Repulsion of energies of delocalized states in a double-layer system in a strong magnetic field

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A double-layer system in a strong perpendicular magnetic field is considered. Assuming a random potential in each layer to be smooth, we calculate the positions of the delocalized states within a wide range of the tunneling strengths. We show that each delocalized state is composed of alternating pieces of equipotentials from different layers. These combined equipotentials form a percolation network, the nodes of which are the regions where equipotentials from different layers nearly touch each other. [S0163-1829(97)00631-0]

I. INTRODUCTION

The origin of the integer quantum Hall effect lies in the peculiar property of the spectrum of a two-dimensional (2D) electron in a perpendicular magnetic field. Namely, there is only a single delocalized state per a disorder-broadened Landau level. Comprehensive numerical studies (see the review, Ref. 1) allowed us to establish how the localization length, $\xi(E)$, diverges as the energy, E, approaches the center of the Landau level: $\xi(E)\propto E^{-\nu}$, where $\nu\approx 2.35$. Currently, the question of interest is to trace the evolution of the localization properties of the electronic spectrum as the third dimension is added.[2–4] The natural realization of this situation is a system of 2D layers stacked in parallel (superlattice). As a tunnel coupling between the neighboring layers increases, the delocalized states in individual layers should smear into a metallic band. The crucial step in understanding of this transformation is the analysis of a double-layer system. Namely, the question to answer is whether a naive picture of the tunneling-induced level repulsion applies to delocalized states in two layers. The related question is about the structure of the wave functions of delocalized states in the presence of tunneling.

Both questions become trivial if a disorder is the same (or strongly correlated) in both layers. Then the tunneling affects only the size-quantization wave functions in the direction perpendicular to the layers. As a result, the change in positions of the delocalized states follows the repulsion of the size-quantization levels. If, in the absence of tunneling, the energies of delocalized states were E_1 and E_2, then with tunneling switched on, the new positions are given by a standard expression

$$\widetilde{E}_{1,2}=\frac{E_1+E_2}{2}\mp\sqrt{\left(\frac{E_1-E_2}{2}\right)^2+t^2}, \qquad (1)$$

where t is the tunnel integral. However, for further going to multilayers, correlated disorder in a double-layer system gives no insight: even if random potentials are correlated in the neighboring layers, they will get uncorrelated for two distant layers. Thus the really interesting case is the case of uncorrelated disorder. By now two groups[5,6] reported the numerical study of the structure of electronic states in a double-layer system with uncorrelated disorder. Surprisingly, it appeared[6] that, with a short-range disorder, even for t as small as 0.1Γ, where Γ is the disorder-induced width of the Landau level, the positions of delocalized states are very close to those with correlated disorder. On the other hand, the case of a smooth disorder, with correlation radius larger than the magnetic length, l, has an advantage that there is a transparent picture of the eigenstates. Namely, they represent cyclotron orbits with a guiding center drifting along equipotential lines and tunneling through the saddle points.[7] In the present paper we extend this picture to the case of a double-layer system and obtain analytical expressions for the energies of delocalized states.

II. THE STRUCTURE OF DELOCALIZED STATES

The crucial difference between the cases of a correlated and of an uncorrelated disorder is that in the latter case the equipotentials from different layers may cross each other after the projection on the same plane. This is illustrated in Fig. 1. As a result of such a crossing, there is a finite probability, $\mathcal{W}$, for an electron to change equipotential (and, correspondingly, the layer). We will show that this probability is given by

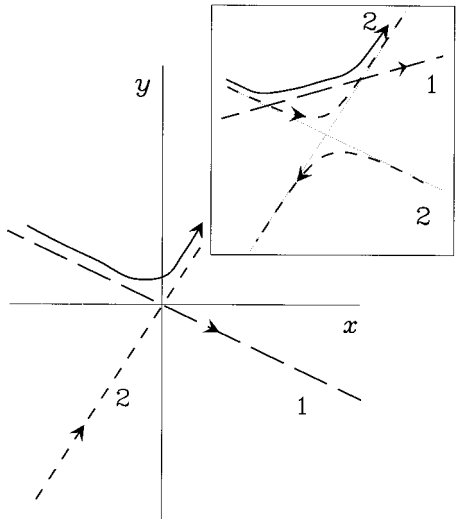

FIG. 1. Crossing of equipotentials from the first (long-dashed lines) and the second (dashed lines) layer after projection on the same plane. The inset shows the bypassing of a saddle point due to switching of equipotentials at the intersections.

