Angular dependence of giant magnetoresistance in magnetic multilayers

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Dependence of the current-in-plane giant magnetoresistance (GMR) in magnetic multilayers on the angle ϕ between the magnetizations of successive magnetic films is analyzed theoretically in the quasiclassical and quantum limits. Spin dependent electron scattering on impurities as well as on interfacial roughness is taken into account. In the quasiclassical limit GMR is shown to vary approximately linearly with $\sin^2(\phi/2)$. In the quantum limit the linear behavior occurs only for symmetrical structures with a crystal electronic potential independent of the electron spin orientation. Deviations from the linear behavior occur when either the crystal potential is spin dependent or the structure is asymmetrical. [S0163-1829(97)08534-2]

I. INTRODUCTION

In most theoretical descriptions of the giant magnetoresistance (GMR) in magnetic multilayers, only parallel and antiparallel orientations of the magnetizations of successive magnetic films were considered.[1–8] Only a few papers have addressed the question of variation of the effect with the angle ϕ between the magnetizations.[9,10] This angular dependence, however, is very interesting for several reasons. As pointed out in Ref. 10, it can give some information on the relative contributions of different kinds of electrons to the electrical conductivity, and particularly to GMR. It can also give an information on the spin dependence of electron potentials.

The problem of angular dependence was first discussed in Ref. 9 for an infinite superlattice with a uniform periodic part of the electron potential (potential in a defect-free system, called here also crystal potential), where a linear variation of GMR with $\sin^2(\phi/2)$ was found. The problem was examined later in more details by Vedyayev *et al.*,[10] who calculated the angular dependence of GMR in a system composed of two magnetic films in direct contact (no nonmagnetic spacer in between). For the crystal potential independent of the electron spin (no potential steps at the interface) the authors found the linear variation of GMR with $\sin^2(\phi/2)$. If, however, some potential bariers occurred at the interfaces (due to the spin dependent crystal potential in their case), significant deviations from the linear behavior were found.

The angular dependence of GMR was also investigated experimentally for both current-in-plane[11–13] (CIP) and current-perpendicular-to-plane[14] (CPP) geometries. In the former case almost a linear dependence of the effect on $\sin^2(\phi/2)$ was found. In the CPP case, however, remarkable deviations from this linear behavior were observed in some systems.[14]

In this paper we consider angular variation of CIP GMR in two opposite limits, i.e., in the quasiclassical limit and in the limit of strong quantum interference of electron waves reflected from interfaces and/or outer surfaces (quantum limit).

Within the quasiclassical approach the angular variation of GMR was introduced approximately in Ref. 15 *via* an angular dependence of transmission coefficients. This simplified description gives correct amplitude of the effect. In the limit of small values of GMR, it also gives reasonable results for the angular dependence. However, exact variation of the effect with the angle ϕ, particularly for larger values of GMR, requires a more accurate analysis. The quasiclassical approach developed here is based on the Boltzmann kinetic equation, with nondiagonal components of the electron distribution function taken into account.

The method used in the quantum limit is a generalization of the formalism developed earlier for collinear configurations of the film magnetizations.[16] This formalism is adapted here to the case where the magnetizations are not collinear, but still in the film plane.

We show in this paper that in the quasiclassical limit GMR varies approximately linearly with $\sin^2(\phi/2)$. Deviations from this behavior occur in the limit of quantum transport, where interference of electron waves reflected from interfaces and/or surfaces leads to formation of sharp quantum energy levels. But even then the linear dependence occurs for symmetrical structures with a crystal potential independent of the spin orientation. Consider first the quasiclassical limit.

II. QUASICLASSICAL LIMIT

We will consider a trilayer in which two ferromagnetic films (corresponding to the layer index $\alpha=1$ and $\alpha=2$) of thicknesses d_1 and d_2 are separated by a nonmagnetic layer of thickness d_0 ($\alpha=0$). However, by imposing perfectly reflecting boundary conditions at the external surfaces (as will be discussed later), one can obtain results which correspond to a superlattice with doubled thickness of the magnetic films.[15]

Electronic structure of the system will be considered in the free-electron approximation for conducting electrons. Consider first the case where the crystal potential is uniform across the structure. Consequently, the conduction electrons do not feel potential barriers at the interfaces and are entirely transmitted when the interfaces are perfectly flat. This is a large simplification as in real structures there is always some misfit of electronic bands at interfaces. However, this approximation is reasonable for s electrons. A general case, where electrons are subject to a spin dependent superlattice potential, will be discussed later.

