# Quantum phase and persistent magnetic moment current and Aharonov-Casher effect in a $s=\frac{1}{2}$ mesoscopic ferromagnetic ring

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(Received 7 March 1997)

The quantum phase and persistent magnetic moment current exist in a $s=1/2$ mesoscopic ferromagnetic ring via the Aharonov-Casher effect at finite temperature because of the excitation and propagation of spin waves. We regarded the spin wave as a kind of boson propagating on a $s=1/2$ ferromagnetic background with a magnetic moment $\boldsymbol{\mu}=-\mu\mathbf{e}_z$. The persistent magnetic moment current is in direct proportion to $k_BT$ at low temperature. [S0163-1829(97)01533-6]

The Berry phase[1] has been studied by many physicists and its concept has been applied in different areas of modern physics since its discovery. This topological phase is an extra phase in addition to a standard dynamic phase acquired by the wave function's cyclic adiabatic evolution in an external parameter space. This discovery helps people acquire a better understanding of different topological effects in quantum mechanics. Aharonov and Anandan[2] abandoned the adiabatic limitation and extended the study of the geometric phase to an unadiabatic system. The simplest examples that illustrate the concept of Berry phase are Aharonov-Bohm (AB) effect[3] and Aharonov-Casher (AC) effect.[4] The two effects are different. The AB effect is produced by the quantum orbital motion of charged particles in mesoscopic rings threaded by a magnetic flux. In other words, it comes from the true gauge-invariant coupling $j_\mu A_\mu$ between the current $j_\mu$ and the electromagnetic vector potential $A_\mu$.[5] So the AB effect can be observed if the magnetic field equals zero on the particle's path. On the other hand, the AC effect is produced by the quantum orbital motion of neutral magnetic moments in mesoscopic rings in an electric field. The original reason is the spin-orbit interaction, a coupling of spin current $j_\mu^{\sigma_l}$ to an effective tensor gauge potential $E_\nu\epsilon_{\mu\nu l}$, where $\epsilon_{\mu\nu l}$ is the antisymmetric tensor and $E_\nu$ is the electric field. So a nonzero electric field is necessary to produce the AC phase in the region where particles are propagating.[5,6] A transparent demonstration of the AB effect is a persistent current in mesoscopic rings threaded by a magnetic field. This was predicted by Büttiker *et al.*[7] in 1983 and proved by Lévy *et al.*[8] in 1990. Balatsky and Altshuler[5] studied that the spin-orbit interaction produces persistent spin and mass currents in a ring via the Aharonov-Casher effect and proposed an experiment in the $^3$He-$A_1$ phase in which this effect leads to the excitation of mass and spin supercurrent. The main reason for producing the persistent currents is that the time-reversal symmetry is destroyed when a magnetic flux penetrates to the loop or an electric field parallels the radial direction of the loop. Recently, many physicists intensified their interest in persistent currents in mesoscopic rings.[9–20]

Most studies on persistent currents via AB effect or AC effect are focused on fermions. The total number of fermions is a constant and they occupy levels from lower to upper at low temperature. The energy of the ground state is $E_{gs}=\Sigma_n^{\text{occupied}}E_n$, where $E_n$ is the energy of the $n$th level, $E_n=(\hbar^2/2mr_0^2)[n-\Phi_{AB}\ (\text{or}\ \sigma_z\Phi_{AC})]^2$ at $T=0$ K. Here $\Phi_{AB}$ and $\Phi_{AC}$ are geometric phases via the AB effect, and AC effect, respectively, and $\sigma_z$ is the Pauli matrix along the $\mathbf{e}_z$ direction. The persistent current is $I\sim\Sigma_n^{\text{occupied}}\partial E_n/\partial\Phi_{AB}$ (or $\Phi_{AC}$) and it is dependent on the total number $N$ even or odd. When $N$ is even, $I\sim\Phi_{AB}$ (or $\Phi_{AC}$) $-1/2$, and when $N$ is odd, $I\sim\Phi_{AB}$ (or $\Phi_{AC}$) because all linear terms of $n$ in $I$ cancel out each other at the second case but not at the first case. At low temperature, the persistent current $I$ is not obviously dependent on the temperature $T$ since all particles contribute to $I$ and only a few of the particles near the Fermi level are adjusted their distributions. In 1995, Wei *et al.*[21] studied the quantum phase of induced dipoles moving in a magnetic field and predicted the possible nonzero superfluid velocity in the ground state of the $^4$He condensate. Though it needs more thorough investigation and study to make clear whether such a superfluid exists,[22,23] Wei *et al.* extended the study on persistent currents to boson systems. $^4$He is a boson system and all particles in the mesoscopic ring condense to the lowest level at $T=0$ K. Therefore, it is different to study its quantum phase and persistent currents compared with fermion systems.

