Stark shifts of excitonic complexes in quantum wells

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We utilize a nonvariational ansatz as the perturbative term to study the quadratic Stark effect of hydrogenic systems in a fractional-dimensional space. Applying a Dalgarno-Lewis-type technique, the ansatz is used to derive an analytical expression for the Stark shifts in quantum wells. Estimates of the energy shifts of negatively charged excitons are in good agreement with recent experimental results, and provide qualitative explanations for their behavior in a weak external electric field. The model is extended to study the Stark shifts of positively charged excitons and biexcitons in quantum wells. [S0163-1829(97)06131-6]

I. INTRODUCTION

The effects of electric fields on hydrogenic systems like excitons have been extensively investigated in semiconductor quantum wells[1–8] for the past ten years. These studies are mainly motivated by technological applications[2,9–11] due to the room-temperature optical spectra[12] of confined excitons with enhanced binding energies. Recent interest[13–18] in charged exciton states (X^- and X^+) have opened up the possibility of measurable stark effects, with the application of weak electric fields. Both X^- and X^+, for which, respectively, an electron is bound to an exciton and a hole is bound to an exciton, have been observed to be stable with binding energies in the range of 1–2 meV, at zero electric field, for III-V semiconductor materials. In one of the first experiments of its type, Shields *et al.*[19] demonstrated that when a weak electric field ($\leqslant$ 10 kV/cm) is applied perpendicularly to the quantum-well layers of a remotely doped 300-Å $GaAs/Al_{0.33}Ga_{0.67}As$, the ground-state charged exciton (X^-) peak energy experiences a measurable redshift. This shift in energy was seen to be less than the redshift experienced by its uncharged exciton counterpart.

In a lower fractional-dimensional space, the anisotropic interaction in an exciton, which is typical of quantum well systems in three-dimensional (3D) space, become isotropic in nature.[20–23] Thus only a single parameter, known as the degree of dimensionality (denoted by α), is needed to incorporate the effect of change in the widths of the well or barrier regions on the strength of the interaction. Our main result in this paper is an analytical expression of the redshift in the energy of an exciton in a finite quantum well, as a function of the dimensionality parameter α, due to a weak external electric field. The electric field is assumed to act perpendicularly to the direction of confinement of the exciton. We do not aim to compete with variational calculations[1–4] or intensive computational methods,[5,6] but to avoid tedious computations in the calculations of the Stark shifts of an exciton in the presence of a weak electric field. Due to the simplicity of the approach adopted here, we are able to extend calculations of the Stark shifts to the cases of the charged exciton and biexciton.

This paper is organized as follows. In Sec. II, we present the nonvariational ansatz, which is determined by the dimensionality of the hydrogenic system subjected to a weak electric field. We also provide the theoretical basis needed to obtain a simple analytical expression for the quadratic Stark shift of an exciton in a finite quantum well. In Sec. III, we compare the Stark shifts in the energy of the ground-state heavy-hole exciton obtained using the model here with the experimental results of Lengyel, Jelly, and Engelmann.[4] In Sec. IV, we study the Stark shifts of the charged exciton and obtain numerical results for the X^- complex in $GaAs/Al_{0.33}Ga_{0.67}As$ quantum wells of width 300 Å. We also provide qualitative results of the effect of a weak electric field on a positively charged exciton X^+. In Sec. V, we extend the model developed in Sec. II to predict the redshifts of biexcitons in quantum wells. In Sec. VI, we present the conclusion of this work.

II. QUADRATIC STARK EFFECT IN A FRACTIONAL-DIMENSIONAL SPACE

We consider a single quantum well, in which a weak external static electric field is applied perpendicular to the well layer. Utilizing an isotropic fractional-dimensional space Laplacian proposed by Stillinger,[24] the relative motion of an exciton in the quantum well can be described by an effective-mass Schrödinger equation

$$H_o\psi(r,\theta)=\left[-\frac{\hbar^2}{2\mu r^{\alpha-1}}\frac{\partial}{\partial r}r^{\alpha-1}\frac{\partial}{\partial r}+\frac{l^2}{2\mu r^2}-\frac{e^2}{4\pi\epsilon\epsilon_o r}\right]\times\psi(r,\theta)$$
$$=(E-E_g)\psi(r,\theta), \tag{1}$$

where μ is the reduced mass of the exciton, $\mathbf{r}$ is the relative distance vector, and ϵ is the dielectric constant in the well region. The term l^2 denotes the angular-momentum operator and α is the dimensionality parameter which describes the degree of anisotropy of the electron-hole interaction, $1\leqslant\alpha\leqslant 3$. The exciton wave function $\psi(r,\theta)$ can be written in a separable form[20,22] $\psi(r,\theta)=R(r)\Phi(\theta)$ for various values of the quantum numbers, n, l, and m.