Conduction electrons at the Fermi level E_F are scattered by impurities distributed inside the films and by rough interfaces and surfaces. Electron scattering by impurities is taken into account by appropriate relaxation times, which in the magnetic films ($\alpha=1,2$) are $\tau_{\alpha\uparrow}$ for spin-majority electrons (electrons with spin parallel to the spin of electrons responsible for the magnetic moment) and $\tau_{\alpha\downarrow}$ for spin-minority electrons, while in the spacer layer it is independent of the electron spin orientation, $\tau_{0\uparrow}=\tau_{0\downarrow}=\tau_0$. In the following we will use the electron mean free paths (MFP's) $\lambda_{\alpha\uparrow(\downarrow)}$ and λ_0 instead of the corresponding relaxation times $\tau_{\alpha\uparrow(\downarrow)}$ and τ_0. To include diffuse scattering due to interface and surface roughness, we assume the same model as in Ref. 15. Within this approach the electron scattering due to roughness is taken into account by spin dependent coefficients of specular transmission through the interfaces and specular reflection from the surfaces.

The electron distribution function, and finally electronic conductivity, can be calculated by solving the corresponding Boltzmann kinetic equation together with the appropriate boundary conditions at all interfaces and surfaces. In a bulk ferromagnet, and in a coordinate system with the quantization axis collinear with the magnetization direction, the electron distribution is described by two diagonal components, $f_\uparrow(y,\mathbf{v})$ and $f_\downarrow(y,\mathbf{v})$, respectively for spin-majority and spin-minority electrons. Here, $\mathbf{v}$ is the electron velocity and the axis y is normal to the films. In magnetic multilayers with noncollinear magnetizations of successive films, electron exchange between the magnetic films leads to nondiagonal components $f_{\uparrow\downarrow}(y,\mathbf{v})$ and $f_{\downarrow\uparrow}(y,\mathbf{v})$ of the distribution function, even in a coordinate system with the local quantization axis (by local quantization axis we mean the axis opposite to the local magnetization direction).

Assume now a quantization axis which is at an angle θ with respect to the local one, but still in the film plane. The distribution function $\mathbf{f}(y,\mathbf{v})$ has then two diagonal components, $f_+(y,\mathbf{v})$ and $f_-(y,\mathbf{v})$, and two nondiagonal ones. $f_{+-}(y,\mathbf{v})$ and $f_{-+}(y,\mathbf{v})$. We use here the notation according to which the spin projection on the local quantization axis is denoted as $\uparrow$ and $\downarrow$, respectively for spin-majority and spin-minority electrons, while projection on a nonlocal or global quantization axis is denoted by $+$ and $-$. The distribution function can be decomposed in two parts, $\mathbf{f}(y,\mathbf{v})=\mathbf{f}_0(\mathbf{v})+\mathbf{g}(y,\mathbf{v})$, where $\mathbf{g}(y,\mathbf{v})$ is a deviation from the equilibrium distribution $\mathbf{f}_0(\mathbf{v})$. The Boltzman kinetic equation for the distribution function $\mathbf{g}(y,\mathbf{v})$ can be now written in the form

$$\frac{\partial \mathbf{g}(y,\mathbf{v})}{\partial y}+\frac{1}{v_y}\,\boldsymbol{\tau}^{-1}\mathbf{g}(y,\mathbf{v})=\frac{eE}{mv_y}\,\frac{\partial f_0}{\partial v_x}\,\mathbf{1}, \tag{1}$$

where E is the electric field applied along the in-plane axis x. e and m denote the electron charge ($e>0$) and electron mass, $\mathbf{1}$ is a 2×2 unit matrix, and

$$\boldsymbol{\tau}^{-1}=\begin{pmatrix} (1/\tau_\uparrow)\cos^2(\theta/2)+(1/\tau_\downarrow)\sin^2(\theta/2) & (1/\tau_\uparrow-1/\tau_\downarrow)\sin(\theta/2)\cos(\theta/2) \\ (1/\tau_\uparrow-1/\tau_\downarrow)\sin(\theta/2)\cos(\theta/2) & (1/\tau_\uparrow)\sin^2(\theta/2)+(1/\tau_\downarrow)\cos^2(\theta/2) \end{pmatrix}. \tag{2}$$

Equations (1) and (2) apply to each layer. Note that $\boldsymbol{\tau}^{-1}$ is diagonal for nonmagnetic film. When writing Eq. (1) it was assumed that $\mathbf{f}_0(\mathbf{v})\approx f_0(\mathbf{v})\mathbf{1}$. Following the standard procedure, Eq. (1) will be solved in each film separately, and the solutions will be matched with the assistance of some boundary conditions. Since it is more convenient to deal with Eq. (1) (and also with the appropriate boundary conditions) in the local coordinate systems, we introduce, as in Ref. 15, a fictitious plane inside the internal layer, say at $y=y_0$, and will use for $y<y_0$ the local quantization axis of the left magnetic film, while for $y>y_0$ the local quantization axis of the right magnetic film.

