# Effect of disorder in specific realizations of multibarrier random systems

Gastón García-Calderón
Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 México, Distrito Federal, Mexico

Roberto Romo and Alberto Rubio
Facultad de Ciencias, Universidad Autónoma de Baja California, Apartado Postal 1880, Ensenada, Baja California, Mexico
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A resonance formalism is used to study the effect of disorder in specific realizations of multibarrier random systems. We solve the periodic case and introduce disorder by allowing random values for the well widths. We analyze the motion of the complex poles of the *S* matrix on the energy plane and calculate the resonant states for systems of fixed length as a function of the disorder strength. Our analysis of the eigenfunctions, the decay widths, and the Thouless criterion allows us to distinguish in general three different types of states: quasilocalized, intermediate, and border states. [S0163-1829(97)08331-8]

## I. INTRODUCTION

Since the seminal work by Anderson,[1] almost four decades ago, localization in disordered systems has been the subject of numerous investigations.[2] One-dimensional systems have been convenient models for both theoretical and numerical investigations on the properties of localized states,[3,4] as, for example, in studies on the localization length. In recent years technical improvements in the fabrication of semiconductor heterostructures have allowed the possibility of designing multibarrier potential profiles almost at will.[5,6] This provides the opportunity to study the properties of electron propagation in multibarrier systems.

Erdös and Herndon pointed out some time ago[7] the convenience of having a link between the transmission amplitude, i.e., the scattering properties, and the one-electron Green function that relates to the eigenfunctions and eigenvalues of the problem. Our approach establishes this link. Previous works on specific systems considered either the eigenfunction or the scattering approaches. Among the former, one finds works where the wave function vanishes at the boundaries of the system,[8] implying that the system is closed and hence that a connection to the transmission problem is not possible. On the other hand, one finds works based on the properties of resonant tunneling, where the transmission coefficient and wave function as functions of the energy are studied.[9] These approaches lacked a definition for the resonance eigenfunctions and eigenvalues associated with the disordered potential. Here it is worth mentioning recent work involving resonant tunneling in connection with quantum dots. In this case, however, Coulomb interaction effects become relevant for an appropriate description of the problem, and the corresponding treatment becomes more involved.[10] On the other hand, it is also appropriate to refer to a number of recent works that addressed the effect of correlated disorder on the properties of localization in one dimension.[11] These works showed that the prevalent notion that in one dimension all states are localized for any amount of disorder does not hold in general.

The purpose of this work is to study the onset and properties of localization in a specific realization of a multibarrier potential profile generated in a random manner. Our approach is based on a resonance formalism that considers the multibarrier system as an open system. This leads to a complex eigenvalue problem, and allows one to connect the problem of transmission scattering with the resonant states and eigenvalues of the system.

In our approach we consider a specific system of length $L$ to study it as a function of disorder. Hence the notion of localization length, that arises from statistical considerations involving an ensemble of systems, does not seem appropriate to characterize the properties of a particular system. Transmission scattering probes the resonant states of a multibarrier system. As is well known, this is exhibited as peaks in a plot of the transmission coefficient versus energy. These peaks reflect the existence of resonant states of the system, and their position in energy is related to the real part of the complex poles of the transmission amplitude. Since the system has a finite length, eventually an electron seated on one of these states decays out of the system with a time scale proportional to the inverse of the imaginary part of the complex pole. Since the transmission amplitude is an element of the *S* matrix, the above complex poles are precisely the complex poles of the *S* matrix of the problem. A very important aspect of our approach is that one may associate a resonant eigenfunction with each of the transmission levels. A convenient way to do this is by exploiting the analytical properties of the outgoing Green function of the problem. Our formalism establishes a connection between the wave solution and the outgoing Green function of the problem along the internal region of the system that includes, as a special case, a connection with the transmission amplitude. It is well known that the effect of disorder on the transmission coefficient causes irregular fluctuations as a function of energy.[2] The effect of disorder on the *S*-matrix poles in one-dimensional chains also modifies their distribution on the complex energy plane.[12] However to our knowledge no treatments have been reported in the literature on the effect of disorder on resonant eigenfunctions and the connection of these functions with the complex poles.

It is well known from numerical calculations of transmis-

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sion scattering on periodic multibarrier systems that the resonant levels group themselves in minibands, the number of levels in each miniband being equal to the number of wells in the system.[13] A periodic multibarrier system may be specified by the set of equal $(N+1)$ barrier heights $\{V_0\}$ with barrier widths $\{b\}$, and $N$ well widths $\{W\}$. Using the resonance formalism discussed below, a given configuration yields a set of $N$ resonant states $\{u_n(x)\}$, along the internal region of the system, and $N$ complex poles $\{\epsilon_n - i\Gamma_n/2\}$, with $n=1,2,\ldots,N$, for each miniband. Without a loss of generality, we shall be concerned in this work with the first miniband of a multibarrier system.

It is of relevance to emphasize that the set of resonant states and the complex poles are analytical functions of the potential profile parameters. This means that a modification of the parameters of the potential modifies both the position of the poles in the complex energy plane and the way the resonant states vary with distance along the internal region of the potential. Following the above considerations, in this work we refer to specific realizations of the potential profile parameters, generated in a random manner, to calculate the corresponding set of resonant states $\{u_n(x)\}$ and complex poles $\{\epsilon_n - i\Gamma_n/2\}$. Moreover we are able to calculate the trajectory that each complex pole follows in the complex plane as the potential profile varies from an ordered to a random configuration, and, similarly, how each resonant state is modified. In this way we are able to study the onset of localization in specific realizations of a multibarrier random system.

