

# Level broadening effects in quantum kinetic equations: Hot luminescence from a quantum wire near the optical-phonon-emission threshold

Michal Rokni and Y. Levinson

*Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*

(Received 12 February 1997)

We consider the quantum kinetic equations in a spatially homogeneous situation for cases in which only level shift and broadening are of importance, while coherence phenomena can be neglected. In this case we can formulate the quantum kinetic equations in a form similar to semiclassical Boltzmann rate equations. We also consider a specific problem: Hot luminescence from a quantum wire near the threshold for optical-phonon emission. The problem is first discussed within the framework of the Boltzmann equation. After pointing out the failure of this description, the Boltzmann equations are generalized to a set of quantum kinetic equations, which in turn are solved in order to describe the luminescence spectra. [S0163-1829(97)07331-1]

## I. INTRODUCTION

The simplest way to describe kinetic phenomena is by the Boltzmann equation (BE). There are, however, cases when the BE cannot be used, and one has to use quantum kinetic equations (QKE's) that are in the simplest case equations for the one-particle Green functions. If the Keldysh technique<sup>1-3</sup> is used, the one-particle Green functions that participate in the description of kinetic processes are the retarded Green function (and its complex conjugate), and the "statistical" Green function, which is simply the field correlator. For free particles, these Green functions are diagonal in the basis of the free-particle states, and are described by the free-particle energies  $\epsilon_\alpha$ , and their occupation numbers  $n_\alpha$ . These are the quantities that appear in the BE for the *interacting* particles in *external fields*.

The description of kinetic phenomena by QKE's instead of BE's results in two effects. The first effect is state mixing—due to interactions and external fields, the Green functions are nondiagonal in the basis of the free particle states. This effect is responsible for coherence phenomena, of which a well-known example is interband polarization described by semiconductor Bloch equations.<sup>4-6</sup>

The second effect is level shift and broadening—the Green functions are nonzero off the energy shell, within some width defined by the interactions and the external fields. Level broadening effects in QKE's for phonons have been studied in Refs. 7 and 8.

The effects of level broadening are of importance in two cases. The first case is that in which level broadening allows scattering processes that are forbidden by energy and momentum conservation. An example of such a case is that of electron-electron scattering in a two-dimensional electron gas, (2DEG), in the presence of a quantizing magnetic field.<sup>9</sup> If one tries to calculate the Auger transition rates responsible for anti-Stokes luminescence, that was observed in Ref. 10, using Fermi's golden rule, one encounters  $\delta$  functions with an argument equal to zero, due to singularities in the density of states; see Ref. 11.

The second case in which level-broadening effects should be considered is that of narrow energy distributions: When

there are small energy scales in the problem that are narrower than the quantum width of the state. A simple example of such a case, that has one small energy scale, is that of an electron distribution, excited by a narrow-band-width laser. The problem becomes more complex when another small energy scale is added; for example, the electrons are excited to the close vicinity of some threshold. A problem of this nature will be dealt with in this paper.

In the time domain, level shift and broadening lead to non-Markovian effects in the scattering processes.<sup>12,13,6</sup> These effects are important in the case of short excitation pulses.<sup>14-17</sup>

In a spatially homogeneous situation, if state mixing induced by coherent excitation can be neglected, and only shift and broadening effects are considered, the QKE's can be presented in a form similar to the BE's. We will give a general recipe for writing such equations, and will use this recipe when considering the following problem: Hot luminescence from a quantum wire (QWR) due to electrons that are excited by a narrow band and noncoherent laser to the close vicinity of the threshold for optical LO-phonon emission.

Optical and transport phenomena in QWR's have been of great interest lately. The great interest in transport phenomena<sup>18-21</sup> is due to the possibility that a 1DEG in a QWR will have the properties of a Luttinger liquid. The Fermi-edge singularity<sup>22</sup> and the possibility of stimulated emission<sup>23</sup> have enlarged interest in the optics of QWR's. The important role played by LO phonons in the trapping of electrons<sup>24</sup> from three-dimensional extended states, into one-dimensional localized states, has been reported in Ref. 25.

Theoretical studies of QWR's were concentrated around elementary excitations of a 1DEG,<sup>26,27</sup> Fermi-edge singularity,<sup>28</sup> excitonic effects,<sup>29-31</sup> electron-phonon scattering rates<sup>32</sup> and interactions,<sup>33,34</sup> relaxation of photoexcited carriers,<sup>35</sup> calculation of envelope states,<sup>36</sup> and conductance of QWR's with self-consistent broadening effects.<sup>37</sup>

## II. QUANTUM KINETIC EQUATIONS

The retarded Green function and the "statistical" Green function which appear in the kinetic equations are defined as

$$G^r(\mathbf{r}_1, \sigma_1, t_1; \mathbf{r}_2, \sigma_2, t_2) \equiv -i\theta(t_1 - t_2) \langle [\hat{\Psi}(\mathbf{r}_1, \sigma_1, t_1), \hat{\Psi}^\dagger(\mathbf{r}_2, \sigma_2, t_2)]_{\pm} \rangle, \quad (1)$$

$$G^s(\mathbf{r}_1, \sigma_1, t_1; \mathbf{r}_2, \sigma_2, t_2) \equiv -i \langle [\hat{\Psi}(\mathbf{r}_1, \sigma_1, t_1), \hat{\Psi}^\dagger(\mathbf{r}_2, \sigma_2, t_2)]_{\mp} \rangle, \quad (2)$$

where  $\hat{\Psi}(\mathbf{r}, \sigma, t)$  are the field operators in the Heisenberg representation. The square brackets with a plus (minus) sign signify an anticommutator (commutator). The upper signs refer to fermions, and the lower signs refer to bosons (this convention is used throughout the paper). The spin degrees of freedom  $\sigma$  will be suppressed from now on, and one can think of them as included in the space coordinates.

In case of interacting particles the Green functions obey the Dyson equation

$$\begin{aligned} \tilde{G}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) &= \tilde{G}^{(0)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) + \int d\mathbf{r}_3 d\mathbf{r}_4 \int dt_3 dt_4 \\ &\times \tilde{G}^{(0)}(\mathbf{r}_1, t_1; \mathbf{r}_3, t_3) \tilde{\Sigma}(\mathbf{r}_3, t_3; \mathbf{r}_4, t_4) \tilde{G}(\mathbf{r}_4, t_4; \mathbf{r}_2, t_2), \end{aligned} \quad (3)$$

where  $\tilde{G}$  is a matrix of Green functions, and  $\tilde{\Sigma}$  is a matrix of self energy functions:<sup>2</sup>

$$\tilde{G} = \begin{pmatrix} 0 & G^a \\ G^r & G^s \end{pmatrix}, \quad \tilde{\Sigma} = \begin{pmatrix} \Sigma^s & \Sigma^r \\ \Sigma^a & 0 \end{pmatrix}. \quad (4)$$

The superscript (0) indicates free-particle Green functions. The advanced Green function  $G^a = G^{r*}$ .

In the case of free particles we can use the free-particle eigenstates  $\psi_\alpha(\mathbf{r})$ , and the corresponding annihilation operators  $\hat{a}_\alpha(t)$ , to write  $\hat{\Psi}(\mathbf{r}, t) = \sum_\alpha \psi_\alpha(\mathbf{r}) \hat{a}_\alpha(t)$ . In this case  $G^{l(0)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$  depends only on the time difference  $t_2 - t_1$ , and can be Fourier transformed in time into  $G^{l(0)}(\mathbf{r}_1, \mathbf{r}_2; \epsilon)$  (the index  $l$  stands for  $r$ ,  $a$ , or  $s$ ). For an orthonormal set of states  $\alpha$ , we can define  $G_\alpha^{l(0)}(\epsilon)$  such that

$$G^{l(0)}(\mathbf{r}_1, \mathbf{r}_2; \epsilon) = \sum_\alpha \psi_\alpha(\mathbf{r}_1) \psi_\alpha^*(\mathbf{r}_2) G_\alpha^{l(0)}(\epsilon). \quad (5)$$

From definitions (1) and (2) we obtain (near the singularities)

$$G_\alpha^{r(0)}(\epsilon) = \left[ \begin{array}{c} \mathcal{P}(\epsilon - \epsilon_\alpha) - i\pi\delta(\epsilon - \epsilon_\alpha) \\ \text{sgn}(\epsilon) \{ \mathcal{P}[\epsilon - \text{sgn}(\epsilon)\epsilon_\alpha] - i\pi\delta[\epsilon - \text{sgn}(\epsilon)\epsilon_\alpha] \} \end{array} \right], \quad (6)$$

and

$$G_\alpha^{s(0)}(\epsilon) = \left[ \begin{array}{c} -2\pi i(1 - 2n_\alpha)\delta(\epsilon - \epsilon_\alpha) \\ -2\pi i(1 + 2n_\alpha)\delta[\epsilon - \text{sgn}(\epsilon)\epsilon_\alpha] \end{array} \right], \quad (7)$$

where  $\epsilon_\alpha$  is the energy of state  $\alpha$ , and  $\mathcal{P}$  is the principal part function. The upper (lower) term in the column corresponds to fermions (bosons). The occupation number of state  $\alpha$ ,  $n_\alpha = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle$ , is not necessarily the equilibrium occupation number.

