Neutron low-index thin-film waveguides with antiresonant high-reflection layers

S. P. Pogossian
CNRS, Laboratoires de Bellevue, EP-95, 1 Place Aristide Briand, F-92195 Meudon, France

A. Menelle
Laboratoire Léon Brillouin (CEA-CNRS), CE Saclay, F-91191, Gif-sur-Yvette, France

H. Le Gall,* J. Ben Youssef, and J. M. Desvignes
CNRS, Laboratoires de Bellevue, EP-95, 1 Place Aristide Briand, F-92195 Meudon, France
(Received 15 October 1996; revised manuscript received 3 March 1997)

Neutron low-index thin-film waveguides and waveguide coupler with antiresonant high-reflection layers are proposed and realized experimentally. In contrast to the conventional thin-film waveguides, neutrons are confined in a region with a refractive index lower than those of the neighboring media. As shown, the in/out coupling of resonant modes is realized by the conventional neutron-reflectometry technique. Original numerical methods are used to evaluate the eigenstates.
[S0163-1829(97)02132-2]

INTRODUCTION

In the recent past a great deal of attention is drawn to thin-film neutron waveguides[1–7] (see for useful references in Ref. 5). A branch of matter wave guided optics is now developed due to the recent experimental observation of a different class of neutron/x-ray thin-film waveguides.[1–4] The confinement of neutrons into thin films can be useful for the study of physical properties of these particles and thin films. The recent progress in thin-film deposition technology permits the production of complex multilayer thin-film structures with a high surface and interface quality. Nevertheless, until now the neutrons guided propagation in thin films is achieved only with poor coupling and guiding efficiency. That is due to the coherence and roughness phenomena which are of high importance for thin films of such dimensions. Actually it is problematic to guide neutrons in most of magnetic thin-film waveguides due to the low-refractive index of the latter.[2,6] We propose in the present paper a low-index waveguide with antiresonant high reflecting layers (ARROW waveguide) in order to obtain efficient coupling and waveguiding of neutrons in thin films with a low refractive index. As demonstrated experimentally these low-index waveguide couplers with a high-reflecting coating are more efficient than those reported previously.[2,4] It will be shown that the ARROW waveguides manifest particular guiding properties and open vast possibilities for practical device applications. Since the refractive index of the air is greater than those of most materials, the neutrons are coupled into these waveguides by a conventional grazing angle reflectometry technique (Fig. 1). Original numerical methods are used to evaluate the eigenstates of these structures.

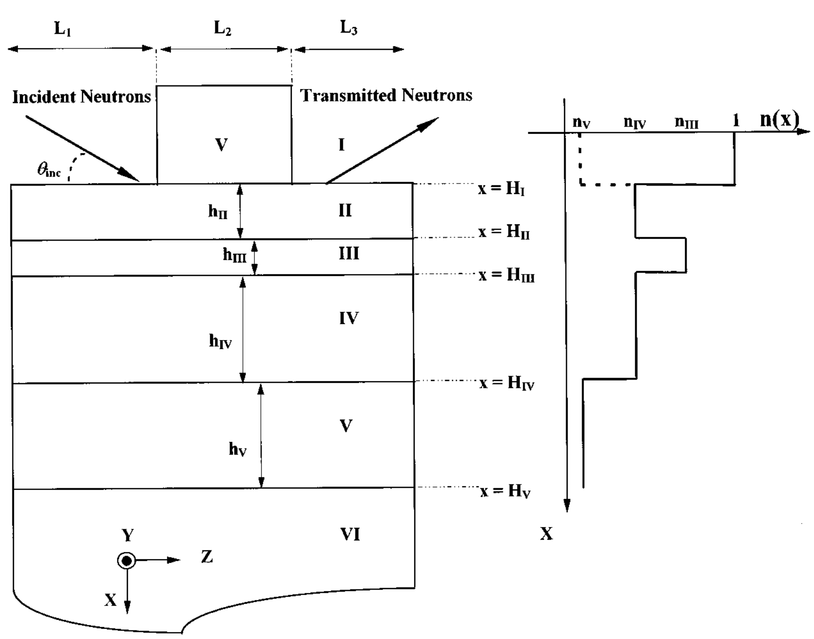

FIG. 1. Schematic of a neutron ARROW waveguide. At the right side the refractive index profile in the coupler (solid line) and waveguide (dotted line) regions is presented.

