Floquet states and intersubband absorption in strongly driven double quantum wells

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The effects of an intense driving field on the linear intersubband-absorption spectrum of a symmetric double quantum well are nonperturbatively calculated within the Floquet theory. The dependence of the absorption spectrum on the intensity of the driving field is qualitatively different for photon energies larger and smaller than the splitting of the excited-state doublet: In the former case the splitting of the two absorption lines originally present in the spectrum of the undriven double quantum well is altered by the intensity of the driving field. For certain intensities, a collapse of the tunnel splitting is predicted. In the latter case photon replicas of the original lines appear in the spectrum of the driven quantum well. The absorption strength of these additional lines depends nonmonotonically on the intensity of the driving field. It is shown that these effects can be interpreted as generalizations of the Rabi or ac Stark splitting, which results from Floquet theory in the limit of the driving frequency being in resonance with the splitting of the excited-state doublet. [S0163-1829(97)07432-8]

I. INTRODUCTION

Semiconductor quantum-well structures strongly coupled to an intense high-frequency field have gained a lot of attention recently. Extensive work on calculating the modifications of the quantum-well energy spectrum induced by the driving field has been performed by several groups.[1–6] Exciting effects like the coherent suppression of tunneling,[7,8] the miniband collapse in quantum-well superlattices,[2] absolute negative conductance, and photon-assisted tunneling in resonant tunneling diodes[9,10] have been predicted and partly experimentally verified.[11–15] However, up to now, mainly transport measurements have been performed in order to test the theoretical results. In this paper we propose probing the ac-field-induced modifications in the quantum-well energy spectrum by intersubband-absorption measurements with a weak probing beam. Compared to transport measurements, optical experiments have the advantage of a higher energy resolution. In addition, transport measurements do not directly measure the spectral function, but rather its convolution with—at least—the supply function in the emitter of the device and the spectral function in the neighboring well.

Surprisingly, calculations of the absorption spectrum of a driven quantum-well system, and its dependence on the intensity of the driving field, have, up to now, existed in the literature only in the rotating-wave approximation (RWA) (Ref. 16) for the limited range of the Rabi splitting regime[17] [$\hbar\omega$ nearly resonant with states of the undriven quantum well and $eFx_{ij}/\hbar\omega \ll 1$, where ω (F) denotes the frequency (amplitude) of the driving field and x_{ij} the x-matrix element between the resonantly coupled states]. By employing Floquet theory,[18] we are able to extend these results to virtually arbitrary intensity and frequency of the driving field, thereby covering parameter regions beyond the applicability of the RWA ($eFx_{ij}/\hbar\omega > 1$) that have become accessible with the advent of powerful free-electron lasers delivering radiation in the THz region. It is shown that in this parameter range exciting effects like the collapse of the tunnel splitting and photon-sideband absorption occur in the intersubband spectrum of a strongly driven double-quantum-well system. It is interesting to note that closely related phenomena have been predicted and observed in atomic physics.[19] By applying Floquet theory, it is possible to treat the suppression of tunnel splitting, photon-sideband absorption, and Rabi splitting in a natural way within the same framework.

II. THEORY AND RESULTS

The Hamiltonian for an electron in a quantum-well structure driven by an intense electromagnetic field has the form $H = H_0 + xeF\cos(\omega t)$, where $H_0 = p^2/2m + V(x)$ is the Hamiltonian for the undriven quantum well. Since H is periodic in time, according to the Floquet theorem,[18] an ansatz

$$\psi(x,t) = \exp(-i\varepsilon t/\hbar)\ \ w(x,t), \tag{1}$$

where w is periodic in time, can be made for the solutions of the time-dependent Schrödinger equation, which then becomes

$$\left(H - \varepsilon - i\hbar\,\frac{\partial}{\partial t}\right) w(x,t) = 0. \tag{2}$$

