beta(T/T_c)$ parameters are plotted in Fig. 5 as a function of the reduced temperature T/T_c in a semilogarithmic diagram. The plot shows that all points fall on the same curve, i.e., it observes a universal behavior. $\beta(T/T_c)$ diverges as T approaches the critical temperature

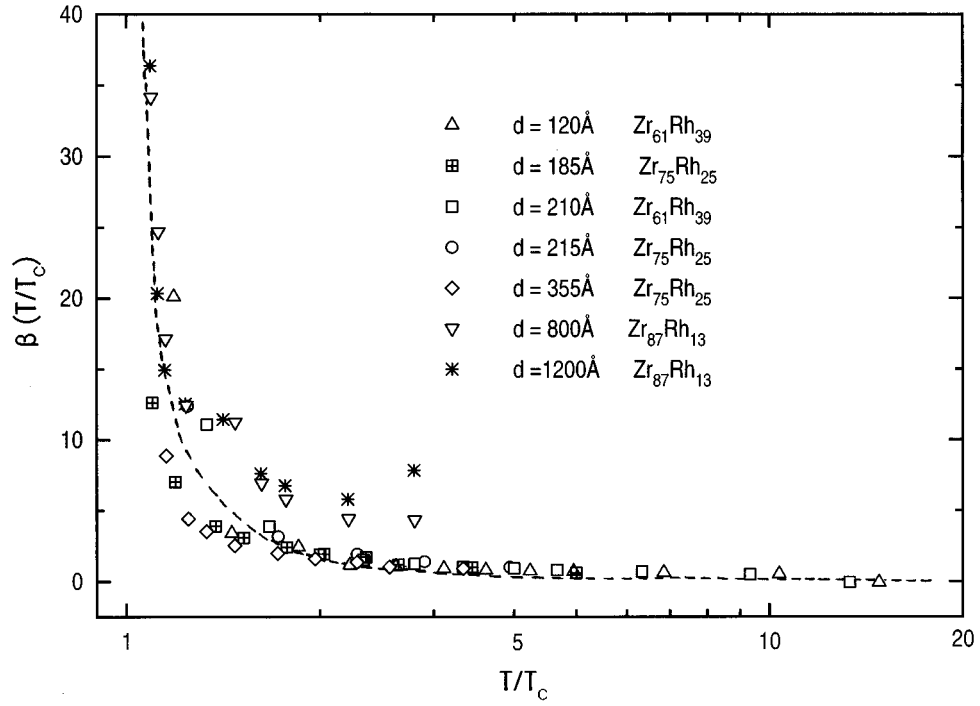

FIG. 5. Dependence of the Larkin parameter $\beta(T/T_c)$ as a function of the reduced temperature T/T_c for the investigated films. The dashed curve is a theoretical plot of $\beta_L(T/T_c)$ after Larkin (Ref. 1).

T_c. For comparison, Fig. 5 also shows the theoretical values of $\beta_L(T/T_c)$ which have been tabulated by Larkin[1] versus the electron coupling constant $g_c^{-1}(T/T_c) = -\ln(T/T_c)$. The dashed curve drawn through the experimental points corresponds to the theoretical values of $\beta_L(T/T_c)$. A comparison shows that there is a very good agreement between the experimental data of $\beta(T/T_c)$ and the theoretical $\beta_L(T/T_c)$. The values of electronic diffusion coefficient D corresponding to the characteristic field H^* is of the same order of magnitude with the values calculated from the upper critical field H_{c2}.The values of the pair-breaking factor δ coming from the fitting procedure on the magnetoresistance data are plotted in Fig. 6 as a function of $\ln(T/T_c)$ for the investigated films. One can see that δ depends on the resistance per square of the films and shows a tendency to increase as the temperature T approaches T_c. These values of δ differ from the corresponding values of δ extracted from the fitting procedure on the temperature dependence of the resistance by a factor of 3. The curves of Fig. 6 are the calculations of pair-breaking parameter $\delta = \pi\hbar/8k_BT\tau_\varphi$ using theoretical expressions for the different scattering times, which will be analyzed below.