$$\mathcal{W}=1-\exp\left(-\frac{2\pi t^2}{e^2|\boldsymbol{\mathcal{E}}_1\times\boldsymbol{\mathcal{E}}_2|l^2}\right), \tag{2}$$

where $\boldsymbol{\mathcal{E}}_1$, $\boldsymbol{\mathcal{E}}_2$ are the values of the local electric field in two layers at the point of intersection. Let us sketch the derivation of Eq. (2). Suppose that two equipotentials, corresponding to the energy E, measured from the center of the nth Landau level, intersect at $\rho=0$. The in-plane potentials $V_1(\boldsymbol{\rho})$, $V_2(\boldsymbol{\rho})$ behave near $\rho=0$ as $V_1(\boldsymbol{\rho})=E+e\boldsymbol{\mathcal{E}}_1\cdot\boldsymbol{\rho}$; $V_2(\boldsymbol{\rho})=E+e\boldsymbol{\mathcal{E}}_2\cdot\boldsymbol{\rho}$. Then the two-component amplitude, $\Psi=(\psi_1,\psi_2)$, to find an electron within the first and the second layer, obeys the Schrödinger equation $\hat{H}\Psi=\hbar\omega(n+\frac{1}{2})\Psi$, where ω is the cyclotron frequency. The matrix Hamiltonian, $\hat{H}$, is defined as

$$\hat{H}=\begin{pmatrix} \dfrac{\left(\hat{\mathbf{p}}+\dfrac{e}{c}\mathbf{A}\right)^2}{2m}+V_1(\boldsymbol{\rho}) & -t \\ -t & \dfrac{\left(\hat{\mathbf{p}}+\dfrac{e}{c}\mathbf{A}\right)^2}{2m}+V_2(\boldsymbol{\rho}) \end{pmatrix}, \tag{3}$$

where $\mathbf{A}=B(-y,x,0)/2$ is the vector potential. The important step is to separate the cyclotron motion and the motion of the guiding center. This is achieved by the following transformation:[8]

$$x=l(X-s), \quad y=-il\left(\frac{\partial}{\partial X}+\frac{\partial}{\partial s}\right), \tag{4}$$

$$\frac{\partial}{\partial x}=\frac{1}{2l}\left(\frac{\partial}{\partial X}-\frac{\partial}{\partial s}\right), \quad \frac{\partial}{\partial y}=-\frac{i}{2l}(X+s). \tag{5}$$

The motion of the guiding center is described by the coordinate X. This suggests to search for the solution of the Schrödinger equation in the form

$$\Psi=\exp\left[-i\frac{\beta_{1,2y}}{2}\left(s-\frac{\beta_{1,2x}}{2}\right)\right]\phi_n\left(s-\frac{\beta_{1,2x}}{2}\right)F_{1,2}(X), \tag{6}$$

where ϕ_n is the eigenfunction of the harmonic oscillator. Then, the equations for $F_{1,2}$ take the form

$$\left(-i\beta_{1y}\frac{\partial}{\partial X}+\beta_{1x}X\right)F_1-\gamma_1F_1=tF_2, \tag{7}$$

$$\left(-i\beta_{2y}\frac{\partial}{\partial X}+\beta_{2x}X\right)F_2-\gamma_2F_2=tF_1, \tag{8}$$

where $\beta_{1,2x}=e\mathcal{E}_{1,2x}l$, $\beta_{1,2y}=e\mathcal{E}_{1,2y}l$, $\gamma_{1,2}=2(\beta_{1x,2x}^2+\beta_{1y,2y}^2)/\hbar\omega$.

In fact, this system is equivalent to the system, describing nonadiabatic transitions between the crossing energy levels in molecules. This problem was first considered more than 60 years ago.[9] Up to a phase factor, the solution of the system (7), (8) can be expressed in terms of parabolic cylinder functions,[11] $D_\nu(\pm\overline{X}\sqrt{2}e^{i\pi/4})$, with ν and $\overline{X}$ given by the following formulas:

$$\nu=-i\frac{t^2}{e^2l^2|\boldsymbol{\mathcal{E}}_1\times\boldsymbol{\mathcal{E}}_2|}=-i\frac{t^2l^2}{|\mathbf{v}_1\times\mathbf{v}_2|\hbar^2}, \tag{9}$$

$$\overline{X}=\sqrt{\frac{|\boldsymbol{\mathcal{E}}_1\times\boldsymbol{\mathcal{E}}_2|}{2\mathcal{E}_{1y}\mathcal{E}_{2y}}}\left[X-\frac{\hbar\omega}{2}\frac{(2n+1)(\mathcal{E}_{1y}-\mathcal{E}_{2y})}{el|\boldsymbol{\mathcal{E}}_1\times\boldsymbol{\mathcal{E}}_2|}\right]. \tag{10}$$

