# Inertial mass of the large polaron

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It is demonstrated that studies on the effective mass of the large polaron, including the pioneering work of Landau and Pecar, contain a mistake of principle: namely, they ignore the spatial dispersion of the lattice polarizability. As a result, the maximum phonon group velocity is zero so that polarization cannot follow the polaron motion. Here we are deriving expressions for the inertial effective mass and for the mass coefficient characterizing the velocity dependence of the polaron energy by taking into account the spatial dispersion of the lattice polarizability. The polaron mass turns out to depend on the polaron velocity and on the minimum phase velocity of phonons and can be more than twice as small or large in comparison with conventional polaron theory predictions. [S0163-1829(97)05733-0]

## I. INTRODUCTION

For more than half a century, theorists and experimentalists have studied large radius polarons as a first example of spontaneous symmetry breaking in a spatially homogeneous system. However, several important problems remain in the physics of polarons. Data on the autolocalized charges dynamics often differed essentially from the predictions of theory.[1] In our previous work[2] it was shown that neglecting the spatial dispersion of the lattice polarizability was the principal mistake of the polaron theory from the time of Landau and Pecar.[3,4] If we take the spatial dispersion into account the polaron velocity turns out to be limited by the relatively small value of the maximum group velocity of phonons. This changes radically our notion about the polaron motion in crystals in comparison with conventional polaron theory.

Now we can say that all work on the calculation of the polaron effective mass which does not take into account the spatial dispersion of the lattice polarizability, beginning with the pioneering work of Landau and Pecar,[3] contains a mistake of principle. The nature of this mistake is purely physical. Indeed, neglecting the spatial dispersion means that group velocity of phonons participating in the carrier autolocalization is equal to zero. But in such a case the polarization ''cloud'' cannot move so that consideration of the moving polaron without taking into account the spatial dispersion makes the theory contradictory. The mathematics applied in the calculations of the polaron effective mass masked this mistake because an impermissible expansion of the function $c(\omega)=\text{const}/(\omega^2-\Omega^2)$ having a pole in a power series of $\omega^2/\Omega^2$ was used.[3] If this expansion is not used the calculation shows that the polarization field following the moving charge is delocalized.[5,6,2]

Taking into account the spatial dispersion of the lattice polarizability shows that the polarization field related to the charge carrier is localized only at velocities $v$ lower than the minimum phase velocity $u$ of relevant optical phonons.[2,5] Thus, the polaron does not exist outside this sufficiently narrow interval of velocities ($v<u$), and only in this interval the polaron effective mass has a sense (whereas at $v>u$ the effective mass of the charge carrier in the medium under consideration is equal to the effective mass of the delocalized carrier). An attempt to calculate the polaron effective mass in the model of nonzero dispersion of phonons was made by Davydov and Enolskii,[6] but they obtained the result only for the case of slight phonon dispersion and low polaron velocity ($u,v\ll 25\times 10^4$ cm s$^{-1}$). Below, taking into account the spatial dispersion of lattice polarizability rigorously, an expression for the polaron effective mass correct for any values of $u$ and $v$ ($v<u$) will be obtained. Moreover, as will be seen, not only the energetic mass coefficient characterizing the change of the polaron energy with its velocity can be obtained as was done earlier, but also the expression for the inertial mass of the large polaron can be derived.

## II. INERTIAL MASS OF THE POLARON

The inertial mass of the polaron can be obtained as the time derivation of the polaron momentum $\mathcal{P}$ [in supposition that $v=v(t)$] divided by $dv/dt$:

$$m_{\text{in}}^{**}=\frac{d\mathcal{P}}{dt}\left(\frac{dv}{dt}\right)^{-1}. \tag{1}$$

The operator of the full momentum of the polaron has the form[7]

$$\hat{\mathcal{P}}=\hat{p}+\int dk\hbar k b_k^{+}b_k, \tag{2}$$

where $\hat{p}$ is an operator of the carrier momentum. The motion of the polaron as a whole is naturally characterized by the average momentum of the carrier since it ignores intrapolaron motion of the carrier. If the effective mass of the carrier in the periodic potential $m^*$ does not differ significantly from the electron mass $m_e$, the carrier average momentum can be written as $\overline{p}=m^*v$. (If the difference between $m^*$ and $m_e$ is essential, the velocity dependence of $m^*$ must be taken into account.) Thus, to find the polaron momentum we must obtain the average momentum of phonons participating in the polaron formation. The sum of two these average momentums gives us a certain value of the full momentum of the polaron $\mathcal{P}$.