In this paper we demonstrate that the AC effect leads to quantum phases and persistent magnetic moment currents when spin waves are propagating in a $S=1/2$ mesoscopic ferromagnetic ring. This study is different from other studies before: (1) The spin wave is a boson excitation and satisfies the rule of Bose-Einstein statistics. Most of them belong to low-energy excitations at low temperature. (2) The total particle number is not conserved. The spin wave is a kind of collective excitation and arises at a finite temperature. Hence its number is dependent on the temperature. (3) Each spin wave has a magnetic moment $-\mu$ when the system is excited to produce it (we assume that all magnetic moments on sites are up at $T=0$ K; it will be explained in the ensuing paragraphs). The quantum phase produced by the AC effect is $\Phi_{AC}=\mu Er_0/\hbar c^2$, where $\mu$ is the magnetic moment, $E$ is the radial electric field in the ring's plane, and $r_0$ is the ring's radius. The persistent current is $I=-\Sigma_{n=0}^{\pm\infty}[f(E_n)/4\pi r_0](\partial E_n/\partial\Phi_{AC})$, where $E_n$ is the energy of the $n$th level, $E_n(\Phi_{AC})=2J\{1-\cos[(n$

 

$-\Phi_{AC})2\pi/N]\}$ and $f(E_n)$ is the distribution function of Bose-Einstein statistics, $f(E_n)=1/(e^{E_n/k_BT}-1)$.

Consider first a one-dimensional $s=1/2$ ferromagnetic ring of radius $r_0$. We describe it using a Heisenberg model on a closed chain of $N$ sites separated by a distance $a=2\pi r_0/N$. The Hamiltonian of this chain can be written as

$$H=-J\sum_l (\mathbf{S}_l\cdot\mathbf{S}_{l+1}+\text{H.c.}), \tag{1}$$

where $J$ denotes the energy of exchange interaction and $\mathbf{S}_l$ stands for the spin operator of site $l$. For ferromagnetic chains, $J>0$. We can rewrite Eq. (1) using $S_l^z, S_l^+, S_l^-$,

$$H=-J\sum_l \left\{\left[S_l^zS_{l+1}^z+\frac{1}{2}(S_l^+S_{l+1}^-+S_l^-S_{l+1}^+)\right]+\text{H.c.}\right\}. \tag{2}$$

A weak magnetic field perpendicular to the plane, $\mathbf{B}=B\mathbf{e}_z$, results in spins of all sites being up. So the ground state is

$$|0\rangle\equiv|\tfrac{1}{2},\tfrac{1}{2}\rangle_1|\tfrac{1}{2},\tfrac{1}{2}\rangle_2\cdots|\tfrac{1}{2},\tfrac{1}{2}\rangle_l|\tfrac{1}{2},\tfrac{1}{2}\rangle_{l+1}\cdots|\tfrac{1}{2},\tfrac{1}{2}\rangle_N, \tag{3}$$