Following the work of Shirley,[20] Eq. (2) can be transformed into an eigenvalue problem for an infinite matix by expanding the function w in a Fourier series $w = \Sigma a_n(x)\exp(-i\omega nt)$ with position-dependent coefficients $a_n(x)$. Expanding these coefficients in the complete set ψ_j^0 of eigenfunctions of H_0 leads to the following expression for $w(x,t)$:

$$w(x,t) = \sum_{n,j} c_{n,j}\exp(-i\omega nt)\,\psi_j^0(x). \tag{3}$$

Inserting Eq. (3) into Eq. (2), multiplying from the left side with $\exp(i\omega mt)(\psi_k^0)^*$, and integrating x over the normalization interval and t over one period $T = 2\pi/\omega$ leads to the infinite matrix eigenvalue equation

$$\sum_{n,j} \{[E_j-(\varepsilon+n\hbar\omega)]\delta_{m,n}\delta_{k,j} + \tfrac{1}{2}eFx_{k,j}(\delta_{m,n-1}+\delta_{m,n+1})\}c_{n,j} = 0, \quad (4)$$

where E_j denotes the eigenenergy of H_0 corresponding to the function ψ_j^0, and $x_{k,j}=\langle\psi_k^0|x|\psi_j^0\rangle$ denotes the x-matrix element.

It is straightforward to show that if ε^0 is an eigenvalue of Eq. (4) with eigenvector $c_{n,j}^0$, then, for all integers l,

$$\varepsilon^l=\varepsilon^0+l\hbar\omega,\ \ c_{n,j}^l=c_{n+l,j}^0 \quad (5)$$

are also eigenenvalues with corresponding eigenvectors. However, from Eqs. (1) and (3) it follows that the same total wave function $\psi(x,t)$ is obtained for all l. This is a direct consequence of the Floquet theorem, and illustrates that in a periodically driven system the ''quasienergies'' ε are only determined up to an integer multiple of the driving frequency $\hbar\omega$, and, therefore, can be mapped into a quasienergy zone with boundaries separated by $\hbar\omega$ (Refs. 18 and 20). The quasienergy zone picture turns out to be very useful for strong driving fields, i.e., for dipole energies $|ex_{ij}F|$ much greater than the splitting $|E_i-E_j|$. However, in the following we will discuss the changes of the intersubband-absorption spectrum induced by fields with maximum dipole energies in the order of magnitude of the splitting. In this case it appears more appropriate to represent the eigenstates by the quasienergies that evolve from the respective eigenenergies for $F=0$, i.e., we use the eigenvalues $\varepsilon^{i,0}$ and eigenvectors $c_{n,j}^{i,0}$ of Eq. (4) that are defined by

$$F\rightarrow 0:\ \ \varepsilon^{i,0}\rightarrow E_i \quad \text{and} \quad |c_{0,i}^{i,0}|^2\rightarrow 1. \quad (6)$$

For the sake of nomenclature in the following, we will refer to this representation as the zero-photon representation (indicated by a 0 in the *upper* pair of indices at ε and c) and to the quasienergies $\varepsilon^{i,l}=\varepsilon^{i,0}+l\hbar\omega$ and the corresponding eigenvectors $c_{n,j}^{i,l}=c_{n+l,j}^{i,0}$ as the l-photon representation. This has to be distinguished from the nth Fourier component of the l-photon representation which is denoted by the n in the *lower* pair of indices of $c_{n,j}^{i,l}$.