 

The Hamilton function of the polarization field taking into account the spatial dispersion of the lattice polarizability has the form[6]

$$H=\frac{2\pi}{c\Omega^2}\int\left[\Omega^2P^2+\left(\frac{\partial\mathbf{P}}{\partial t}\right)^2-u^2\mathbf{P}\nabla_{\mathbf{r}}^2\mathbf{P}\right]d^3\mathbf{r}, \quad (3)$$

where $\mathbf{P}$ is the polarization vector, $\Omega$ is a frequency of the logitudinal optic phonons in the center of the Brilloin zone, $c$ is inverse effective dielectric permittivity: $c=1/\varepsilon_\infty-1/\varepsilon_0$, and $u$ is an asymptotic value of the minimum phase velocity of phonons and the maximum group velocity of ones. It is obvious that the character of spatial dispersion of the medium suspectibility in Eq. (3) is equivalent to the following phonon-dispersion law: $\Omega^2(k)=\Omega^2+u^2k^2$. By expansion of the polarization vector in Fourier series, Eq. (3) can be transformed to the following:

$$H=\frac{2\pi}{c\Omega^2}\int[\Omega^2(k)\mathbf{P}_k\mathbf{P}_{-k}+\dot{\mathbf{P}}_k\dot{\mathbf{P}}_{-k}]\frac{d^3\mathbf{k}}{(2\pi)^3}, \quad (4)$$

where $\mathbf{P}_k$ is the Fourier component of the polarization vector. Let us introduce the variable $\mathbf{T}_k=\beta\dot{\mathbf{P}}_k$ (generalized momentum[7]), with $\beta=4\pi/(c\Omega^2)$; then Eq. (3) takes the form

$$H=\int\left[\frac{\beta}{2}\Omega^2(k)\mathbf{P}_k\mathbf{P}_{-k}+\frac{1}{2\beta}\mathbf{T}_k\mathbf{T}_{-k}\right]\frac{d^3\mathbf{k}}{(2\pi)^3}. \quad (5)$$

To carry out the transition to a quantum Hamiltonian the generalized coordinate and momentum $\mathbf{P}_k,\mathbf{T}_k$ are substituted with the corresponding operators $\hat{P}_k,\hat{T}_k$. Keeping in mind the form of the Hamiltonian in the filling numbers representation–$H=\int d^3\mathbf{k}[\hbar\Omega(k)b_k^+b_k+1/2]$–the operators $\hat{P}_k,\hat{T}_k$ can be easily related with the operators for the creation and annihilation of the longitudinal optical phonons with the wave vector $k$:

$$b_k^+=\frac{\Omega(k)\beta}{2\hbar(2\pi)^3}\hat{P}_k-i\frac{1}{2\hbar(2\pi)^3\Omega(k)\beta}\hat{T}_k,$$

$$b_k=\frac{\Omega(k)\beta}{2\hbar(2\pi)^3}\hat{P}_{-k}+i\frac{1}{2\hbar(2\pi)^3\Omega(k)\beta}\hat{T}_{-k}. \quad (6)$$

Now, using Eqs. (6) the operator of the momentum of phonons $\hat{p}_{\rm ph}=\int\hbar\mathbf{k}b_k^+b_kd^3\mathbf{k}$ can be expressed as follows:

$$\hat{p}_{\rm ph}=\int\hbar\mathbf{k}\left[\frac{\Omega(k)\beta}{2\hbar}\hat{P}_k\hat{P}_{-k}+\frac{1}{2\hbar\Omega(k)\beta}\hat{T}_k\hat{T}_{-k}+\frac{i}{2\hbar}\hat{P}_k\hat{T}_{-k}-\frac{i}{2\hbar}\hat{P}_{-k}\hat{T}_k\right]\frac{d^3\mathbf{k}}{(2\pi)^3}. \quad (7)$$

