Anomalous behavior at a superconducting quantum critical point

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(Received 3 April 1997)

Motivated by pressure experiments on UBe_{13} and $Tl_2Ba_2CaCu_2O_8$, we discuss low-temperature effects of the pairing interaction at a superconducting quantum critical point in a *clean* system. We point out that measurements at this quantum critical point can provide a diagnostic tool to mark out non-BCS mechanisms of superconductivity. [S0163-1829(97)03133-0]

Current experimental studies keep multiplying the variety of examples of non-Fermi-liquid (NFL) behavior in solid state physics. Many heavy fermion compounds[1] and high-temperature superconductors[2] fall into yet do not exhaust this growing class of materials. In many cases the origins of NFL behavior remain controversial. However, the growing body of experimental evidence[3] confirms that in some itinerant magnets such as MnSi and $ZrZn_2$ the unusual low-temperature behavior appears due to closeness to the quantum critical point. Indeed, the existing theory[4,5] predicts singular behavior in this case, and strong phenomenological arguments have been advanced for similar explanation of NFL behavior of certain heavy fermion systems such as $U_{0.2}Y_{0.8}Pd_3$ and $UCu_{3.5}Pd_{1.5}$.[6]

The problem arises whether NFL behavior can appear near a zero-temperature superconducting transition. It has been studied extensively in cases when the transition temperature T_c is suppressed to zero by perturbations which violate the time reversal symmetry—such as magnetic field or magnetic impurities.[7] However, given both the time reversal and the translational invariance, the question finds simple answer: in the framework of the BCS theory[8] pairing interaction cannot give rise to any anomalous behavior close to the normal-to-superconducting transition at $T_c=0$. The reason is that the finite-temperature BCS instability occurs for an arbitrarily weak effective attraction between electrons. The transition temperature T_c of a BCS superconductor turns into zero only at the zero value of the coupling constant λ, when pairing interaction vanishes together with all its manifestations. It is important to note that this also holds for ''unconventional'' superconductors with nonzero orbital momentum of Cooper pairs.

In this paper, we would like to point out that C-odd superconducting pairing[9] in a *clean* system is a model example for which T_c turns into zero at a finite value of λ. Hence singular contributions of the pairing interaction to the thermodynamic and transport properties at $T_c=0$ do appear, as opposed to clean BCS or even ''unconventional'' superconductors. By C-odd we refer to the parity of the gap function under the charge conjugation C. Asymptotically close to the Fermi surface this symmetry operation turns a particle into a hole and is realized as reflection in the Fermi surface: $\delta k\hat{n}\leftrightarrow-\delta k\hat{n}$. Here δk denotes the deviation of the momentum from the Fermi surface along the local normal $\hat{n}$. For the sake of convenience, hereafter we will refer to the C-odd pairing as to ''odd.''

Indeed, at first sight ''odd'' pairing appears to be a quite exotic and unlikely possibility. However, as pointed out in Refs. 9 and 10, disappearance of the gap at the Fermi surface affords superconductivity in a system with *strong* Coulomb repulsion, where conventional types of pairing are rendered impossible.

Experimentally, our restriction of the problem to the ''clean'' case is supported by the observation of the large negative pressure coefficient dT_c/dP in clean UBe_{13} (Ref. 11) and, more recently, in clean $Tl_2Ba_2CaCu_2O_8$ (Ref. 12) at high pressures. Were it possible to actually suppress T_c to zero, observation of nontrivial low-temperature behavior at $T_c=0$ would point at a very peculiar nature of superconductivity in these compounds.

We will write down the effective interaction in the ''odd'' channel and will calculate the leading low-temperature corrections to the specific heat and the conductivity at $T_c=0$. To make the presentation self-contained, we will briefly repeat the main steps of the derivation of the ''odd'' pairing state.[9] For illustrative purposes, we will restrict ourselves to a toy model in which the pairing is driven by separable attractive interaction $L_0(\xi_1,\xi_2)$ of quasiparticles with energies ξ_1 and ξ_2, which are both lower than certain cut-off ω_c:[9]

$$L_0(\xi_1,\xi_2)=\lambda\begin{cases} s(\xi_1/\omega_c)s(\xi_2/\omega_c), & |\xi_1|,|\xi_2|<\omega_c, \\ 0, & |\xi_1|,|\xi_2|>\omega_c, \end{cases}$$

where $s(x)$ is an odd function, linear for $x\ll 1$. Such an attraction may arise[13] as an antisymmetric part of a typical boson-mediated interaction:

$$V(\xi_1,\xi_2)\propto[\omega_k^2-(\xi_1-\xi_2)^2]^{-1},$$

where ω_k is the boson energy. The symmetric part of the interaction must be repulsive and strong enough to suppress ''even'' pairing, as argued by Mila and Abrahams.[9] This repulsive part may be due to interactions other then those which generate the attraction. In this case the gap equation

$$\Delta(\xi)=-N\int d\eta L_0(\xi,\eta)\,\frac{\Delta(\eta)}{2\sqrt{\eta^2+\Delta^2(\eta)}}\times\tanh\frac{\sqrt{\eta^2+\Delta^2(\eta)}}{2T}$$

admits only a nontrivial solution

$$\Delta(\xi_p) = \alpha(T) s(\xi_p),$$

which is odd in quasiparticle energy ξ_p. Here N is the quasiparticle density of states at the Fermi surface. The gap is isotropic, independent of frequency, and vanishes linearly at the Fermi surface. Thus the ground state is gapless, although it still manifests long-range order and is translationally invariant.

The dependence of T_c on λ can be obtained by taking the limit $\alpha(T)\to 0$ in the gap equation, which results in

$$1 = \lambda N \int_0^{\omega_c} \frac{d\xi}{\xi}\, s^2(\xi) \tanh \frac{\xi}{2T_c}.$$

Introducing the dimensionless coupling constant $g \equiv \lambda N$, one finds that T_c becomes equal to zero when $g = g_c$ such that

$$1 = g_c \int_0^{\omega_c} \frac{d\xi}{\xi}\, s^2(\xi).$$

Expanding the gap equation in the vicinity of $T_c = 0$ and $g = g_c$, one finds the transition temperature dependence on g in the limit $(g-g_c)/g_c \ll 1$:

$$T_c \propto \omega_c \sqrt{(g-g_c)/g_c}.$$

The separability of $L_0(\xi,\xi')$ allows one to easily find the effective pairing interaction by summing up the ladder series in the Cooper channel. This summation modifies $L_0(\xi,\xi')$ by introducing the denominator $1-\lambda\Pi(q,\Omega)$, describing the pair propagation. In accordance with,[9]

$$\Pi(q,\Omega) = T \sum_{\nu,p} s^2(\xi_p) G(\nu,p) G(\Omega-\nu, p-q).$$

Here $G(\nu,p)$ is a single-electron Green's function and ξ_p is the electron spectrum in the absence of pairing. The sums are taken over the Matsubara frequency ν and momentum p, while q and Ω denote the total momentum and Matsubara frequency of the pair.

Presence of $s^2(\xi_p)$ makes $\Pi(0,0)$ finite; thus, the effective interaction $L(q,\Omega)$ has a pole only if $\lambda > \lambda_c = 1/\Pi(0,0)$, in full agreement with the result of the gap equation analysis. Evaluating $\Pi(q,\Omega)$ on the line $\lambda = \lambda_c$ in the (T,λ) plane, one finds for $T,\Omega, vq \ll \omega_c$:

$$L(q,\Omega) = \frac{L_0(\xi_1,\xi_2)}{a\left(\frac{T}{\omega_c}\right)^2 + \frac{1}{6}\left(\frac{vq}{\omega_c}\right)^2 + b|\Omega| \frac{N'}{N} + \left(\frac{\Omega}{\omega_c}\right)^2 \ln\frac{\omega_c}{\Lambda}}. \tag{1}$$

Hereafter v is the Fermi velocity, a and b are dimensionless nonuniversal constants of the order of 1, $\Lambda = \max\{|\Omega|, vq, T\}$ and N' is the energy derivative of the density of states at the Fermi surface.

For a generic band structure $(N' \neq 0)$, at the lowest energies the superconducting quantum critical point in an ''odd'' superconductor falls into the $z=2$ universality class, in agreement with the hypothesis advanced by Hertz[4] regarding the nature of a normal-to-superconducting transition at $T_c = 0$. However, as the frequency exceeds $T^* \sim \omega_c^2 N'/N \ll \omega_c$, the crossover to ''almost'' $z=1$ takes place. Generally, the assumption $\omega_c N'/N \ll 1$ is equivalent to $\omega_c \ll \epsilon_F$, where ϵ_F is the Fermi energy. In a simpler language, this can be described as a crossover of the effective pairing interaction from quasidiffusive propagation at lowest frequencies to ''almost'' (up to the logarithm) soundlike propagation at higher frequencies. Such a crossover is by no means unusual.[14] However, the logarithmic factor in (1) is quite peculiar; as we will show below, it modifies observable physical properties in the temperature region $T^* < T < \omega_c$.