It is worth noting that for a symmetric quantum-well system $(H-i\hbar\partial/\partial t)$ is invariant under the operation S_p: $(x\rightarrow -x, t\rightarrow t+T/2)$. As a consequence, a complete set of solutions of Eq. (2) can be found among Floquet functions w with either even or odd parity under S_p. Since in the expansion of the Floquet function w [Eq. (3)] the wave functions of the undriven quantum well with even [$j=1$ (ground state), 3...) and odd ($j=2,4...$) parity with respect to space inversion are used as a basis, the coefficients $c_{n,j}$ must satisfy the following relations:

$$w,\text{even parity:} \quad c_{n,j}=0 \quad \text{for } n+j \text{ even}, \quad (7a)$$

$$w,\text{odd parity:} \quad c_{n,j}=0 \quad \text{for } n+j \text{ odd}. \quad (7b)$$

For a given symmetry of the Floquet functions, half of the expansion coefficients $c_{n,j}$ vanish according to Eq. (7). For the Floquet functions in the zero-photon representation, the operation S_p becomes equivalent to the space inversion $(x\rightarrow -x)$ for vanishing F. Therefore, for a symmetric quantum-well system, the zero-photon Floquet functions have the same parity with respect to S_p as the corresponding basis functions ψ_j^0 have with respect to space inversion.

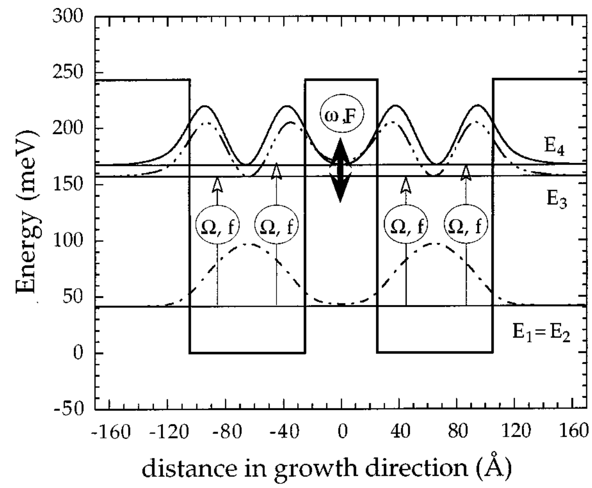

FIG. 1. Schematic energy diagram of the coupled GaAs/$Al_{0.3}Ga_{0.7}As$ quantum wells under consideration. The eigenenergies and the squared moduli of the wave functions of the undriven system are shown for the ground- (E_1,E_2) and excited-state doublets (E_3,E_4). The transitions induced by the weak probing field (amplitude f frequency Ω) are indicated by the thin arrows, whereas the thick double arrow schematically shows the intense driving field (amplitude F; frequency ω, $F\gg f$, $\omega\ll\Omega$).

In the following we consider two symmetric quantum wells separated by a thin barrier. The ground states in the adjacent quantum wells (E_1,E_2) are assumed not to be coupled through the barrier and, consequently, are degenerate $(\Delta_{12}=E_2-E_1=0)$. Furthermore, we assume that the first excited states (E_3,E_4) are separated from the ground-state doublet by approximately 100 meV. Due to the higher energy of these states, they are coupled by tunneling, and, therefore, are split by an energy $\Delta_{34}=E_4-E_3$ assumed to be in the order of 5–10 meV. Such a situation, for example, is realized by two 80 Å GaAs/$Al_{0.3}Ga_{0.7}As$ quantum wells separated by a 50 Å barrier, the energy diagram of which is schematically shown in Fig. 1. In this structure, virtually all the intersubband oscillator strength is contained in $E_1\rightarrow E_4$ and $E_2\rightarrow E_3$ transitions (indicated by the thin arrows in Fig. 1). The driving frequency $\hbar\omega$ is assumed to be much smaller than the intersubband transition energies, i.e., on the order of magnitude of the tunnel splitting Δ_{34} (sketched by the thick arrow in Fig. 1). Under these conditions, the coupling of the ground- and excited-state doublets due to the driving field can be safely neglected for realistic field strengths F. Therefore, Eq. (4) can be separated into two blocks, one describing the ground-state doublet ($j,k\in[1,2]$ in Eq. (4)), the other one the excited-state doublet ($j,k\in[3,4]$ in Eq. (4)).