After an inverse substitution of the generalized coordinate and momentum $\mathbf{P}_k,\mathbf{T}_k$, instead of the operators $\hat{P}_k,\hat{T}_k$ in Eq. (7), we obtain the average momentum of phonons of the polaron polarization "cloud" $\overline{p_{\rm ph}}$. Let us choose a coordinate system with the $z$ axis along the polaron velocity $\mathbf{v}$. Then the generalized momentum has the form $\mathbf{T}_k=ik_zv\beta\mathbf{P}_k$. Taking into account that $\mathbf{P}_{-k}=-\mathbf{P}_k$ the average phonon momentum takes the form

$$\overline{p_{\rm ph}}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}\beta\Omega(k)}{2}\mathbf{P}_k^2\left[-1-2\frac{k_zv}{\Omega(k)}-\left(\frac{k_zv}{\Omega(k)}\right)^2\right]. \quad (8)$$

It must be noted that only the $z$ projection of $\overline{p_{\rm ph}}$ ($\overline{p_{{\rm ph}\,z}}$) differs from zero. Besides, only integration of the term with even powers of $k_z$ gives a nonzero contribution to $\overline{p_{{\rm ph}\,z}}$:

$$\overline{p_{{\rm ph}\,z}}=-\beta v\int\frac{d^3\mathbf{k}}{(2\pi)^3}k_z^2P_k^2. \quad (9)$$

To obtain the Fourier component of the polarization vector $\mathbf{P}_k$ we used the ordinary relation between the polarization vector $\mathbf{P}(\mathbf{r},t)$ and the polarization charge $\rho(\mathbf{r},t)$. The last can be expressed through the square of the carrier wave function in the polaron $\psi(\mathbf{r},t)$ using a Green function[2] $G(\mathbf{r},t)$:

$$\rho(\mathbf{r},t)=ec\Omega^2\int G(\mathbf{r}-\mathbf{r}',t)\psi^2(\mathbf{r}')d^3\mathbf{r}',$$

so that the polarization vector will take the form

$$\mathbf{P}(\mathbf{r},t)=\frac{ec\Omega^2}{4\pi}\nabla_{\mathbf{r}}\int\frac{G(\mathbf{r}_1-\mathbf{r}_2,t)\psi^2(\mathbf{r}_2,t)d^3\mathbf{r}_1d^3\mathbf{r}_2}{|\mathbf{r}-\mathbf{r}_1|}. \quad (10)$$

The form of the Green function $G(\mathbf{r},t)$ has been obtained in Ref. 5:

$$\underset{i=1,2}{G_i(\mathbf{r},t)}=\left\{\begin{array}{l}\dfrac{\exp(-\Omega_i[(z-vt)^2/\beta_{1i}^2+r^2]^{1/2}/u_i)}{4\pi u_i^2\beta_{1i}[(z-vt)^2/\beta_{1i}^2+r^2]^{1/2}},\quad v<u_i,\quad \beta_{1i}^2=1-v^2/u_i^2\\[2ex] \dfrac{\cos(\Omega_i[(z-vt)^2/\beta_{2i}^2-r^2]^{1/2}/u_i)}{2\pi u_i^2\beta_{2i}[(z-vt)^2/\beta_{2i}^2-r^2]^{1/2}},\quad v>u_i,\quad \left\{\begin{array}{l}z-vt<0,\\ r<|z-vt|/\beta_{2i}\end{array}\right.\\[2ex] 0,\quad v>u_i,\quad \left\{\begin{array}{l}z-vt<0,\\ r>|z-vt|/\beta_{2i},\end{array}\right.\quad z-vt>0,\quad \beta_{2i}^2=v^2/u_i^2-1.\end{array}\right\}. \quad (11)$$

As it is seen from Eq. (11), at the velocity $v=u$ there is a radical reconstruction of the Green function that results, as it was demonstrated in our previous work[2] in the fact that the localized state cannot exist at $v>u$. The expression for the Green function Eq. (11) was also obtained in Ref. 6. Unfortunately, the authors of Ref. 6 have not used the rigorous expression for $G(\mathbf{r},t)$ in their calculation of the polaron effective mass (they used for this purpose the simplified Pecar's-like expression that completely distorted the expression for the polaron effective mass,[6] in comparison with what we shall obtain below.)