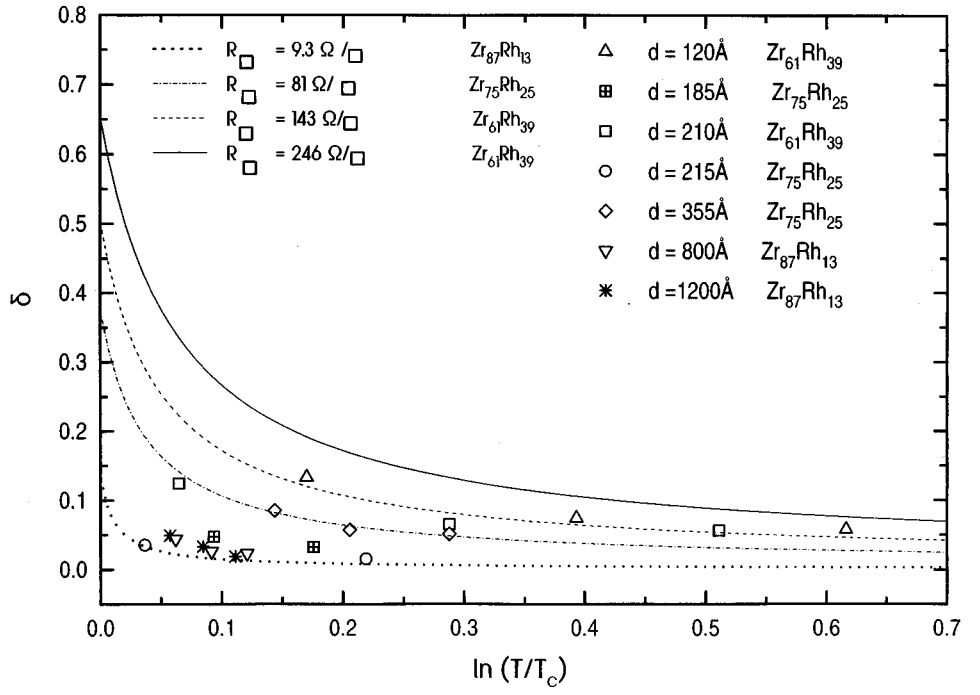

FIG. 6. The pair-breaking parameter δ as a function of $\ln(T/T_c)$ of all investigated films.

An interesting consequence of the fitting is the determination of the characteristic field H_φ and the corresponding phase-breaking rate τ_φ^{-1}. As mentioned in the introduction the phase-breaking rate $\tau_\varphi^{-1}$is given by Eq. (4), i.e., $\tau_\varphi^{-1} = \tau_{\rm in}^{-1}(T) + 2\tau_s^{-1}$, where the temperature-independent part is the spin-flip scattering rate due to the presence of residual magnetic impurities, while the temperature-dependent part deals with inelastic-scattering processes in relation with electron-phonon, electron-electron, and electron-fluctuation interactions. The temperature-dependent rate $\tau_{\rm in}^{-1}(T)$ is expected to be the sum of the terms due to inelastic electron-phonon, electron-electron, and electron-fluctuation scattering mechanisms, namely:

(i) In the case of electron-phonon scattering the dimensionality of the investigated films plays a decisive role, with respect to phonon propagation, i.e., the comparison of the physical dimensions of the films with the phonon wavelength $\lambda_{\rm ph} = h v_s / k_B T$, where v_s is the velocity of sound. In order to estimate the dimensionality of the present samples with respect to the electron-phonon scattering it is necessary to know the phonon wavelength $\lambda_{\rm ph} = h v_s / k_B T$ at temperature T and consequently the sound velocity v_s of the alloy system $Zr_{1-X}Rh_X$. Unfortunately there is no data on V_S for this system in the literature. An estimate of the sound velocity is made using the formula $v_s(Zr_{1-x}Rh_x) = (1-x)v_s(Zr) + xv_s(Rh)$, where $v_s(Zr) = 4360$ m/s is the sound velocity of pure Zr and $v_s(Rh) = 6190$ m/s is the sound velocity of pure Rh.[34] This compromise gives a sound velocity of about 4820 m/s for our samples ($x = 0.25$). On the other hand, the Debye temperature $\Theta_D = 191$ K corresponds to a transverse phonon velocity $v_T \sim 1750$ m/s or to a longitudinal velocity $v_L \sim 3500$ m/s. A sound velocity of 4820 m/s at $T = 5$ K corresponds to a phonon wavelength of about 460 Å. This suggests that the present samples are between 2D and 3D and in the dirty limit ($l \ll \lambda_{\rm ph}$).