For each of the two doublets under consideration, Eq. (4) can be separated into equations for the even and odd Floquet functions by using Eq. (7), rearranging lines and columns, and setting the origin of the energy scale to the mean value of the doublet energies:

$$\begin{pmatrix} \ddots & \mu & 0 & & & & \\ \mu & \pm\delta+2-\tilde{\varepsilon} & \mu & 0 & & & \\ 0 & \mu & \mp\delta+1-\tilde{\varepsilon} & \mu & 0 & & \\ & 0 & \mu & \pm\delta-\tilde{\varepsilon} & \mu & 0 & \\ & & 0 & \mu & \mp\delta-1-\tilde{\varepsilon} & \mu & 0 \\ & & & 0 & \mu & \pm\delta-2-\tilde{\varepsilon} & \mu \\ & & & & 0 & \mu & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ c_{-2}^{\pm} \\ c_{-1}^{\pm} \\ c_{0}^{\pm} \\ c_{1}^{\pm} \\ c_{2}^{\pm} \\ \vdots \end{pmatrix} = 0. \tag{8}$$

In Eq. (8), the abbreviations $\mu = eFx_{jk}/2\hbar\omega$, $\delta = |\Delta_{jk}|/2\hbar\omega$, and $\tilde{\varepsilon} = \varepsilon/\hbar\omega$ are used. The upper (lower) signs in Eq. (8) correspond to Floquet functions with even (odd) parity. In addition, the index j of the Fourier components $c_{n,j}$ has been omitted since, for a symmetric two-level system, the specification of the parity and frequency (n) uniquely determines the expansion of the Floquet function [Eq. (4)] in the sense of Eq. (7).

For the ground-state doublet, $\Delta_{12} = 0$ according to our assumption. In this case, the eigenvalues and eigenvectors of Eq. (8) are readily obtained to be[21]

$$\varepsilon^{\pm,m} = m\hbar\omega, \quad c_n^{\pm,m} = J_{n+m}\left(\frac{eFx_{12}}{\hbar\omega}\right), \tag{9}$$

where J_n denotes the nth-order Bessel function of the first kind. Therefore, in the zero-photon representation ($m=0$), the two Floquet functions evolving from the ground-state doublet are given by

$$w^{+(-),0}(x,t) = \sum_{n=-\infty}^{\infty} [J_{2n}(\alpha_{12}) e^{-i2n\omega t} \psi_{1(2)}^0(x) + J_{2n+1}(\alpha_{12}) e^{-i2(n+1)\omega t} \psi_{2(1)}^0(x)], \tag{10}$$

where $\alpha_{12} = eFx_{12}/\hbar\omega$. These results are similar to those obtained, for example, by Holthaus[1] for strongly driven coupled quantum wells.

For the excited-state doublet (E_3, E_4), Eq. (8) cannot be solved analytically since the splitting Δ_{34} is finite. Therefore, in the following, results obtained by numerically diagonalizing Eq. (8) are presented. In these calculations the infinite matrix in Eq. (8) has to be truncated. As a consequence, the infinite series of eigenvalues spaced by $\hbar\omega$ [defined in Eq. (5)] is broken, and the eigenvalues of those states, for which the coupling to the truncated states would be significant, will deviate from the equally spaced quasienergy ladder. This can be used to check the error induced by truncating Eq. (8) *a posteriori*: for the range of field strengths F such that the difference between the quasienergies of the zero-photon representation and the adjacent quasienergies belonging to the same ladder, i.e., the quasienergies of the ±1-photon representation) does not deviate significantly from $\pm\hbar\omega$, the error due to the truncation of Eq. (8) is negligible. In our calculations, interactions via up to ten photons [corresponding to 21 Fourier components in Eq. (8)] were included, resulting in an error in the difference between the eigenvalues of the zero- and one-photon representation of less than 0.1% of $\hbar\omega$ for fields up to $eFx_{ij}/\hbar\omega < 15$.