NEUTRON ANTIREFLECTING LEAKY WAVEGUIDES

The neutrons waveguiding in a thin film is a lossless propagation of the neutron waves along the guide by successful total reflections from the interfaces of surrounding media. It should be noted that as a necessary condition for neutrons waveguiding (lateral resonance), the refractive index of the guiding film should be higher than those of the surrounding media. In many practical problems, however, the neutrons have to propagate in a medium with a refractive index lower than those of the neighboring media.[6,7] That is the case of most of magnetic materials which have a low refractive index for up polarization (up polarized neutrons are those having their spin parallel to the magnetization direction which itself is in the plane of the guiding film). For this purpose many alternative solutions are available. The simplest one is the low-index leaky waveguide structure,[4,8] where the guiding film with a low refractive index is confined between two neighboring media with higher refractive indices. At incident grazing angles the reflectivity coefficient is high from the boundary with the high refractive index for notable refractive index difference of these media. The reflectivity coefficient can be very close to one. In order to increase further the reflection coefficient, two quarter-wave films with high and low refractive indices are deposited between the guiding film of low refractive index and the air of high refractive index. Thus, the thicknesses of the quarter-wave films are chosen to be antiresonant for the given propagation constant of the quasiguided mode,[19] while the thickness of the guiding film satisfies the resonance condition. This kind of low-index waveguide with high reflecting coatings is known in integrated optics as an ARROW waveguide which we have found many interesting applications in semiconductor integrated optics.[9] The main block of ARROW waveguide is thus the low refractive index guiding film[9–13] separated from the air of higher-refractive index by several high reflecting quarter-wave layers. In conventional ARROW waveguides only one pair of high reflecting layers is used which consists of two different quarter-wave films having a large refractive index difference. That provides high reflectivity at each interface of the ARROW structure. The number of high reflecting films can be increased infinitely. Each additional quarter-wave layer increases further the reflectivity coefficient. On the one hand, the greater the number of quarter wave films the higher the reflection coefficient. On the other hand, the roughness induced scattering losses will increase with the number of high reflecting quarter-wave films. In general, any possible array of quarter-wave coatings with arbitrary arrangement may be used for that purpose by providing a high reflectivity at each interface. One possibility is the use of a large number of identical antiresonant blocks with each block consisting of two high reflecting quarter-wave films having a large refractive index difference. If the roughness-induced scattering losses are small (high-quality multilayers), then such a structure will exhibit properties specific to periodic structures thus yielding to the concept of Bragg composite waveguides.[14–16] A guided propagation via total reflections can be achieved by confining the guiding film between two periodic Bragg reflecting quarter-wave mirrors. These mirrors with an infinite number of periods are designed to realize the guided propagation within the stop bands of the periodic media.

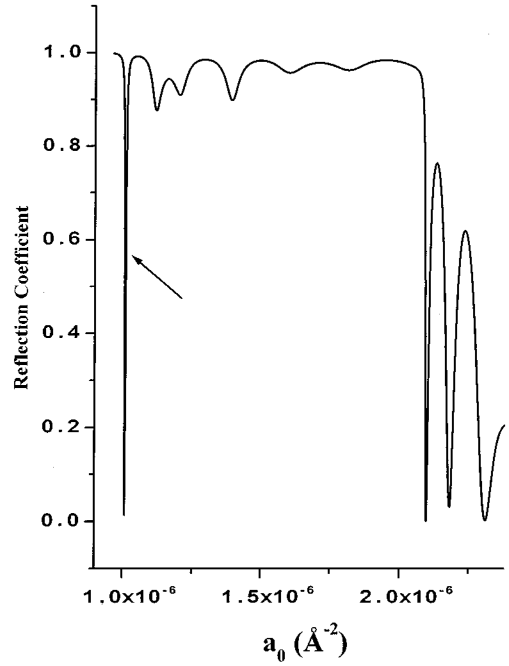

FIG. 2. Reflectivity coefficient for the experimental ARROW waveguide given in Table I. The resonance absorption deep corresponding to the fundamental guided mode is shown by an arrow.

The reflection and refraction of neutrons in a medium is described by an index of refraction. For nonabsorbing materials the refractive index can be written in the following form:

$$n_i^2(\lambda) = 1 - \lambda^2 \delta_i, \tag{1}$$

where δ_i is a constant independent from wavelength and depends only on the physical properties of ith medium. For most of materials $(n_i - 1)$ is of the order of 10^{-6}.

In integrated optics the air has the smallest refractive index and is used for the confinement of the optical energy in similar optical ARROW waveguides from one side.[9–13] In neutronics[6] in contrast to the integrated optics, the refractive index of the air is higher than that of most materials. That opens an interesting possibility, since the fundamental guided mode of a neutron ARROW waveguide can be excited by a conventional neutron-reflectometry technique. That is basically different from the use of ARROW waveguides in integrated optics. The fundamental mode of the neutron ARROW waveguide schematized in Fig. 1 may be observed as a reflection deep in the grazing reflectivity curve (Fig. 2) or by the transmitted guided wave technique (Fig. 6).[2,4] In Fig. 2 we bring in the reflectivity theoretical curve for a neutron ARROW waveguide where the ARROW guided fundamental mode is represented as a reflectivity deep. The parameters of the latter waveguide have been determined by the inverse reflectivity calculation and are given in Table I.

The experimental ARROW guiding structure is schematized in Fig. 1. In the direction of the Z axis the ARROW waveguide is composed of three main parts: In and out ARROW waveguide coupler with lengths, respectively, denoted

TABLE I. In the first column the parameters of the experimental ARROW waveguide are given obtained by the fit to the reflectivity data. In the second column the first two resonances are evaluated from the above theory. In the third and fourth columns the positions and half-height width of two Lorentzian curves fitted to two resonance peaks observed by a transmitted beam technique.