Based on the above-mentioned arguments for the dimensionality, namely that the films are in the intermediate range between two and three dimensions (neglecting the coupling scattering of the film with substrate), one can take for the inelastic-scattering rate a relation given by Raffy *et al.*,[11] which is valid for the dirty two-dimensional limit ($l, d \ll \lambda_{\rm ph}$):

$$\tau_{\rm in(e\text{-}ph)}^{-1} = 14\pi^2 \zeta(2) \lambda \omega_D \left(\frac{T}{\Theta_D}\right)^3 = 8.147 \times 10^8 \ ({\rm s}^{-1}\,{\rm K}^{-3}) \lambda T^3, \qquad (11)$$

where $\zeta(2)$ is the two-dimensional Riemann's zeta function. The numerical coefficient of this equation is calculated using for Debye temperature Θ_D and frequency ω_D the values 191 K and 2.5×10^{13} s^{-1}, respectively. The Debye temperature Θ_D is determined from the coefficient β of the cubic term of the molar specific heat for amorphous Zr_3Rh alloy.[26] The electron-phonon coupling constant λ can be estimated from McMillan expression[35] for the T_c of strong-coupling superconductors assuming the effective Coulomb repulsion parameter μ^* to be 0.1. The λ values of the present films are given in Table I.

(ii) For the inelastic electron-electron scattering in a two-dimensional dirty limit, i.e., with thermal diffusion length $L_T = (\hbar D / k_B T)^{1/2}$ larger than the thickness d of the films ($L_T \gg d$), Al'tshuler, Aronov, and Khmelnitsky[36] and Fukujama and Abrahams[37] have found for the scattering rate the expression:

$$\tau_{\rm in(e\text{-}e)}^{-1} = \left(\frac{e^2}{2\pi\hbar^2}\right) R_\square k_B T \ln\left[\frac{\pi\hbar}{e^2 R_\square}\right] = c_2 T. \qquad (12)$$

Using as diffusion constant D the values, which are derived from Einstein relating, one obtains values of L_T which are listed in Table I. A comparison between L_T and the thickness d shows that most of the samples are in an intermediate range between two and three dimensions with respect to electron-electron interaction.

(iii) At temperatures very close to T_c, Brenig and co-workers[38,39] have suggested that the existence of the superconducting fluctuations can affect the inelastic-scattering rate. According to Brenig *et al.*[38] this term arises from the inelastic process associated with the recombination of electrons into superconducting pairs and is given by

$$\tau_{\rm in(e\text{-}fl)}^{-1} = \left(\frac{e^2}{2\pi\hbar^2}\right) R_\square k_B T \left[\frac{2\ln 2}{\ln(T/T_c) + C_2}\right] = C_1 T \frac{1}{\ln(T/T_c) + C_2}, \qquad (13)$$

where $C_2 = 4\ln 2/[\{[\ln(\pi\hbar/e^2R_\square)]^2 + 128\hbar/e^2R_\square\}^{1/2} - \ln(\pi\hbar/e^2R_\square)]]$. Equation (13) means that for $T \gg T_c$, the $\ln(T/T_c)$ term is large and the electron-fluctuation scattering is very small in comparison to electron-phonon and electron-electron scattering processes. However as the temperature approaches T_c, the $\ln(T/T_c)$ term goes to zero, the electron-fluctuation rate $\tau_{\rm (e\text{-}fl)}^{-1}$ exceeds the inelastic electron-electron scattering rate $\tau_{\rm in(e-e)}^{-1}$ and becomes the dominating process.