Using the asymptotics of the D functions,

$$D_\nu(X)\sim X^\nu e^{-(1/4)X^2}, \quad |X|\to\infty, \quad \left(-\frac{3\pi}{4}<\arg X<\frac{3\pi}{4}\right), \tag{11}$$

$$D_\nu(X)\sim X^\nu e^{-(1/4)X^2}-\frac{(2\pi)^{1/2}}{\Gamma(-\nu)}e^{-i\pi\nu}X^{-\nu-1}e^{(1/4)X^2},$$

$$|X|\to\infty, \quad \left(-\frac{5\pi}{4}<\arg X<-\frac{\pi}{4}\right), \tag{12}$$

one can see that the right behavior at $X\to+\infty$ for the function $F_n(X)$ (no reflected wave), is ensured by the following choice:

$$F_n(X)\propto D_\nu(\pm\overline{X}\sqrt{2}e^{i\pi/4}). \tag{13}$$

The asymptotics for F at $\overline{X}\to\infty$ and $\overline{X}\to-\infty$ differ by a factor $\exp(i\pi\nu)$. With ν given by Eq. (9), the probability of changing the equipotential after crossing, $\mathcal{W}=1-|F(-\infty)|^2/|F(\infty)|^2$, takes the form of Eq. (2).

The Eq. (2) sets a relevant scale for t. Indeed, the typical value of the electric field can be estimated as $\mathcal{E}_{1,2}\sim\Gamma/eR_c$, where $R_c\gg l$ is the correlation radius of the random potential. Then it follows from Eq. (2) that the crossover value of t is $\sim\Gamma l/R_c$. For smaller t the probability of retaining equipotential after crossing is close to 1. Conversely, for $t>\Gamma l/R_c$ practically each intersection results in the change of the equipotential. It is important that the crossover value of t is much smaller than Γ, which means that the density of states is unaffected by tunneling at such t. Note also that the applicability of Eq. (2) is limited by the condition $t<\Gamma$. This is because Eq. (2) was derived assuming that $\mathcal{E}_1$, $\mathcal{E}_2$ are constant. On the other hand, if $t>\Gamma l/R_c$, so that F_1 and F_2 in Eqs. (7), (8) are of the same order, the characteristic X is $\sim t/e\mathcal{E}_{1,2}l$. Hence, the characteristic length where the ''interaction'' between the equipotentials occurs is $\sim Xl\sim t/e\mathcal{E}_{1,2}$. Since this length must be smaller than R_c, we arrive at the condition $t<\Gamma$. The opposite case $t>\Gamma$ is transparent, since the repulsion of the size-quantization levels becomes a dominant factor, and the positions of the delocalized states are given by Eq. (1).

Thus, we have established that within a wide region $\Gamma l/R_c<t<\Gamma$, each crossing of equipotential lines from different layers leads to the change of the layer, in which the electron moves. As a result, the saddle points of the random potentials $V_1(\boldsymbol{\rho})$ and $V_2(\boldsymbol{\rho})$ in the layers, which played a crucial role for delocalization[7] at $t=0$, become irrelevant in this region: *a typical saddle point is bypassed due to switching of equipotentials* (see the inset in Fig. 1). Thus, the structure of the delocalized states must be completely different from that at $t=0$, when the wave function was concentrated within a magnetic length from a percolating equipotential.[7] To establish this structure, consider first the energy, E, deep

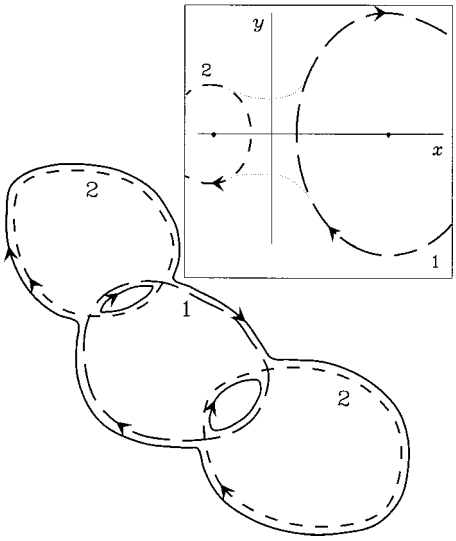

FIG. 2. A cluster formed by equipotentials from different layers after projection on the same plane. Full lines show the resulting trajectories of the electron motion. The inset depicts an effective saddle point formed by equipotentials from different layers after projection on the same plane.

in the tail of the Landau level: $E<E_1<E_2$. Then the equipotentials $V_1(\boldsymbol{\rho})=E$ and $V_2(\boldsymbol{\rho})=E$, being projected on the same plane, represent a set of isolated circles. As E moves up, the equipotentials, corresponding to different layers, start to overlap and form clusters, as it is shown in Fig. 2. Our main observation is that, due to switching of equipotentials at intersections, *the motion of an electron within a cluster occurs either inside the cluster or along its boundary* (see Fig. 2). In the latter case, the envelope trajectory consists of alternating pieces of equipotentials from different layers. As E further increases, the average size of a cluster grows and so does the envelope trajectory. Finally, at some $E=\widetilde{E}_1$ critical clusters merge, so that the envelope trajectories form an infinite equipotential, i.e., the classical percolation occurs. Thus, the energy $\widetilde{E}_1$ corresponds to the lower delocalized state. The upper delocalized state, $\widetilde{E}_2$, emergies in quite a similar way, if we first consider the energy $E>E_2>E_1$, and then gradually move it downwards. It is obvious from symmetry that $\widetilde{E}_1+\widetilde{E}_2=E_1+E_2$.