Due to the specific form of Eq. (10) the Fourier component of the polarization vector $\mathbf{P}_k$ is the composition of the Fourier components of functions $\nabla_{\mathbf{r}}(1/|\mathbf{r}|)$, $G(\mathbf{r},t)$, and $\psi^2(\mathbf{r},t)$:

$$\mathbf{P}_k=\frac{ec\Omega^2}{4\pi}\,\frac{4\pi i\mathbf{k}}{k^2+k_z^2}\,\frac{1}{k_z^2(v^2-u^2)-k^2u^2-\Omega^2}\,\psi_k^2, \quad (12)$$

where ${\psi^2}_k$ is the Fourier component for the square of the polaron wave function.

Now, using Eq. (12) in Eq. (9) we obtain the expression for the average momentum of phonons forming the polaron polarization cloud:

$$\overline{p}_{\mathrm{ph}\,z}=\frac{4\pi\beta v e^2c^2\Omega^4}{(2\pi)^3}\int_0^\infty\frac{kdkk_z^2dk_z}{k^2+k_z^2}\times\frac{(\psi_k^2)^2}{(k_z^2(v^2-u^2)-k^2u^2-\Omega^2)^2}. \quad (13)$$

At last, the inertial mass of the polaron $m_{\mathrm{in}}^{**}$ determined in accordance with Eq. (1) has the form

$$m_{\mathrm{in}}^{**}=m^*+\frac{2e^2c\Omega^2}{\pi u^4}\int_0^\infty\frac{kdkk_z^2dk_z}{k^2+k_z^2}\times\frac{k_z^2[1+3(v^2/u^2)]+k^2+\Omega^2/u^2}{(k_z^2(1-v^2/u^2)+k^2+\Omega^2/u^2)^3}\,(\psi_k^2)^2. \quad (14)$$

At polaron velocities $v$ low in comparison with the minimum phase velocity of phonons $u$ ($v\ll u$), the polaron inertial mass can be approximated by the following expression:

$$m_{\mathrm{in},v\ll u}^{**}=m^*+\frac{2e^2c\Omega^2}{\pi u^4}\int_0^\infty\frac{kdkk_z^2dk_z}{k^2+k_z^2}\times\frac{k_z^2[1+6(v^2/u^2)]+k^2+\Omega^2/u^2}{(k_z^2+k^2+\Omega^2/u^2)^3}\,(\psi_k^2)^2. \quad (15)$$

## III. VELOCITY DEPENDENCE OF THE POLARON ENERGY

All the previous works on the polaron effective mass studied not inertial polaron mass but rather the mass coefficient characterizing the change of the polaron energy with the change of its velocity. Let us obtain an expression for this mass coefficient (which we shall also call an energetic mass of the polaron).

The Hamilton function for the system of a charge carrier and polarizable medium has the form[6]

$$H_0+H_{\mathrm{pol}}+H_{\mathrm{int}}=\int d^3\mathbf{r}\left\{\nabla_{\mathbf{r}}\psi^2+\frac{2\pi}{c\Omega^2}\left[\Omega^2\mathbf{P}^2+\left(\frac{\partial\mathbf{P}}{\partial t}\right)^2-u^2\mathbf{P}\nabla_{\mathbf{r}}^2\mathbf{P}\right]-\mathbf{PD}\right\}, \quad (16)$$

$$\mathbf{D}=-e\nabla_{\mathbf{r}}\int\psi^2(\mathbf{r},t)\frac{d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|},$$

where $\psi(\mathbf{r},t)$ is the carrier wave function. Let us consider a change of energy of the polarization and of the interaction between carrier and polarization $H_{\mathrm{pol}}+H_{\mathrm{int}}$ with the velocity. By taking into account the equation of motion for $\mathbf{P}$, this part of the Hamilton function (16) has the form

$$H_{\mathrm{pol}}+H_{\mathrm{int}}=-\frac{1}{2}\int\mathbf{PD}d^3\mathbf{r}+\frac{2\pi}{c\Omega^2}\int d^3\mathbf{r}\left[\left(\frac{\partial\mathbf{P}}{\partial t}\right)^2-\mathbf{P}\,\frac{\partial^2\mathbf{P}}{\partial t^2}\right]. \quad (17)$$