As we shall see below, the effect of disorder on the complex poles of the propagator is to diminish their width, so, though most of the states increase their lifetime considerably, they still decay out of the system with time and hence do not satisfy Anderson criteria for localization,[1] which impose an absence of decay for localized states. On the other hand, we shall find that most resonant states become confined within a region of the system due to the effect of disorder, and more interestingly that they reasonably satisfy Thouless criteria for localization, for these states become insensitive to changes on the boundaries of the system.[3] Thus we shall see that, under disorder, the resonant states of finite-length multibarrier systems are not strictly localized, though they share some of their properties. For the above reasons we shall refer to them as quasilocalized states. In general, for a given disorder strength we find states that have a decay width smaller than those of the periodic case but large compared to the quasilocalized states. As we shall encounter, they are sensitive to changes on the boundaries of the system. We shall refer to them as intermediate states. In addition, as will be found, the behavior of the decay widths of the system as a function of the disorder strength always shows the formation of two states that acquire very large decay widths, yet they are confined near the edges of the system. These states are also very sensitive to changes in the boundaries of the system, and we shall call them border states.

The paper is organized as follows: in Sec. II we present an account of the resonance formalism. Section III deals with a number of numerical examples, and the discussion of the results of our investigation. Finally, concluding remarks are given in Sec. IV.

## II. RESONANCE FORMALISM

Although this formalism has been presented elsewere,[13] for completness we will recount it here. Consider an electron of mass $m$ and energy $E$ approaching from $x<0$ on a potential of arbitrary shape $V(x)$ defined in the region extending from $x=0$ to $x=L$, and vanishing outside. The solution $\psi(x)$ of the Schrödinger equation outside the interval $(0,L)$ is of the form $\psi(x)=\exp(ikx)+r(E)\exp(-ikx)$ for $x<0$ and $\psi(x)=t(E)\exp(ikx)$ for $x>L$, where $t(E)$ and $r(E)$ are, respectively, the transmission and the reflection amplitudes, and $k=[2mE]^{1/2}/\hbar$ is the corresponding wave number. The wave function $\psi(x)$ can be written in terms of the outgoing Green propagator in the form[14]

$$\psi(x)=2ikG^{+}(0,x;E), \quad 0\leqslant x\leqslant L. \tag{2.1}$$

Evaluating the above equation for $x=L$, leads to a useful relationship between the transmission amplitude and the propagator, namely,

$$t(E)=2ikG^{+}(0,L;E)\exp(-ikL). \tag{2.2}$$

In general, for a miniband consisting of $N$ states it was shown in Ref. 13 that one may write the function $G^{+}(x,x';E)$ as an expansion that involves the corresponding $N$ resonant states of the system plus a background contribution $B(E)$;[13] moreover, it was shown[13] that the background $B(E)$ is negligible for the energies within the miniband, that is,

$$G^{+}(x,x';E)=\sum_{n=1}^{N}\frac{u_n(x)u_n(x')}{k^2-k_n^2}, \quad 0<(x,x')\leqslant L. \tag{2.3}$$

The quantities $k_n^2$ are the complex eigenvalues of the Schrödinger equation,

$$\frac{d^2u_n(x)}{dx^2}+[k_n^2-U(x)]u_n(x)=0, \tag{2.4}$$

for the eigenfunctions $u_n(x)$ and $U(x)=2mV(x)/\hbar^2$. These complex eigenvalues are related to the poles of the Green function on the complex energy plane in the usual way, namely, $E_n=\hbar^2k_n^2/2m=\epsilon_n-i\Gamma_n/2$. It is well known that $\epsilon_n$ and $\Gamma_n$ describe, respectively, the position and the width of the resonance. The functions $u_n$ obey purely outgoing boundary conditions at the points $x=0$ and $x=L$, i.e.,

$$\left(\frac{du_n(x)}{dx}\right)_{x=0}=-ik_nu_n(0), \tag{2.5a}$$

$$\left(\frac{du_n(x)}{dx}\right)_{x=L}=ik_nu_n(L), \tag{2.5b}$$

and their normalization condition is,

$$\int_0^L u_n^2(x)dx+\frac{i}{2}\left[\frac{u_n^2(0)}{k_n}+\frac{u_n^2(L)}{k_n}\right]=1. \tag{2.6}$$

From Eqs. (2.2) and (2.3), and the definition of the transmission coefficient $T(E)$, we obtain[13]

$$T(E)=|t(E)|^2=\sum_{n=1}^{N} T_n+\sum_{n<m}^{N} T_{nm}, \tag{2.7}$$

where $T_n(E)$ is a Breit-Wigner expression for the transmission in the vicinity of the $n$th resonance, and $T_{nm}(E)$ represents the interference between the $n$th and the $m$th resonances. The expressions for $T_n(E)$ and $T_{nm}(E)$ are given by

$$T_n(E)=\frac{k^2}{a_n^2}I_n^2\frac{\Gamma_n^0\Gamma_n^L}{(E-\epsilon_n)^2+\left(\frac{1}{2}\Gamma_n\right)^2} \tag{2.8}$$

and

$$T_{nm}(E)=2C_{nm}(\Gamma_n^0\Gamma_n^L\Gamma_m^0\Gamma_m^L)^{1/2} \times \mathrm{Re}\left[\frac{\exp(i\phi_{nm})}{(E-\epsilon_n+i\Gamma_n/2)(E-\epsilon_m-i\Gamma_m/2)}\right], \tag{2.9}$$