ARROW waveguide parameters	Values of resonant wavelength of two first modes evaluated by the above theory	Experimental resonant wavelengths obtained by transmitted beam technique	Half-height widths of experimental resonance peaks
$\delta_I=0.0$ Å^{-2}	$\lambda_0=6.95$ Å	$\lambda_0=7.38$ Å	$\Delta\lambda_0=0.13$ Å
$\delta_{II}=9.55\times10^{-7}$ Å^{-2}			
$\delta_{III}=6.68\times10^{-7}$ Å^{-2}	$\lambda_1=6.60$ Å	$\lambda_1=6.81$ Å	$\Delta\lambda_1=0.18$ Å
$\delta_{IV}=9.55\times10^{-7}$ Å^{-2}			
$\delta_V=2.07\times10^{-6}$ Å^{-2}			
$h_{II}=1075$ Å			
$h_{III}=430$ Å			
$h_{IV}=2000$ Å			

by L_1 and L_3 and the ARROW waveguide denoted by L_2. The guiding film is denoted by IV and its refractive index and thickness by n_{IV} and h_{IV}, respectively. Since the roughness-induced scattering losses are dramatically dependent on the number of interfaces of high reflecting coating, the number of quarter-wave films, which separate the guiding film from the air, is reduced to a minimum, i.e., to two. Indeed, the minimum number of quarter-wave layers should be two but not one as expected, because the guiding film has an index of refraction smaller than that of the air. In consequence two quarter-wave films are required in order to provide high reflectivity at each interface simultaneously. The two quarter-wave films are denoted by II and III. The refractive indices and thicknesses of latter films are denoted by n_{II}, n_{III} and h_{II}, h_{III}, respectively. The air is denoted by I and the substrate by VI. A buffer layer with the lowest refractive index (n_V) separates the ARROW guiding structure from the substrate in order to provide total reflection from the interface (IV,V). The buffer layer is quite thick (3000 Å) in order to provide complete optical isolation of the guide from the substrate (VI) for the incident angle corresponding to the resonant mode. The neutrons in the guiding film undergo total reflection from the interface with the buffer layer (V) and reflect almost totally from the interface with a quarter-wave film (III). At the right side of Fig. 1 the refractive index profile of a neutron ARROW waveguide is schematized.

For some critical value of the propagation constant, the phase change over one zigzag beam path equals to 2π and a resonant flux of incident neutrons into the guiding film occurs:

$$2q_{IV}h_{IV}-2\tan^{-1}(p_V/q_{IV})-\pi=2\pi m, \tag{2}$$

where $q_{IV}=(k^2n_{IV}{}^2-\beta_m{}^2)^{1/2}$ and $p_V=(\beta_m{}^2-k^2n_V{}^2)^{1/2}$ are transverse propagation constants of the guiding film (IV) and buffer layer (V), respectively. β_m is the lateral propagation constant of the mth guided mode with $m=0,1,2\cdots$. For the fundamental mode $m=0$. $k=2\pi/\lambda$, where λ is the incident wave wavelength. The propagation constant of mth mode is related to the neutrons incident angle θ_{inc} by the following relation: $n_I\cos(\theta_{\text{inc}})=\beta_m/k$. As was proposed in Ref. 5, the resonance condition and the transverse propagation constants can be written in an elegant wavelength independent form by representing β_m in the following form:

$$(\beta_m/k)^2=1-\lambda^2a_m. \tag{3}$$

Then taking into account Eqs. (1) and (3), Eq. (2) takes the following form:

$$2h_{IV}(a_m-\delta_{IV})^{1/2}-\tan^{-1}[\{(\delta_V-a_m)/(a_m-\delta_{IV})\}^{1/2}]/\pi$$
$$=m+1/2. \tag{4}$$

The constant a_m defined from Eq. (4) are thus wavelength independent and are determined by geometrical size of thin film (h_{IV}) and the physical properties of the guiding structure (δ_i). It is important to note that all transverse propagation constants $q_{IV}=2\pi(a_m-\delta_{IV})^{1/2}$ and $p_V=2\pi(\delta_V-a_m)^{1/2}$ become also wavelength independent. That gives a physical signification to Eq. (3) by showing that the guided and nonguided [Eq. (1)] propagation exhibit the same dispersion properties. For the fixed incidence angle the resonant wavelengths of guided modes are related to a_m by the help of the following relation:

$$\lambda_m=\theta_{\text{inc}}/(a_m)^{1/2} \tag{5}$$

In order to achieve maximal reflectivity from the interface (III,IV), the thicknesses of two quarter-wave layers are chosen such as to satisfy the antiresonance condition for the propagation constant corresponding to the resonant fundamental guided mode, i.e., for the mode with $m=0$. Antiresonance requires the total phase change in each quarter-wave film to be equal to π, thus fulfilling the destructive interference condition. Approximately the antiresonance condition for the fundamental mode can be written in the following form:

$$h_i(a_0-\delta_i)^{1/2}=\tfrac{1}{4}, \tag{6}$$

where $i=II,III$.