In the case of interacting particles in external fields, if one neglects coherence effects, the Green functions are still diag-

onal in  $\alpha$ , but the energy levels are broadened and shifted. Let us first look at the time invariant case (systems under dc conditions). The level shift  $\Delta$  and the level broadening  $\Gamma$  are defined as

$$\Delta_\alpha(\epsilon) \equiv \text{Re}\Sigma_\alpha^r(\epsilon), \quad \Gamma_\alpha(\epsilon) \equiv -2 \left[ \frac{1}{\text{sgn}(\epsilon)} \right] \text{Im}\Sigma_\alpha^r(\epsilon), \quad (8)$$

where  $\Sigma_\alpha^r$  is the retarded self-energy of particles in state  $\alpha$ . The column has the same meaning as in expression (6).

The retarded Green function is an outcome of the Dyson equation (3), that is written as

$$G_\alpha^r(\epsilon) = G_\alpha^{r(0)}(\epsilon) + G_\alpha^{r(0)}(\epsilon) \Sigma_\alpha^r(\epsilon) G_\alpha^r(\epsilon) \quad (9)$$

in the representation of the  $\alpha$  states. Using definitions (8) we obtain, from Eq. (9) (for positive frequencies),

$$G_\alpha^r(\epsilon) = \mathcal{P}[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon) ] - i\pi\delta[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)], \quad (10)$$

where

$$\mathcal{P}[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon) ] = \frac{\epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)}{[\epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)]^2 + [\Gamma_\alpha(\epsilon)/2]^2}, \quad (11)$$

is the ‘‘smeared’’ and ‘‘shifted’’ principal value, and

$$\delta[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon) ] = \frac{\Gamma_\alpha(\epsilon)/2\pi}{[\epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)]^2 + [\Gamma_\alpha(\epsilon)/2]^2} \quad (12)$$

is the ‘‘smeared’’ and ‘‘shifted’’  $\delta$  function.

Comparing expression (10) with expression (6), we see that  $G_\alpha^r(\epsilon)$  is in fact a generalization of  $G_\alpha^{r(0)}(\epsilon)$ , where the level shift and level width have been introduced into the principal value and  $\delta$  functions. We therefore write  $G_\alpha^s(\epsilon)$  as a generalization of  $G_\alpha^{s(0)}(\epsilon)$ , thus defining the occupation functions  $n_\alpha(\epsilon)$

$$G_\alpha^s(\epsilon) = -2\pi i [1 \mp 2n_\alpha(\epsilon)] \delta[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)]. \quad (13)$$

Expressions (10) and (13) were written for the case of positive frequencies, which will be assumed from now on. In case of negative Boson frequencies one has to multiply the retarded Green function and  $\epsilon_\alpha + \Delta_\alpha(\epsilon)$  in these expressions by  $-1$ .

The definitions above agree with the relation between the statistical Green function and the retarded Green function at thermal equilibrium,

$$G_\alpha^s(\epsilon) = 2i [1 \mp 2n_T(\epsilon)] \text{Im}G_\alpha^r(\epsilon), \quad (14)$$

where  $n_T(\epsilon)$  is the equilibrium distribution function (Fermi or Bose-Einstein). The smeared  $\delta$  function obeys the normalization

$$\int d\epsilon \delta[\Gamma_\alpha(\epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)] = 1, \quad (15)$$

and the occupation numbers are obtained from the occupation functions in the following manner:

$$n_\alpha = \int d\epsilon \delta[\Gamma_\alpha(\epsilon)|\epsilon - \epsilon_\alpha - \Delta_\alpha(\epsilon)]n_\alpha(\epsilon). \quad (16)$$

Relations (14) – (16) can be obtained from the Lehmann representation of the Green functions. These relations are valid only if the one-particle Green functions can be considered as diagonal in  $\alpha$ , in other words, in the absence of coherence effects.

As a result of definitions (8) and (13), the three unknown functions  $\text{Re}G^r$ ,  $\text{Im}G^r$ , and  $iG^s$  have been replaced by a different set of three unknown functions:  $\Gamma_\alpha(\epsilon)$ ,  $\Delta_\alpha(\epsilon)$ , and  $n_\alpha(\epsilon)$ .

If the system is not under dc conditions, one can define a new pair of time variables: the relative time  $t_{12} = t_1 - t_2$ , and the ‘‘center of mass’’ time  $\bar{t}_{12} = (t_1 + t_2)/2$ . Only slow processes will be considered so that all quantities vary in  $\bar{t}$  with some time scale that is large compared to the inverse characteristic energy of the particles. In such a case one can still use the formalism given above for the Green functions, but with  $\bar{t}$  as a parameter of the problem, in addition to  $\epsilon$  (see, for example, Ref. 2).

For slow processes,

$$G_\alpha^r(\bar{t}, \epsilon) = G_\alpha^{r(0)}(\bar{t}, \epsilon) + G_\alpha^{r(0)}(\bar{t}, \epsilon) \Sigma_\alpha^r(\bar{t}, \epsilon) G_\alpha^r(\bar{t}, \epsilon). \quad (17)$$

Again  $G_\alpha^r(\bar{t}, \epsilon)$  can be written as a generalization of  $G_\alpha^{r(0)}(\epsilon)$ , in the same manner as above, but now  $\bar{t}$  is a parameter which appears in all functions. Thus expressions (10) and (13) for  $G_\alpha^r$  and  $G_\alpha^s$  can be used with the three unknown functions  $\Gamma_\alpha(\bar{t}, \epsilon)$ ,  $\Delta_\alpha(\bar{t}, \epsilon)$ , and  $n_\alpha(\bar{t}, \epsilon)$  that depend on  $\bar{t}$  as well as on  $\epsilon$ .

The known procedure for obtaining a kinetic equation for  $n_\alpha(\bar{t}, \epsilon)$  is to apply the operator  $\hat{G}_{\alpha 2}^{-1*} - \hat{G}_{\alpha 1}^{-1}$  to the Dyson equation for  $G_\alpha^s(\bar{t}, t_{12})$ ,<sup>3</sup> where the operator  $\hat{G}_{\alpha i}^{-1}$  is equal to  $i\partial/\partial t_i - \epsilon_\alpha$  for electrons, and  $\partial^2/\partial t_i^2 + \epsilon_\alpha^2$  for photons and phonons (we use the convention  $\hbar = 1$  throughout the paper). The slow variation in  $\bar{t}$  should be considered and then the equation should be Fourier transformed in the time difference coordinates. This procedure leads to the QKE

$$\begin{aligned} \frac{\partial}{\partial \bar{t}} G_\alpha^s(\bar{t}, \epsilon) &= -2 \Sigma_\alpha^s(\bar{t}, \epsilon) \text{Im} G_\alpha^r(\bar{t}, \epsilon) \\ &+ 2 \text{Im} \Sigma_\alpha^r(\bar{t}, \epsilon) G_\alpha^s(\bar{t}, \epsilon). \end{aligned} \quad (18)$$

From now on the ‘‘bar’’ over  $t$  will be dropped, with the understanding that all time variables are in fact center-of-mass time variables. Equation (18) for photons and phonons includes a factor of  $\epsilon/\epsilon_\alpha$  that multiplies the time derivative on the left-hand side. Since we are interested in off-shell energies that are close to the on-shell energy, this factor can be taken to be 1.