The discrete bound-state exciton energies E_{ex} are given by

$$E_{ex}=E_g-\frac{R}{\left(n+\frac{\alpha-3}{2}\right)^2}, \tag{2}$$

where E_g is the band-gap energy, $n=1,2,\ldots$ is the principal quantum number, and R_y is the effective Rydberg given by

$$R_y=\frac{13\,600}{\epsilon^2}\frac{\mu}{m_e}\ \text{meV}, \tag{3}$$

where m_e is the free-electron mass. Likewise, the bound exciton state radii $a_{\rm ex}$ are given by

$$a_{\rm ex}=\left(n+\frac{\alpha-3}{2}\right)^2 a_{\rm ex}^{\rm 3D} \tag{4}$$

where the three-dimensional exciton radius $a_{\rm ex}^{\rm 3D}$ is given by

$$a_{\rm ex}^{\rm 3D}=0.53\epsilon\frac{m_e}{\mu}\ \text{Å}. \tag{5}$$

We now introduce the α-dimensional perturbative term due to the weak external field as

$$h=eF(\alpha-2)a_{\rm ex}^{3-\alpha}r^{\alpha-2}\cos\theta, \tag{6}$$

where F is the electric-field intensity. The ansatz term in Eq. (6) can be considered as an anisotropic perturbative potential term that is added to the unperturbed Hamiltonian H_o in Eq. (1) when a weak external electric field is switched on. The form of the ansatz satisfies the dimensionality condition, and reduces to the well-known[1] value of 0 at the zero well width limit, i.e., when $\alpha=2$. The term also yields the expected forms in the exact three-dimensional limits ($\alpha=3$), and its validity in the quasi-two-dimensional case will be justified in Sec. III, by comparison with experimental results obtained in Ref. 4.

The shift in the ground-state exciton energy due to an external electric field, which is in fact a hydrogen atom in an electric-field problem, can be obtained using second-order nondegenerate perturbation theory,[25]

$$\Delta E_{\rm ex}(\alpha)=\sum_{n\neq m}\frac{|\langle n|h|m\rangle|^2}{E_n^{(0)}-E_m^{(0)}}=\sum_{m\neq 0}\frac{|\langle 0|h|m\rangle|^2}{E_n^{(0)}-E_m^{(0)}} \tag{7}$$

$$=e^2F^2(\alpha-2)a_{\rm ex}^{3-\alpha}\sum_{m\neq 0}\langle 0|r^{\alpha-2}\cos\theta|m\rangle\langle 0|Q|m\rangle \tag{8}$$

$$=e^2F^2(\alpha-2)a_{\rm ex}^{3-\alpha}[\langle 0|(r^{\alpha-2}\cos\theta)Q|m\rangle - \langle 0|r^{\alpha-2}\cos\theta|0\rangle\langle 0|Q|0\rangle] \tag{9}$$

where $|0\rangle$ labels the ground state, and $|m\rangle$ all the other excited states of the exciton. The operator Q appears as a result of the simplification of Eq. (7) to Eq. (8) by the Dalgarno-Lewis technique.[26] Thus Eq. (9) can be reduced to a simple form once the form of the operator Q is determined. Here we use a modified form of Q as[25]

$$Q=-\frac{\mu a_{\rm ex}}{\hbar^2}S(\alpha)\left(\frac{r}{2}+a_{\rm ex}\right)r\cos\theta, \tag{10}$$

where $S(\alpha)$ is a term which scales the operator, Q, in an α-dimensional space:

$$S(\alpha)=10^{3\alpha-9}. \tag{11}$$

It is to be noted that Eq. (11) is used within the context of a fractional-dimensional space. This establishes a crucial relationship between α and the operator, Q, and therefore the final results of the redshift are strongly dependent on the form of $S(\alpha)$. At $\alpha=3$, Q reduces to the well-known form as utilized in the case of the isotropic three-dimensional hydrogen atom.[25]