Despite the fact that for delocalized states at $E=\widetilde{E}_1,\widetilde{E}_2$ the real saddle points are irrelevant, the localization properties of the states with energies close to either $\widetilde{E}_1$ or $\widetilde{E}_2$ are described by a *single-channel* network model, proposed by Chalker and Coddington[7] (see also Ref. 11). In their model the cells of the network are closed equipotentials, while the nodes are the saddle points separating them. It is obvious that in our case the role of cells is played by the envelope trajectories (like the one shown in Fig. 2). Less obvious is that a region, where two different envelope trajectories come close and *nearly touch each other* acts exactly as a saddle point. Namely, as energy (and, thus, the degree of their overlap) is swept within a narrow interval, $\Delta\ll\Gamma$, the result of passing of an electron through this region changes from reflection (retaining the equipotential) to transmission (switching the equipotential). This is illustrated in the inset in Fig. 2. The derivation of the transmission coefficient for an effective saddle point is given in the Appendix. It has the same form as for a conventional saddle point[8]

$$T=\frac{1}{1+\exp\left[\dfrac{\pi(E-E_0)}{\Delta}\right]}, \tag{14}$$

where E_0 is approximately the energy at which two trajectories touch each other and Δ is typically of the order of $\Gamma^{3/2}l^2/t^{1/2}R_c^2$ and is much smaller than Γ. The parameter Δ defines the localization length for energies close to $\widetilde{E}_1,\widetilde{E}_2$. Indeed, since the quantum mechanical description of the transmission through the region of touching becomes important when $|E-\widetilde{E}_1|<\Delta$, the size of a ''unit cell'' of a network is $\sim R_c(\Gamma/\Delta)^{4/3}$, where 4/3 is the critical exponent for the classical percolation. Then we have

$$\xi\sim R_c\left(\frac{\Gamma}{\Delta}\right)^{4/3}\left(\frac{\Delta}{|E-\widetilde{E}_{1,2}|}\right)^{\nu}. \tag{15}$$

III. POSITION OF DELOCALIZED STATES

We turn now to the calculation of the positions of the delocalized states. This calculation reduces to the following percolation problem. Consider an auxiliary 2D plane. For a fixed energy, E, we ascribe black color to a point, $\boldsymbol{\rho}$, if at least one of the values $[V_1(\boldsymbol{\rho})+E_1]$, $[V_2(\boldsymbol{\rho})+E_2]$ is smaller than E. Otherwise, the point $\boldsymbol{\rho}$ is white. Then the energy $\widetilde{E}_1$ is the critical energy at which the percolation over white regions switches to the percolation over black regions. In other words, if we define an auxiliary random potential, $\widetilde{V}(\boldsymbol{\rho})=\min\{V_1(\boldsymbol{\rho})+E_1,V_2(\boldsymbol{\rho})+E_2\}$, then $\widetilde{E}_1$ represents the percolation threshold for $\widetilde{V}(\boldsymbol{\rho})$. Since the distribution functions of $V_1(\boldsymbol{\rho})$, $V_2(\boldsymbol{\rho})$ are Gaussian, $f(V)\propto\exp(-V^2/\Gamma^2)$, one can easily calculate the distribution function and the average of $\widetilde{V}$,

$$\widetilde{f}(\widetilde{V})=-\frac{1}{\pi}\frac{\partial}{\partial\widetilde{V}}\left\{\Phi\left[\frac{\widetilde{V}-E_1}{\Gamma}\right]\Phi\left[\frac{\widetilde{V}-E_2}{\Gamma}\right]\right\}, \tag{16}$$

$$\langle\widetilde{V}\rangle=E_1-\frac{\Gamma e^{-[(E_1-E_2)^2/2\Gamma^2]}}{\sqrt{2\pi}}+\frac{E_2-E_1}{\sqrt{\pi}}\Phi\left[\frac{E_2-E_1}{\sqrt{2}\Gamma}\right], \tag{17}$$