In order to obtain equations for  $\Gamma_\alpha(t, \epsilon)$  and  $\Delta_\alpha(t, \epsilon)$  one has to substitute an expression for the retarded self-energy function, in terms of the one-particle Green functions, into the definitions of  $\Gamma$  and  $\Delta$ , Eq. (8). These equations are in fact the imaginary and the real parts of Eq. (17). This yields coupled, self-consistent equations for  $\Gamma$  and  $\Delta$ , that are also

$n$  dependent. It is important to stress that the occupation function depends on time explicitly, while the dependence of  $\Gamma$  and  $\Delta$  on time is only through their dependence on  $n$ . Therefore in order to find  $n$  one has to write a kinetic equation that will include time evolution through time derivatives. The equations for the level shift and width will include their dependence on time only through the appearance of  $n$ , and will not include time derivatives.

If one uses the self-consistent Born approximation for the self-energy functions, one finds that the equations obtained for  $n_\alpha(t, \epsilon)$ ,  $\Gamma_\alpha(t, \epsilon)$ , and  $\Delta_\alpha(t, \epsilon)$  could have been obtained easily by applying a few generalizations to the BE for  $n_\alpha$ . In the transfer from the BE to the QKE, the number of unknowns increases from one ( $n$ ), to three ( $n, \Gamma$  and  $\Delta$ ); therefore apart from the kinetic equation we require two more equations for the unknown functions  $\Gamma_\alpha(t, \epsilon)$  and  $\Delta_\alpha(t, \epsilon)$  that appear in the equation for  $n_\alpha(t, \epsilon)$ .

Here we present a recipe which allows one to go from the BE for  $n_\alpha(t)$  to the QKE for  $n_\alpha(t, \epsilon)$ , and from the decay term in the Boltzmann equation (which is defined below) to an equation for  $\Gamma_\alpha(t, \epsilon)$ . An equation for  $\Delta_\alpha(t, \epsilon)$  can be obtained from the equation for  $\Gamma_\alpha(t, \epsilon)$  using Kramers-Kronig relations.

The BE, in a spatially homogeneous situation, can be written as

$$\begin{aligned} \frac{\partial}{\partial t} n_\alpha &= \sum_{\beta, \gamma, \dots} |M_{\alpha, \beta, \gamma, \dots}|^2 [(1 \mp n_\alpha) n_\beta (1 \mp n_\gamma) \dots \\ &- n_\alpha (1 \mp n_\beta) n_\gamma \dots] 2\pi \delta(\epsilon_\alpha - \epsilon_\beta + \epsilon_\gamma \dots) \equiv S. \end{aligned} \quad (19)$$

This equation describes the evolution in time of the occupation number of particles in state  $\alpha$ , due to interactions with particles in states  $\beta, \gamma, \dots$ , with matrix elements  $M_{\alpha, \beta, \gamma, \dots}$ . Different states can also mean different particles. The right-hand side of the equation includes  $\delta$  functions that are responsible for energy conservation.

There are many cases in which it is convenient to think of the collision integral  $S$  as composed of ‘‘scattering in’’ events and ‘‘scattering out’’ events:  $S = S_{\text{in}} - S_{\text{out}}$ . The first term on the right-hand side of Eq. (19) is the scattering-in term: It describes processes in which particles enter state  $\alpha$  due to the interaction with other particles. This term can be written as  $S_{\text{in}} = (1 \mp n_\alpha) \mathcal{G}_\alpha$ , where  $\mathcal{G}_\alpha$  includes all the terms on the right-hand side of the equation that do not multiply  $n_\alpha$ . We refer to  $\mathcal{G}_\alpha$  as the ‘‘generation’’ term. The second term is the scattering-out term: It describes processes in which particles leave state  $\alpha$ . This term is usually written as  $S_{\text{out}} = n_\alpha / \tau_\alpha$ , where  $\tau_\alpha$  is the scattering-out time from state  $\alpha$ .

In the context of the generalization of the BE to a QKE, it is more convenient to think of the collision integral  $S$  as made up of ‘‘generation’’ and ‘‘decay’’ terms:  $S = \mathcal{G}_\alpha - \Gamma_\alpha n_\alpha$ , where the decay term  $-\Gamma_\alpha n_\alpha$  includes all the terms containing  $n_\alpha$ . Thus  $\Gamma_\alpha = \pm \mathcal{G}_\alpha + 1/\tau_\alpha$  is the total decay rate of particles in state  $\alpha$ . It is the *total* decay rate, and not only the scattering-out rate, that is related to the retarded self-energy.<sup>12</sup> The general form for the decay rate  $\Gamma_\alpha$ , as it is deduced from Eq. (19), is

$$\Gamma_\alpha = \sum_{\beta, \gamma, \dots} |M_{\alpha, \beta, \gamma, \dots}|^2 [\pm n_\beta (1 \mp n_\gamma) \dots + (1 \mp n_\beta) n_\gamma \dots] 2\pi \delta(\epsilon_\alpha - \epsilon_\beta + \epsilon_\gamma \dots). \quad (20)$$

In a dc situation, for the occupation numbers one obtains

$$n_\alpha = \mathcal{G}_\alpha / \Gamma_\alpha. \quad (21)$$

In order to transform the BE into a QKE, one should apply the following rules. To the left hand side of Eq. (19), apply  $n_\alpha(t) \rightarrow [n_\alpha(t, \epsilon) \mp \frac{1}{2}] \delta[\Gamma_\alpha(t, \epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(t, \epsilon)]$ , where  $\epsilon$  is the off-shell energy of particles in state  $\alpha$ . To the right-hand side of the equation apply the following steps.

(i) The occupation numbers for particles in states  $\alpha, \beta, \dots$  are replaced with the occupation functions, that depend on the off-shell energies  $\epsilon, \epsilon' \dots$  corresponding to the on-shell energies  $\epsilon_\alpha, \epsilon_\beta \dots$ , respectively:  $n_\alpha(t) \rightarrow n_\alpha(t, \epsilon)$ ,  $n_\beta(t) \rightarrow n_\beta(t, \epsilon') \dots$

(ii) The summand under the sum over all states  $\beta, \gamma \dots$  is multiplied by the product of the smeared  $\delta$  functions for particles in these states with the corresponding off-shell energies  $\epsilon', \epsilon'' \dots$ , and integrated over the off-shell energies:

$$\sum_{\beta, \gamma, \dots} \rightarrow \sum_{\beta, \gamma, \dots} \int d\epsilon' d\epsilon'' \dots \delta[\Gamma_\beta(t, \epsilon') | \epsilon' - \epsilon_\beta - \Delta_\beta(t, \epsilon')] \delta[\Gamma_\gamma(t, \epsilon'') | \epsilon'' - \epsilon_\gamma - \Delta_\gamma(t, \epsilon'')] \times \dots$$

(iii) The energy conservation  $\delta$  function of the on-shell energies  $\epsilon_\alpha, \epsilon_\beta \dots$  is replaced with an energy conservation  $\delta$  function of their corresponding off-shell energies  $\epsilon, \epsilon' \dots$ :  $\delta(\epsilon_\alpha \dots) \rightarrow \delta(\epsilon \dots)$ .

(iv) Multiply the entire right-hand side by  $\delta[\Gamma_\alpha(t, \epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(t, \epsilon)]$ .

In order to obtain an equation for  $\Gamma_\alpha(t, \epsilon)$ , one should apply the first three steps of the generalization rules for the right-hand side of the equation to the expression for  $\Gamma_\alpha$ , Eq. (20).

The QKE can be written generally in the form

$$\frac{\partial}{\partial t} \left\{ \left( n_\alpha(t, \epsilon) \mp \frac{1}{2} \right) \delta[\Gamma_\alpha(t, \epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(t, \epsilon)] \right\} = \{ \mathcal{G}_\alpha(t, \epsilon) - \Gamma_\alpha(t, \epsilon) n_\alpha(t, \epsilon) \} \times \delta[\Gamma_\alpha(t, \epsilon) | \epsilon - \epsilon_\alpha - \Delta_\alpha(t, \epsilon)]. \quad (22)$$

The function  $\mathcal{G}_\alpha(t, \epsilon)$  contains all the terms on the right-hand side of the QKE that do not multiply  $n_\alpha(t, \epsilon)$ . It is the generating term—the term in the kinetic equation that is responsible for the particle generation rate. Equation (22) shows clearly that the particle level width  $\Gamma_\alpha(t, \epsilon)$  is in fact the total particle decay rate. Note that due to Eq. (15) the integration of Eq. (22) over  $\epsilon$  results in the disappearance of the  $\mp 1/2$  factor from the left-hand side. The occupation function in the time-independent case is given by

$$n_\alpha(\epsilon) = \mathcal{G}_\alpha(\epsilon) / \Gamma_\alpha(\epsilon). \quad (23)$$

The representation of the QKE in terms of  $n$ ,  $\Gamma$ , and  $\Delta$  is more convenient than the representation in terms of Green