where $|\tfrac{1}{2},\tfrac{1}{2}\rangle_l$ labels the spin of site $l$ is in state $|S_l=\tfrac{1}{2},S_l^z=\tfrac{1}{2}\rangle$, and the total spin and energy are $S_{|0\rangle}^z=\Sigma_l\langle 0|S_l^z|0\rangle=N/2$ and $E_{|0\rangle}=-JN/2$, respectively. Consider a low-excitation state, such as the spin of site $l$ inverses from $|\tfrac{1}{2},\tfrac{1}{2}\rangle$ to $|\tfrac{1}{2},-\tfrac{1}{2}\rangle$. The state of the system can be written as

$$|1\rangle\equiv|\tfrac{1}{2},\tfrac{1}{2}\rangle_1|\tfrac{1}{2},\tfrac{1}{2}\rangle_2\cdots|\tfrac{1}{2},-\tfrac{1}{2}\rangle_l|\tfrac{1}{2},\tfrac{1}{2}\rangle_{l+1}\cdots|\tfrac{1}{2},\tfrac{1}{2}\rangle_N. \tag{4}$$

It will evolve to

$$|1\rangle'\equiv|\tfrac{1}{2},\tfrac{1}{2}\rangle_1|\tfrac{1}{2},\tfrac{1}{2}\rangle_2\cdots|\tfrac{1}{2},\tfrac{1}{2}\rangle_l|\tfrac{1}{2},-\tfrac{1}{2}\rangle_{l+1}\cdots|\tfrac{1}{2},\tfrac{1}{2}\rangle_N \tag{5}$$

by the action of the term $S_l^+S_{l+1}^-$ in Eq. (2) on it. Hence this spin's inversion propagates in the lattice instead of being fixed on one site. That is to say, the inversion of single spins is shown by exchange interactions between sites in the form of collective excitation.

When the ring is put into an electric field $\mathbf{E}$, the inversing spin will obtain an additional phase—a quantum phase $\theta$ (or $-\theta$) besides a dynamic phase when it hops from the $l$th site to the $(l+1)$th site [or to the $(l-1)$th site], and the Hamiltonian will be rewritten as

$$H=-J\sum_l \{[S_l^zS_{l+1}^z+\tfrac{1}{2}(e^{i\theta}S_l^+S_{l+1}^-+e^{-i\theta}S_l^-S_{l+1}^+)]+\text{H.c.}\}. \tag{6}$$

Here $\theta$ is decided by

$$\theta=\frac{1}{\hbar}\int_l^{l+1}\boldsymbol{\mu}\times\frac{\mathbf{E}}{c^2}\cdot d\mathbf{l}, \tag{7}$$

where $\boldsymbol{\mu}$ is the magnetic moment of the inversing spin.

Using operators of spin waves in momentum space, $b_{\mathbf{k}}$ and $b_{\mathbf{k}}^\dagger$, which satisfy the Boson's commutation relation $[b_{\mathbf{k}},b_{\mathbf{k}'}^\dagger]=\delta_{\mathbf{kk}'}$ and $[b_{\mathbf{k}},b_{\mathbf{k}'}]=[b_{\mathbf{k}}^\dagger,b_{\mathbf{k}'}^\dagger]=0$,[24,25] we can obtain a diagonalized Hamiltonian of the ring,

$$H=\sum_k E_k(\theta)b_k^\dagger b_k, \tag{8}$$

where $E_k(\theta)$ is the dispersion of spin waves,

$$E_k(\theta)=2J[1-\cos(ka-\theta)]. \tag{9}$$

According to cyclic boundary condition, we can get $k=2\pi n/Na$ ($N$ is the number of sites in the ring, and $n=0,\pm1,\pm2,\ldots$). Hence the energy of spin waves can be expressed as

$$E_n(\theta)=2J\left[1-\cos\left(\frac{2\pi}{N}(n-\Phi_{AC})\right)\right], \tag{10}$$

where $\Phi_{AC}$ is the sum of quantum phase via the AC effect which an inversing spin gets when it runs around the ring once, $\Phi_{AC}=N\theta/2\pi$. The average number of the $n$th level is $f(E_n)=\langle b_n^\dagger b_n\rangle_T=1/(e^{E_n/k_BT}-1)$. When the system is excited to produce a spin wave, the total spin changes to $S_{|1\rangle}^z=\Sigma_l\langle 1|S_l^z|1\rangle=\Sigma_l\langle 0|b_kS_l^zb_k^\dagger|0\rangle=N/2-1=S_{|0\rangle}^z-1$ from $S_{|0\rangle}^z=N/2$. Therefore the spin wave can be considered as a kind of boson propagating on a $s=1/2$ ferromagnetic background with a magnetic moment $\boldsymbol{\mu}=-\mu\mathbf{e}_z$.