Figure 2(a) shows the result of the calculation for the $n = 0, \pm1$ Fourier components of the eigenvector corresponding to the third level in the zero-photon representation ($\varepsilon^{3,0}$) as a function of the strength F of the driving field. In

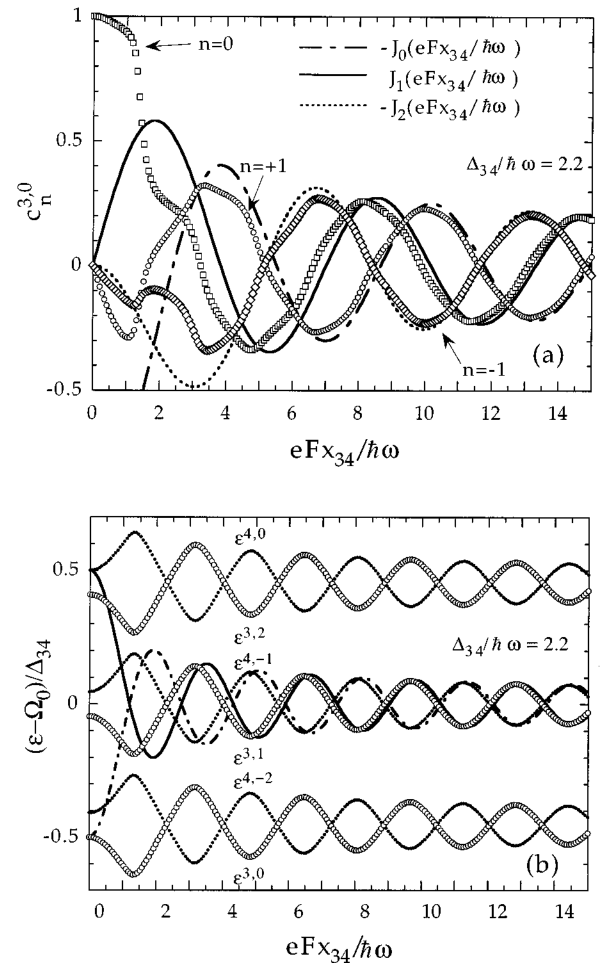

FIG. 2. (a) Numerical solution for the $n=-1$ (diamonds), $n=0$ (squares), and $n=+1$ (circles) Fourier components of the eigenvector corresponding to the third level in the zero-photon representation. For intense driving fields, these solutions can be approximated by the Bessel functions shown in the plot by the dash-dotted, full, and dotted lines. In the calculation $\Delta_{34}/\hbar\omega = 2.2$ was assumed. (b) Part of the quasienergy ladder calculated for $\Delta_{34}/\hbar\omega = 2.2$. Shown are only those quasienergies that form a ladder with the quasienergies evolving from E_3 (circles) and E_4 (dots). For intense driving fields, the dependence of the quasienergies on the driving field can be approximated by $\pm J_0(2eFx_{34}/\hbar\omega)$, as shown by the full and dash-dotted lines.

the calculations, $\Delta_{34}/\hbar\omega = 2.2$ is assumed. Figure 2(a) shows that the coefficients strongly deviate from the Bessel function behavior that is obtained for vanishing splitting according to Eq. (9). However, for very strong fields ($eFx_{34}/\hbar\omega > 7$) the $n = -1$, 0, and 1 components approach the Bessel functions $-J_2$, J_1, and $-J_0$ [shown by the dotted, full, and dash-dotted lines in Fig. 2(a), respectively]. In this regime of field strength, $\Delta_{34} \ll eFx_{34}$ and therefore, can be neglected in Eq. (8) (strong driving limit). As a consequence, the Bessel functions given by Eq. (9) are approximate solutions of Eq. (8) for intense driving fields. [Setting $m = -1$ in Eq. (9) and using $J_{-n} = (-1)^n J_n$ results (up to the phase factor factor -1) in the Bessel functions shown in Fig. 2(a)]. Following the work of Holthaus,[1] in the strong driving limit the corrections of the quasienergies due to Δ_{34} being finite can then be obtained in first-order perturbation theory:

$$\varepsilon^{\pm,m} = m\hbar\omega \mp (-1)^m \frac{\Delta_{34}}{2} J_0(2\alpha). \tag{11}$$