where $a_n=\mathrm{Re}k_n$, $C_{nm}=k^2I_nI_m/(a_na_m)$, and $I_n=\int_0^L|u_n(x)|^2dx$, and the phase $\phi_{nm}$ is given by $\phi_{nm}=[\phi_n(0)+\phi_n(L)-\phi_m(0)-\phi_m(L)]$. The relevant quantities $\Gamma_n^0$ and $\Gamma_n^L$ are, respectively, the partial decay widths through the boundaries of the system at $x=0$ and $x=L$, and are defined as[13]

$$\Gamma_n^0=\hbar\frac{\hbar a_n}{m}\frac{|u_n(0)|^2}{I_n} \tag{2.10}$$

and

$$\Gamma_n^L=\hbar\frac{\hbar a_n}{m}\frac{|u_n(L)|^2}{I_n}, \tag{2.11}$$

where $a_n$ is the real part of the complex wave number $k_n=[2mE_n]^{1/2}/\hbar=a_n-ib_n$. The total decay width $\Gamma_n$ and the partial decay widths $\Gamma_n^L$ and $\Gamma_n^0$ are related through the equation

$$\Gamma_n=\Gamma_n^0+\Gamma_n^L. \tag{2.12}$$

The decay width $\Gamma_n$ yields the lifetime $\tau_n$ of the resonant state,

$$\tau_n=\hbar/\Gamma_n. \tag{2.13}$$

The above follows from the well-known time dependence of resonant states, namely,

$$u_n(x,t)=u_n(x)e^{-i\epsilon_n t/\hbar}e^{-\Gamma_n t/2\hbar}. \tag{2.14}$$

## III. RESULTS AND THEIR DISCUSSION

### A. Effect of disorder on systems with overlapping or isolated levels

In this section we shall study the effects of disorder on the resonance parameters of small multibarrier systems. Our aim is to investigate how the degree of disorder affects systems characterized by overlapping or nonoverlapping (isolated) resonance levels. As discussed below, the systems with isolated resonance levels require much less disorder strength to show noticeable changes than the systems sustaining overlapping resonances.

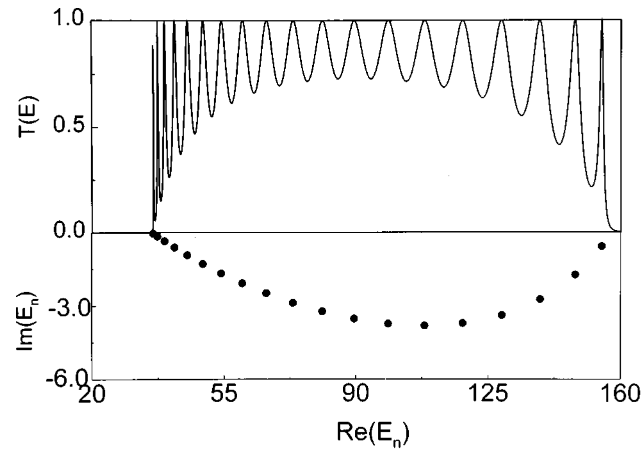


FIG. 1. Plot of the transmission coefficient $T(E)$ vs energy $E$ for a symmetric multibarrier system of 19 wells, with parameters as discussed in the text, that leads to overlapping resonances. Also shown are the corresponding complex poles of the propagator forming the ''necklace'' distribution characteristic of a periodic case.

Consider two periodic structures, each with 20 barriers (19 wells), such that the corresponding potential parameters lead to different resonance widths. One of the systems is formed by barriers thinner than the well widths, and as a consequence the resonances are broad and hence overlap with each other. The parameters of this potential profile are barrier heights $V_0=0.25$ eV, barrier widths $b_n=10$ Å, and well widths $W_0=50$ Å. Figure 1 shows a plot of transmission coefficient $T(E)$ vs the energy $E$ for this system. Also shown is the corresponding distribution of the complex poles of the propagator on the energy plane. As can be seen, the poles distribute, forming a ''necklace'' shape. The other system consists of thicker barriers. The corresponding parameters of the potential for this case are barrier heights $V_0=0.3$ eV, barrier widths $b_n=50$ Å, and well widths $W_0=50$ Å. As illustrated by Fig. 2 the transmission coeffi-

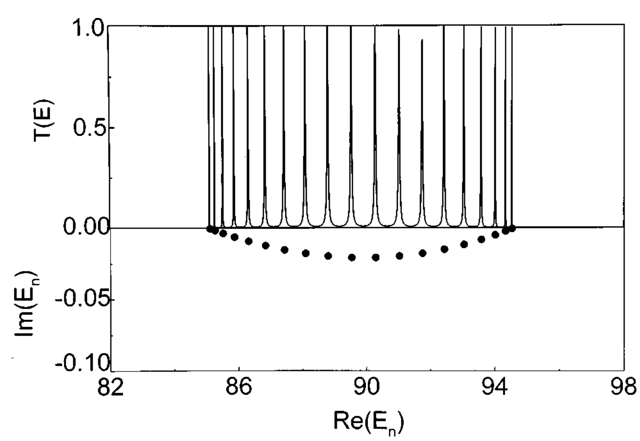


FIG. 2. Plot of the transmission coefficient $T(E)$ vs energy $E$ for a symmetric multibarrier system of 19 wells, with parameters as discussed in the text, that leads to nonoverlapping resonances. Also shown are the corresponding complex poles of the propagator forming the necklace distribution characteristic of a periodic case.