In the local coordinate systems general solutions for the four components of the distribution function can be found very easily from Eq. (1). The appropriate constants, which occur in those solutions, can be then determined from boundary conditions at the interfaces and surfaces. In a local coordinate system the boundary conditions for transmission across the βth ($\beta=1,2$) interface between the ferromagnetic and nonmagnetic films are

$$\mathbf{g}^{\pm}(y=y_\beta\pm 0)=\mathbf{T}_\beta\mathbf{g}^{\pm}(y=y_\beta\mp 0), \tag{3}$$

where $+$ and $-$ at the distribution function $\mathbf{g}$ correspond respectively to $v_y>0$ and $v_y<0$. For clarity of notation, the argument $\mathbf{v}$ of the distribution function has been omitted in the above formulas. Apart from this, $y=y_\beta-0$ and $y=y_\beta+0$ mean that the distribution function is taken respectively on the left and right sides of the βth interface ($y_1=d_1$ and $y_2=d_1+d_0$). In Eq. (3) $\mathbf{T}_\beta$ is defined as

$$\mathbf{T}_\beta=\begin{pmatrix} T_{\beta\uparrow} & 0 \\ 0 & T_{\beta\downarrow} \end{pmatrix}, \tag{4}$$

with $T_{\beta\uparrow}$ and $T_{\beta\downarrow}$ being the coefficients of specular transmission across the interface, respectively, for spin-majority and spin-minority electrons.

At the left ($y_l=0$) and right ($y_r=d_1+d_0+d_2$) outer surfaces we use the Fuchs-type boundary conditions,

$$\mathbf{g}^{+}(y=y_l)=\mathbf{P}_l\mathbf{g}^{-}(y=y_l), \tag{5}$$

$$\mathbf{g}^{-}(y=y_r)=\mathbf{P}_r\mathbf{g}^{+}(y=y_r), \tag{6}$$

where

$$\mathbf{P}_{l(r)}=\begin{pmatrix} p_{l(r)\uparrow} & 0 \\ 0 & p_{l(r)\downarrow} \end{pmatrix}, \tag{7}$$

with $p_{l(r)\uparrow}$ and $p_{l(r)\downarrow}$ being the corresponding coefficients of specular reflection, respectively for spin-majority and spin-minority electrons.

At the fictitious plane at $y=y_0$ within the nonmagnetic film, the boundary conditions are

$$\mathbf{g}^{\pm}(y=y_0+0)=\mathbf{R}^{-1}(\phi)\mathbf{g}^{\pm}(y=y_0-0)\mathbf{R}(\phi). \tag{8}$$

Since the distribution functions on the right and left sides of the fictitious plane are written in different local coordinate systems, we matched them after rotating the spin quantization axis on one side by the angle ϕ between the magnetizations. This rotation is described by the matrix

$$\mathbf{R}(\phi)=\begin{pmatrix} \cos(\phi/2) & -\sin(\phi/2) \\ \sin(\phi/2) & \cos(\phi/2) \end{pmatrix}. \tag{9}$$

Equation (8) does not depend explicitly on y_0. For calculations one can simply assume y_0 at one of the interfaces.

In the following we will consider two cases: (i) when the GMR effect is of bulk origin, and (ii) when it is due to interfacial scattering. In the former case we assume some spin asymmetry in the relaxation times (or equivalently in MFP's) in the magnetic films, while transmission coefficients across the interfaces are assumed to be independent of the electron spin. In the latter case, on the other hand, MFP's are independent of the spin orientation, while $T_{\beta\uparrow}\neq T_{\beta\downarrow}$. To simulate a superlattice we assume in both cases perfectly reflecting boundary conditions at the external surfaces, $p_{l(r)\uparrow(\downarrow)}=1$.

Numerical results for the magnetoresistance normalized to its maximum value, GMR(ϕ)/GMR($\phi=\pi$), are shown in Figs. 1 and 2, respectively for the case (i) and case (ii), as described above. Different curves in Fig. 1 correspond to different values of the electron MFP λ_0 in the nonmagnetic spacer. For all curves, however, $\lambda_{\alpha+}/\lambda_{\alpha-}=5$ and $2\lambda_0/(\lambda_{\alpha+}+\lambda_{\alpha-})=1$ was assumed for both $\alpha=1$ and $\alpha=2$. GMR varies almost linearly with $\sin^2(\phi/2)$ and only a small difference between the curves corresponding to $\lambda_0=100$ Å (solid line) and $\lambda_0=1000$ Å (dashed line) can be noticed. One has, however, bear in mind the fact that for large MFP's the quasiclassical description ceases to be valid.

In Fig. 2 the magnetoresistance is of interfacial origin. As in Fig. 1, different curves correspond to different values of λ_0. However, $\lambda_{\alpha+}=\lambda_{\alpha-}=\lambda_0$ for $\alpha=1,2$ and $T_{\beta\uparrow}/T_{\beta\downarrow}=2$ for $\beta=1,2$ was now assumed for all curves. Again, GMR varies linearly with $\sin^2(\phi/2)$, although now the difference between the curves for $\lambda_0=100$ Å and $\lambda_0=1000$ Å is more remarkable than in Fig. 1. Thus, independently of the origin, GMR varies almost linearly with $\sin^2(\phi/2)$ for a uniform electronic potential.