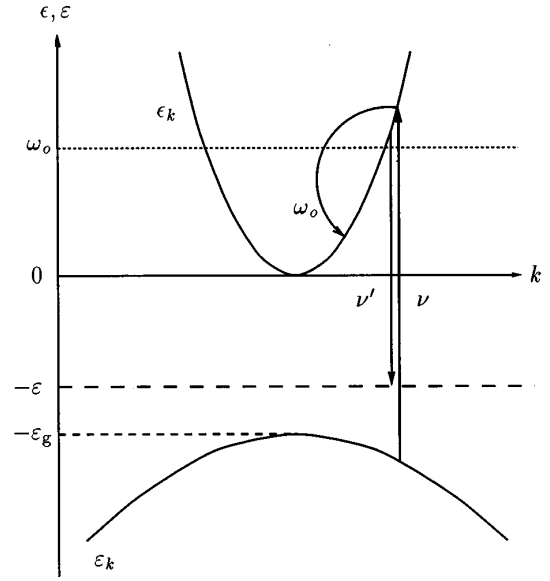


FIG. 1. Hot luminescence from a QWR: Electrons are excited from the highest valence subband  $\epsilon_k$  to the lowest conduction subband  $\epsilon_k$ , via photons  $\nu$ . These electrons relax to the bottom of the band emitting LO phonons  $\omega_0$ . The hot luminescence  $\nu'$  is due to recombination of electrons from the vicinity of the threshold for phonon emission (dotted line) with holes in an impurity level  $-\epsilon$  (large dashed line).

functions, since one can clearly see all physical processes, and use the physical intuition that one gained from the BE in order to simplify the QKE.

### III. LUMINESCENCE FROM A QUANTUM WIRE

We now use the formalism given in Sec. II in order to deal with the specific problem of hot luminescence from a QWR. The case in which electrons are excited from the highest valence subband to the lowest conduction subband via photons, and then relax via LO-phonon emission (see Fig. 1), is considered. All other conduction and valence subbands are neglected (in contrast to Refs. 35 and 28). We consider a situation in which electrons are excited by a narrow-band noncoherent laser, just above the threshold for optical-phonon emission  $\epsilon_k = \omega_0$ , and describe the hot luminescence due to the recombination of electrons with holes in an impurity level. The laser spot along the QWR is taken to be large, and electron diffusion along the wire is neglected, so that the excitation can be considered homogeneous along the QWR. It is assumed that there is some mechanism due to which the electrons leave the bottom of the conduction subband, so that there is no electron accumulation there, but this recombination can be neglected in the balance equations near the threshold. The spin degree of freedom will be completely disregarded.

Unless specified otherwise we will assume that the electrons are excited above the threshold, since we are mainly interested in luminescence due to these electrons. Below the threshold the electrons relax to the bottom of the band by emitting acoustical phonons. The relaxation rate due to acoustical phonons,  $\tau_{ac}^{-1}$ , is much smaller than that due to optical phonons, and can be neglected above the threshold.

The QWR runs along the  $z$  direction, so that the electron wave functions are localized in the  $xy$  plane, and free waves in the  $z$  direction. The wave function in the conduction band can be written as  $e^{ikz}\phi^e(\mathbf{r})/L^{1/2}$ , where  $L$  is the wire normalization length, and  $\mathbf{r}$  is a vector in the  $xy$  plane. The electron energy is  $\epsilon_k = k^2/2m_e$ . All energies are measured from the bottom of the conduction band. The holes in the valence band have wave functions of the form  $e^{ipz}\phi^h(\mathbf{r})/L^{1/2}$  and energy  $\epsilon_g + \epsilon_p$ , with  $\epsilon_p = p^2/2m_h$ .

The exciting photons are taken to be plane waves, and the photon frequency is  $\nu = \nu_{\mathbf{f}} = c|\mathbf{f}|$  ( $\mathbf{f}$  is the photon momentum and  $c$  is the light velocity). The LO phonons are three dimensional and have a flat dispersion law  $\omega_{\mathbf{q}} = \omega_o$ .

The impurities are distributed randomly in the bulk. The wave function of a hole in impurity  $i$  is of the form  $\psi(\mathbf{r} - \mathbf{r}_i, z - z_i)$ , where  $\mathbf{r}_i$  and  $z_i$  are the coordinates of the impurity position.

In order to analyze the hot luminescence, we neglect light polarization effects for simplicity, and assume that the wire and the crystal are cylindrically symmetric (an assumption that would not fit  $V$ -grooved wires nor the assumption of Ref. 36). Thus the wave functions of the emitted photons are

$$\chi_{f,m,n}(\mathbf{r}, z) = \left( \frac{1}{\pi R^2 L |J_{m+1}(\kappa_{m,n} R)|^2} \right)^{1/2} e^{ifz} e^{im\varphi} J_m(\kappa_{m,n} r), \quad (24)$$

where  $R$  is the normalizing radius and  $L$  the normalizing length of the crystal, and  $z$ ,  $\varphi$ , and  $r$  are the cylindrical polar coordinates.  $J_m$  are the Bessel functions, and  $(\kappa_{m,n} R)$  is the  $n$ th zero of  $J_m$ . The luminescence photon frequency is given by  $\nu' = \nu_{f,m,n} = c(f^2 + \kappa_{m,n}^2)^{1/2}$  (where the prime signifies this frequency is of luminescence, and not of excitation). We neglect all excitonic effects (these are treated for the case of optical absorption in one-dimensional semiconductors in Refs. 29–31).

We begin by writing down BE's for the electrons in the conduction band, and for the luminescence photons. It is shown why this description fails, and one has to turn to QKE's. We will then employ the generalization rules in order to obtain QKE's from the BE's, and use these QKE's in the analysis of the luminescence.

### A. Semiclassical description

We assume a low temperature and a weak excitation level. At equilibrium, when there is no excitation, all the electrons are in the valence band, and there are no phonons [ $N(\omega_o) = 0$ ], due to the low temperature. In other words, there are no electrons in the conduction band [ $n(k) = 0$ ], no holes in the valence band [ $\bar{n}(p) = 0$ ], and the impurity level is fully occupied by holes [ $\bar{n}(\epsilon) = 1$ ].

The excitation creates electrons in the conduction band and holes in the valence band. Most of the excited electrons relax to the bottom of the conduction band, emitting LO phonons, but a small fraction of them recombine with holes in the impurity level, producing hot luminescence. By the assumption of weak excitation we mean that  $N(\omega_o)$ ,  $n(k)$ , and  $\bar{n}(p)$  are small. The luminescence would be even weaker than the excitation, and therefore  $\bar{n}(\epsilon)$  remains close

to 1. Due to the weak excitation the luminescence is spontaneous, and the occupation numbers for the luminescence photons are also small.

In the derivation of the BE's, terms that are nonlinear in the electron occupation numbers will be neglected. We will also neglect nonequilibrium contributions to the phonon and hole (both in the valence band and in the impurity level) occupation numbers.

The interaction of the electrons with the luminescence field is neglected in the balance equation for the electrons, in comparison with their interaction with the excitation field, due to the weakness of the excitation. In the case of cw excitation the balance equation for the electrons is

$$0 = \sum_{\mathbf{f}} \sum_p |M_{k,p,\mathbf{f}}^{\text{exc}}|^2 2\pi \delta(\nu_{\mathbf{f}} - \epsilon_k - \epsilon_g - \epsilon_p) \times [1 - n(k)] N(\mathbf{f}) - \sum_{\mathbf{q}} \sum_{k'} |M_{k,k',\mathbf{q}}^{e\text{-LO}}|^2 2\pi \delta(\epsilon_k - \epsilon_{k'} - \omega_o) n(k), \quad (25)$$

where  $M^{\text{exc}}$  is the matrix element for the electron-exciting-photon interaction, and  $M^{e\text{-LO}}$  is the matrix element of the electron-LO-phonon interaction.