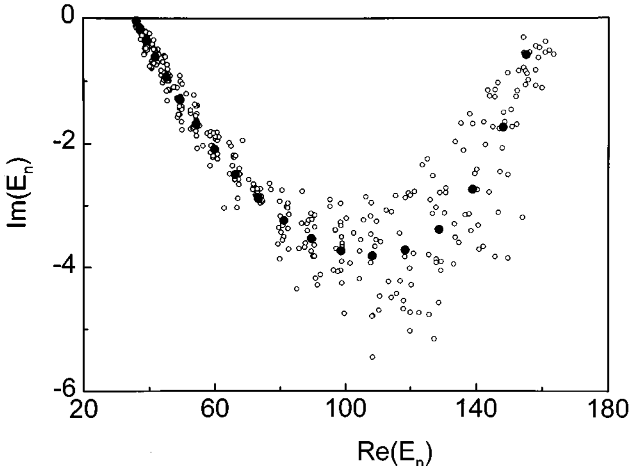


FIG. 3. Distribution of the poles of the propagator on the complex energy plane for 20 random systems generated from the periodic system with overlapping resonances. The black dots correspond to the periodic case.

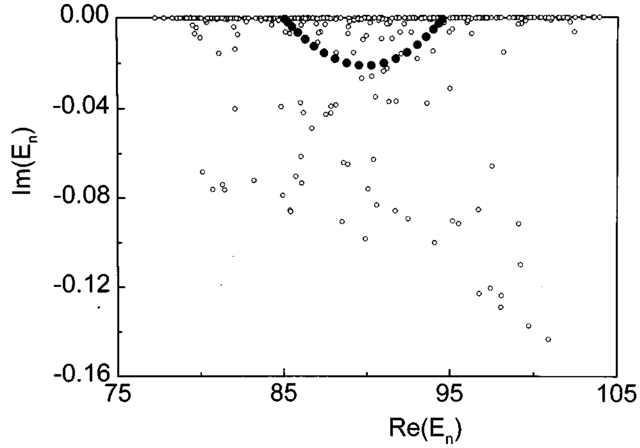


FIG. 4. Distribution of the poles of the propagator on the complex energy plane for 20 random systems generated from the periodic system with nonoverlapping resonances. The black dots correspond to the periodic case.

cient shows resonances that are sharp and isolated, leading to a nonoverlapping situation. Also shown is the corresponding pole distribution on the energy plane, forming also a necklace as in the previous case. Notice that, although the nonoverlapping resonance system has a total length much larger than the overlapping resonance system, the miniband energy width of the former is much narrower (by a factor of 10) than that of the latter. The necklace-shaped pole distribution on the energy complex plane shown by Figs. 1 and 2 is typical of periodic systems.

We now introduce disorder in the potential profiles by letting the well widths assume random values $W_k$ within an interval centered at $W_0$ of width $2\Delta W$, with $\Delta W$ the strength of the disorder. That is, each $W_k$ is generated randomly between $(W_0-\Delta W)$ and $(W_0+\Delta W)$. As an example we shall consider a disorder strength that guarantees that the systems under study are not destroyed; that is, the changes on the values of the well widths are such that the number of wells remain the same. Hence we consider $\Delta W=10\%$ of $W_0$, i.e., $\Delta W=5$ Å, consequently, the well widths vary at most between 45 and 55 Å. Figure 3 shows the pole positions for a set of 20 random systems generated from the periodic structure with overlapping resonances referred to in Fig. 1. Similarily, Fig. 4 shows the corresponding poles for a set of 20 random systems generated from the periodic structure with sharp resonances displayed in Fig. 2. In both Figs. 3 and 4 we have included, as a reference, the pole positions of the corresponding periodic structures (full circles). Note that the pole distributions in Figs. 3 and 4 differ considerably. In the case of overlapping resonances the poles are scattered around the necklace of the periodic configuration, but they still remain close to it. Conversely, in the case of sharp resonances, a dramatic effect is observed. Here the same amount of disorder has moved most of the poles upward, closer to the real energy axis, which implies a longer lifetime $\tau=\hbar/\Gamma$ for the resonant states that is due to its confinement inside the structure. This behavior is quite interesting, since it implies a tendency to localization for such states, as will be discussed in Sec. III B. To see this kind of behavior for the overlapping case would require a much higher disorder strength.

In symmetrical structures the partial decay widths are equal, thus from Eq. (2.12) they satisfy the relationship $\Gamma_n^0=\Gamma_n^L=\Gamma_n/2$. Hence a plot of $\Gamma_n^L$ vs $\Gamma_n^0$ for such a structure would be a set of points lying on a straight line with slope 1 that passes through the origin. Any deviation from such a line implies that there is some asymmetry in the potential profile. In Figs. 5 and 6 we illustrate the plots of $\Gamma_n^L$ vs $\Gamma_n^0$ for the two sets of 20 random systems considered above. In both figures the symmetrical situation is represented by full circles that fall in the previously described straight line. Figure 5 corresponds to the case with overlapping resonances, and Fig. 6 to that of sharp isolated resonances. In the former case one sees some deviation from the straight line, as shown by the cloud of empty circles. One sees, however, that these circles are still located in the neighborhood of the straight line. In the latter case, for isolated resonances, the situation is quite different. The points are now uncorrelated with the straight line, and most of them have moved toward the origin, implying that $\Gamma_n^0$ and $\Gamma_n^L$ tend to zero as the disorder