The real part of $\Pi(q,\Omega)$ at real frequencies follows directly from (1) after the substitution $|\Omega| \to i\Omega$ and does not require any additional calculation:

$$1 - \lambda\ \mathrm{Re}\Pi(q,\Omega) = a\left(\frac{T}{\omega_c}\right)^2 + \frac{1}{6}\left(\frac{vq}{\omega_c}\right)^2 - \left(\frac{\Omega}{\omega_c}\right)^2 \ln\frac{\omega_c}{\Lambda}.$$

Calculating corrections to the specific heat and conductivity requires knowledge of the imaginary part of $L^{-1}(q,\Omega+i0)$. At $\Omega < T^*$ this imaginary part comes in a straightforward way from the term $ib\Omega N'/N$ in (1), while at $\Omega > T^*$ it can be found by evaluating the imaginary part of $\Pi(q,\Omega+i0)$:

$$\mathrm{Im}\Pi(q,\Omega+i0) \propto \begin{cases} \left[\left(\frac{\Omega}{\omega_c}\right)^2 + \frac{1}{3}\left(\frac{vq}{\omega_c}\right)^2\right], & T \ll \Omega, vq < \Omega, \\ \Omega^3/(\omega_c^2 vq), & T \ll \Omega < vq, \\ \frac{\Omega}{T}\left[\left(\frac{\Omega}{\omega_c}\right)^2 + \frac{1}{3}\left(\frac{vq}{\omega_c}\right)^2\right], & \Omega, vq \ll T, \\ \frac{\Omega}{vq}\left(\frac{T}{\omega_c}\right)^2, & \Omega \ll T < vq. \end{cases}$$

Hereafter Re and Im denote the real and the imaginary parts, respectively. In two spatial dimensions, $\mathrm{Im}\Pi(q,\Omega)$ differs from the above expressions only by numerical values of the coefficients.

Corrections to the specific heat and the conductivity can be evaluated separately in the ''low-temperature'' region, $T \ll T^*$, and in the ''high-temperature region'' $T^* \ll T \ll \omega_c$. In the former, one can neglect the term $(\Omega/\omega_c)^2 \ln[\omega_c/\Lambda]$ in (1), while in the latter it is the term $b|\Omega|N'/N$ which has to be omitted.

The specific heat correction follows in a straightforward way from the contribution of Gaussian fluctuations of the pairing interaction to the free energy.[15] In the ''low-temperature'' region $T \ll T^*$, the leading singular correction to the specific heat coefficient is

$$\frac{\Delta C}{T} \sim \frac{1}{\epsilon_F}\left(\frac{\omega_c}{\epsilon_F}\right)^3 \sqrt{\frac{T}{\epsilon_F}}.$$

In the ''high-temperature'' region $T^* \ll T \ll \omega_c$, one finds a rather weak albeit nonanalytic correction $\Delta C/T \propto T^2/\ln(\omega_c/T)$.

The leading correction to the conductivity can be estimated by computing the Aslamazov-Larkin graph,[16] describing conduction of ''superconducting fluctuations'' as of particles with the propagator $L(q,\omega)$. The formula, expressing the Aslamazov-Larkin correction through $L(q,\omega)$, is identical to (7a) of Ref. 17 up to the constant coefficient and reads

$$\Delta\sigma=\sum_p p^2\int \frac{dz}{4\pi T}\frac{1}{\sinh^2\frac{z}{2T}}[\mathrm{Im}L(p,z+i0)]^2.$$

Evaluating this expression at $T\ll T^*$, one finds a $\sqrt{T}$ correction, which is quite unusual for a clean system:

$$\Delta\sigma(T)\sim\left(\frac{\omega_c}{\epsilon_F}\right)^3\sqrt{\frac{T}{\epsilon_F}}.$$

At $T^*\ll T\ll\omega_c$, the correction turns out to behave like $T/\ln^4(\omega_c/T)$. The above ''high-temperature'' correction is also singular and has much stronger temperature dependence than the leading T^2 term of a Fermi liquid. The Maki-Thompson correction[18] turns out to be less singular: at $T\ll T^*$ it behaves as $T^{3/2}$ in three dimensions and as $T\ln T$ in two.

For completeness, we would also like to specify the results for two spatial dimensions. At $T\ll T^*$ similar calculations lead to a $T\ln T$ specific heat correction and to a strongly divergent $1/T$ correction to the conductivity. At $T^*\ll T\ll\omega_c$, the correction to the specific heat coefficient behaves as $T/\ln(\omega_c/T)$. However, the conductivity correction is much more singular: $\Delta\sigma(T)\propto 1/\ln^4(\omega_c/T)$.