The question arises now, whether a similar behavior also takes place in the case of a spin dependent superlattice potential. To check this point the method described above has been extended to the model of Hood *et al.*[17] which takes into account electron-band misfit at the interfaces. Assume that

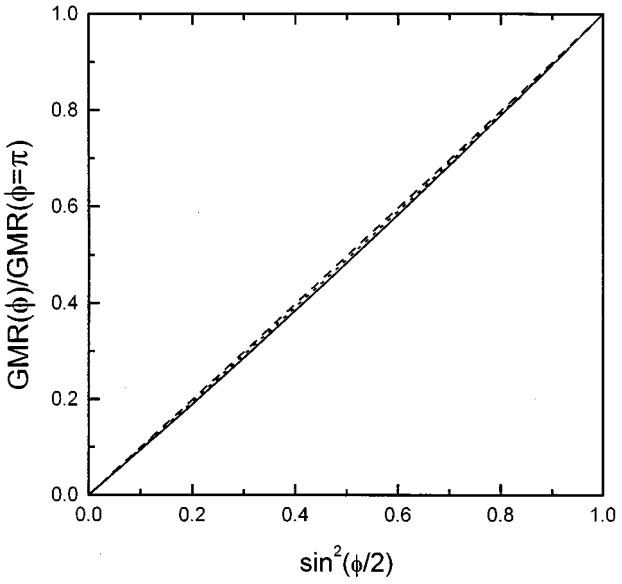

FIG. 1. Angular dependence of GMR for a superlattice with a uniform electronic potential across the structure. Different curves correspond to different values of λ_0; $\lambda_0=100$ Å (solid line), $\lambda_0=300$ Å (dotted line), and $\lambda_0=1000$ Å (dashed line). In each case, however, $\lambda_{1\downarrow}/\lambda_{1\uparrow}=\lambda_{2\downarrow}/\lambda_{2\uparrow}=5$ and $(\lambda_{1\uparrow}+\lambda_{1\downarrow})/2=(\lambda_{2\uparrow}+\lambda_{2\downarrow})/2=\lambda_0$ was assumed. The other parameters are $E_F=4$ eV, $T_{1\uparrow}=T_{1\downarrow}=T_{2\uparrow}=T_{2\downarrow}=1$ (no roughness), $d_0=20$ Å, while the magnetic films are 100 Å thick.

the electron potential in the magnetic films ($\alpha=1,2$) is equal to $U_{\alpha\uparrow}$ for spin-majority electrons and $U_{\alpha\downarrow}$ for spin-minority ones, while in the nonmagnetic spacer it is equal to U_0 for both spin orientations. The corresponding results are shown in Fig. 3 for three different values of the potential barriers at

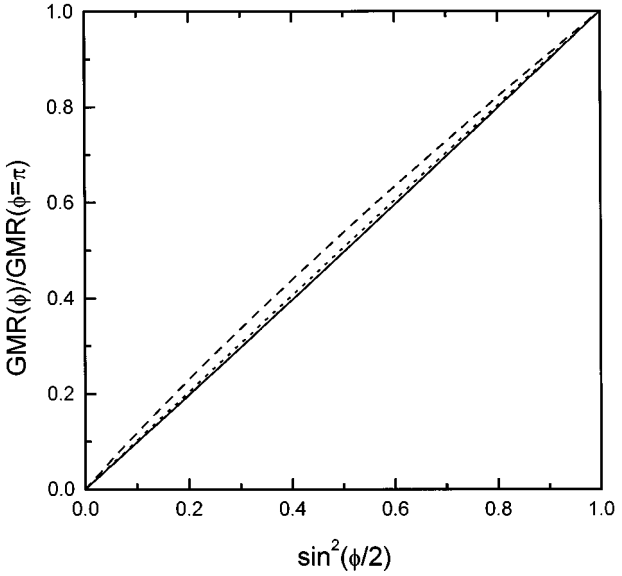

FIG. 2. Angular dependence of GMR for a superlattice with a uniform electronic potential for $\lambda_0=100$ Å (solid line), $\lambda_0=300$ Å (dotted line), and $\lambda_0=1000$ Å (dashed line). In each case, however, $\lambda_{1\uparrow}=\lambda_{1\downarrow}=\lambda_{2\uparrow}=\lambda_{2\downarrow}=\lambda_0$ was assumed. The other parameters are $E_F=4$ eV, $T_{1\uparrow}=T_{2\uparrow}=0.5$, $T_{1\downarrow}=T_{2\downarrow}=1$, $d_0=20$ Å, while the magnetic films are 100 Å thick.