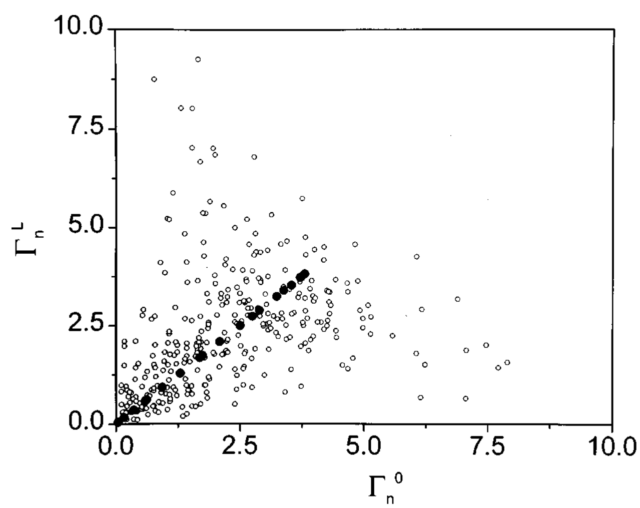


FIG. 5. Plot of partial decay widths $\Gamma_n^L$ vs $\Gamma_n^0$ for the 20 random generated from the overlapping periodic case. The full dots correspond to the periodic case.

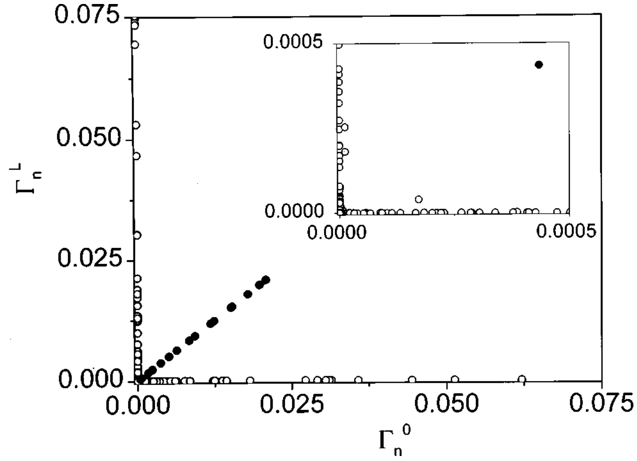


FIG. 6. Plot of partial decay widths $\Gamma_n^L$ vs $\Gamma_n^0$ for the 20 random systems generated from the nonoverlapping periodic case. The full dots correspond to the periodic case. The inset shows the region very close to the origin to emphasize the large number of states with partial widths very close to zero.

increases. Note in the inset of Fig. 6 the great accumulation of points around the origin. The diminution of both $\Gamma_n^0$ and $\Gamma_n^L$ for the state $n$ means that its total width $\Gamma_n$ diminishes as well, and hence its lifetime $\tau=\hbar/\Gamma_n$ increases substantially compared with the symmetric situation. However the partial decay widths do not in general tend to smaller values in the same way. As we shall discuss below, this difference is reflected in the values of the transmission coefficient peak.

### B. Study of a specific random potential profile

Let us consider the 20-barrier symmetrical system that produces isolated sharp resonances like that depicted by Fig. 2. Let us choose a specific realization of the 20-barrier potential profile generated previously in a random fashion, namely, one of those whose poles are shown in Fig. 4. The corresponding distribution of complex poles of the propagator on the energy plane (empty circles) is shown in Fig. 7. That figure also shows the necklace of poles corresponding to the symmetric case (full circles), and the set of trajectories that each pole followed from the symmetric to the final random configuration. The specific final random configuration is characterized by the set of positive or negative values $\{\delta W_k\}$, which represent, for each $k$ ($k=1,2,\ldots,19$), the full deviation that the width of the $k$th well experiences in going from the symmetric to the final configuration. Then the trajectories are generated using the expression $W_k(\alpha)=W_0+\alpha\delta W_k$, with $\alpha$ varying from 0 to 1, and the unity value corresponds to the final configuration. Note that the trajectories joining the symmetric with the chosen final configuration are not unique.

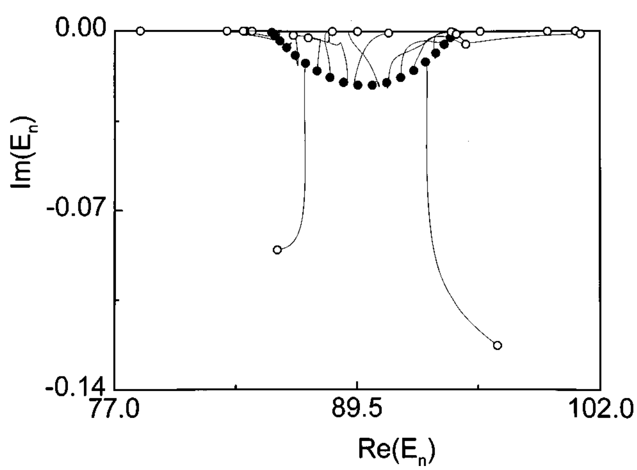


FIG. 7. Plot of the motion of the complex poles for a specific configuration of 20 barriers as a function of the disorder strength $\Delta W=5$ Å. Full circles represent the periodic case, while open circles refer to the final configuration $\alpha=1$, as discussed in the text.

Figure 7 reveals that most of the poles move close to the real axis, which implies that $\Gamma_n$ becomes very small. However, two of the poles move away from the axis ($n=6$ and 15), which implies that $\Gamma_n$ increases.