where $\Phi(x)=\int_x^\infty dx\,e^{-x^2}$ is the error function. It appears that $\widetilde{f}$ is symmetric with respect to $\langle\widetilde{V}\rangle$ with very high accuracy (in the ''worst'' case $E_1=E_2$ the asymmetry at half width is smaller than one percent). Thus, we can identify $\langle\widetilde{V}\rangle$ with $\widetilde{E}_1$. Note also, that an intuitive criterion that at $E=\widetilde{E}_1$ the areas of the white and black regions are equal, leads to a different condition, but, numerically, yields the values of $\widetilde{E}_1$ very close to $\langle\widetilde{V}\rangle$. One can view Eq. (17) as an analog of Eq. (1) for the case of smooth potentials in the layers and $\Gamma>t>\Gamma l/R_c$. For $E_1=E_2$ the splitting of delocalized states $\widetilde{E}_1,\widetilde{E}_2$ is equal to 0.8Γ. For arbitrary E_2-E_1 it is shown in Fig. 3. Note that the correlation between $V_1(\boldsymbol{\rho})$ and $V_2(\boldsymbol{\rho})$ can be easily incorporated into the above calculation. Certainly it will lead to the reduction of the splitting.

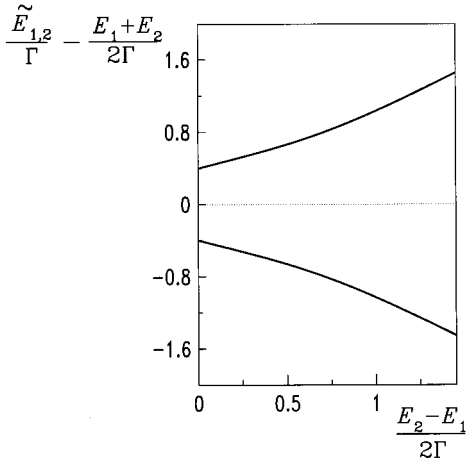

FIG. 3. The energies of delocalized states are plotted as functions of the ''bare'' asymmetry (E_2-E_1) between the layers.

IV. CONCLUSION

In conclusion, we have calculated the energies of the delocalized states in a double-layer system with a smooth uncorrelated disorder, for a wide range of the tunneling parameter, $\Gamma l/R_c<t<\Gamma$. For the case of two identical layers, these energies are shown schematically in Fig. 4 as a function of tunneling strength. We have shown that for uncorrelated disorder the dependence of $\widetilde{E_1},\widetilde{E_2}$ on the tunneling parameter t has a wide plateau. Note that this prediction can, in principle, be tested experimentally on a *single* sample, since, as it was shown by Hu and MacDonald,[12] t can be effectively tuned (suppressed) by applying a parallel magnetic field. It follows from our consideration that the description of a double-layer system, based on a two-channel network model[6] with tunneling, causing the mixing of the channels, becomes inadequate

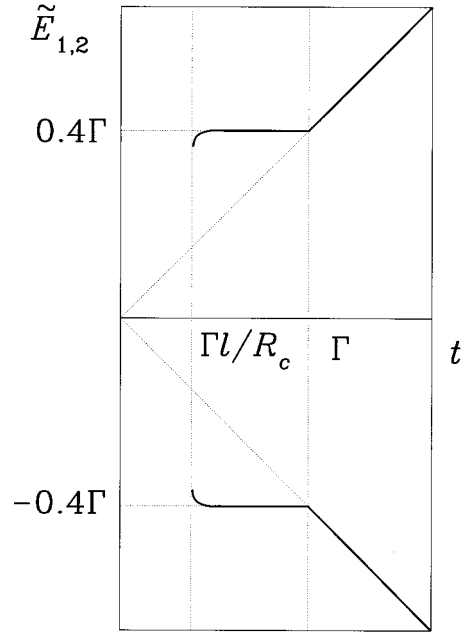

FIG. 4. Schematic plot of the energy positions of delocalized states for two identical layers $(E_1=E_2)$, at different values of interlayer coupling.

even at relatively weak tunneling. Instead, the tunneling leads to the formation of *two independent networks* with nodes being the regions of touching of equipotentials from different layers.

As t becomes smaller than $\Gamma l/R_c$, the above picture of electronic states is not valid anymore. A typical crossing of equipotentials would not cause the change of the plane for the electron motion. We can present only a plausible argument about the evolution of the positions of delocalized states in this limit. Note that, although a typical crossing is not efficient for $t<\Gamma l/R_c$, the regions of touching of equipotentials from different planes still act as saddle points. It is shown in Appendix that the corresponding condition on t is $t>\Gamma(l/R_c)^{4/3}$. Thus, in the domain $\Gamma l/R_c>t>\Gamma(l/R_c)^{4/3}$, these effective saddle points would couple electronic states belonging to closed equipotentials from different planes. As a result, the delocalized states would occur at energies $\pm E_c^{(1)}$, at which the size of a closed equipotential, $R_c(\Gamma/E_c^{(1)})^{4/3}$, is big enough to have ~ 1 effective saddle point somewhere on its perimeter. It is obvious that $E_c^{(1)}\ll E_c$. Then the problem again reduces to a single-channel network model with the cells of the network being closed equipotentials, alternatingly, from the first and from the second layer, and the nodes being the effective saddle points.