The polarization vector in Eq. (17) is expressed in conformity with Eq. (10). Due to the specific type of integral in Eq. (10) the expression for $H_{\mathrm{pol}}+H_{\mathrm{int}}$ will simplify after the transition to Fourier components. If we choose the coordinate system with the $z$ axis along the polaron velocity $\mathbf{v}$, Eq. (16) can be overwritten in the following form:

$$H_{\mathrm{pol}}+H_{\mathrm{int}}=\frac{1}{2}\int\mathbf{P}_k\mathbf{D}_k\,\frac{d^3\mathbf{k}}{(2\pi)^3}+\frac{4\pi}{c\Omega^2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,k_z^2v^2P_k^2, \quad (18)$$

where the fact that $\mathbf{D}_{-k}=-\mathbf{D}_k$, $\mathbf{P}_{-k}=-\mathbf{P}_k$ is taken into account. Using Eq. (12) and the corresponding expression for $\mathbf{D}_k$ we transform Eq. (18) to the following expression:

$$H_{\mathrm{pol}}+H_{\mathrm{int}}=-\frac{e^2c\Omega^2}{\pi u^2}\int_0^\infty\frac{kdkdk_z}{k^2+k_z^2}\times\frac{k_z^2[1-3(v^2/u^2)]+k^2+\Omega^2/u^2}{(k_z^2(1-v^2/u^2)+k^2+\Omega^2/u^2)^2}\,(\psi_k^2)^2. \quad (19)$$

The mass coefficient characterizing the change of the polaron energy with its velocity obtained from Eq. (19) as $m_{\mathrm{en}}^{**}=2[E(v)-E(0)]/v^2$ where $E$ is the polaron energy has the form:

$$m_{\mathrm{en}}^{**}=\frac{2e^2c\Omega^2}{\pi u^2v^2}\int_0^\infty\frac{kdkdk_z}{k^2+k^2_z}\,(\psi_k^2)^2\left\{\frac{1}{k_z^2+k^2+\Omega^2/u^2}-\frac{k_z^2[1-3(v^2/u^2)]+k^2+\Omega^2/u^2}{(k_z^2(1-v^2/u^2)+k^2+\Omega^2/u^2)^2}\right\}. \quad (20)$$

The expression for $m_{\mathrm{en}}^{**}$ at polaron velocities, low in comparison with the minimum phase velocity of phonons $u$ ($v\ll u$), is easily obtained from Eq. (19):

$$m_{\mathrm{en},v\ll u}^{**}=\frac{2e^2c\Omega^2}{\pi u^4}\int_0^\infty\frac{kdkk_z^2dk_z}{k^2+k_z^2}\,\frac{(\psi_k^2)^2}{(k_z^2+k^2+\Omega^2/u^2)^2}. \quad (21)$$