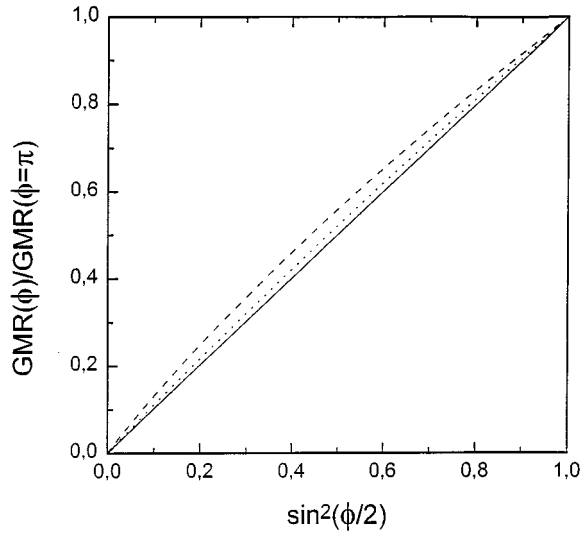

FIG. 3. Angular dependence of GMR for a superlattice with a spin dependent electronic potential in the magnetic films; $U_{1\uparrow}=U_{2\uparrow}=0$, $U_{1\downarrow}=U_{2\downarrow}=1$ eV (solid line), $U_{1\uparrow}=U_{2\uparrow}=1$ eV, $U_{1\downarrow}=U_{2\downarrow}=2$ eV (dotted line), and $U_{1\uparrow}=U_{2\uparrow}=1$ eV, $U_{1\downarrow}=U_{2\downarrow}=3$ eV (dashed line). The other parameters are $U_0=0$, $E_F=4$ eV, $(\lambda_{1\uparrow}+\lambda_{1\downarrow})/2=(\lambda_{2\uparrow}+\lambda_{2\downarrow})/2=\lambda_0=300$ Å, $\lambda_{1\downarrow}/\lambda_{1\uparrow}=\lambda_{2\downarrow}/\lambda_{2\uparrow}=5$, $d_0=20$ Å, while the magnetic films are 100 Å thick. The interfaces are assumed perfectly flat.

the interfaces. The interfaces were assumed there perfectly flat and the GMR effect was due to spin dependent impurity scattering. It is evident that large spin dependent potential barriers produce only weak deviations from the perfectly linear dependence. The same dependence occurs also for trilayer structures with arbitrary rate of specular electron reflection at the outer surfaces. Thus, the almost linear behavior seems to be a general feature of the quasiclassical limit.

III. QUANTUM LIMIT

Let us analyze now how the quantum effects modify the angular dependence. To do this we consider the same trilayer structure as in the preceding section and assume that the outer surfaces are perfectly flat. The corresponding single electron Hamiltonian H is of the form

$$H=-\frac{\hbar^2}{2m}\nabla^2+U(y,\boldsymbol{\sigma})+V(\mathbf{r},\boldsymbol{\sigma}). \tag{10}$$

The first term is the usual kinetic term, which is diagonal in the spin space. The second term, $U(y,\boldsymbol{\sigma})$, is the periodic part of the electronic potential (crystal potential). The last term in Eq. (10) is the scattering potential of impurities and interface roughness, which is assumed in the form

$$V(\mathbf{r},\boldsymbol{\sigma})=\sum_{i\alpha} v_\alpha(\boldsymbol{\sigma})\,\delta(\mathbf{r}-\mathbf{r}_{i\alpha})-(-1)^\beta \times\sum_\beta h_\beta(\mathbf{R})V_\beta^{\mathrm{eff}}(\boldsymbol{\sigma})\,\delta(y-y_\beta^c(\mathbf{R})). \tag{11}$$

In the first term on the right side of Eq. (11) $v_\alpha(\boldsymbol{\sigma})$ denotes the spin dependent scattering potential of an impurity located inside the αth layer and the index i runs over all impurities distributed inside this film. The impurity potential $v_\alpha(\boldsymbol{\sigma})$ is assumed to be diagonal in the spin space when the quantization axis is colinear with the local magnetization direction in the magnetic films, with respectively $v_{\alpha+}$ for the majority electrons and $v_{\alpha-}$ for the minority electrons. The scattering potential of impurities in the spacer layer is assumed independent of the electron spin, $v_{0+}=v_{0-}\equiv v_0$. In the second term of Eq. (11) $h_\beta(\mathbf{R})$ describes deviation of the βth interface from the perfectly flat plane $y=y_\beta$, with $\mathbf{R}$ denoting the in-plane position vector. By definition, $\langle h_\beta\rangle=0$ and $\eta_\beta=\langle h_\beta^2\rangle^{1/2}$ is the rms roughness amplitude. For simplicity, the same exponential form of the correlation function $G(R/\xi)$ will be assumed for both interfaces. The correlation length ξ_β can generally be different for both interfaces. Apart from this, $V_\beta^{\mathrm{eff}}=V_\beta^{(1)}(\boldsymbol{\sigma})+V_\beta^{(2)}(\boldsymbol{\sigma})$, where $V_\beta^{(1)}(\boldsymbol{\sigma})=U_\beta(\boldsymbol{\sigma})-U_0(\boldsymbol{\sigma})$ is the spin dependent potential step at the βth interface and $V_\beta^{(2)}(\boldsymbol{\sigma})$ is a scattering potential which effectively takes into account s-d scattering induced by the interface roughness.[16] Both components $V_\beta^{(1)}(\boldsymbol{\sigma})$ and $V_\beta^{(2)}(\boldsymbol{\sigma})$ are diagonal in the local coordinate systems. Finally, $y_\beta^c(\mathbf{R})=y_\beta+(1/2)h_\beta(\mathbf{R})$ is the position center of the interfacial scattering potential.