To estimate the magnitude of $E_c^{(1)}$ note that the perimeter of critical equipotential scales with energy as:[13] $\mathcal{L}(E)\sim R_c(\Gamma/E)^{7/3}$. The probability that two equipotentials from different layers would come close and form an effective saddle point with height within the interval $(E-\Delta,E+\Delta)$ can be estimated as $\Delta/\Gamma\sim(\Gamma/t)^{1/2}(l/R_c)^2$. Remember that Δ is the energy scale of the transmission coefficient in Eq. (14). Then the condition to find $E_c^{(1)}$ can be written as $(\mathcal{L}(E_c^{(1)})/R_c)(\Delta/\Gamma)\sim 1$, and it yields $E_c^{(1)}\sim\Gamma(\Gamma/t)^{3/14}(l/R_c)^{6/7}$.

In the paper we demonstrate that when the tunnel integral exceeds the characteristic value $\Gamma l/R_c$, the critical exponent, ν, of the localization length is the same as for $t=0$. We also argue that the description based on a single-channel network model (and thus $\nu=7/3$) is applicable within the interval $\Gamma l/R_c>t>\Gamma(l/R_c)^{4/3}$. Therefore if the doubling of ν established in Ref. 6 occurs, this may happen only for $t<\Gamma(l/R_c)^{4/3}$. However, it seems more likely that the description in terms of a single-channel network model with neighboring cells belonging to different layers applies even at very small t. The reason why we anticipate this is the following. The solution of the model problem described in Appendix shows that if two minima of the random potential in different layers are located anomalously close to each other [y_1,y_2 in Eqs. (A1), (A2) are anomalously small], then the formation of the effective saddle point becomes possible *even at very small* t. Certainly, the smaller t is, the more sparse these saddle points are. At $t=0$ the localization length increases in each layer as $\xi(E)\propto E^{-7/3}$, and each localized state consists of many cells of the intralayer network, separated by saddle points. Then, if t is finite and very small, at some $E=E_c^{(2)}$ there will be ~ 1 effective saddle point per perimeter of a localized state in each layer. At this energy (as well as at $E=-E_c^{(2)}$) the states in two layers would form a new network with a much larger unit cell $\sim\xi(E_c^{(2)})$. Then the localization length would behave as $(E-E_c^{(2)})^{-7/3}$ with a

prefactor much bigger than that for a single layer. Certainly, this scenario is only hypothetical.

Note that there is a significant difference between our picture and the one outlined by Sørensen and MacDonald.[6] The line of argument in Ref. 6 is as follows. In the absence of a disorder an electron residing initially, say, in the first layer, would oscillate between the layers with a period $\tau=2\pi\hbar/t$. When the disorder is present, the electron drifts within the layer with velocity v_1. It was assumed in Ref. 6 that the change of the layers would most probably occur after an electron travels the distance $l_{\rm dr}=v_1\tau=2\pi\hbar v_1/t \sim\Gamma l^2/tR_c$. In our picture the possibility for an electron in the first layer to tunnel depends on the actual topography of the random potential in the second layer. In contrast to Ref. 6, in our picture the change of the layers occurs locally, at the intersections of equipotentials with the same energy. At the same time in the regime of strong coupling, $t>\Gamma l/R_c$, that we considered, the distance (in the vicinity of an intersection), over which the change of the layers takes place, is $\sim tR_c/\Gamma$ and it is much larger than $l_{\rm dr}$. Our picture also differs from that by Laikhtman and Menashe.[4] Similarly to Ref. 6, they assume that the process of changing layers, during the drift along the equipotential, occurs homogeneously, but they get a different estimate for the characteristic length of travel within a given layer. Their estimate, $R_c^2\Gamma/lt$, is larger than tR_c/Γ and, correspondingly, larger than $l_{\rm dr}$.

Experimentally it might be hard to realize both, smooth disorder within each layer and the absence of correlation between the layers. However, for our arguments to apply, it is sufficient that the correlation between the layers is not absolute. In other words, it should be allowed for two equipotentials, corresponding to the same energy, to be displaced by more than a magnetic length. Then our picture remains valid, but the correlation would lead to the shift of the percolation threshold to an energy much closer to the center of the Landau level than in the case of uncorrelated disorder. This implies that the ''splitting'' of the delocalized states in the regime when the coupling between equipotentials is already strong would still remain much smaller than Γ, i.e., the plateau shown in Fig. 4 would become lower, and hence, narrower.