In order to evaluate Eq. (9), we use the ground exciton state $\psi(r,\theta)$ in α dimensional space as:

$$\psi(r,\theta)=F(\alpha)\exp\left(-\frac{2}{\alpha-1}\right)\left(\frac{r}{a_{\rm ex}}\right), \tag{12}$$

where $F(\alpha)$ is given by

$$F(\alpha)=\left[\frac{2^{3(\alpha-1)}\Gamma\left[\frac{\alpha}{2}\right]^2\Gamma\left[\frac{\alpha-1}{2}\right]}{\pi^{(\alpha+1)/2}\Gamma[\alpha-1]^2(\alpha-1)^{\alpha+1}}\frac{1}{a_B^\alpha}\right]^{1/2}, \tag{13}$$

and the spatial integral relation in α-D space:[20]

$$\int_{\alpha\rm D}d\mathbf{r}=\frac{2\pi^{(\alpha-1)/2}}{\Gamma\left[\frac{\alpha-1}{2}\right]}\int_0^\infty r^{\alpha-1}dr\int_0^\pi d\theta\sin^{\alpha-2}\theta. \tag{14}$$

An explicit expression for the energy shift, ΔE, can be evaluated using Eqs. (10)–(14) in Eq. (9):

$$\Delta E_{\rm ex}(\alpha)=-G(\alpha)\frac{\mu e^2 a_{\rm ex}^4}{\hbar^2}F^2, \tag{15}$$

where $G(\alpha)$ can be obtained as

$$G(\alpha)=H(\alpha)10^{3\alpha-9}\frac{(\alpha-2)(\alpha-1)^{\alpha-1}\Gamma\left(\frac{\alpha}{2}\right)^2}{2^{\alpha+2}\pi\Gamma(\alpha-1)^2} \times\left(\frac{\Gamma(2\alpha)}{2}+\frac{4\,\Gamma(2\alpha-1)}{\alpha-1}\right), \tag{16}$$

where $\Gamma[x]$ is the Euler's gamma function, and the integral $H(\alpha)$ is given by

$$H(\alpha)=\frac{\pi^{1/2}\Gamma\left[\frac{\alpha-1}{2}\right]}{2\Gamma\left[\frac{\alpha}{2}\right]}-\frac{\pi\Gamma[\alpha-1]}{2^{\alpha-1}\Gamma\left[\frac{\alpha+2}{2}\right]\Gamma\left[\frac{\alpha-2}{2}\right]}. \tag{17}$$

Both expressions in Eqs. (16) and (17) can be easily evaluated using MATHEMATICA.[27]

With the field strength F, measured by 1 (a.u.)$=m_e^2e^5/\hbar^4=5.142\times10^9$ (V/cm), the energy shift $\Delta E_{\rm ex}$, in Eq. (15) can be reduced to

$$\Delta E_{\rm ex}(\alpha)=-1.31G(\alpha)\frac{\mu}{m_e^*}(a_{\rm ex}\times10^8\ \text{m}^{-1})^4F^2\ \text{meV}, \tag{18}$$

where F and $a_{\rm ex}$ are given in kV/cm and Å, respectively, and m_e^* is the effective electron mass. It is clear from Eq. (18) that only a single quantity, the dimensionality α, needs to be determined in order to obtain information about the shifts in the exciton energy. The expression in Eq. (15) reduces to the

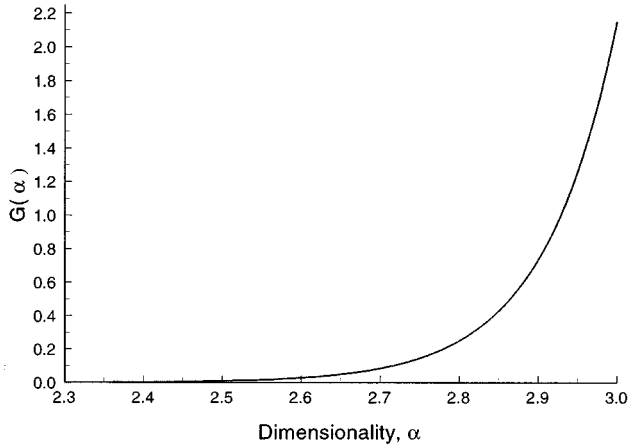

FIG. 1. Plot of $G(\alpha)$ vs α using $\mu = 0.04$, $a_{ex}^{3D} = 170$ Å.

well-known form of $\Delta E_{ex} = -\frac{9}{4}(\mu e^2 a_{ex}^4/\hbar^2)F^2$ for three-dimensional ($\alpha = 3$) hydrogenic systems. Figure 1 shows the sharp increase of $G(\alpha)$ with α, which implies a decrease in the Stark effect as the dimensionality of the hydrogenic system is lowered.

III. COMPARISON WITH EXPERIMENTAL RESULTS

In this section, we compare the Stark shifts in the ground-state heavy-hole exciton energy computed using Eq. (18) with the experimental results of Ref. 4 obtained using $GaAs/Al_{0.32}Ga_{0.68}As$ quantum wells of widths between 5 and 20 nm. We only consider the shifts in the exciton energy for electric fields up to 100 kV/cm, well within the regime of a weak external electric field. Hence Eq. (18), which is based on the second-order perturbation theory, remains valid for comparison with the experimental results.