Formulas (3), (4), and (6) are of great importance in order to estimate roughly the thickness and propagation constant of the fundamental mode. As was mentioned above, on the one hand the thickness of two quarter-wave films are chosen conveniently providing the highest reflectivity for the resonant propagation constant of ARROW guided modes. That condition is fulfilled for the fundamental guided mode. The other high-order modes are effectively filtered out by high leakage losses.[10] On the other hand, the propagation constant of the resonant mode can be evaluated exactly, when the refractive indices and thicknesses of all media are known. Moreover, the fundamental low loss guided mode is leaky and therefore the leakage losses are dramatically dependent on the thickness of the quarter-wave films. Therefore a more refined analysis is required for the exact design and control of all parameters of this structure. In order to describe the propagation in a leaky guiding structure where the energy transverse distribution changes along the propagation length, the radiation mode expansion is to be used.[10] The initial transverse distribution of the field determines completely its further evolution. For that purpose the initial field is represented as an integral of all radiation modes. Since there is no energy exchange between them (they form a complete set of proper functions) each radiation mode propagates with unchanged amplitude but with the phase changing along the propagation length proportional to its propagation constant. At a distance L the phase of each mode acquires a different value. By integrating the field at the distance L all over the radiation modes we obtain the total field. If this approach is used for the description of the propagation in ARROW waveguides, then evidently the initial form of the wave packet becomes of crucial importance for the evaluation of leakage and guiding properties. In the case when the initial beam shape and the ARROW waveguide excitation function are not specified, that may lead to ambiguities or even controversies as analyzed in Refs. 11–13. Since the guided modes of ARROW waveguides are leaky, the conventional complex pole analysis and Abeles well-known matrix method were used by most of authors to evaluate the propagation constants and optimize the thicknesses of quarter-wave films.[10–13,17,18] The Chilwell and Hodgkinson's approximation,[18] mainly used for this kind of optimization, was shown previously by us to give nonphysical solutions for strong leakage.[19] Based on resonance considerations another method was proposed in order to circumvent the infinitely growing wave solutions due to the complex propagation constant approximation.[19] The highly monochromatic neutron plane-wave incident at fixed angle θ_{inc} on the surface of the ARROW waveguide coupler shown in Fig. 1 has a well-defined momentum. Then the propagation constant of a quasiguided mode in the ARROW waveguide coupler can be associated with the resonant wavelength which gives the maximal amplitude of the plane wave in the guiding film (IV). The mathematics used in the present paper are similar to that of Ref. 19. Briefly we recall the bases of the method. The incident wave in I, II, and III, and IV is of a sinusoidal form with unknown phases and amplitudes, which should be determined from the boundary conditions. From the buffer layer (V) total reflection occurs, so the field is of an exponentially decaying form with only one unknown amplitude coefficient. By applying the conventional boundary conditions for the field and its first derivative, all unknown phases of sinusoidal functions are determined as functions of wavelength for the given propagation constant. The field amplitude coefficients can be determined as functions of incident neutron wavelength if the amplitude of incident neutron wave in the air (I) is supposed to be known (may be put equal to one). Then the wavelengths, for which the amplitude coefficient in the guiding film (IV) passes through a maximum, are considered as guided modes of the ARROW waveguide. The amplitude coefficient as a function of the incident neutron wavelength is of Lorenzian-like shape. The half-height width of the latter can be associated with the attenuation coefficient. A brief description of the mathematics is given in Appendix A. A more detailed theory is in Ref. 19. It was shown previously that the subtle difference between the propagation constants and fields of the quasiguided mode in the in/out couplers (L_1, L_3) and the truly guided mode in the waveguide (L_2) can be neglected.[2,4] So in this paper we have not taken into account the scattering and the reflection at the boundaries of the guide with in/out couplers. In ARROW waveguides the guiding film and the quarter-wave film II are made of the same material so they have the same refractive index. Then from formulas (3), (4), and (6) it can be shown that for the fundamental mode the thickness of the quarter-wave film II should be approximately half the thickness of the guiding film IV. Thus for the given thickness of the guiding film, the propagation constant and the leakage losses depend only on the refractive index and the thickness of the quarter-wave film III. In Figs. 3 and 4 we have plotted the behavior of the propagation constant and the half-height width of the resonance as a function of the thickness of the quarter-wave film III. It can be noticed that the optimal thickness for the minimum leakage determined from Eqs. (3), (4), and (6) does not coincide with that obtained by the present approach. The difference is of the order of 20%. The same concerns the propagation constant which gives a smaller difference of the order of 0.05%.

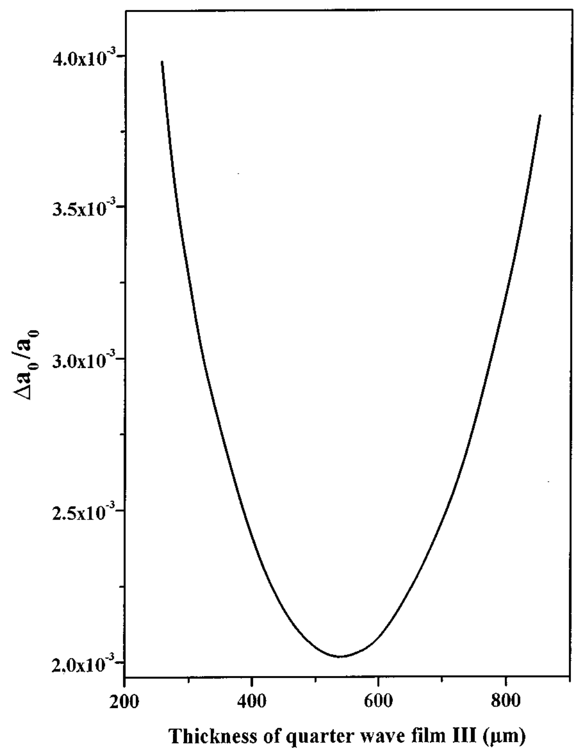

FIG. 3. Plot of the half-height width of the guided mode resonance as a function of the thickness of the quarter-wave film III (Fig. 1) evaluated by the theory (Ref. 19).