Figure 8 shows the plots of $|u_n(x)|^2$ vs $x$ for a subset of the states of the system corresponding to the final configuration. The states displayed are $n=4$, 6, 7, 15, and 17. We choose these states because they are representative of the different types of states that arise in the system due to disorder. As discussed below, we distinguish three different types of states based on the behavior of the poles, particularly the values of the total decay widths, and the Thouless criterion on the insensitivity to a change in the boundaries of the system. Figure 8(a) corresponds to the periodic case, where all states are extended, while Fig. 8(b) displays the corresponding states for the disordered case. As discussed below, states $n=4$ and 17 are confined in a short region in the midsection of the system, and we will refer to them as quasilocalized states. They are characterized by partial decay widths that are both very small. States $n=6$ and 15 that concentrate, respectively, on the left and right edges of the system, will be denoted as border states. They are characterized by having one of the partial decay widths much larger than the other. The total decay width of each border state is much larger than the corresponding one for the periodic case (see Fig. 7) The third type of state, exemplified by $n=7$, corresponds to an intermediate situation between extended and quasilocalized states, and will be called an intermediate state. Here the partial decay widths are smaller than the corresponding values for the periodic case, but larger than those of the quasilocalized case. These states are still sensitive to a variation on the boundaries of the system.

The above considerations are consistent with the qualitative criterion given by Thouless that characterizes localized states by their independence of a variation on the boundaries of the system.[3] Indeed, as discussed below, the quasilocalized states $n=4$ and 17 are characterized by their independence on the particulars of the boundaries of the system. On the other hand, border states are very sensitive to small changes on the edges of the system, namely, states $n=6$ and 15 are sensitive to changes on the left and right edges, respectively. According to the Thouless criterion, this fact implies that border states, though confined to one of the edges of the system, cannot be considered quasilocalized states. Finally, intermediate states are characterized in a precise way because they are in fact sensitive to variations in the boundaries of the system.

In order to illustrate the Thouless criterion, we proceed to modify the potential profile in the left- and right-hand ex-

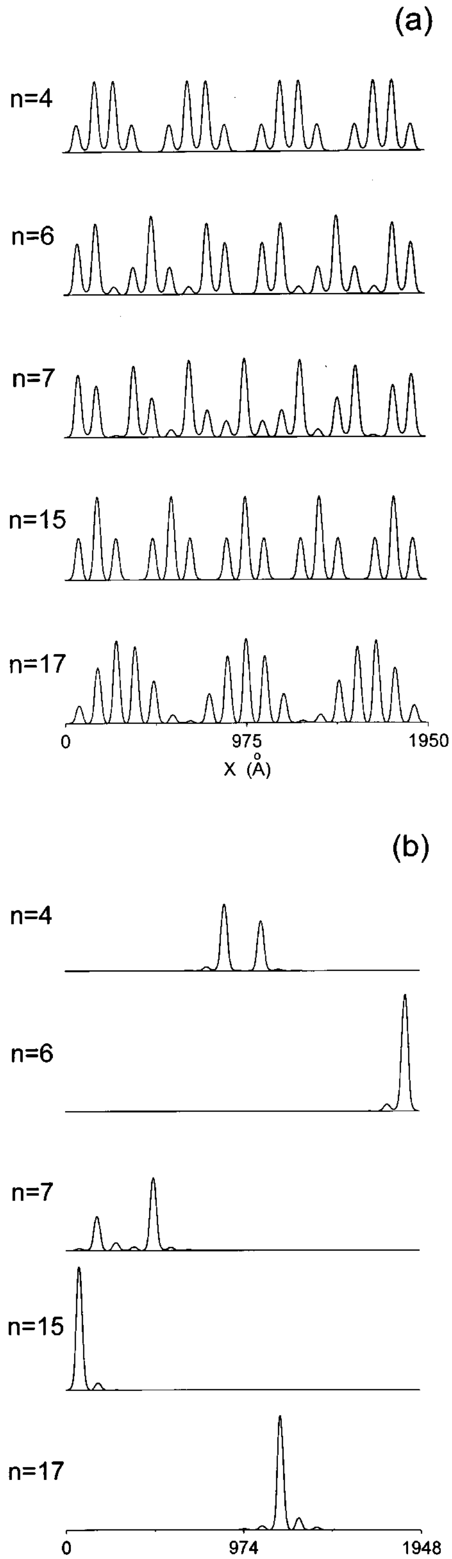


FIG. 8. Plot of $|u_n(x)|^2$ along the internal interaction region for different values of $n$, as discussed in the text. (a) refers to the periodic case, while (b) corresponds to the disordered situation.