Figure 2 shows experimental values[4] of the shift of the exciton energy as a function of the applied electric field. The figure also shows the the best quadratic fit through the experimental points, calculated using Eq. (18). The corresponding values of α_{ex} used for the quadratic fit are also provided in the figure.

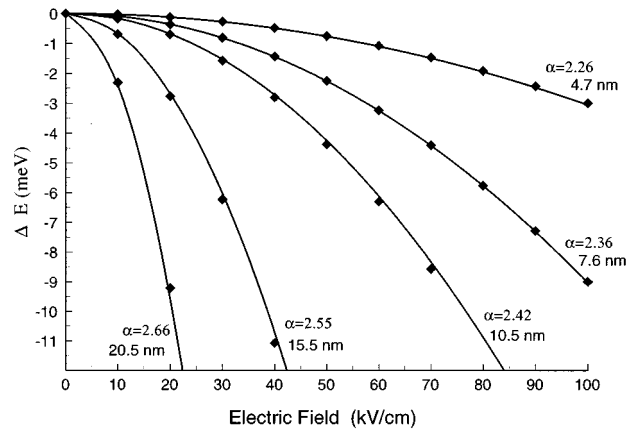

FIG. 2. Diamond-shaped points denote experimental values (Ref. 4) of the shift of the exciton energy as a function of the applied electric field at the indicated well widths. The solid line is the best quadratic fit through the experimental points, calculated using Eq. (18).

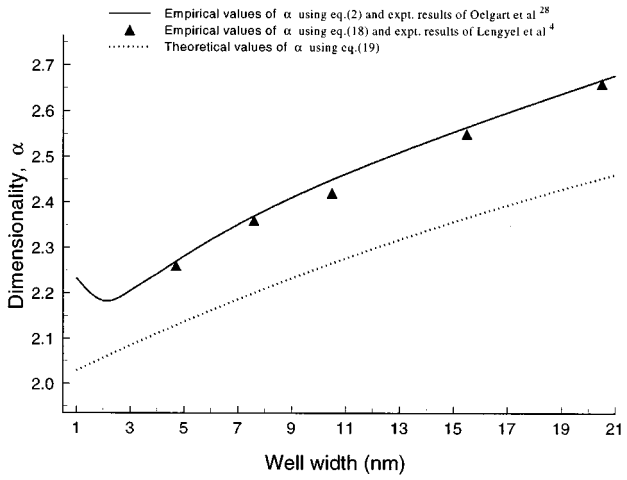

FIG. 3. Comparison of α_{ex} values obtained in Fig. 2 at the given well widths, with the best empirical estimates of α_{ex}. The empirical estimates of α_{ex} are obtained using Eqs. (2) and (3) and experimental values (Ref. 28) of exciton binding energies as a function of the well width.

In Fig. 3, we compare the α_{ex} values obtained in Fig. 2, at the given well widths, with the best empirical estimates of α_{ex}. The empirical estimates of α_{ex} are obtained using Eqs. (2) and (3) and experimental values[28] of exciton binding energies as a function of the well width. This method of determining α_{ex} yields the most reliable values[29] for the dimensionality of the confined exciton. It is to be noted that it is almost impossible to obtain accurate exciton binding energies, and hence reliable values for α_{ex}, via theoretical methods. This is due to various factors[30] like valence-band-mixing effects, the nonparabolicity of the conduction band, screening effects, and coupling between excitons from different subbands. Also, apart from the numerical difficulties linked to the complicated nature of the valence Hamiltonian[31] in an electric field, there is still a lack of knowledge of accurate values for the bulk valence parameters. One also needs to take into account the conditions under which the experimental binding energies are determined.

However, in the absence of experimental results, rough estimates of α_{ex} can generally be determined using[21]

$$\alpha_{ex} = 3 - \exp\left(-\frac{L_w}{2a_{ex}^{3D}}\right). \tag{19}$$

In Fig. 3, the values of α_{ex} calculated using Eq. (19) and $a_{ex}^{3D} = 170$ Å[1] are shown as dotted lines. The values are generally less (≤ 0.2) than the empirical values of α_{ex} for a fixed well width. This is attributed to the various factors mentioned in the previous paragraph.

Figure 3 shows that there is good agreement between the estimates of α computed using two independent approaches and two different sets of experimental results.[4,28] This clearly justifies the form of ansatz introduced in Eq. (6), and highlights the important role of a single parameter α in determining the Stark shifts of excitons in weak electric fields.