Moreover, using similar arguments, it can be shown that, in the limit $\Delta_{34} < \hbar\omega$, Eqs. (9) and (11) are good approximations even for low amplitudes of the driving field.

In Fig. 2(b), the quasienergy ladder in the energy range $-\Delta_{34}/2$ to $\Delta_{34}/2$, as obtained by numerical diagonalization of Eq. (8) for $\Delta_{34}/\hbar\omega = 2.2$, is shown as a function of the driving field strength. The quasienergies belonging to the same ladder as the one evolving from the level E_3 (E_4) of the undriven well are shown by circles (dots), and are labeled by $\varepsilon^{3,n}$ ($\varepsilon^{4,n}$). Also shown in Fig. 2(b) by the broken and full lines are the approximate quasienergies calculated from Eq. (11) for $m = 0$. As in the case of the Fourier components, since $\Delta_{34}/\hbar\omega = 2.2$ only for very strong fields ($eFx_{34}/\hbar\omega > 7$), Eq. (11) becomes a good approximation to the numerical solution of Eq. (8).

In the remaining part of this paper, the intersubband transition probability from the ground-state doublet (E_1, E_2) to the excited-state doublet (E_3, E_4) and its dependence on the driving field is calculated. Such transitions can be induced by a weak probing field with a frequency on the order of the energy difference of the two doublets. The interaction of a strongly driven quantum-well system with a probing field can be described in the dipole approximation by adding the term $H_{\text{int}} = \text{exf} \cos(\Omega t)$ ($f \ll F$) to the Hamiltonian H. The transition rates P_{if} (proportional to the linear absorption coefficient) from an initial state ψ_i to the final state ψ_f [$\psi_{i(f)} = \exp(-i\varepsilon_{i(f)} t/\hbar) w_{i(f)}(x,t), i \in \{1,2\}, f \in \{3,4\}$] induced by this interaction are then calculated in first order in H_{int} by time-dependent perturbation theory. The relevant matrix elements are given by

$$\langle w_f(x,t) | x | w_i(x,t) \rangle = x_{f,i} \sum_N{}' \exp(-iN\omega t) \sum_n c_n^f c_{n+N}^i, \tag{12}$$

where the apostrophe at the left summation symbol indicates that the summation index N runs only over even (odd) integers for Floquet functions w_f, w_i with opposite (equal) parity with respect to S_P. In addition, in deriving Eq. (12), for the symmetric quantum-well structure under consideration we used $x_{32} \approx x_{41} \equiv x_{f,i}$. With Eq. (12), Fermi's golden rule for the transition rates P_{if} becomes

$$P_{\text{if}}(\Omega) \propto \sum_{N=-\infty}^{\infty}{}' \delta(\varepsilon^i - \varepsilon^f + N\hbar\omega + \hbar\Omega) \left| \sum_n c_n^f c_{n+N}^i \right|^2. \tag{13}$$

Needless to say, $P_{\text{if}}(\Omega)$ is independent of the representation, as can be shown by using Eq. (5). In the following we will discuss the dependence of the quantum-well absorbance on the frequency and intensity of the driving field as it is gradually increased from zero. Therefore, the zero-photon representation is the natural representation to work with. Note that for the transition processes discussed below the numbers given for N refer to this representation.