The persistent magnetic moment current is

$$I=-\sum_{n=0}^{\pm\infty}\frac{f(E_n)}{4\pi r_0}\frac{\partial E_n}{\partial\Phi_{AC}}. \tag{11}$$

For a ring of radius $r_0\sim0.01$ mm, in a radial external electric field $\mathbf{E}\sim10^7\mathbf{e}_r$ V/m, if the magnetic moment of the inversing spin, $\boldsymbol{\mu}\sim-\mu_B\mathbf{e}_z$ ($\mu_B$ is the Bohr magnetic moment), the quantum phase $\Phi_{AC}=\mu Er_0/\hbar c^2\sim10^{-4}$. The energy of the lowest level is $E_0(\theta)=2J[1-\cos(2\pi\Phi_{AC}/N)]\approx J(2\pi\Phi_{AC}/N)^2$; the second is $E_1(\theta)=2J\{1-\cos[2\pi(1-\Phi_{AC}/N)]\}\approx J(2\pi/N)^2$. So $E_1(\theta)/E_0(\theta)\sim\Phi_{AC}^2\sim10^{-8}\ll1$. Extending such approximations to all levels, we can obtain $E_{n\neq0}\ll E_0$. Hence $f(E_{n\neq0})\ll f(E_0)$, it means that most spin waves occupy the lowest level. Therefore the persistent current of the mesoscopic ring is approximately equal to that due to spin waves occupying the lowest level,

$$I\approx-\frac{f(E_0)}{4\pi r_0}\frac{\partial E_0}{\partial\Phi_{AC}}. \tag{12}$$

If the distance of the two nearest sites $a\sim1$ Å, the total number of sites $N=2\pi r_0/a\sim10^6$ and $E_0\sim10^{-20}$ eV $\ll k_BT$ ($k_BT\approx10^{-4}$ at $T=1.0$ K), and so $f(E_0)\approx k_BT/E_0$. Hence the persistent magnetic moment current is in direct proportion to the temperature, $I\approx-k_BT/2\pi r_0\Phi_{AC}$, because the total number of spin waves is proportional to the temperature.

In summary, we studied the quantum phase and persistent magnetic moment current produced by the AC effect when spin waves are propagating in an $s=1/2$ mesoscopic ferromagnetic ring in an external electric filed. We regarded spin waves as a kind of boson propagating on a $s=1/2$ ferromagnetic background with a magnetic moment $\boldsymbol{\mu}=-\mu\mathbf{e}_z$. The total number of spin waves is not a constant and dependent on the temperature. Most of spin waves occupy the lowest level at low temperature and the number of them is propor tional to the temperature. So the persistent magnetic moment current is dominated by these spin waves which occupy the lowest level and are proportional to the temperature. It will decrease to zero while the temperature drops to $T=0$ K. In this paper we extended the study of the quantum phase and persistent magnetic moment current to a boson system, the total number of which is not conservative at nonzero temperature. We think that this quantum phase and persistent magnetic moment current can be observed by investigating the transportation properties along the direction which is perpendicular to the ring at a finite temperature because this magnetic moment current must affect them. In fact, once an electric current penetrates a ferromagnetic plane, such a quantum phase and magnetic moment current must occur because this electric current must produce a radial electric field in the plane and all inversing spins must couple with this field.

One of authors (Z.L.C.) acknowledges useful discussions with Dr. Y. N. Xie, Dr. T. Li, and Dr. M. Xiao. This work was supported by the NSF of China, the Foundation of 863 Project, and the National Education Foundation of China.

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