Figure 3 also shows that α_{ex} decreases as the well width decreases, and reaches a minimum value at a critcal well width before increasing again. This is mainly due to the effect of the spreading of the electron and hole wave function into the barrier regions, as the well width is decreased. Recent experimental results by Yuan *et al.*[32] has confirmed a similar crossover effect, where the heavy-hole exciton was shown to be best confined at a well width around ~ 20 Å. The Stark shifts of excitons are expected to show a similar crossover effect when smaller well widths ($\leqslant 35$ Å) are investigated.

IV. STARK SHIFTS OF THE NEGATIVELY CHARGED EXCITON X^- IN QUANTUM WELLS

Although theoretical calculations[33,34] of charged excitons in quantum wells have shown good agreement with experimental results, they are variational, and involve intensive computations. The accuracy of the results also depend on the parameters chosen to represent the trial wave functions. In an earlier paper, we simplified the approach of determing the binding energies of X^- and X^+ in quantum wells by modeling the charged exciton complex as a hydrogenic system in a fractional-dimensional space.[18] In this section, we apply Eq. (18) to the hydrogenic model of the charged exciton complex, to determine their shifts in weak electric fields.

The binding energy Eb_{X^-} of a negatively charged exciton, in the presence of an electric field F, can be written as[33]

$$Eb_{X^-}(F)=E_g(F)+E_{ex}(F)-E_{X^-}(F),$$
$$\Delta Eb_{X^-}(F)=\Delta E_{ex}(F)-\Delta E_{X^-}(F). \tag{20}$$

Equation (20) means that the shift in the binding energy of X^-, $\Delta Eb_{X^-}(F)$, is essentially equal to the difference in the redshifts of the exciton (ΔE_{ex}) and charged exciton (ΔE_{X^-}). Both ΔE_{ex} and ΔE_{X^-} can be easily determined using Eq. (18), once their dimensionalities α_{ex} and α_{X^-1} are determined accurately.

In order to obtain numerical values for the Stark effects of X^-, it is important to estimate the best value of α_{ex} and α_{X^-1}, so that comparison can be made with the experimental result[19] of ΔE_{X^-} [Eq.(18)] performed using remotely doped GaAs/$Al_{0.33}Ga_{0.67}$As quantum wells of width 300 Å. Using the three-dimensional effective Rydberg energy R_y and Bohr radius a_{ex} as 3.1 meV and 170 Å,[1] respectively, for the heavy-hole exciton, we obtain $\alpha_{ex}=2.75$ by using Eq. (2) and the binding energy Eb_{ex}, of 4 meV for an exciton in GaAs/$Al_{0.33}Ga_{0.67}$As quantum wells of width 300 Å.

The dimensionality α_{X^-1} can be determined using a simple analytical form that was recently[18] derived for Eb_{X^-}:

$$Eb_{X^-}=\left[\epsilon_r^2\left(1+\frac{2\sigma+1}{\sigma^2+4\sigma+2}\right)^{-1}-1\right]Eb_{ex}, \tag{21}$$

where ϵ_r is the ratio of the dielectric constant of the exciton to that of the charged exciton, X^- and $\sigma=m_e^*/m_h^*$. Due to the lack of knowledge of accurate values for ϵ_r and σ of the X^- complex, we have used the experimental value of $Eb_{X^-}\approx 0.95$ meV at $F=0$, as obtained by Shields *et al.*[19] in quantum wells of width 300 Å. Substituting $Eb_{ex}=4$ meV into Eq. (21), we obtain $\sigma=0.73$ and $\epsilon_r=1.34$. Using Eqs. (5) and (19), we obtain the radius of the charged exciton as $a_{X^-}=340$ Å and dimensionality $\alpha_{X^-}=2.36$. A more accurate estimate of α_{X^-} can be obtained by adding a corrective value of 0.15 (see Fig. 3) to α_{X^-}:

$$\alpha_{X^-}\approx 2.36+0.15=2.51. \tag{22}$$

Substituting $\alpha_{ex}=2.75$ and $\alpha_{X^-}=2.51$, for wells of width 300 Å, into Eq. (18), we obtain

$$\Delta E_{ex}(F)=-0.062F^2 \text{ meV}, \tag{23}$$

$$\Delta E_{X^-}(F)=-0.051F^2 \text{ meV}, \tag{24}$$

where F is given in units of kV/cm. A plot of ΔE_{ex} and ΔE_{X^-}, as well as their difference, $Eb_{X^-}(F)$, is shown in Fig. 2. The figure shows that our calculated results are consistent with the experimental results obtained by Shields *et al.*[19] It is interesting to note that the energy shifts of the exciton and charged exciton complex X^- are about 5–6 times as large as the shift in the binding energy of the extra electron, given by $Eb_{X^-}(F)$.