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APPENDIX

To derive the transmission coefficient for an effective saddle point, let us assume for concreteness that the two equipotentials that nearly touch each other have their origin in two displaced potential minima in each layer (Fig. 5)

$$V_1(x,y)=\frac{m\Omega_1^2}{2}[(y-y_1)^2+x^2]+V_{10} \approx\frac{m\Omega_1^2}{2}[x^2-2y_1y+y_1^2]+V_{10}, \tag{A1}$$

$$V_2(x,y)=\frac{m\Omega_2^2}{2}[(y+y_2)^2+x^2]+V_{20} \approx\frac{m\Omega_2^2}{2}[x^2+2y_2y+y_2^2]+V_{20}, \tag{A2}$$

where $(y_1,0)$ and $(-y_2,0)$ are the positions of the minima, V_{10},V_{20} are the heights, and Ω_1,Ω_1 are the curvatures. We neglect the terms $\sim y^2$ in Eqs. (A1), (A2) since the relevant y appears to be small. The condition that two equipotentials come close to each other at $x=0,\ y=0$, can be expressed as

$$\frac{m\Omega_1^2y_1^2}{2}+V_{10}\approx\frac{m\Omega_2^2y_2^2}{2}+V_{20}\approx E. \tag{A3}$$

Then the system of equations for the amplitudes Ψ_1 and Ψ_2, can be written as

$$\left[\frac{\hat p_x^2}{2m}+\frac{m(\omega^2+\Omega_1^2)}{2}\left(x-\frac{\omega}{\omega^2+\Omega_1^2}\frac{\hat p_y}{m}\right)^2\right]\Psi_1 +\left(\frac{\Omega_1^2}{2m\omega^2}\hat p_y^2-m\Omega_1^2y_1y\right)\Psi_1 -\left[E-\hbar\omega\left(n+\frac{1}{2}\right)-\frac{m\Omega_1^2y_1^2}{2}-V_{10}\right]\Psi_1 =t\Psi_2, \tag{A4}$$

$$\left[\frac{\hat p_x^2}{2m}+\frac{m(\omega^2+\Omega_2^2)}{2}\left(x-\frac{\omega}{\omega^2+\Omega_2^2}\frac{\hat p_y}{m}\right)^2\right]\Psi_2 +\left(\frac{\Omega_2^2}{2m\omega^2}\hat p_y^2+m\Omega_2^2y_2y\right)\Psi_2 -\left[E-\hbar\omega\left(n+\frac{1}{2}\right)-\frac{m\Omega_2^2y_1^2}{2}-V_{20}\right]\Psi_2 =t\Psi_1. \tag{A5}$$

We will search for a solution in the following form:

$$\Psi_1=\int dk\,A(k)e^{iky}\phi_n\left(x-\frac{\omega^2}{\omega^2+\Omega_1^2}l^2k\right), \tag{A6}$$

$$\Psi_2=\int dk\,B(k)e^{iky}\phi_n\left(x-\frac{\omega^2}{\omega^2+\Omega_2^2}l^2k\right). \tag{A7}$$

Upon substituting Eqs. (A6), (A7) into (A4), (A5), we get a system of equation for the functions A and B,

$$-im\Omega_1^2y_1\frac{\partial A}{\partial k}+a(k)A(k)=tc(k)B(k), \tag{A8}$$

$$+im\Omega_2^2y_2\frac{\partial B}{\partial k}+b(k)B(k)=tc(k)A(k), \tag{A9}$$

where $c(k)$ is the overlap integral

$$c(k)=\int dx\,\phi_n\left(x-\frac{\omega^2 l^2 k}{\omega^2+\Omega_1^2}\right)\phi_n\left(x-\frac{\omega^2 l^2 k}{\omega^2+\Omega_2^2}\right), \tag{A10}$$

and the coefficients $a(k)$ and $b(k)$ are defined as

$$a(k)=\frac{m\Omega_1^2 l^4 k^2}{2}-\left(E-\frac{m\Omega_1^2 y_1^2}{2}-V_{10}\right), \tag{A11}$$

$$b(k)=\frac{m\Omega_2^2 l^4 k^2}{2}-\left(E-\frac{m\Omega_2^2 y_2^2}{2}-V_{20}\right). \tag{A12}$$

It can be easily seen that $c(k)=1$ for $\Omega_1=\Omega_2$ and the correction in the case when the two frequencies are different, is proportional to $(\Omega_1^2-\Omega_2^2)^2/\omega^4$. Since the the random potential is smooth, we can neglect this correction and set $c(k)=1$. The system Eqs. (A8), (A9) can be reduced to a single second-order differntial equation, say, for $A(k)$,

$$\frac{d^2A}{dk^2}+i\left(\frac{a}{m\Omega_1^2 y_1}-\frac{b}{m\Omega_2^2 y_2}\right)\frac{dA}{dk}+\left[\frac{ab-t^2}{m^2\Omega_1^2\Omega_2^2 y_1 y_2}+\frac{i}{m\Omega_1^2 y_1}\frac{da}{dk}\right]A=0. \tag{A13}$$