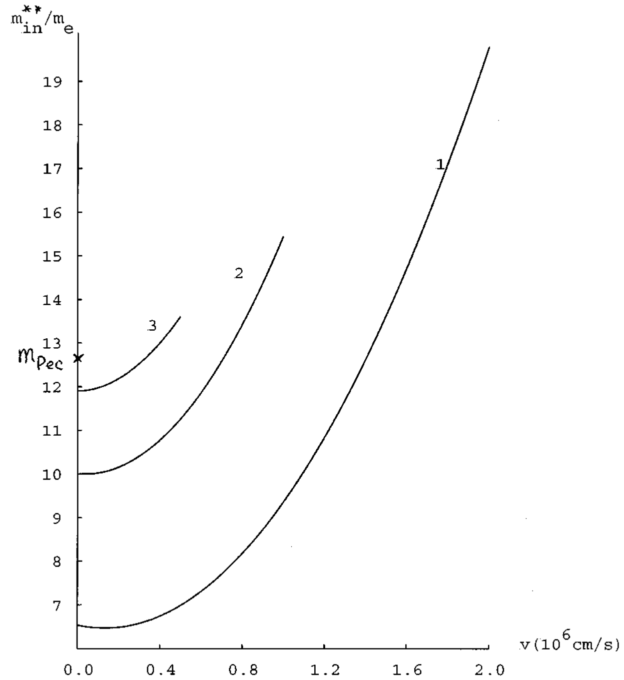


FIG. 1. The inertial effective mass of the polaron as a function of its velocity for different values of the minimum phase velocity of phonons: curves 1, 2, and 3 correspond to $u_1 = 2\times 10^6$ cm s$^{-1}$, $u_2 = 10^6$ cm s$^{-1}$, and $u_3 = 5\times 10^5$ cm s$^{-1}$ accordingly. Other medium parameters are taken as follows $\Omega = 360$ cm$^{-1}$, $c = 0.27$, $m^*/m_e = 1$.

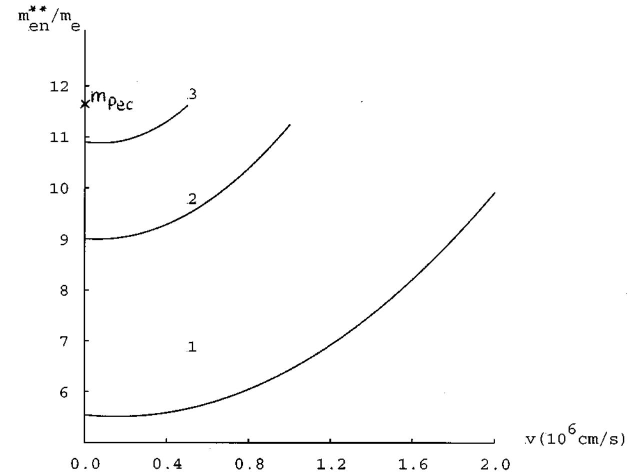


FIG. 2. The energetic mass coefficient characterizing the velocity dependence of the polaron energy as a function of the polaron velocity for different values of the minimum phase velocity of phonons: curves 1, 2, and 3 correspond to $u_1 = 2\times 10^6$ cm s$^{-1}$, $u_2 = 10^6$ cm s$^{-1}$, and $u_3 = 5\times 10^5$ cm s$^{-1}$ accordingly. Other medium parameters are taken as follows $\Omega = 360$ cm$^{-1}$, $c = 0.27$, $m^*/m_e = 1$.

## IV. DISCUSSION

As it is seen from Eqs. (14) and (20) the effective mass of the large polaron (as inertial as ''energetic'') depends not only on the phonon frequency $\Omega$, inverse effective dielectric permittivity $c$, and the ratio $m^*/m_e$ as it was usually considered,[1,3,8] but also on the dispersion of phonons participating in the carrier localization (more exactly, on the minimum phase velocity $u$ of phonons) and on the polaron velocity $v$. Moreover, the energy of the polaron at rest also depends on the minimum phase velocity of phonons [as it is seen from Eq. (19)], which is natural since the characteristic size of the polarization charge distribution generated by the moving point charge [$G(\mathbf{r},t)$, Eq. (11)] is proportional to $u/\Omega$.

Figures 1 and 2 demonstrate the inertial mass and the ''energetic'' mass of the polaron accordingly calculated in conformity with Eqs. (14) and (20) as functions of the polaron velocity $v$ for different values of the minimum phase velocity of phonons $u$. Curve 1 in Fig. 1 and curve 1 in Fig. 2 show $m^{**}_{\rm in}(v)$ and $m^{**}_{\rm en}(v)$ accordingly for the minimum phase velocity of phonons $u = 2\times 10^6$ cm s$^{-1}$; curve 2 in Fig. 1 and in Fig. 2 gives $m^{**}_{\rm in}(v)$ and $m^{**}_{\rm en}(v)$, respectively, for $u = 10^6$ cm s$^{-1}$; and at last, curve 3 in Fig. 1 and in Fig. 2 demonstrate $m^{**}_{\rm in}(v)$ and $m^{**}_{\rm en}(v)$, respectively, for $u = 5\times 10^5$ cm s$^{-1}$. The medium parameters were taken as follows: $\Omega = 360$ cm$^{-1}$, $c = 0.27$, $m^*/m_e = 1$. Pecar's wave function[4] $\psi(r,t) = \alpha^{3/2}(7\pi)^{-1/2}(1+\alpha r)\exp(-\alpha r)$ was used where $\alpha$ is determined by the minimization of the functional of the polaron energy. At considered values of $u$ ($u < 2\times 10^6$ cm s$^{-1}$), the value of parameter $\alpha$ corresponding to the minimum of the functional of the polaron energy, taking into account the polarizability spatial dispersion, does not differ considerably from the one ($\alpha = 0.97c$) obtained by Pecar[4] in the model with zero group velocity of phonons. This is, obviously, owing to the fact that at such values of $u$ the increase of the polaron radius caused by the spatial dispersion of polarizability is not essential. The Fourier component $\psi_k^2$ of the square of Pecar's wave function has the form