An approximate value of the electric field at which it becomes difficult to resolve the spectral splitting between the exciton and charged exciton peaks is determined using

$$\Delta Eb_{X^-}(F)\simeq Eb_{X^-}(F=0). \tag{25}$$

Using $Eb_{X^-}(F=0)\approx 0.95$ meV and Eq. (24), we obtain $F\simeq 9.3$ kV/cm, which agrees well with the experimental result ($9.4\leqslant F\leqslant 10.8$) of Shields *et al.*[19]

The hydrogenic model of a charged exciton has a larger effective bohr radius, and hence a smaller dimensionality when compared to an exciton in a quantum well of the same well width. This results in a smaller redshift of X^- in comparison to the uncharged exciton, as is shown in Fig. 4. Thus

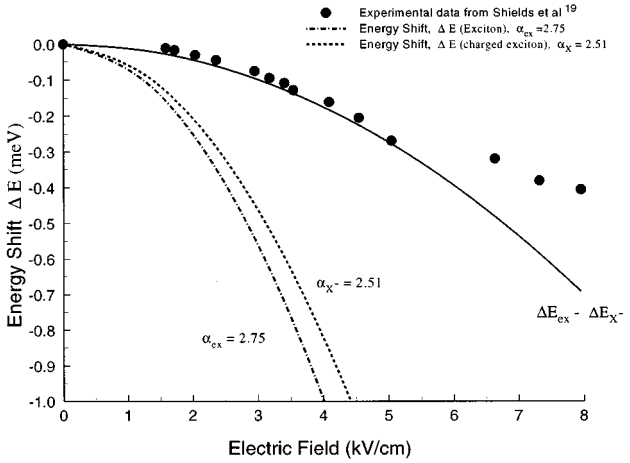

FIG. 4. Plot of the redshift of X^- and the uncharged exciton as a function of the electric field. The difference in their energy shifts is compared with the experimental data of Shield *et al.* (Ref. 19).

the spectral splitting between the exciton and charged exciton peaks decreases as the electric-field intensity increases. It should be noted that the quadratic fit through the experimental points based on our model breaks down for electric field, $F \geqslant 5.5$ meV. This may be caused by the high sensitivity of the binding energy of X^- to extrinsic electric fields in the experimental setup.[19]

It may be desirable to mention the behavior of the positively charged exciton X^+, which has been observed to be stable[14] in quantum wells. Theoretical calculations[18,33] have shown the binding energy of X^+ to be slightly higher than that for X^-, so that $\alpha_{X^+} \leqslant \alpha_{X^-}$. Consequently, the energy shifts experienced by X^+ are expected to be less than that for X^-. It may be difficult to resolve the spectral splitting between the exciton and positively charged exciton peaks, for electric fields $F \geqslant 5$ kV/cm in wells of width 300 Å.

V. STARK SHIFTS OF BIEXCITONS IN QUANTUM WELLS

Biexcitons are bound two-exciton states, and their role in quantum wells have been the subject of much recent interest[35–38] recently. Biexciton states are generally reached by two-electron excitations from the ground state, or by two-photon excitation from the ground state and their binding energy $Eb_{\rm biex}$ is determined using:

$$Eb_{\rm biex} = 2E_{\rm ex} - E_{\rm biex}. \tag{26}$$

Recent works have shown that the biexciton has an anisotropic character in two-and three-dimensional systems,[38,39] and can effectively be represented by a hydrogenic system in a fractional-dimensional space. Thus the Stark shifts of biexcitons in quantum wells can be easily determined using Eq. (18), once an accurate estimate for the dimensionality of biexciton is determined.

The Stark shifts of the biexciton can be determined using a recently derived result[38] of the ratio of the binding energy of the biexciton ($Eb_{\rm biex}$) to that of the exciton:

$$\frac{Eb_{\rm biex}}{Eb_{\rm ex}} \approx 0.228. \tag{27}$$

Using Eq. (27) in Eq. (15), we obtain

$$\Delta E_{\rm biex}(\alpha_{\rm biex}) = \frac{\mu_{\rm biex}}{\mu_{\rm ex}} \left(\frac{a_{\rm biex}}{a_{\rm ex}} \right)^4 \Delta E_{\rm ex}(\alpha_{\rm biex}). \tag{28}$$