Let us first look at the electron-exciting-photon matrix element. In this interaction one photon of wave vector  $\mathbf{f}$  is absorbed, and a hole of momentum  $p$  and an electron of momentum  $k$  are created. The interaction, in second quantized form, is given by

$$\int d\mathbf{r} \int dz \frac{ep_o}{m_o c} \hat{\Psi}^{e\dagger}(\mathbf{r}, z) \hat{A}^{\text{exc}}(\mathbf{r}, z) \hat{\Psi}^{h\dagger}(\mathbf{r}, z). \quad (26)$$

The mass  $m_o$  is the bare electron mass, and the constant  $p_o$  is the "bare" electron momentum operator (in the direction of the photon polarization, assumed linear for simplicity), sandwiched between the valence-band top and the conduction-band bottom Bloch wave functions.  $\hat{\Psi}^e(\mathbf{r}, z)$  and  $\hat{\Psi}^h(\mathbf{r}, z)$  are the electron and hole field operators, respectively. The photon field operator is given by

$$\hat{A}^{\text{exc}}(\mathbf{r}, z) = \sum_{\mathbf{f}} \left( \frac{2\pi c^2}{V\nu_{\mathbf{f}}} \right)^{1/2} e^{i\mathbf{f}_{\perp} \cdot \mathbf{r} + ifz} (\hat{a}_{\mathbf{f}} + \hat{a}_{-\mathbf{f}}^{\dagger}), \quad (27)$$

where  $\mathbf{f}_{\perp}$  and  $f$  are the components of  $\mathbf{f}$  in the  $xy$  plane, and in the  $z$  direction, respectively,  $\hat{a}_{\mathbf{f}}$  is the annihilation operator, and  $V$  is the normalization volume of the crystal in which the quantum wire is embedded.

In order to find  $M^{\text{exc}}$ , one should sandwich interaction (26) between the states  $\langle n(k) = 1, \bar{n}(\epsilon_p) = 1, N_{\mathbf{f}} = 0 |$  and  $| n(k) = 0, \bar{n}(\epsilon_p) = 0, N_{\mathbf{f}} = 1 \rangle$ . This is equal to

$$M_{k,p,\mathbf{f}}^{\text{exc}} = \frac{ep_o}{m_o c} \left( \frac{2\pi c^2}{V\nu_{\mathbf{f}}} \right)^{1/2} \frac{1}{L} \int d\mathbf{r} \int dz \times e^{-i(k+p)z} \phi^{e*}(\mathbf{r}) \phi^{h*}(\mathbf{r}) e^{i\mathbf{f}_{\perp} \cdot \mathbf{r} + ifz}. \quad (28)$$

The photon momentum is small and so is  $\mathbf{r}$ , since it is limited by the wire cross section; therefore  $e^{i\mathbf{f}_{\perp} \cdot \mathbf{r}} = 1$ . Integrating over the  $z$  coordinates, the square of the matrix element is

$$|M_{k,p,\mathbf{f}}^{\text{exc}}|^2 = \left( \frac{ep_o}{m_o c} \right)^2 \frac{2\pi c^2}{V\nu_{\mathbf{f}}} \left| \int d\mathbf{r} \phi^e(\mathbf{r}) \phi^h(\mathbf{r}) \right|^2 \delta_{f,k+p}. \quad (29)$$

In the electron-LO-phonon interaction, one phonon of momentum  $\mathbf{q}$  is emitted, an electron  $k$  is annihilated, and an electron  $k'$  is created. The interaction of the electrons with the polarization created by the LO phonons is given by

$$\int d\mathbf{r} \int dz \frac{4\pi e \gamma}{|\mathbf{q}|} \hat{\Psi}^{e\dagger}(\mathbf{r},z) \hat{B}(\mathbf{r},z) \hat{\Psi}^e(\mathbf{r},z), \quad (30)$$

where  $\gamma$  is some interaction constant (see, for example, Refs. 38 and 39), and the phonon field operator is

$$\hat{B}(\mathbf{r},z) = \sum_{\mathbf{q}} \left( \frac{1}{2V\rho\omega_o} \right)^{1/2} e^{i\mathbf{q}_{\perp} \cdot \mathbf{r} + iqz} (\hat{a}_{\mathbf{q}} + \hat{a}_{-\mathbf{q}}^{\dagger}). \quad (31)$$

The components of  $\mathbf{q}$  are defined in the same manner as those of  $\mathbf{f}$ ,  $\hat{a}_{\mathbf{q}}$  is the annihilation operator, and  $\rho$  is the reduced mass per unit cell in the lattice.

The matrix element  $M^{e\text{-LO}}$  can be found by sandwiching interaction (30) between the states  $\langle n(k)=0, n(k')=1, N(\omega_o)=1 |$  and  $| n(k)=1, n(k')=0, N(\omega_o)=0 \rangle$ . Carrying out the integration over the  $z$  coordinates, the square of the matrix element is

$$|M_{k,k',\mathbf{q}}^{e\text{-LO}}|^2 = \frac{1}{|\mathbf{q}|^2} \frac{4\pi\alpha\omega_o^{3/2}}{V(2m_e)^{1/2}} \left| \int d\mathbf{r} |\phi^e(\mathbf{r})|^2 e^{-i\mathbf{q}_{\perp} \cdot \mathbf{r}} \right|^2 \delta_{k-k',q}, \quad (32)$$

where we exchanged  $\gamma$  for the known Fröhlich constant  $\alpha$ , using the relation  $\gamma^2 = (\rho\omega_o^2/4\pi)(2\omega_o/m_e)^{1/2}(\alpha/e^2)$  (see Ref. 39).

Substituting  $|M^{\text{exc}}|^2$  and  $|M^{e\text{-LO}}|^2$  into Eq. (25), we can sum over  $p$  and  $\mathbf{q}$ . Taking the normalizing volumes to infinity and thus exchanging the sums with integrals, we obtain

$$\begin{aligned} 0 = & \int \frac{d\mathbf{f}}{(2\pi)^3} |M^{\text{exc}}(\nu_{\mathbf{f}})|^2 2\pi \delta(\nu_{\mathbf{f}} - \epsilon_k - \epsilon_g - \epsilon_{f-k}) \\ & \times [1 - n(k)] N(\mathbf{f}) \\ & - \int \frac{dk'}{2\pi} |M^{e\text{-LO}}(k-k')|^2 2\pi \delta(\epsilon_k - \epsilon_{k'} - \omega_o) n(k), \quad (33) \end{aligned}$$

where

$$|M^{\text{exc}}(\nu_{\mathbf{f}})|^2 = \frac{2\pi e^2 p_o^2}{m_o^2 \nu_{\mathbf{f}}} \left| \int d\mathbf{r} \phi^e(\mathbf{r}) \phi^h(\mathbf{r}) \right|^2, \quad (34)$$

$$|M^{e\text{-LO}}(k-k')|^2 = \frac{1}{2\pi} \frac{4\pi\alpha\omega_o^{3/2}}{(2m_e)^{1/2}} \sigma(k-k'), \quad (35)$$

and

$$\begin{aligned} \sigma(k-k') = & \int d\mathbf{r} \int d\mathbf{r}' |\phi^e(\mathbf{r})|^2 |\phi^e(\mathbf{r}')|^2 \\ & \times K_0(|k-k'| |\mathbf{r}-\mathbf{r}'|). \quad (36) \end{aligned}$$

The zeroth-order Bessel function  $K_0$  is a result of the integration over  $\mathbf{q}_{\perp}$ .

Since the matrix elements (34) and (35) are smooth functions of their arguments, and we are interested only in processes which involve electrons that were excited close to the threshold, we may take them at the threshold values. The threshold value of  $\nu_{\mathbf{f}}$  is  $\bar{\nu} = \epsilon_g + (1+\eta)\omega_o$ , where  $\eta = m_e/m_h$ . This is the frequency that will excite electrons from the valence band to the threshold in the conduction band. The threshold value of  $|k-k'|$  is  $q_o = (2m_e\omega_o)^{1/2}$ .

The electron wave functions that appear in  $\sigma$  limit the spatial integration to  $r \leq a$ , where  $a$  is the wire width; therefore we can estimate that  $\sigma(q_o) \propto |\ln(q_o a)|$  for  $q_o a \ll 1$  and  $\sigma(q_o) \propto 1/(q_o a)^2$  for  $q_o a \gg 1$ . For GaAs,  $q_o = 2.5 \times 10^6 \text{ cm}^{-1}$ , thus for most wire widths we are in the regime of  $q_o a \gg 1$ , in which  $\sigma$  is expected to decrease with increasing wire cross section. The constant  $\alpha\sigma(q_o) \equiv \alpha^*$  is the effective Fröhlich constant in the one-dimensional case. According to Ref. 32,  $\alpha^* \approx 0.1$  (which is of the same order as the bulk Fröhlich constant) for a wire of cross section  $200 \times 100 \text{ \AA}^2$ , while, according to Ref. 35,  $\alpha^* \approx 0.02$  for a wire of cross section  $300 \times 100 \text{ \AA}^2$ .