To find the electronic conductivity within the formalism described in Ref. 16, we have to calculate first electronic states (miniband edges) for an arbitrary angle between the magnetizations. This can be done in a similar way as in Ref. 18, where the electronic states for perpendicular orientation of the film magnetizations where calculated. It is convenient to use the local coordinate systems. The corresponding scattering potentials are then diagonal in the spin space. The appropriate Schrödinger equation was solved first in each layer separately and the relevant constants were determined from the normalization condition and the standard boundary conditions of continuity of the wave function and its first derivative across the interfaces. For simplicity, we assume infinite potential walls at the external surfaces. The eigenstates have the general form

$$\boldsymbol{\Psi}_{\mu\mathbf{k}}(\mathbf{r})=\begin{pmatrix}\psi_{\mu+}(y)\\ \psi_{\mu-}(y)\end{pmatrix}\exp(i\mathbf{k}\cdot\mathbf{R}), \tag{12}$$

where μ is the miniband index and $\mathbf{k}$ is the in-plane wave vector. The corresponding eigenenergy is $\epsilon_{\mu\mathbf{k}}=\epsilon_\mu+\hbar^2k^2/2m$.

In the Born approximation the transition probability $P_{\mu\mu'}(\mathbf{k},\mathbf{k}')$ is given by

$$P_{\mu\mu'}(\mathbf{k},\mathbf{k}')=\frac{2\pi}{\hbar}\,|\langle\boldsymbol{\Psi}_{\mu\mathbf{k}}|V(\mathbf{r},\boldsymbol{\sigma})|\boldsymbol{\Psi}_{\mu'\mathbf{k}'}\rangle|^2. \tag{13}$$

On calculating the above transition probabilities and following Refs. 16 and 19 one arrives at the following expression for the electronic conductivity:

$$g_\parallel=\frac{e^2\hbar^3}{2m^2L}\sum_{\mu=1}^{N}\sum_{\mu'=1}^{N}Q_\mu^2Q_{\mu'}^2[\mathbf{C}^{-1}(E_F)]_{\mu\mu'}, \tag{14}$$

where N is the number of occupied minibands, Q_μ is the in-plane Fermi wave vector corresponding to the μth miniband, and the matrix $\mathbf{C}(E_F)$ is given by the formula

$$[\mathbf{C}(E_F)]_{\mu\mu'} = \delta_{\mu\mu'} Q_\mu^2 \sum_{\nu=1}^{N} \left[2\pi \sum_{\alpha=1}^{3} n_\alpha^{\mathrm{imp}} D_\alpha^{\mu\nu} + \sum_{\beta=1}^{2} L_\beta^{\mu\nu} (\eta_\beta \xi_\beta)^2 \int_0^{2\pi} d\theta\, F(\xi_\beta Q_{\mu\nu}) \right] - Q_\mu Q_{\mu'} \sum_{\beta=1}^{2} L_\beta^{\mu\mu'} (\eta_\beta \xi_\beta)^2 \times \int_0^{2\pi} d\theta \cos\theta F(\xi_\beta Q_{\mu\mu'}). \quad (15)$$

The factors $D_\alpha^{\mu\nu}$ are defined as

$$D_\alpha^{\mu\nu} = \int_{(d_\alpha)} dy [v_{\alpha+} \psi_{\mu+}^{(\alpha)}(y) \psi_{\nu+}^{(\alpha)}(y) + v_{\alpha-} \psi_{\mu-}^{(\alpha)}(y) \psi_{\nu-}^{(\alpha)}(y)]^2, \quad (16)$$

where the integration is over y ranging the αth layer and $\psi_{\mu\pm}^{(\alpha)}(y)$ are the spinor components in the local coordinate systems. The other two terms in Eq. (15) result from scattering on both interfaces, with $L_\beta^{\mu\mu'}$ defined as

$$L_\beta^{\mu\mu'} = \sum_{j=1,2} \left[V_{\beta+}^{(j)} \left(\psi_{\mu+}^{(\beta)}(y_\beta) \psi_{\mu'+}^{(\beta)}(y_\beta) + \frac{\eta_\beta^2}{4} \psi_{\mu+}^{\prime(\beta)}(y_\beta) \psi_{\mu'+}^{\prime(\beta)}(y_\beta) \right) + V_{\beta-}^{(j)} \left(\psi_{\mu-}^{(\beta)}(y_\beta) \psi_{\mu'-}^{(\beta)}(y_\beta) + \frac{\eta_\beta^2}{4} \psi_{\mu-}^{\prime(\beta)}(y_\beta) \psi_{\mu'-}^{\prime(\beta)}(y_\beta) \right) \right]^2 \quad (17)$$

for $\beta = 1,2$. It is assumed here that the potentials $V^{(1)}(\boldsymbol{\sigma})$ and $V^{(2)}(\boldsymbol{\sigma})$ scatter incoherently. Finally, $F(\xi Q_{\mu\mu'})$ in Eq. (15) is the Fourier transform of the correlation function $G(R/\xi)$, with $Q_{\mu\mu'} = (Q_\mu^2 + Q_{\mu'}^2 - 2Q_\mu Q_{\mu'} \cos\theta)^{1/2}$.