The term with the first derivative can be eliminated by the following substitution:

$$A(k)=\exp\left[-\frac{i}{2}\int_{-\infty}^{k}dk'\left(\frac{a(k')}{m\Omega_1^2 y_1}-\frac{b(k')}{m\Omega_2^2 y_2}\right)\right]\mathcal{A}(k), \tag{A14}$$

after which Eq. (A13) takes the form

$$\frac{d^2\mathcal{A}}{dk^2}+\left[\frac{1}{4}\left(\frac{a}{m\Omega_1^2 y_1}+\frac{b}{m\Omega_2^2 y_2}\right)^2-\frac{t^2}{m^2\Omega_1^2\Omega_2^2 y_1 y_2}+\frac{il^4 k}{2}\left(\frac{1}{y_1}+\frac{1}{y_2}\right)\right]\mathcal{A}=0. \tag{A15}$$

It is convenient to introduce the following notations:

$$\frac{2}{y_0}=\frac{1}{y_1}+\frac{1}{y_2},\quad \Omega_0^2=\Omega_1\Omega_2\frac{\sqrt{y_1 y_2}}{y_0}, \tag{A16}$$

$$\varepsilon_0=\frac{\Omega_0^2 y_0}{2\Omega_1^2 y_1}\left[E-\frac{m\Omega_1^2 y_1^2}{2}-V_{10}\right]+\frac{\Omega_0^2 y_0}{2\Omega_2^2 y_2}\times\left[E-\frac{m\Omega_2^2 y_2^2}{2}-V_{20}\right]. \tag{A17}$$

Using the definitions of a and b, Eq. (A15) becomes

$$\frac{d^2\mathcal{A}}{dk^2}+\left[\left(\frac{\varepsilon_0}{m\Omega_0^2 y_0}-\frac{l^4k^2}{2y_0}\right)^2-\left(\frac{t}{m\Omega_0^2 y_0}\right)^2+\frac{il^4k}{y_0}\right]\mathcal{A}=0. \tag{A18}$$

Equation (A18) has the form of the Schrödinger equation with a complex ''potential energy.'' However, not all the terms in the ''potential energy'' are relevant. This becomes obvious if we introduce the following rescaling of the argument

$$k=\frac{1}{l}\left(\frac{m\Omega_0^2 y_0^2}{t}\right)^{1/4}z. \tag{A19}$$

Then Eq. (A18) takes the form

$$\frac{d^2\mathcal{A}}{dz^2}+\left[-z^2\frac{\varepsilon_0}{t}+\frac{\varepsilon_0^2-t^2}{t^2\alpha^2}+\frac{1}{4}z^4\alpha^2+iz\alpha\right]\mathcal{A}=0, \tag{A20}$$

where we have introduced the dimensionless parameter

$$\alpha=\frac{l}{y_0}\left(\frac{m\Omega_0^2 y_0^2}{t}\right)^{3/4}. \tag{A21}$$

This parameter can be estimated as follows. The typical value of y_0 is $\sim R_c$; the curvature Ω_0 can be found from the condition:[14] $m\Omega_0^2 R_c^2\sim\Gamma$. Then we have $\alpha\sim(l/R_c)(\Gamma/t)^{3/4}$. But within the domain we are interested in, t is larger than $\Gamma l/R_c$. Then we get $\alpha<(l/R_c)^{1/4}\ll 1$. This allows us to drop the last two terms in the potential energy. It can be also seen that the effective saddle point corresponds to $|\varepsilon_0+t|\ll t$. Indeed, under this condition Eq. (A20) takes the form

$$\frac{d^2\mathcal{A}}{dz^2}+\left[z^2-2\frac{(\varepsilon_0+t)}{t\alpha^2}\right]\mathcal{A}=0, \tag{A22}$$

which is the equation describing the scattering from the inverted parabolic potential. The expression for the transmition coefficient for this potential is well known,[15]

$$T(E)=\frac{1}{1+\exp\left[\dfrac{2\pi(\varepsilon_0+t)}{t\alpha^2}\right]}. \tag{A23}$$

We see that the characteristic energy scale for the change of the transmission coefficient is

$$\Delta\sim t\alpha^2\sim\frac{\Gamma^{3/2}}{t^{1/2}}\left(\frac{l}{R_c}\right)^2. \tag{A24}$$

Using the above estimates, the condition $\alpha\ll 1$, which guarantees that the region of nearly touching of two equipotentials acts as a saddle point, can be rewritten as $t\gg\Gamma(l/R_c)^{4/3}$. Note that Δ *decreases* with increasing t, reflecting the fact that the larger t is, the closer the equipotentials should approach each other in order to form the effective saddle point.

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