We now perform the integrations which appear in Eq. (33). We begin with the generating term  $\mathcal{G}(\epsilon_k)$ , the free term [one that does not include  $n(k)$ ] in the balance equation (33) for the electron occupation number. This term is physically the generation rate of electrons in the conduction band. Since  $f \ll k$ , we can approximate  $\epsilon_{f-k} \approx \epsilon_{-k}$ . Using  $d\mathbf{f} = \nu_{\mathbf{f}}^2 d\nu_{\mathbf{f}} d\Omega_{\mathbf{f}}/c^3 \approx \bar{\nu}^2 d\nu_{\mathbf{f}} d\Omega_{\mathbf{f}}/c^3$ , where  $d\Omega_{\mathbf{f}}$  stands for the solid angle increment, we obtain

$$\begin{aligned} & \int \frac{d\mathbf{f}}{(2\pi)^3} |M^{\text{exc}}(\bar{\nu})|^2 2\pi \delta(\nu_{\mathbf{f}} - \epsilon_k - \epsilon_g - \epsilon_{-k}) N(\mathbf{f}) \\ & = 2\pi |M^{\text{exc}}(\bar{\nu})|^2 D(\bar{\nu}) \langle N(\epsilon_k + \epsilon_g + \epsilon_{-k}) \rangle \equiv \mathcal{G}(\epsilon_k), \quad (37) \end{aligned}$$

where  $D(\bar{\nu}) = \bar{\nu}^2/(2\pi^2 c^3)$ , is the photon density of states, and

$$\langle N(\epsilon_k + \epsilon_g + \epsilon_{-k}) \rangle = \int \frac{d\Omega_{\mathbf{f}}}{4\pi} N(\mathbf{f}) \Big|_{\nu_{\mathbf{f}} = \epsilon_k + \epsilon_g + \epsilon_{-k}} \quad (38)$$

is the angular average of  $N(\mathbf{f})$ , for photons of frequency  $\epsilon_k + \epsilon_g + \epsilon_{-k}$  that create electrons with energy  $\epsilon_k$ .

We consider a narrow-band photon excitation field such that the angular average of the occupation number is given by

$$\langle N(\nu) \rangle = I \left( \frac{c}{\nu} \right)^3 \frac{\Delta\nu_o/2\pi}{(\nu - \nu_o)^2 + \Delta\nu_o^2/4}, \quad (39)$$

where  $I$  is the excitation field energy density,  $\Delta\nu_o$  is the spectral width, and  $\nu_o$  is the central frequency. We are interested in a narrow-band excitation close to the threshold; therefore  $\Delta\nu_o \ll \omega_o$ , and the detuning (the difference between the central excitation frequency and the threshold frequency) is small:  $\tilde{\nu}_o = \nu_o - \bar{\nu} \ll \omega_o$ .

The integrations that appear in the expression for the decay rate  $\Gamma(\epsilon_k)$ , the term that includes all coefficients of  $-n(k)$  in the kinetic equation (33), will now be carried out.

The first contribution to  $\Gamma(\epsilon_k)$  is that of the photons, and is equal to the generating term (37). The second contribution to  $\Gamma(\epsilon_k)$  is due to LO phonons,

$$\begin{aligned}\Gamma_{\text{LO}}(\epsilon_k) &= \int \frac{dk'}{2\pi} |M^{e\text{-LO}}(q_o)|^2 2\pi \delta(\epsilon_k - \epsilon_{k'} - \omega_o) \\ &= 2\alpha^* \omega_o \left( \frac{\omega_o}{\epsilon_k - \omega_o} \right)^{1/2}.\end{aligned}\quad (40)$$

The photon contribution to the decay rate is negligible compared to the phonon contribution, since radiative processes are slow compared to nonradiative processes. Therefore  $\Gamma(\epsilon_k)$  can be written as

$$\Gamma(\epsilon_k) = \Gamma_{\text{LO}}(\epsilon_k) = \left( \frac{\Gamma_c^3}{\epsilon_k - \omega_o} \right)^{1/2}, \quad (41)$$

where

$$\Gamma_c = (2\alpha^*)^{2/3} \omega_o. \quad (42)$$

The meaning of the energy scale  $\Gamma_c$  will be explained at the end of this section. For  $\alpha^* = 0.02$ ,<sup>35</sup> one obtains  $\Gamma_c = 4$  meV, thus  $\Gamma_c \ll \omega_o = 36$  meV.

Below the threshold, where  $\Gamma_{\text{LO}} = 0$ , the acoustical-phonon contribution to  $\Gamma$  is important. Since the latter is a smooth function near the threshold, a good approximation for  $\Gamma$  is  $\Gamma(\epsilon_k) = \tau_{\text{ac}}^{-1} = \text{const}$  at  $\epsilon_k < \omega_o$ .

The occupation number of the electrons that were excited above the threshold is given by expression (21), where  $\mathcal{G}$  and  $\Gamma$  are given by expressions (37) and (41) respectively. It is clear from the expressions above that  $n(k)$  depends on  $k$  through  $\epsilon_k$  only.

We now turn our attention to the kinetic equation for the luminescence photons,

$$\begin{aligned}\frac{\partial}{\partial t} N(f, m, n) &= \sum_k |M_{k,f,m,n}^{\text{lum}}|^2 2\pi \delta(\nu_{f,m,n} - \epsilon_k - \varepsilon) \\ &\times \{ [1 + N(f, m, n)] n(\epsilon_k) \bar{n}(\varepsilon) \\ &- N(f, m, n) [1 - n(\epsilon_k)] [1 - \bar{n}(\varepsilon)] \},\end{aligned}\quad (43)$$

which for  $\bar{n}(\varepsilon) = 1$  and in case of spontaneous luminescence is

$$\frac{\partial}{\partial t} N(f, m, n) = \sum_k |M_{k,f,m,n}^{\text{lum}}|^2 2\pi \delta(\nu_{f,m,n} - \epsilon_k - \varepsilon) n(\epsilon_k). \quad (44)$$

$M^{\text{lum}}$  is the matrix element for the luminescence photon-electron interaction. The time derivative in the equation above is kept in order to clarify what is the luminescence source.

Let us first look at the matrix element that describes the recombination of an electron with a hole from a specific impurity. We will then have to sum over all impurities and average over all impurity configurations. In this process a photon of quantum numbers  $f$ ,  $m$ , and  $n$  is emitted, and an electron  $k$  and a hole in impurity  $i$  are destroyed. The interaction is given by

$$\int d\mathbf{r} \int dz \frac{ep_o}{m_o c} \hat{\Psi}^e(\mathbf{r}, z) \hat{A}^{\text{lum}}(\mathbf{r}, z) \hat{\Psi}^h(\mathbf{r}, z), \quad (45)$$

where

$$\begin{aligned}\hat{A}^{\text{lum}}(\mathbf{r}, z) &= \sum_{f,m,n} \left( \frac{2\pi c^2}{\nu_{f,m,n}} \right)^{1/2} [\chi_{f,m,n}(\mathbf{r}, z) \hat{a}_{f,m,n} \\ &+ \chi_{f,m,n}^*(\mathbf{r}, z) \hat{a}_{f,m,n}^\dagger]\end{aligned}\quad (46)$$

is the field operator of the luminescence photons.