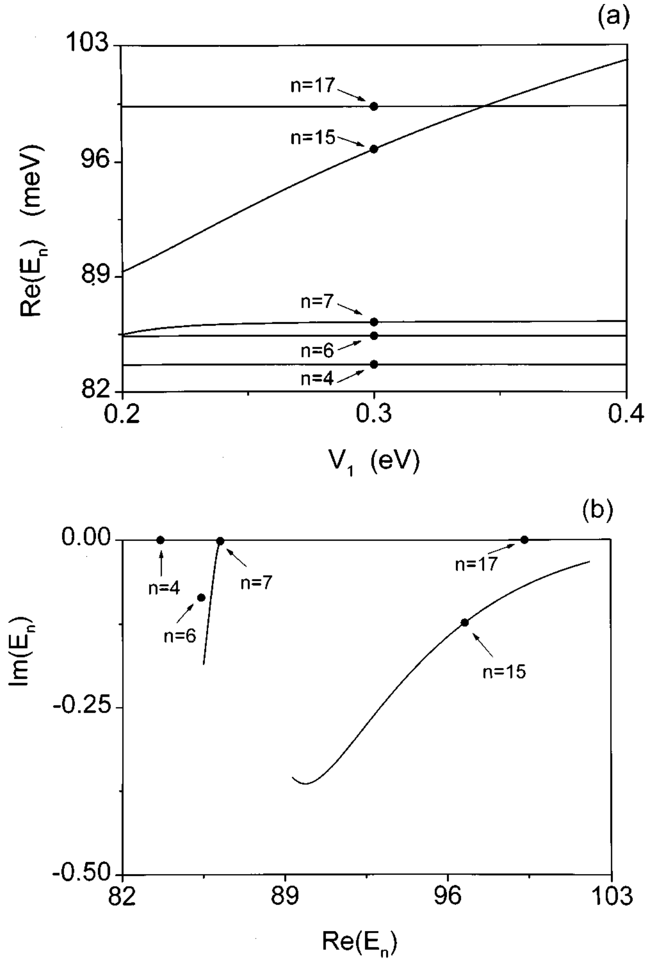


FIG. 9. (a) Plot of the resonance positions $\epsilon_n = \mathrm{Re}(En)$ as the height $V_2$ of the last barrier is varied from 0.2 to 0.4 eV for the states $n=4$, 6, 7, 15, and 17. (b) Plot of the positions of the resonance poles on the complex energy plane for the same states.

trema of the final random structure whose poles (empty circles) are shown in Fig. 7. We modify the height of the first barrier $V_1$ from 0.2 to 0.4 eV. The result of such a variation is shown in Figs. 9(a) and 9(b). Figure 9(a) illustrates a plot of the resonance positions $\epsilon_n = \mathrm{Re}(E_n)$ vs $V_1$ for the states $n=4$, 6, 7, 15, and 17; the full circles represent the initial values of the energy. Figure 9(b) displays the corresponding resonance poles in the complex energy plane for the same states; the full circles show the initial positions of the poles. We note that the intermediate state $n=7$ and the border state $n=15$ exhibit a dependence on this perturbation, while the other states remain insensitive to it. A modification of the height of the last barrier leads to similar results. There, instead, the other border state $n=6$ is altered strongly. The relevant point here is that the states $n=4$ and 17, those that were identified as quasilocalized, were insensitive to both perturbations, as stated by the Thouless criterion. Similar results were obtained with modifications of the barrier widths $b_1$ and $b_{20}$, as well as with modifications of the well widths $w_1$ and $w_{19}$, or combinations of them.

It is worth mentioning that the resonant phase $\phi_n(x)$ associated with the state $u_n(x)$, namely, $\phi_n(x) = \tan^{-1}[\mathrm{Im}(u_n(x))/\mathrm{Re}(u_n(x))]$ does not display any relevant feature due to the effect of disorder. We analyzed $\phi_n(x)$ as a function of $x$ along the internal region of the system, and obtained that it does not allow one to distinguish among the extended, quasilocalized, border and intermediate states. This seems to indicate that the phase coherence of resonant

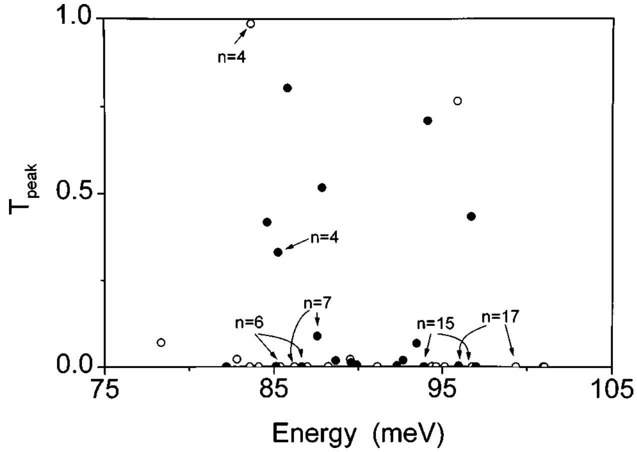


FIG. 10. Plot of the peak value of the transmission coefficient $T_{\rm peak}$ vs the incident energy $E$ for disorder strengths $\Delta W = 2.5$ Å (full circles) and $\Delta = 5$ (open circles). The arrows point to the transmission peaks of states $n = 4$, 6, 7, 15, and 17.

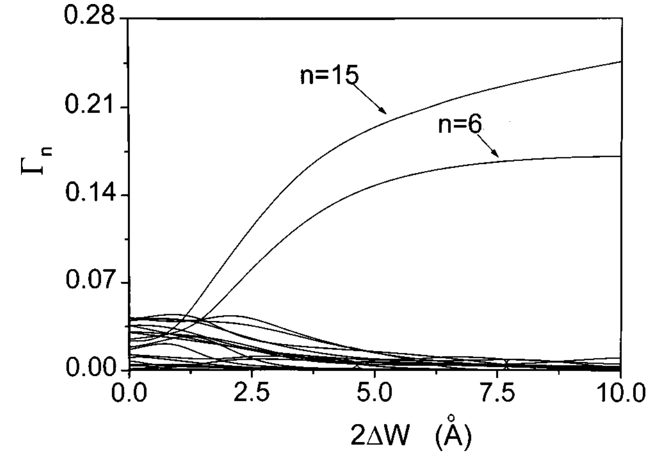


FIG. 11. Plot of the total decay width $\Gamma_m$ vs twice the disorder strength $\Delta W$ for the set of states of the specific configuration discussed in the text. One sees that most resonant widths tend to zero. However, two states $n = 6$ and 15 have increasing widths with disorder. They correspond to the border states of the system.

states is not destroyed by the effect of disorder, a result consistent with studies of the phase correlations of the wave solution $\psi(k,x)$ at resonance energy.[15]

Let us now examine the transmission coefficient peaks for the different states of the system. It may be shown, from Eq. (2.8), that for an isolated resonance the factor $k^2 I_n^2/a_n^2$ becomes unity, and hence the transmission peak depends only on the partial decay widths $\Gamma_n^0$ and $\Gamma_n^L$, namely,

$$T_n(E=\epsilon_n) = T_{\rm peak} = \frac{4\Gamma_n^0 \Gamma_n^L}{\Gamma_n^2}, \tag{3.1}$$

where $\Gamma_n$ is given by Eq. (2.12).