In Fig. 3 the transition rates $P_{\text{if}}(\Omega)$ calculated according to Eq. (13) and summed over the degenerate initial-state doublet of the coupled quantum wells under consideration are shown as a function of the strength of the driving field (F) and the photon energy of the probe field (Ω) for four different ratios of $\Delta_{34}/\hbar\omega$. (In Fig. 3, Ω_0 denotes the average of the transition energies $E_4 - E_1$ and $E_3 - E_2$. The transition rate is grey scale coded with black indicating high transition rates.) Figure 3 shows that for frequencies greater than the splitting of the excited states in the undriven system ($\Delta_{34}/\hbar\omega = 0.8$ in Fig. 3), the absorption spectrum mainly consists of two lines for all intensities of the driving field. These strong absorption lines evolve continuously from the absorption lines for $F = 0$, showing that only the $N = 0$ terms in Eq. (13) contribute significantly to the absorption spectrum. The splitting of the absorption lines closely follows the Bessel function behavior of Eq. (11) (even for small fields), and, therefore, it can be suppressed at $2eFx_{34}/\hbar\omega$ equal to the zeros of the Bessel function J_0: as shown in Fig. 3, only one absorption line remains in the spectrum for these field strengths. This suppression of the tunnel splitting is analogous to the miniband collapse predicted by Holthaus.[1,2]

Decreasing the driving frequency to $\Delta_{34}/\hbar\omega = 1.2$ leads to a splitting of the two absorption lines for moderate fields ($eFx_{34}/\hbar\omega \leq 1$; see the upper right panel of Fig. 3). The two additional absorption lines emerging for finite F are separated by $\pm\hbar\omega$ from the absorption lines that evolve from the two lines present at $F = 0$ [i.e., they correspond to the $N = \pm 1$ terms in Eq. (13)]. For $eFx_{34}/\hbar\omega \ll 1$, this splitting is the well-known Rabi or ac Stark splitting. However, the formalism chosen in this work is not restricted to small driving fields and, therefore, we are able to calculate the Rabi splitting and its dependence on the field strength beyond the RWA for all magnitudes of the driving field: The upper right panel of Fig. 3 shows that for increasing field the oscillator strength is gradually transferred to the $N = \pm 1$ transitions. Finally, for very intense fields, these are the only transitions that remain in the calculated absorption spectrum. Their transition energy oscillates about and decays toward the arithmetic mean of the transition energies for $F = 0$. In analogy to the dc Stark effect (i.e., a strong dc field decouples the double quantum well, so that both wells finally absorb at the