Since the role of bulk scattering was considered in Ref. 10, we will restrict the following numerical calculations mainly to systems where the interfacial scattering is dominant. When considering the problem numerically, one has to take into account variation of the Fermi level with the angle between the magnetizations. This variation takes place only for spin dependent crystal electron potential. The Fermi level should be then adjusted to keep the areal electron density constant. However, the difference between the case with constant particle number and constant chemical potential μ is rather small, so in the following numerical calculations we will assume constant μ.

Consider first a symmetrical structure and let us begin with the situation when the electronic potential in the magnetic films is independent on the spin orientation. Assume for simplicity a quantum well in the spacer layer, which is independent of the spin orientation, i.e., $U_{1+} = U_{1-} = U_{2+} = U_{2-} \equiv U > U_0 = 0$. The geometrical roughness can generate then GMR only via the component $V^{(2)}(\boldsymbol{\sigma})$, provided it is spin dependent. When the potential is uniform across the total system ($U = 0$), then—as shown by Zhang *et al.*[9] in the limit of infinite superlattice—MR varies linearly with $\sin^2(\phi/2)$. This is also the case in the trilayer structure considered here, as shown in Fig. 4 by the dotted line. For $U > 0$ there is still almost a linear dependence of the effect on $\sin^2(\phi/2)$, as shown in Fig. 4 by the solid line. Thus, the electron potential which is independent of the electron spin orientation leads to approximately linear dependence of MR on $\sin^2(\phi/2)$ in symmetrical trilayers. It is noteworthy that in that case the electronic levels are independent of the angle between the magnetizations.

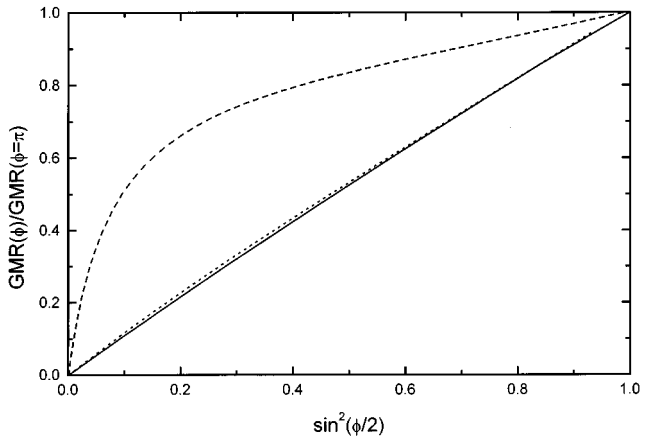

FIG. 4. Angular dependence of GMR generated by interfacial scattering in a symmetrical trilayer with $d_1 = d_2 = 20$ Å, $d_0 = 9$ Å, $\eta_1 = \eta_2 = 2$ Å, $\xi_1 = \xi_2 = 2$ Å, and $\mu = 3.04$ eV. The other parameters are $U_{1+} = U_{2+} = U_{1-} = U_{2-} = U_0 = 0$, $V_{1+}^{(2)} = V_{2+}^{(2)} = 0.5$ eV, $V_{1-}^{(2)} = V_{2-}^{(2)} = 1$ eV (dotted line); $U_{1+} = U_{2+} = U_{1-} = U_{2-} = 0.5$ eV, $U_0 = 0$, $V_{1+}^{(2)} = V_{2+}^{(2)} = 0.5$ eV, $V_{1-}^{(2)} = V_{2-}^{(2)} = 1$ eV (solid line); $U_{1+} = U_{2+} = 0.5$ eV, $U_{1-} = U_{2-} = 0.9$ eV, $U_0 = 0$, $V_{1+}^{(2)} = V_{2+}^{(2)} = V_{1-}^{(2)} = V_{2-}^{(2)} = 0$ (dashed line).

Consider now the case when the electronic potential in the magnetic films is spin dependent. This, consequently, leads to spin dependent potential steps at the interfaces. Now, GMR can be generated by the interfacial roughness not only via the potential $V^{(2)}(\boldsymbol{\sigma})$, but also via $V^{(1)}(\boldsymbol{\sigma})$. The corresponding results are shown by the dashed line in Fig. 4. There is a large deviation from the linear dependence on $\sin^2(\phi/2)$. Thus, spin dependent potential barriers at the interfaces generate deviations from the linear behavior, as shown first by Vedyayev *et al.*[10] and more recently by Wang *et al.*[20] Note that now the energy levels depend on the angle between the magnetizations.

When the structure is asymmetrical then deviations from the linear dependence can occur also for a uniform electron potential. Two such situations are shown in Fig. 5. The dashed line corresponds to the case where the scattering potentials are symmetrical but the magnetic films differ in thicknesses. The solid line, on the other hand, corresponds to the case where the thicknesses are the same, but the scattering potentials at the two interfaces are asymmetrical. In both cases there are remarkable deviations from the linear variation.