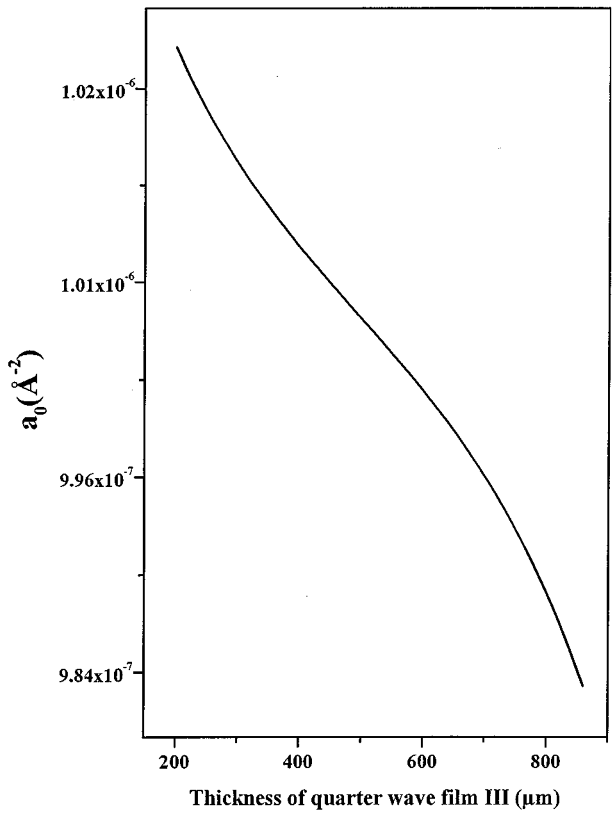

FIG. 4. Plot of the associated effective refractive index a_0 as a function of the thickness of the quarter-wave film III (Fig. 1) evaluated by the theory (Ref. 19).

As was shown above the propagation constant of a neutron ARROW waveguide is obtained by requiring the field amplitude coefficient in the guiding film to be maximal. This procedure requires the evaluation of the phase and amplitude constants of all layers by recursion formulas. In the case when the refractive index of the guiding film (IV) is not constant but has a continuous distribution, it should be dissected into many thinner films in which the refractive index is considered to be constant. In this case the definition of the quasiguided mode has to be modified slightly. A quasiguided mode then corresponds to the state when the intensity function in the guiding region composed of the stack of dissected thin films achieves its maximal value. Analogous to Ref. 19, the intensity function is defined as the sum of squares of all amplitude coefficients of dissected thin films constituting the guiding region divided by the amplitude coefficient of the field in the air. In the same way the antiresonance condition can be formulated for antiresonant guiding blocks with continuous refractive index distribution. Thus in the latter case the refractive index is analogously dissected into many thin films where the refractive index is considered to be constant. Then the antiresonant condition requires the sum of squares of individual amplitude coefficients to be minimum. If the number of antiresonant quarter-wave films is large or in the case of continuous refractive index distribution, the above approach may be substituted by a more efficient algorithm. The main inconvenience of the above method applied to the multilayer ARROW waveguides comes from the need of evaluation of the phase and the amplitude coefficient in each layer recursively from the boundary matching of the fields and their first derivatives. It will be more efficient to implement such a method which automatically takes into account the boundary matching of the wave function and its first derivative. Such a method is the Numerov's algorithms[20] which permits for the given incident angle and wavelength, the field to be directly evaluated from the wave equation for any refractive index distribution. In the case of ARROW waveguide the Numerov's algorithm can be started in using the known exponentially decreasing form of the field in the total reflection region. Thus for the implementation of the latter method for ARROW and Bragg waveguides, first the field in the total reflection region should be evaluated for a given arbitrary amplitude coefficient. Then the field in other regions is evaluated directly from the wave equation. By determining the maximal values of the field in the guiding film (IV) and in the air (I), their ratio is evaluated. For the propagation constants corresponding to the quasiguided resonant modes that ratio passes through a maximum. In Appendix B we bring in all necessary formulas for the implementation of Numerov's algorithm for this problem. In Fig. 5 the field is evaluated by Numerov's algorithm for the experimental waveguide whose parameters are given in Table I.