The matrix element of the interaction above between the states  $\langle N(f, m, n) = 1, n(\epsilon_k) = 0, \bar{n}(\varepsilon) = 0 |$  and  $| N(f, m, n) = 0, n(\epsilon_k) = 1, \bar{n}(\varepsilon) = 1 \rangle$  for holes in impurity  $i$  is

$$\begin{aligned}M_{k,f,m,n}^{\text{lum}}(i) &= \frac{ep_o}{m_o c} \left( \frac{2\pi c^2}{\pi R^2 L^2 \nu_{f,m,n} |J_{m+1}(\kappa_{m,n} R)|^2} \right)^{1/2} \\ &\times \int d\mathbf{r} \int dz e^{ikz} \phi^e(\mathbf{r}) \psi(\mathbf{r} - \mathbf{r}_i, z - z_i) \\ &\times e^{-ifz - im\varphi} J_m^*(\kappa_{m,n} r).\end{aligned}\quad (47)$$

We now sum the matrix element squared over all impurities and average over all impurity configurations. This yields

$$\begin{aligned}|M_{k,f,m,n}^{\text{lum}}|^2 &= \left( \frac{ep_o}{m_o c} \right)^2 \frac{2\pi c^2}{\pi R^2 L^2 \nu_{f,m,n} |J_{m+1}(\kappa_{m,n} R)|^2} \\ &\times \sum_i \frac{1}{V} \int d\mathbf{r}_i \int dz_i \left| \int d\mathbf{r} \int dz e^{ikz} \phi^e(\mathbf{r}) \right. \\ &\times \left. \psi(\mathbf{r} - \mathbf{r}_i, z - z_i) e^{-ifz - im\varphi} J_m^*(\kappa_{m,n} r) \right|^2.\end{aligned}\quad (48)$$

Carrying out the integrations over the  $z$  coordinates, we obtain

$$\begin{aligned}|M_{k,f,m,n}^{\text{lum}}|^2 &= \left( \frac{ep_o}{m_o c} \right)^2 \frac{2\pi c^2}{\pi R^2 L^2 \nu_{f,m,n} |J_{m+1}(\kappa_{m,n} R)|^2} \\ &\times \sum_i \frac{1}{V} \int d\mathbf{r}_i \left| \int d\mathbf{r} \phi^e(\mathbf{r}) \psi(\mathbf{r} - \mathbf{r}_i, -k) \right. \\ &\times \left. e^{-im\varphi} J_m^*(\kappa_{m,n} r) \right|^2,\end{aligned}\quad (49)$$

where  $\psi(\mathbf{r}, k) = \int dz e^{-ikz} \psi(\mathbf{r}, z)$ . The second argument of  $\psi$ ,  $f - k$ , was replaced by  $-k$  because  $f \ll k$ .

Since the impurities are randomly distributed in the bulk, the average over all impurity configurations of the product of the hole wave functions that appear in Eq. (49) can depend only on the difference between their coordinates. Thus we define

$$\mathcal{L}(\mathbf{r} - \mathbf{r}', -k) \equiv \int d\mathbf{r}_i \psi(\mathbf{r} - \mathbf{r}_i, -k) \psi^*(\mathbf{r}' - \mathbf{r}_i, k).$$

The summation over impurities divided by the normalizing volume renders a factor of  $n_{\text{imp}}$ —the impurity spatial density. The averaged matrix element squared can then be written as

$$\begin{aligned}
|M_{k,f,m,n}^{\text{lum}}|^2 &= \left(\frac{ep_o}{m_o c}\right)^2 \frac{2\pi c^2}{\pi R^2 L v_{f,m,n} |J_{m+1}(\kappa_{m,n} R)|^2} n_{\text{imp}} \\
&\times \int d\mathbf{r} \int d\mathbf{r}' \phi^e(\mathbf{r}) \phi^{e*}(\mathbf{r}') \mathcal{L}(\mathbf{r}-\mathbf{r}', -k) \\
&\times e^{-im(\varphi-\varphi')} J_m^*(\kappa_{m,n} r) J_m(\kappa_{m,n} r'). \quad (50)
\end{aligned}$$

The matrix element squared is much larger for  $m=0$  than for other values of  $m$ , since  $r$  and  $r'$  are constrained to a small region in the  $xy$  plane (due to the electron wave functions), and  $J_0$  is the only Bessel function which is finite at  $r \rightarrow 0$ ,  $J_0(r)|_{r \rightarrow 0} = 1$ . Thus the matrix element squared is proportional to  $\delta_{m,0}$ . From now on the index  $m$  is omitted with the understanding that we are dealing only with  $m=0$ .

The matrix element squared is a smooth function of  $v_{f,n}$  and  $k$ , therefore these can be substituted with their threshold values. The threshold values of  $v_{f,n}$  and  $k$  are  $\bar{v}' = \varepsilon + \omega_o$  and  $k_o = (2m_e \omega_o)^{1/2}$ , respectively. The kinetic equation for the luminescence photons can then be written as

$$\frac{\partial}{\partial t} N(f,n) = \int \frac{dk}{2\pi} |M_n^{\text{lum}}(k_o)|^2 2\pi \delta(v_{f,n} - \varepsilon_k - \varepsilon) n(\varepsilon_k), \quad (51)$$

where the matrix element squared is

$$\begin{aligned}
|M_n^{\text{lum}}(k_o)|^2 &= \left(\frac{ep_o}{m_o c}\right)^2 \frac{2\pi c^2}{\pi R^2 L \bar{v}} |J_1(\kappa_n R)|^2 n_{\text{imp}} L \\
&\times \int d\mathbf{r} \int d\mathbf{r}' \phi^e(\mathbf{r}) \phi^{e*}(\mathbf{r}') \mathcal{L}(\mathbf{r}-\mathbf{r}', -k_o). \quad (52)
\end{aligned}$$

Substituting  $n(\varepsilon_k)$  into Eq. (51), and integrating over  $k$ , we obtain

$$\frac{\partial}{\partial t} N(f,n) = |M_n^{\text{lum}}(k_o)|^2 \left(\frac{2m_e}{v_{f,n} - \varepsilon}\right)^{1/2} \frac{\mathcal{G}(v_{f,n} - \varepsilon)}{\Gamma(v_{f,n} - \varepsilon)}. \quad (53)$$

The luminescence source can be characterized by the spectral dependence of  $E(v') dv'$ , the energy of the emitted field of all spectral modes within the interval  $dv'$  per unit length of the wire per unit time. Multiplying the generating term of the luminescence photons [the right hand side of Eq. (53)] by  $\delta(v' - v_{f,n}) v' dv'/L$  and summing over  $f$  and  $n$ , we obtain

$$\begin{aligned}
E(v') dv' &= \frac{v'}{L} \sum_n \sum_f |M_n^{\text{lum}}(k_o)|^2 \\
&\times \left(\frac{2m_e}{v_{f,n} - \varepsilon}\right)^{1/2} \frac{\mathcal{G}(v_{f,n} - \varepsilon)}{\Gamma(v_{f,n} - \varepsilon)} \delta(v' - v_{f,n}) dv'. \quad (54)
\end{aligned}$$

The variable  $v_{f,n}$  in the square root can be replaced with its threshold value  $\varepsilon + \omega_o$ , due to the smoothness of this function. Taking the normalization volume (that appears in

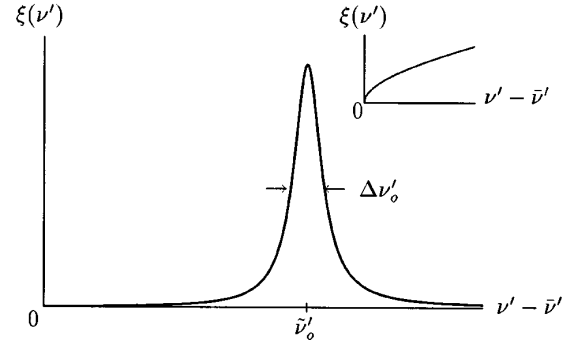


FIG. 2. The behavior of  $\xi(v')$  as a function of  $v'$  for  $\bar{v}_o \gg \Delta v_o$  ( $\eta = \frac{1}{3}$  and  $\bar{v}_o = 8\Delta v_o$ ), where  $\bar{v}'_o = \bar{v}_o/(1+\eta)$  and  $\Delta v'_o = \Delta v_o/(1+\eta)$ . An enlargement of its behavior for  $v' - \bar{v}' \leq \bar{v}'_o$  is shown in the inset.

the matrix element) to infinity, the sums over  $f$  and  $n$  are transformed into integrals. Performing the integration, we obtain the final result

$$E(v') = C v' D(v') \xi(v'), \quad (55)$$

where

$$\begin{aligned}
C &= \left(\frac{ep_o}{m_o c}\right)^2 \frac{2\pi c^2}{\bar{v}'} n_{\text{imp}} \int d\mathbf{r} \int d\mathbf{r}' \phi^e(\mathbf{r}) \phi^{e*}(\mathbf{r}') \\
&\times \mathcal{L}(\mathbf{r}-\mathbf{r}', -k_o) \quad (56)
\end{aligned}$$

is a constant, and

$$\begin{aligned}
\xi(v') &= \left(\frac{2m_e}{\omega_o}\right)^{1/2} \frac{\mathcal{G}(v' - \varepsilon)}{\Gamma(v' - \varepsilon)} = \left(\frac{2m_e}{\omega_o}\right)^{1/2} \frac{2\pi |M^{\text{exc}}(\bar{v})|^2 D(\bar{v})}{\Gamma(v' - \varepsilon)} \\
&\times \langle N[(1+\eta)(v' - \varepsilon) + \varepsilon_g] \rangle, \quad (57)
\end{aligned}$$

is essentially the product of the electron generation rate and the electron lifetime at the corresponding energy.