Equation (28) can be further simplified using the simple ratios[38]

$$\frac{\mu_{\rm biex}}{\mu_{\rm ex}} = \frac{2}{3}, \tag{29}$$

$$\frac{a_{\rm biex}}{a_{\rm ex}} = 2.74. \tag{30}$$

In order to obtain numerical results, we consider the case of the biexciton in quantum wells of width 300 Å. Using Eqs. (19), (23), (28), (29), and (30), we obtain

$$\alpha_{\rm biex} \simeq 2.28, \tag{31}$$

$$\Delta E_{\rm biex} = -0.016F^2 \ \text{meV}, \tag{32}$$

$$\Delta Eb_{\rm biex} = \Delta E_{\rm ex} - \Delta E_{\rm biex} = -0.046F^2 \ \text{meV}. \tag{33}$$

Bearing in mind that our calculated value of $\alpha_{\rm biex}$ is probably underestimated, the calculated value of $\Delta Eb_{\rm biex}$ in Eq. (33) would differ by some factors (1.5–2) from the expected shift. Nevertheless, the qualitative gross features in the shift of the biexciton energy with the electric field is expected to remain unaltered.

Due to the larger Bohr radius of the biexciton ($\sim$ 470 Å), it experiences a smaller redshift in comparison to the charged exciton complex X^-. Thus $\Delta Eb_{\rm biex}$ is expected to be larger than ΔEb_{X^-}, i.e., the binding forces of a biexciton would be more sensitive to an external electric field than that in a charged exciton. We are unable to obtain experimental values of the biexciton at this moment in time to see this trend.

An approximate value of the electric field at which it becomes difficult to resolve the spectral splitting between the exciton and biexciton peaks can be determined using

$$\Delta Eb_{\rm biex}(F) \simeq Eb_{\rm biex}(F=0). \tag{34}$$

With $Eb_{X^-}(F=0) \approx$ 1 meV,[40] we obtain $F \simeq$ 4.7 kV/cm in a well of width 300 Å, which is about half of that in the case of the charged exciton.

VI. CONCLUSION

We used a flexible and computationally simple method of determining the Stark shifts of hydrogenic systems like the exciton, and some of its complexes in quantum wells. We calculated the redshifts of the exciton and its charged exciton complex in quantum wells of width 300 Å, for the sake of comparison with the experimental results of Shield *et al.*[19] The calculations can be easily done for quantum wells of any well widths. We discussed two possible ways (Sec. III) of determining the crucial parameter α which is needed to determine the redshifts in a quantum well of a known width.

It should be noted that, for large well widths ($\geqslant$ 300 Å), there is the possibility of optical nonlinearity due to exciton-exciton interactions.[41] Also, effects arising out of electron-electron and electron-hole interactions[42] may be important to consider when determining the dimensionality of exciton complexes. It is thus likely that these factors may add a small percentage of error to our calculated values of the Stark shifts of the charged exciton and biexciton.

In conclusion, in this paper, we have presented a method of determining the shifts in the energy of the exciton and its complexes. Our calculated energy shifts are in qualitative as well as quantitative agreement with known experimental results. This work may have importance in future experimental work involving exciton complexes in weak electric fields.