SAMPLE PREPARATION AND EXPERIMENTAL RESULTS

The experimental setup is described in Fig. 1. The ARROW waveguide structure: Cu/Cr/Al/Cr/Cu glass was produced by the rf sputtering. It was prepared from the deposition of the first 3000-A-thick Cu buffer layer on the float glass substrate. The 2000-Å-thick Cr guiding film was deposited on the Cu buffer layer followed by the first 430-Å-thick Al quarter-wave film. Then the second 1075-Å-thick Cr quarter-wave film was deposited on the latter. On the Cr quarter-wave film another 3000-Å-thick and $L_2 = 6$-mm-long ''hat'' was deposited in order to provide true waveguiding in the ''hat'' covered region by total reflections and observe the waveguiding by a transmitted beam technique.[1,2,4] The two extremities of the sample with lengths $L_1 = L_3 = 10$ mm ensure the incident neutrons in and out coupling, respectively. The sample was characterized by neutrons reflectivity measurements using time of flight reflectometer Eros installed on the Orphée reactor at the Laboratoire de Léon Brillouin at Saclay which was described previously.[2] The measurements were performed at incidence angle $\theta_{inc} = 0.4°$. The values of refractive indices and thicknesses of Cr and Al layers obtained by the fit to reflectivity data coincide fairly well with their values ''estimated'' from the deposition time considerations. These values are summarized in Table I. As shown in Fig. 6, two transmitted resonant intensity peaks were observed. It should be noted that in our experiment the incidence angle of neutron beam is fixed at $\theta_{inc} = 0.4°$ and the neutron wavelength is varied.[2,4] The experimental data was fitted to a sum of two Lorentzian curves. The first resonance is centered at $\lambda_0 = 7.38$ Å with a full width at half maximum $\Delta\lambda_0 = 0.13$ Å and the second resonance is centered at $\lambda_1 = 6.81$ Å with a full width at half maximum $\Delta\lambda_1 = 0.18$ Å.

In our simulations the incident neutron wave was considered to be a plane wave which is in reality rather of Gaussian distribution. In Fig. 2 the waveguide is considered to have

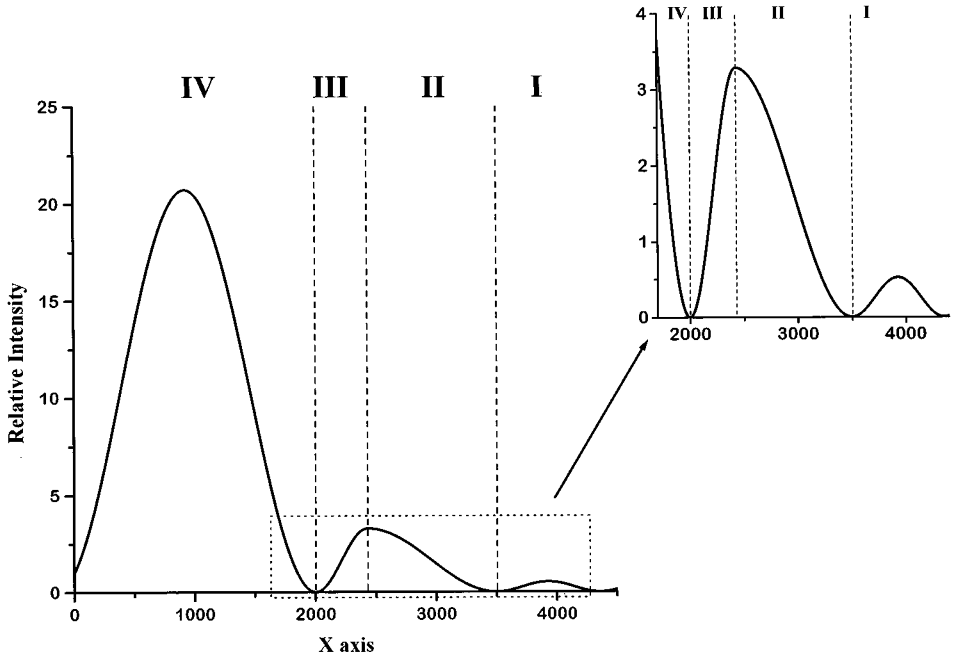

FIG. 5. Transverse distribution of $\Psi^2(x)$. The amplitude of the wave function in air is supposed to be equal to one. By dotted lines the limits of different films are shown. The transverse distribution between $1700 < x < 4500\ \text{Å}^2$ is shown under $6\times$ magnification in order to visualize the leakage into the air (IV).

perfect boundaries. This is why a sharp resonance is seen. In fact, as discussed previously[1,2] the neutron guided propagation is very sensitive to the imperfections of all interfaces, the coherence, and the coupling process which all contribute to the extra broadening of the observed resonances. The experimentally measured half widths of resonances in Fig. 6 are much larger than that of an ideal structure which is mainly due to the imperfections of all interfaces. This confirms once more why only one pair of antiresonant high–reflection quarter-wave layers should be deposited on the guiding film. As can be seen from Table I the position of the first and second resonance are also slightly shifted from the values evaluated by the method described in the present paper. It may be due to the refractive index Cr which cannot be established with a high accuracy by an inverse reflectivity calculation. It should be noted that the transmitted resonance is stronger than those obtained previously in conventional waveguides.[2,4]

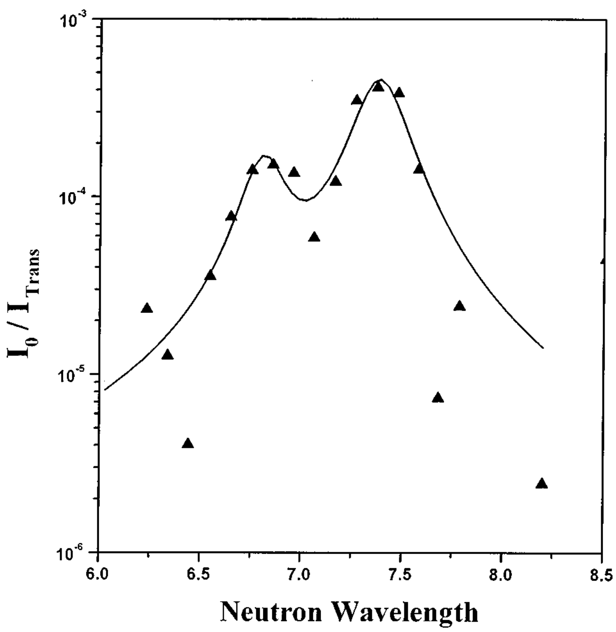

FIG. 6. The intensity ratio of incident and transmitted beams is given as a function of incident neutrons wavelength. The solid line curve is a fit curve to a sum of two Lorentzian shapes.