$$\psi_k^2 = \frac{16\alpha^4}{7\pi}\frac{1}{(4\alpha^2+k^2+k_z^2)^2} \times\left(1+\frac{96\alpha^4-4\alpha^2(k^2+k_z^2)-(k^2+k_z^2)^2}{(4\alpha^2+k^2+k_z^2)}\right). \quad (22)$$

It can be seen from Eqs. (15) and (21) that for any fixed value of $u$ the limits at $v = 0$ of the inertial mass and ''energetic'' mass of the polaron coincide. This can also be seen from Figs. 1 and 2. By increasing the polaron velocity both its inertial mass and ''energetic'' mass rise owing to the deformation of the polarization charge distribution. Indeed, as seen from Eq. (11) the higher the minimum phase velocity of phonons $u$ the greater the degree of spreading the polarization charge distribution in the polaron. At the polaron velocity $v = 0$ this distribution has spherical symmetry but with an increase of the velocity $v$, it experiences increasing contraction in the direction of the motion (resembling the relativistic contraction). This contraction results in increasing the polarization field energy and, consequently, in increasing the polaron effective mass. The inertial mass rises with $v/u$ more rapidly in comparison with the energetic mass. This situation resembles the behavior of two masses of a relativistic particle (although, of course, the increase in polaron masses with $v/u$ is much smaller than for relativistic particle masses).

Figures 1 and 2 demonstrate that the greater phonon dispersion corresponds to the smaller low-velocity limit of the polaron mass. This effect results naturally from the spreading the polarization charge distribution with an increase in phonon dispersion (or $u$ value) demonstrated by Eq. (11). As a result the energy of the polarization field and, consequently, the polaron effective mass decrease with increasing $u$. The limit at $u \Rightarrow 0$ of the both polaron masses corresponding to the case $v \ll u$ [the limit at $u \Rightarrow 0$ of Eqs. (15) and (21)] has the form

$$\lim_{u \Rightarrow 0, v \ll u} m^{**} = \frac{2e^2c}{\pi\Omega^2} \int \frac{kdkk_z^2dk_z}{k^2+k_z^2} (\psi_k^2)^2. \tag{23}$$

It is easy to show that this limit coincides (as in any physics problem) with the limit $v \Rightarrow 0$, $u=0$ which was considered by Landau and Pecar[3,4] (although, as we discussed in the introduction the polaron states with the velocities $v > u = 0$ cannot exist.)

The point on the axis $v=0$ in Figs. 1, 2 with the notation $m_{\text{Pec}}^{**}$ shows the value of the polaron mass predicted by Landau and Pecar's formula[3,4] for the considered medium parameters. As seen from the figures, due to the strong dependence of the polaron mass on the minimum phase velocity of phonons it can be twice as small or more in comparison with one predicted by the conventional polaron theory ($m_{\text{Pec}}^{**}$). On the other hand, due to the polaron mass dependence on its velocity, the inertial mass of the polaron can be twice as large or more than the mass predicted by the polaron theory. This new notion (and the fact that polarons exist only in the interval of their velocities $v < u$) must change strongly the predictions of the polaron theory for experimentally observable values and effects, as well as notions about the value of the bipolaron effective mass that may be important for the bipolaron theory in high-temperature superconductivity.


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