ACKNOWLEDGMENT

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[1] J. A. Brum and G. Bastard, Phys. Rev. B **31**, 3893 (1985).
[2] D. A. B. Miller, D. S. Chemla, T. C. Damen, A. C. Gossard, W. Wiegmann, T. H. Wood, and C. A. Burrus, Phys. Rev. Lett. **53**, 2173 (1984).
[3] P. J. Stevens, M. Whitehead, G. Parry, K. Woodbridge, IEEE J. Quantum Electron. **24**, 2007 (1988).
[4] G. Lengyel, K. W. Jelly, and R. W. H. Engelmann, IEEE J. Quantum Electron. **26**, 296 (1990).
[5] S. J. Kim, Y. T. Oh, S. K. Kim, T. W. Kang, and T. W. Kim, J. Appl. Phys. **77**, 2486 (1995).
[6] C. Y.-P. Chao and S. L. Chuang, Phys. Rev. B **48**, 8210 (1994).
[7] A. Latge, N. Porras-Montenegro, M. de Dios-Leyva, and L. E. Oliveira, Phys. Rev. B **53**, 10 160 (1996).
[8] G. Vonplessen, T. Meier, M. Koch, J. Feldmann, P. Thomas, S. W. Koch, E. O. Gobel, K. W. Goossen, J. M. Kuo, and R. F. Kopf, Phys. Rev. B **53**, 13 688 (1996).
[9] J. Singh, *Physics of Semiconductors and their Heterostructures* (McGraw-Hill, New York, 1993), p. 661.
[10] I. Kotaka, K. Wakita, K. Kawano, M. Asai, and M. Naganuma, Electron. Lett. **27**, 2162 (1991).
[11] A. Bandyopadhyay and P. K. Basu, IEEE J. Quantum Electron. **32**, 1048 (1996).
[12] M. Sugawara, T. Fujii, M. Kondo, K. Kato, K. Damen, S. Yamazaki, and K. Nakajima, Appl. Phys. Lett. **53**, 2290 (1988).
[13] A. J. Shields, J. L. Osborne, M. Y. Simmons, M. Pepper, and D. A. Ritchie, Phys. Rev. B **52**, R5523 (1995); A. J. Shields, M. Pepper, M. Y. Simmons, and D. A. Ritchie, *ibid.* **52**, 7841 (1995).
[14] J. L. Osborne, A. J. Shields, M. Pepper, F. M. Bolton, and D. A. Ritchie, Phys. Rev. B **53**, 13 002 (1996).
[15] G. Finkelstein, H. Shtrikman, and I. Bar-Joseph, Surf. Sci. **362**, 357 (1996); Phys. Rev. Lett. **74**, 976 (1995); Phys. Rev. B **53**, R1709 (1995).
[16] A. Ron, H. W. Yoon, M. D. Sturge, A. Manassen, E. Cohen, and L. N. Pfeiffer, Solid State Commun. **97**, 741 (1996).
[17] S. R. Ryu, W. Y. Yu, L. P. Fu, Z. X. Jiang, A. Petrou, B. D. McCombe, and W. Schaff, Surf. Sci. **362**, 363 (1996).
[18] A. Thilagam, Phys. Rev. B **55**, 7804 (1997).
[19] A. J. Shields, F. M. Bolton, M. Y. Simmons, M. Pepper, and D. A. Ritchie, Phys. Rev. B **55**, 1970 (1997).
[20] X. F. He, Phys. Rev. B **43**, 2063 (1991).
[21] H. Mathieu, P. Lefebvre, and P. Christol, Phys. Rev. B **46**, 4092 (1992).
[22] P. Lefebvre, P. Christol, and H. Mathieu, Phys. Rev. B **48**, 17 308 (1993).
[23] P. Lefebvre, P. Christol, H. Mathieu, and S. Glutsch, Phys. Rev. B **52**, 5756 (1995).
[24] F. H. Stillinger, J. Math. Phys. (N.Y.) **18**, 1224 (1977).
[25] E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1970), p. 424.
[26] A. Dalgarno and J. T. Lewis, Proc. R. Soc. London, Ser. A **233**, 70 (1955).
[27] S. Wolfram, *Mathematica: A System for Doing Mathematics by Computer* 2nd ed. (Addison-Wesley, Reading, MA, 1991).
[28] G. Oelgart, M. Proctor, D. Martin, F. M. Genaud, F. K. Reinhart, B. Orschel, L. C. Andreani, and H. Rhan, Phys. Rev. B **49**, 10 456 (1994).
[29] A. Thilagam, J. Singh, and P. Stulik, Sol. Energy Mater. Sol. Cells (to be published).
[30] L. C. Andreani and A. Pasquarello, Phys. Rev. B **42**, 8928 (1990).
[31] P. Debernardi and P. Fasano, IEEE J. Quantum Electron. **29**, 2741 (1993).
[32] Z. L. Yuan, Z. Y. Xu, Weikun Ge, J. Z. Xu, and B. Z. Zheng, J. Appl. Phys. **79**, 424 (1996).
[33] B. Stebe and A. Ainane, Superlattices Microstruct. **5**, 545 (1989).
[34] A. I. Bobrysheva, M. V. Grodetskii, and V. T. Zyykov, J. Phys. C **16**, 5723 (1983).
[35] S. Adachi, T. Miyashita, S. Takeyama, Y. Takagi, A. Tackeuchi, and M. Nakayama, Phys. Rev. B **55**, 1654 (1997).
[36] Y. Fu, M. Willander, E. L. Ivchenko, and A. A. Kiselev, JETP Lett. **64**, 795 (1996).
[37] K. B. Ferrio and D. G. Steel, Phys. Rev. B **54**, R5231 (1996).
[38] J. Singh, D. Birkedal, V. G. Lyssenko, and Phys. Rev. B **53**, 15 909 (1996).
[39] G. Vektaris, J. Chem. Phys. **101**, 3031 (1994).
[40] R. C. Miller, D. A. Kleinman, A. C. Gossard, and O. Munteanu, Phys. Rev. B **25**, 6545 (1982).
[41] E. Hanamura, Phys. Rev. B **37**, 1273 (1988).
[42] J. J. Palacios, D. Yoshioka, and A. H. MacDonald, Phys. Rev. B **54**, 2296 (1996).