DISCUSSION

The ARROW waveguides can find an immediate application in neutron waveguide polarizers. As previously shown by us the selective property of the waveguiding with respect to the magnetic moment in thin-film magnetic films can be used for the polarization of the incident neutrons. The main obstacle for this kind of polarizer by using conventional total reflection waveguide structures was shown to be the smallness of the refractive index of magnetic materials.[6] This is why the ARROW waveguides are excellent substitutes for polarizer application since in contrast to the conventional waveguides, the guiding region is the one with a low refractive index surrounded from both sides by nonmagnetic materials with a higher refractive index. Thus any magnetic thin film can be used as the guiding film of an ARROW waveguide.

In conclusion we have proposed and observed experimentally low-index thin-film neutron waveguides with antiresonant high reflecting layers which allow the neutrons to be confined in thin films with refractive indices smaller than those of neighboring media. Efficient numerical methods were used for the design and modelization of these waveguides.

APPENDIX A

Consider a multilayer ARROW waveguide structure schematized in Fig. 1. In ARROW waveguides the field of the guided resonant mode is of sinusoidal form in all layers except the buffer layer denoted by V. From the interface of the latter total reflection occurs. The buffer layer thickness is quite large with respect to the Goos-Hänchen penetration depth, so it is considered semi-infinite. In consequence the filed is of exponentially decreasing form in $x>H_{IV}$. Thus the field can be written

$$\Psi(x)=\begin{cases} A_I\cos(q_I x-\phi_I), & x<H_I \\ A_i\cos(q_i x-\phi_i), & H_{i-1}<x<H_i \\ A_V\exp[-p_V(x-H_{IV})], & x>H_{IV}, \end{cases} \tag{A1}$$

where $H_{i+1}=H_i+h_{i+1}$, $H_n=\Sigma^n{}_{i=II}h_i$, $H_I=0$, $q_i=2\pi(a_m-\delta_i)^{1/2}$, $p_V=2\pi(\delta_V-a_m)^{1/2}$, $i=I$, II, III, IV, and a_m lies in the following interval $\min(\delta_i)<a_m<\max(\delta_i)$. The values of amplitude and phase constants A_i and ϕ_i will be determined from the boundary conditions at the boundaries $x=H_I$, H_{II}, H_{III}, and H_{IV}. The values of ϕ_i are determined from the continuity condition of the logarithmic derivative of the wave function: $L(x)=[d\Psi(x)/dx]/\Psi(x)$. Thus

$$L(x)=\begin{cases} -q_I\tan(q_I x-\phi_I), & x<H_I \\ -q_j\tan(q_j x-\phi_j), & H_{j-1}<x<H_j \\ -p_V, & x>H_{IV}. \end{cases} \tag{A2}$$

The boundary-value matching gives the following recursive formulas for ϕ_i:[19]

$$\phi_i=q_iH_i-\tan^{-1}\{(q_{i+1}/q_i)\tan[q_{i+1}H_i-\phi_{i+1}]\}, \tag{A3}$$

with $\phi_{IV}=q_{IV}H_{IV}-\tan^{-1}(p_V/q_{IV})$,

Thus for the given propagation constant a_m, the values of all phase coefficients are easily obtained by the above recursive formulas. Once all values of ϕ_i are obtained, the square value of the ratio of the amplitudes in two adjacent layers can be obtained by the continuity condition of the wave function at the boundaries $x=H_I$, H_{II}, H_{III}, and H_{IV}:

$$\gamma_i=(A_{i+1}/A_i)^2=1+[q_i^2/q_{i+1}^2-1]\sin^2(q_iH_i-\phi_i). \tag{A4}$$

The resonance condition is equivalent to the requirement that the intensity ratio I_r

$$I_r=(A_{IV}/A_I)^2=\gamma_I\gamma_{II}\gamma_{III} \tag{A5}$$

passes through a maximum for the given propagation constant. Indeed the I_r is a function of a_m having a maximum at quasiguided resonance positions:

$$dI_r/da_m=0,\ \ d^2I_r/da_m{}^2<0. \tag{A6}$$

This is the definition of the propagation constant of a quasiguided mode of the ARROW waveguide in the coupler region. Thus the propagation constant corresponds to the resonant enhancement of the incident field in the guiding film of the ARROW waveguide. This is also the main difference with the theories discussed previously.[8,11–13,18]

For the given interval of incident angles corresponding to the ARROW waveguiding the incident neutron beam is reflected totally from the medium $x>H_{IV}$. Consequently the reflection coefficient

$$R=\exp\{-|\phi_I-\phi^*{}_I|\} \tag{A7}$$

is equal to one since for total reflection ϕ_I is a real quantity and its imaginary component is zero. In order to visualize the resonant modes the refractive index of the guiding film, which in our case is the film with $i=IV$, is supposed to be complex with a very small imaginary component. For the propagation constants close to that of the resonant quasiguided modes, a resonant flow into the guiding film takes place. Since we introduced a small, but nonzero imaginary component in the refractive index of the guiding film, the resonant wavelengths will appear as absorption deeps in the reflectivity curve as shown in Fig. 2.