To conclude, we point out that low-temperature measurements near the superconducting quantum critical point can provide a diagnostic tool to mark out unusual (non-BCS) mechanisms of superconductivity. We consider a toy model of ''odd'' pairing in a *clean* system and show how quantum critical behavior clearly distinguishes it from a BCS superconductor. In the former the fluctuation corrections give $\sqrt{T}$ temperature dependences of conductivity and the specific heat coefficient. This is a much stronger temperature dependence than that of a *clean* Fermi liquid system. To the contrary, in a BCS superconductor the fluctuation corrections are absent altogether since the coupling constant vanishes at the quantum critical point.

As envisaged by Hertz,[4] under general circumstances the effective pairing interaction at lowest frequencies falls into the $z=2$ universality class. At higher frequencies, the crossover to $z=1$ regime takes place.

It is important to note that the above model of ''odd'' pairing exemplifies a system in which anomalous low-temperature behavior coexists with a perfect Fermi liquid, since pair fluctuations decouple from the elementary Fermi excitations due to disappearance of the gap at the Fermi surface.[19] This means that unusual low-temperature thermodynamics and transport cannot serve as a proof of the Fermi liquid breakdown, unless quasiparticle lifetime has been probed.

I am indebted to P. Coleman for suggesting the topic of the superconducting quantum critical point and for discussions related to this work. I would also like to thank E. Abrahams, N. Andrei, J. Bennetto, G. Kotliar, and J. Moreno for their critical reading of this article and helpful suggestions, and A. Rylyakov and A. Sengupta for useful comments.

[1] H. v. Löhneysen, Physica B **206&207**, 101 (1995).
[2] M. Gurvitch and A. T. Fiory, Phys. Rev. Lett. **59**, 1337 (1987); L. Forro *et al.*, *ibid.* **65**, 1941 (1990).
[3] C. Pfleiderer *et al.*, Physica B **199&200**, 634 (1994); **206&207**, 847 (1995); F. M. Grosche *et al.*, *ibid.* **206&207**, 20 (1995).
[4] John A. Hertz, Phys. Rev. B **14**, 1165 (1976).
[5] A. J. Millis, Phys. Rev. B **48**, 7183 (1993).
[6] A. M. Tsvelik and M. Reizer, Phys. Rev. B **48**, 9887 (1993)
[7] See, e.g., K. Maki, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969).
[8] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[9] M. H. Cohen, Phys. Rev. Lett. **12**, 664 (1964); S. Nakajima, Prog. Theor. Phys. **32**, 871 (1964); K. Miyake, T. Matsuura, and H. Jichu, *ibid.* **72**, 652 (1984); P. W. Anderson, J. Phys. Chem. Solids **52**, 1313 (1991); F. Mila and E. Abrahams, Phys. Rev. Lett. **67**, 2379 (1991); E. Abrahams, J. Phys. Chem. Solids **53**, 1487 (1992); Q. P. Li and R. Joynt, Mod. Phys. Lett. B **6**, 1145 (1992); M. Dobroliubov *et al.*, Europhys. Lett. **26**, 141 (1994); E. Z. Kuchinskii, A. I. Posazhennikova, and M. V. Sadovskii, JETP **80**, 324 (1995).
[10] G. Baskaran *et al.* (unpublished); Mihir Arjunwadkar *et al.* (unpublished).
[11] J. W. Chen *et al.*, in *Proceedings of the XVIIth International Conference on Low-Temperature Physics*, edited by U. Eckern (North-Holland, Amsterdam, 1984), p. 325; M. C. Aronson *et al.*, Phys. Rev. Lett. **63**, 2311 (1989).
[12] N. Môri *et al.*, J. Phys. Soc. Jpn. **59**, 3839 (1990); see also N. E. Moulton *et al.*, Phys. Rev. B **44**, 12632 (1991); V. Z. Kresin *et al.*, *ibid.* **53**, 11831 (1996).
[13] E. Abrahams (private communication).
[14] For example, density fluctuations of water cross over from diffusive propagation at lowest frequencies to soundlike propagation at higher frequencies.
[15] See, e.g., A. M. Tsvelik, *Quantum Field Theory in Condensed Matter Physics* (Cambridge University Press, Cambridge, UK, 1995).
[16] L. G. Aslamazov and A. I. Larkin, Sov. Phys. Solid State **10**, 1104 (1968).
[17] A. G. Aronov, S. Hikami, A. I. Larkin, Phys. Rev. B **51**, 3880 (1995)
[18] K. Maki, Prog. Theor. Phys. **39**, 897 (1968); **40**, 193 (1968); R. S. Thompson, Phys. Rev. B **1**, 327 (1970).
[19] I am indebted to E. Abrahams for pointing this out.