For the total phase-breaking process, which includes the electron-fluctuation, the electron-electron, electron-phonon, and other temperature-independent scattering mechanisms, a dephasing rate can be applied given by the following form:

$$\tau_\varphi^{-1} = \tau_{\rm in}^{-1} + A = C_1 T \frac{1}{\ln(T/T_c) + C_2} + C_3 T + C_4 T^3 + A. \qquad (14)$$

A contains the spin-flip scattering rate. The term A is obtained by making a best fit to the experimental data of τ_φ^{-1} for temperatures far above T_c. Using only the linear, the cubic, and the constant terms of Eq. (14), where the fluctuation term is negligible. The values of A are collected in Table II. Figure 7 shows in a double logarithmic plot the inelastic part $\tau_{\rm in}^{-1}$ of the phase-breaking rate τ_φ^{-1} as a function of the temperature for all investigated films. The broken curves in Fig. 7 represent the best fits of the inelastic part of Eq. (14). Table II contains the ratios between the prefactors C_1, C_2, C_3, and C_4 determined from this analysis and the theoretical values. The comparison of these coefficients shows that electron-phonon coefficients C_4 for the thinner films are nearly close to the theoretical values for the 2D dirty limit, while the electron-fluctuation and electron-electron coefficients C_1 and C_3 are thickness and composition dependent and deviate considerably from the theoretical values by a

TABLE II. Inelastic scattering fitting parameters.

d (Å)	A	C_1^{exp}/C_1^{th}	C_2^{exp}/C_2^{th}	C_3^{exp}/C_3^{th} (2D) dirty	C_4^{exp}/C_4^{th} (2D) dirty
120	0.39	3.84	2.08	11.00	1.02
185		5.63	0.14	31.55	1.55
210	0.25	9.77	1.24	10.77	1.57
215	0.85	7.10	0.07	0.23	2.31
355	0.94	8.56	2.95	28.79	1.67
800		82.3	0.70	82.7	8.1
1200		112.8	0.80	86.7	9.9

factor which lies between 5 and 10 and 0.2 and 30, respectively. In contradiction for the two thicker films the corresponding experimental values of C_1 and C_3 are 100 times larger than the theoretical.

Table II exhibits also the fact that the cutoff parameters C_2 of Brenig *et al.*[38] for electron-fluctuation interaction for all films are nearly in agreement with the theory. The mentioned large deviations of the experimental coefficients C_1 and C_3 from the theoretical values can be attributed to the fact that the present films are two-component systems with different compositions and probably are inhomogeneous in contradiction to the simple systems like Al films,[28] in which there is good agreement between experimental and theoretical values of the C_1 and C_3.

Apart from this fact the measurements show that in the present system all three inelastic-scattering processes coexist. The solid straight line in Fig. 7 represents the T^3 behavior of the electron-phonon scattering process, which indicates that at high enough temperatures the inelastic rates approach the electron-phonon curve. As the temperature decreases the τ_{in}^{-1} curve deviates from the T^3 dependence due to the activation of the two-dimensional electron-electron scattering, which is linear in T as mentioned above. The τ_{in}^{-1} rates decrease monotonically as the temperature decreases until $T=4$ K, where in lower temperatures, as T approaches T_c, they start to increase. This is consistent with the existence of electron-fluctuation inelastic scattering near T_c.

As is mentioned above, if one uses for the diffusion constant D the values of Table I then the thermal length $L_T=(\hbar D/k_BT)^{1/2}$ compares to d at 5 K. This means that the present disordered films are between two and three dimensions with respect to electron-electron interaction.

On the other hand for the inelastic electron-electron scattering in 3D metals Schmidt[40] has shown that the scattering rate is given by the expression

$$\tau_{in(e\text{-}e)}^{-1}=\frac{\pi}{8}\left(\frac{k_B^2}{\hbar\varepsilon_F}\right)T^2+\frac{\sqrt{3}}{2}\left(\frac{k_B}{k_Fl}\right)^{3/2}\frac{T^{3/2}}{\hbar\sqrt{\varepsilon_F}}, \tag{15}$$

where ε_F is the Fermi energy, k_F is the Fermi wave number, and l is the mean free path. The T^2 term in Eq. (15) describes the Landau electron-electron scattering mechanism and dominates in the pure case, while the $T^{3/2}$ term dominates in the strong disorder limit. Using for $\varepsilon_F=4$ eV (Ref 31), and for $k_Fl\sim4$ the above expression yields

$$\tau_{in(e\text{-}e)}^{-1}=(1.11\times10^6\ T^2+6.58\times10^7\ T^{3/2})\ \ s^{-1}\,K^{-2}. \tag{16}$$