APPENDIX B

In this appendix we recall briefly the main formulas of Numerov's method and apply the latter for the evaluation of the wave functions in neutron ARROW waveguides. Consider the following one-dimensional Schrödinger equation:

$$d^2\Psi(x)/dx^2=f(x)\Psi(x), \tag{B1}$$

where $f(x)$ is determined by the given scattering potential. In the case of neutron ARROW waveguide $f(x)$ is a piecewise constant function taking on different values in each layer of the structure. Numerically $\Psi(x)$ can be obtained by the Numerov's formula:

$$\begin{aligned}\Psi(x)=\{&[1-T(x+h)]\Psi(x+h)\\ &+[1-T(x-h)]\Psi(x-h)\}/[2+10T(x)]\\ &+\cdots O(h^6),\end{aligned} \tag{B2}$$

where $T(x)=h^2f(x)/12$.

This is a finite difference formula of very high precision (up to h^6) which is convenient to use in the wave function calculations for ARROW waveguides. This algorithm is efficient if the wave function is known in two points with a precision of the same order, i.e., not less than h^6. In the case of ARROW waveguides the wave function form is known to be of exponentially decreasing form in $x>H_{IV}$, i.e., is known analytically. Moreover, the value of one of the amplitude coefficients may be chosen arbitrarily. That can chosen to be the amplitude coefficient of the exponential function. Thus by arbitrarily choosing the exponential function amplitude, the latter become determined in the interval $x>H_{IV}$ (Fig. 5) [see Eq. (A1)]. Then by evaluating the exponential function: $\Psi(H_{IV})=A_V$, and $\Psi(H_{IV}+h)=A_V\exp(-p_Vh)$ at $x=H_{IV}$ and $x=H_{IV}+h$, respectively, where h is the iteration step defined from the precision considerations, the value of $\Psi(x)$ at $x=nh$, $(n=1,2,\cdots)$ can be determined from formula (B2). By evaluating the maximal values of the wave function in the intervals $H_{III}<x<H_{IV}$ and $x<H_I$ the value of I_r is determined as the square of their ratio.

*Electronic address: legall@cnrs-bellevue.fr

[1] Y. P. Feng, C. F. Majkrzak, S. K. Sinha, D. G. Wiesler, H. Zhang, and H. W. Deckman, Phys. Rev. B **49**, 10 814 (1994).
[2] S. P. Pogossian, A. Menelle, H. Le Gall, J. M. Desvignes, and M. Artinian, Phys. Rev. B **53**, 14 359 (1996).
[3] L. J. Norton, E. J. Kramer, R. A. L. Jones, F. S. Bates, H. R. Brown, G. P. Felcher, and R. Kleb, J. Phys. (France) II **4**, 367 (1994).
[4] S. P. Pogossian, A. Menelle, H. Le Gall, J. Ben Youssef, and M. Desvignes, J. Appl. Phys. **81**, 4281 (1997).
[5] S. P. Pogossian and H. Le Gall, Opt. Commun. **114**, 235 (1994).
[6] S. P. Pogossian, H. Le Gall, and A. Menelle, J. Magn. Magn. Mater. **152**, 305 (1996).
[7] S. P. Pogossian, H. Le Gall, J. M. Desvignes, and A. Menelle, J. Phys. D **29**, 2570 (1996).
[8] R. Ulrich and W. Prettl, Appl. Phys. **1**, 55 (1973).
[9] M. A. Duguay, Y. Kokubun, and T. L. Koch, Appl. Phys. Lett. **49**, 13 (1986).
[10] T. Baba, Y. Kokubun, T. Sakaki, and K. Iga, J. Lightwave Technol. **LT-6**, 1440 (1988).
[11] W. Jiang, J. Chrostowski, and M. Fontaine, Opt. Commun. **72**, 180 (1989).
[12] J. Kubica, D. Uttamchandani, and B. Culshaw, Opt. Commun. **78**, 133 (1990).
[13] W. Huang, R. M. Shubair, A. Nathan, and Y. L. Chow, J. Lightwave Technol. **LT-10**, 1015 (1992).
[14] A. Y. Cho, A. Yariv, and P. Yeh, Appl. Phys. Lett. **30**, 471 (1977).
[15] P. Yeh and A. Yariv, Opt. Commun. **19**, 427 (1976).
[16] F. Lederer, U. Trutschel, and C. Waechter, J. Opt. Soc. Am. A **8**, 1536 (1991).
[17] F. Abèles, Ann. Phys. (Paris) **5**, 590 (1950).
[18] J. Chilwell and I. Hodgkinson, J. Opt. Soc. Am. A **1**, 742 (1984).
[19] S. P. Pogossian and H. Le Gall, J. Mod. Opt. **42**, 1725 (1995).
[20] J. M. Blatt, J. Comput. Phys. **1**, 382 (1967).