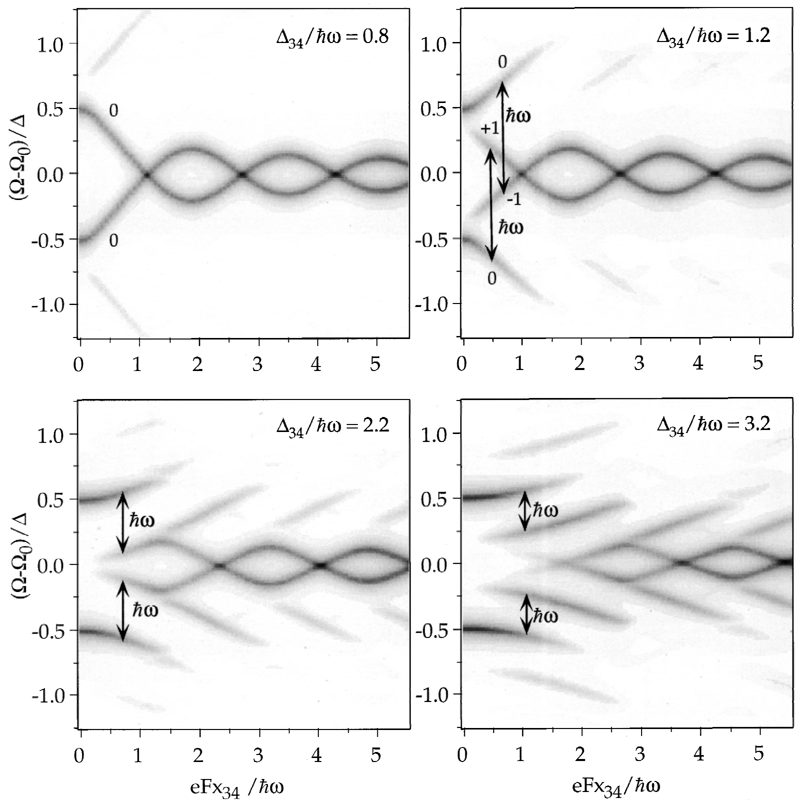

FIG. 3. Grey-scale contour plot of the transition rates P_{if} summed over the initial state doublet (black indicating high transition rates) as a function of the driving field F and of the photon energy of the probing field (Ω) for four different ratios $\Delta_{34}/\hbar\omega$. (Ω_0 denotes the mean energy difference between the ground- and excited-state doublets of the undriven system.) The arrows indicate the energy difference corresponding to a driving field photon. The integers in the upper panels refer to the terms N in Eq. (13) (in the zero-photon representation) that correspond to the respective absorption lines (see the text for further discussion). In the lower two panels these labels have been omitted for clarity.

same energy), this behavior can be interpreted as the quantum wells becoming gradually decoupled by the intense ($eFx_{34}/\hbar\omega \gg 1$) ac driving field.

The lower two panels in Fig. 3 show that decreasing ω further leads to an even richer structure in the calculated absorption spectra as more photon replicas of the original lines (photon sidebands) appear in the absorption spectra: For example, in the panel corresponding to $\Delta_{34}/\hbar\omega = 3.2$, significant oscillator strength is redistributed among as much as six absorption lines in the range $1 < eFx_{34}/\hbar\omega < 2$. It is evident here that Rabi splitting and photon sideband absorption are closely related, the latter being a generalization of the former for nonresonant driving. However, for the largest fields included in the calculations, again only two absorption lines dominate the calculated absorption spectra, indicating the decoupling of the quantum wells by the driving field.

The increased complexity of the absorption spectrum for driving frequencies $\hbar\omega < \Delta_{34}$ can be understood in the following way: Figure 3 shows that for these frequencies the quasienergies evolving from the excited-state doublet are pushed *apart* as the driving field is gradually increased from zero [note that, according to Eq. (9), the quasienergies evolving from the ground-state doublet are independent of F. Therefore, the quasienergies evolving from the excited-state doublet display the same dependence on F as the transition energies shown in Fig. 3]. Consequently, for certain field strengths F, these levels are pushed into multiphoton resonances producing strong mixing and hence the enrichment of the (Fourier) spectrum shown in Fig. 3. Since for driving frequencies $\hbar\omega > \Delta_{34}$ the quasienergies evolving from the excited-state doublet are pushed *together* (see Fig. 3), multiphoton resonances cannot occur and, therefore, no sideband absorption results.

III. CONCLUSIONS

We have calculated the changes in the absorption spectrum of a symmetrically coupled double-quantum-well system induced by a strong driving field. By using the Floquet theory, we were able to numerically calculate these changes in a nonperturbative way for arbitrary frequency and intensity of the driving field. Depending on the relative magnitudes of the splitting of the excited states and the intensity and frequency of the driving field, both the collapse of the splitting and the appearance of multiphoton replicas of the original absorption lines including ac Stark splitting has been predicted within one unified approach. Experimentally, double-quantum-well systems with $\Delta_{34} \approx 10$ meV and $x_{34} \approx 80$ Å are well within the capability of III-V molecular-beam-epitaxy growth. High-frequency driving fields with field strength on the order of 10 kV/cm can, for example, be generated by the free-electron lasers at the University of California at Santa Barbara. Therefore, the phenomena predicted in this work should be observable, and might open a possibility to investigate the dynamics of strongly driven coupled quantum wells by optical means.

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