The first term T^2 is not consistent with the present results, while the prefactor 6.58×10^7 of the $T^{3/2}$ term is much too small in comparison to the experimental values ($6\times10^8-8\times10^{10}$). From the inelastic-scattering rate with respect to the electron-electron interaction it can be concluded that the present films behave like 2D systems, although the two-dimensional condition $L_T=(\hbar D/k_BT)^{1/2}\gg d$ is not valid fully as mentioned above. The nonvalidity of the 2D condition $L_T=(\hbar D/k_BT)^{1/2}\gg d$ in the present samples can be attributed to the inhomogeneities of the present films. We propose that additional interface scattering[41] of the electrons at the grain boundaries of these inhomogeneities reduces the diffusion constant D and also the thermal length L_T and so invalidates the 2D condition. It is worth mentioning here,

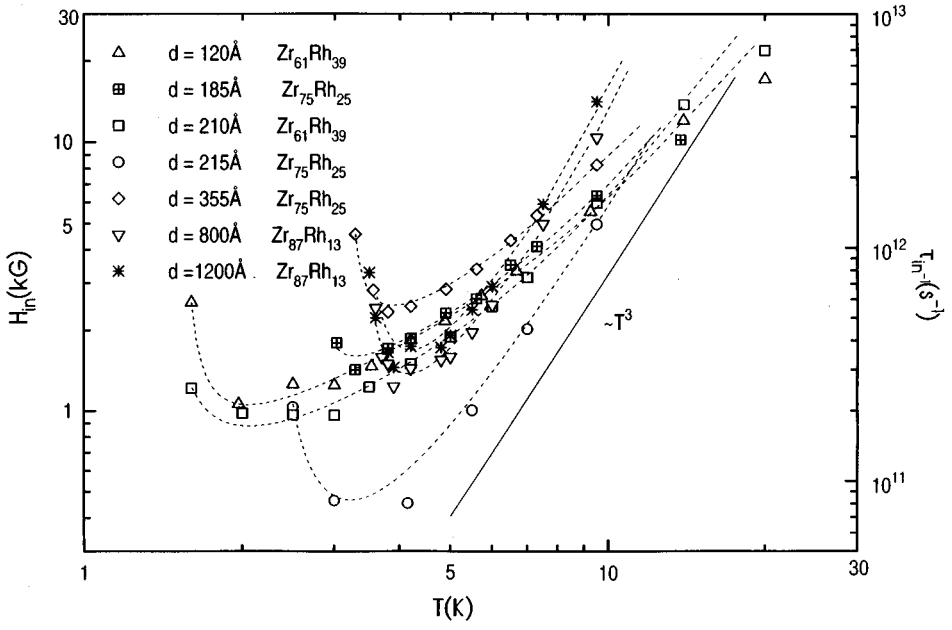

FIG. 7. The temperature dependence of characteristic field H_{in} and the inelastic-scattering rate τ_{in}^{-1} of various thickness films. The dashed curves represent best fits of the inelastic part of Eq. (14). The solid straight line corresponds to the electron-phonon contribution [Eq. (11)].

that if this additional temperature-independent interface scattering process is assumed to exist, then the rate A must be the sum between the spin-flip part and the part resulting from this scattering.

V. CONCLUSION

In the present investigation, low-temperature magnetoresistance data have been used for superconducting $Zr_{1-x}Rh_x$ to determine the Larkin parameter $\beta_L(T/T_c)$ and the magnitude and temperature dependence of the inelastic-scattering rate $\tau_{\rm in}^{-1}$. The positive magnetoresistance of the present system is well described by the existing theories for the weak localization in the presence of a strong spin-orbit coupling, Aslamazov-Larkin, and Maki-Thompson fluctuations. The inelastic-scattering rate $\tau_{\rm in}^{-1}$ is determined as a function of temperature and is in qualitative agreement with the theories which include electron-phonon, electron-electron, and electron-fluctuation inelastic scattering.

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