For the curves shown in Fig. 4 the maximum value of GMR corresponds to $\phi = \pi$. However, this is not a general

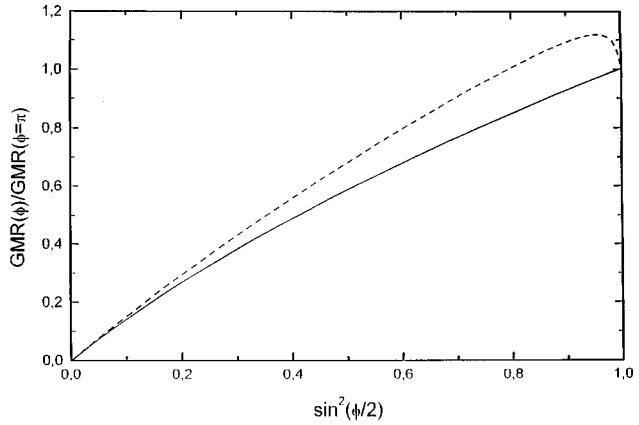

FIG. 5. Angular dependence of GMR generated by interfacial scattering in a trilayer for $\eta_1=\eta_2=2$ Å, $\xi_1=\xi_2=2$ Å, $\mu=3$ eV, and $U_{1+}=U_{2+}=U_{1-}=U_{2-}=U_0=0$. The other parameters are $V_{1+}^{(2)}=V_{2+}^{(2)}=0.5$ eV, $V_{1-}^{(2)}=V_{2-}^{(2)}=0.9$ eV, $d_1=20$ Å, $d_2=15$ Å, and $d_0=9$ Å (dashed line); $V_{1+}^{(2)}=0.5$ eV, $V_{2+}^{(2)}=0.2$ eV, $V_{1-}^{(2)}=0.6$ eV, $V_{2-}^{(2)}=0.9$ eV, $d_1=d_2=20$ Å, and $d_0=9$ Å (solid line).

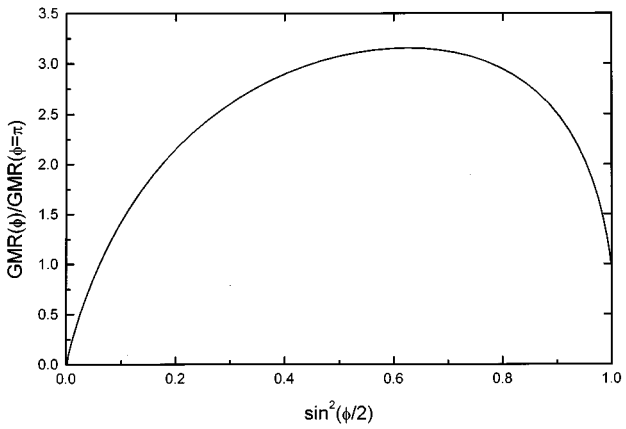

FIG. 6. Angular dependence of GMR generated by interfacial scattering in an asymmetrical trilayer for $d_1=d_2=20$ Å, $d_0=9$ Å, $\eta_1=\eta_2=2$ Å, $\xi_1=\xi_2=2$ Å, $\mu=3$ eV, $U_{1+}=U_{2+}=0.2$ eV, $U_{1-}=1$ eV, $U_{2-}=0.3$ eV, $U_0=0$, and $V_{1+}^{(2)}=V_{2+}^{(2)}=V_{1-}^{(2)}=V_{2-}^{(2)}=0$.

rule as shown already by Vedyayev *et al.*,[10] and what is also visible in Fig. 5 (dashed curve). Here, we point out an interesting situation, when the scattering potentials of both interfaces have the same sign of the spin asymmetry, but differ significantly in the magnitude of this asymmetry—it is large at one interface and significantly smaller at the second interface. The corresponding results are shown in Fig. 6, where GMR is normalized to its value at $\phi=\pi$. The maximum occurs now at $\phi\neq\pi$ and there is a large decrease of GMR when antiferromagnetic alignment is approached. In the ferromagnetic configuration the electrons in one of the two spin channels are weakly scattered, which leads to a large conductivity. In the antiparallel configuration there is also one spin channel where the electrons are scattered rather weakly, although stronger than in the ferromagnetic configuration. For oblique alignment, on the other hand, all electrons are subject to the large scattering potential at one of the interfaces, which results in a large decrease of the conductivity.

IV. SUMMARY

We have calculated the angular variation of the giant magnetoresistance in magnetic layered structures in the quasiclassical and quantum limits. In a general case, the quasiclassical transport leads to a linear dependence of GMR on $\sin^2(\phi/2)$.[21] The situation is qualitatively different in the quantum limit. In that case the linear behavior occurs only for symmetrical structures with the crystal potential independent of the spin orientation. Significant deviations from the linear dependence of the effect on $\sin^2(\phi/2)$ were found either in asymmetrical structures or for spin dependent electron potential.

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