It follows from Eq. (3.1) that the peak value depends on the relative values of the corresponding two partial decay widths, and is independent of the value of the full decay width $\Gamma_n$. When the partial decay widths are equal, the maximum value of the transmission peak is achieved, i.e., $T_{\rm peak} = 1$, and as they become different the transmission peak may tend to a very small value. Figure 10 shows the peak values of the transmission coefficient for the 19 states of the system for two values of the disordered strength. The full circles correspond to $\Delta W =$ 2.5 Å, while the open circles refer to $\Delta W = 5$ Å. Notice that, for a given strength, the large differences among the peak values ranging from negligible ones up to the maximum $T = 1$. For example, for strength $\Delta W = 5$ Å (open circles), for the quasilocalized state $n = 4$, we obtain a peak value close to 1, while for the quasilocalized state $n = 17$ we obtain a value of the order of $10^{-5}$. We see also in Fig. 10 that for a given state $n$ the value of the transmission peak is also affected by the disorder strength. For example, for the state $n = 4$, the value of $T_{\rm peak}$ for disorder strength $\Delta W = 2.5$ Å is less than 0.5. The partial decay widths for the border states are very different. For example, for state $n = 6$ $\Gamma_6^0 \ll \Gamma_6^L$ and state $n = 15$, $\Gamma_{15}^0 \gg \Gamma_{15}^L$. The resulting high value of $\Gamma_6^L$ is an effect of the right edge of the structure, in which $|u_n(L)|^2$ is relatively high compared with $|u_n(0)|^2$. Similarly, the high value of $\Gamma_{15}^0$ is due to the left edge, where $|u_n(0)|^2$ is much larger than $|u_n(L)|^2$. Therefore, from Eq. (3.1), the transmission peaks for both border states are always negligible, as exhibited by Fig. 10 for the two disordered strengths considered. This is in contrast with the behavior of $T_{\rm peak}$ for quasilocalized and intermediate states where $T_{\rm peak}$ can acquire any value.

The resonance widths $\Gamma_n$ for the quasilocalized states are much smaller than those corresponding to the periodic case. Therefore the lifetimes of the quasilocalized states are much longer. This means that a particle incident on the system with an energy close to $\epsilon_n$, for one of these states, will be trapped inside the structure for a relatively long-time interval, of the order $\tau_n \approx \hbar/\Gamma_n$. On the other hand, since border states have a very large value of the total decay width, they have an extremely short lifetime. The peculiar behavior of the border states is a consequence of the finite length of the system.

To complete our discussion, we consider the effect on the properties of the resonant states when the strength of disorder is varied. Figure 11 shows the variation of the total decay width as a function of the disorder strength: The value of $2\Delta W$ is increased from zero to 10.0. It can be seen how most of the states tend to localize as the disorder increases except for two of them. In effect, we know that a signature of localization is a small value of the total width. Figure 11 demonstrates quite clearly which states tend to localize, namely, those for which $\Gamma_n$ diminishes as the disorder is increased. In the same manner the graph reveals the states that will not localize, namely, the border states $n = 6$ and 15. Note that for very slight disorder most of the states remain extended, and that, as the disorder increases, border and quasilocalized states arise.

## IV. CONCLUDING REMARKS

A main result of this work is that in a system of finite length in general one finds quasilocalized, border, and intermediate states. This is contrary to the prevalent view that in one dimension all states are localized independently of the degree of disorder. We indeed observe a tendency toward localization in most states. However, the fact that the system is open implies that the decay widths remain finite, and therefore it is not surprising that the Anderson criterion of localization is not fulfilled. Nevertheless we find, in agreement with the Thouless criterion for localization, that quasilocalized states are insensitive to changes in the boundaries of the system, while intermediate and border states are not. Border states seem to have no counterpart in infinite or cyclic systems. Our approach associates with each state a resonant function whose complex eigenvalue corresponds to a $S$-matrix pole of the problem, and allows us to relate the scattering properties, i.e., transmission, with the eigenvalue problem. We found that the distribution of the complex poles on the energy plane is very sensitive to the strength of the disorder, and allows us to discern among the different types of states. We found that a disorder strength of only 10% of the value of the well width for the periodic case already has dramatic effects on the properties of the states and the motion of the complex poles. By increasing the disorder strength, one eventually reaches situations where the number of wells of the system may change. For example, for a disorder strength $\delta W = 50$, one or several wells may disappear, leading to a different system. We have left these ''breakup'' systems out of the present study. However, for disorder strengths before that breakup limit we still found, in general, the three types of states mentioned above. It is of interest to stress that partial decay widths play a relevant role in characterizing the different types of states as well as the values of the transmission peaks. Finally, we expect that the properties of the different types of states discussed here might be accessible in experiments involving short-period superlattices, where controlled disorder can be introduced by variations of the growth parameters.

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