Since  $v' D(v')$  is a smooth function of  $v'$ , the spectral distribution of the luminescence is given by  $\xi(v')$ . Let us first consider a narrow-band excitation that does not overlap the threshold, i.e.,  $\bar{v}_o \gg \Delta v_o$ . In this case  $\xi(v')$  is a peaked function of  $v'$ , reproducing the shape of the excitation. It is of a Lorentzian shape of width  $\Delta v'_o = \Delta v_o/(1+\eta)$ , centered at  $\bar{v}' + \bar{v}'_o$ , where  $\bar{v}'_o = \bar{v}_o/(1+\eta)$  (see Fig. 2):

$$\xi(v') \propto \left(\frac{\bar{v}_o}{(1+\eta)^3 \Gamma_c^3}\right)^{1/2} \frac{\Delta v'_o/(2\pi)}{(v' - \bar{v}' - \bar{v}'_o)^2 + (\Delta v'_o/2)^2}. \quad (58)$$

The rescaling of  $v'$  compared to  $v$  follows from the obvious relations (see Fig. 1)  $\varepsilon_k + \varepsilon_k = v - \varepsilon_g$  and  $v' = \varepsilon + \varepsilon_k$ .

Only close to the threshold, when  $v' - \bar{v}' \leq \bar{v}'_o/(1+\eta)$ , the spectral behavior of the luminescence differs from that of the excitation and is given by (see the inset of Fig. 2)

$$\xi(v') \propto \frac{\Delta v_o}{2\pi \bar{v}_o^2 \Gamma_c^{3/2}} (v' - \bar{v}')^{1/2}. \quad (59)$$



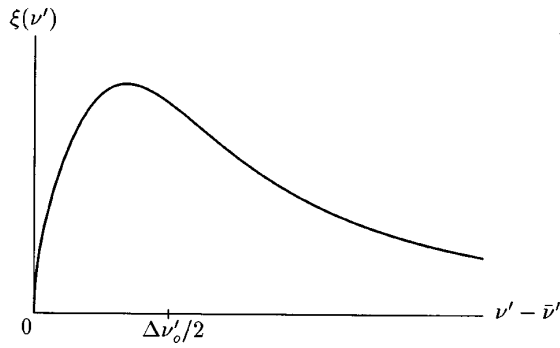


FIG. 3. The behavior of  $\xi(\nu')$  as a function of  $\nu'$  for  $\Delta\nu_o \gg \tilde{\nu}_o$  ( $\eta = \frac{1}{3}$  and  $\Delta\nu_o = 8\tilde{\nu}_o$ ), where  $\Delta\nu'_o = \Delta\nu_o/(1+\eta)$ .

When the narrow-band excitation overlaps the threshold, i.e.,  $\tilde{\nu}_o \ll \Delta\nu_o$ , the spectral distribution of the luminescence is very different from that of the excitation (see Fig. 3):

$$\xi(\nu') \propto \frac{2}{\pi \Delta\nu_o \Gamma_c^{3/2}} (\nu' - \bar{\nu}')^{1/2} \quad \text{when } \nu' - \bar{\nu}' \ll \frac{\Delta\nu'_o}{2}, \quad (60)$$

and

$$\xi(\nu') \propto \frac{\Delta\nu_o}{2\pi(1+\eta)^2 \Gamma_c^{3/2}} (\nu - \bar{\nu}')^{-3/2} \quad \text{when } \nu' - \bar{\nu}' \gg \frac{\Delta\nu'_o}{2}. \quad (61)$$

If one is interested in luminescence due to electrons that were excited close to but *below* the threshold, via an excitation that is centered *above* the threshold, i.e.,  $\nu' - \bar{\nu}' < 0$  and  $\tilde{\nu}_o > 0$ , then  $\Gamma(\nu' - \varepsilon)$  in expression (57) should be replaced by  $\tau_{ac}^{-1}$ . In this case  $\xi(\nu')$  has the same Lorentzian form as Eq. (58) with the amplitude  $[\tilde{\nu}_o / ((1+\eta)^3 \Gamma_c^3)]^{1/2}$  replaced by  $\tau_{ac} / (1+\eta)$ .

One can expect that the luminescence distribution  $\xi(\nu')$ , derived from the BE, is not correct for frequencies  $\nu'$  close to the threshold  $\bar{\nu}'$ . Luminescence of such frequencies is due to electrons with energies  $\epsilon_k$  near the threshold, that have a large width  $\Gamma(\epsilon_k)$  [see Eq. (41)], while the BE assumes that the width of the states is small compared to other energy scales of the problem. In our case one of these energy scales is  $\epsilon_k - \omega_o$ . Equating the width with this energy scale,  $\epsilon_k - \omega_o = \Gamma(\epsilon_k)$ , one finds an additional energy scale of the problem (and a critical decay rate), that is given by  $\Gamma_c$  that has been defined before Eq. (42).

When  $\epsilon_k - \omega_o \leq \Gamma_c$ , we have  $\Gamma(\epsilon_k) \geq \epsilon_k - \omega_o$ , and the assumption of the BE breaks down. One would therefore expect that for  $\epsilon_k - \omega_o \leq \Gamma_c$  in the description of the excited electrons, and for  $\nu' - \bar{\nu}' \leq \Gamma_c$  in the description of the hot luminescence, the results predicted by the Boltzmann equation would fail. We will learn from the QKE's that in fact the situation is more complicated.

## B. Quantum description

As we saw in Sec. III A, the Boltzmann description may fail for electrons excited to the vicinity of the threshold and hence for luminescence photons emitted by these electrons. Therefore, close to the threshold, the electrons and the luminescence photons have to be treated using QKE's. These QKE's will be written by applying the generalization rules to Eqs. (33) and (51). It is assumed that the energies of the holes, the phonons, and the excitation photons are not broadened.

In the generalization to the QKE's only those terms that were kept in the Boltzmann description will be retained. Threshold values of the arguments of the matrix elements will be substituted as was done in the Boltzmann description, due to the smoothness of the matrix elements.

The QKE for the electrons is

$$0 = \int \frac{d\mathbf{f}}{(2\pi)^3} |M^{\text{exc}}(\bar{\nu})|^2 2\pi \delta(\nu_{\mathbf{f}} - \epsilon - \varepsilon_g - \varepsilon_{-k}) N(\mathbf{f}) - \int \frac{dk'}{2\pi} \int d\epsilon' \delta[\Gamma(k', \epsilon')] |\epsilon' - \epsilon_{k'} - \Delta\epsilon(k', \epsilon')] \times |M^{e\text{-LO}}(q_o)|^2 2\pi \delta(\epsilon - \epsilon' - \omega_o) n(k, \epsilon), \quad (62)$$

where  $\epsilon$  is the electron off-shell energy,  $n$  is the electron occupation function, and  $\Gamma$  and  $\Delta\epsilon$  are the electron-energy-level width and shift, respectively.

The coefficients of  $-n(k, \epsilon)$  in Eq. (62) can be recognized as the electron level width due to LO phonons,

$$\Gamma_{\text{LO}}(k, \epsilon) = \int \frac{dk'}{2\pi} \int d\epsilon' \delta[\Gamma(k', \epsilon')] |\epsilon' - \epsilon_{k'} - \Delta\epsilon(k', \epsilon')] \times |M^{e\text{-LO}}(q_o)|^2 2\pi \delta(\epsilon - \epsilon' - \omega_o), \quad (63)$$

and the total electron width is given by

$$\Gamma(k, \epsilon) = \Gamma_{\text{LO}} + 1/\tau_{ac}. \quad (64)$$

The photon contribution to the level width was neglected (as it was neglected in the Boltzmann description of the decay term).

One can see that the generating term in Eq. (62) is equal to expression (37), the generating term of the electrons' BE, with the on-shell energy  $\epsilon_k$  replaced by the off-shell energy  $\epsilon$  (in the exciting photon occupation number)

$$\mathcal{G}(k, \epsilon) = 2\pi |M^{\text{exc}}(\bar{\nu})|^2 D(\bar{\nu}) \langle N(\epsilon + \varepsilon_g + \varepsilon_{-k}) \rangle. \quad (65)$$

The occupation function  $n(k, \epsilon)$  is found by substituting  $\mathcal{G}$  and  $\Gamma$  into Eq. (23). The generating term  $\mathcal{G}(k, \epsilon)$  depends on  $k$  through  $\varepsilon_{-k} = \eta\epsilon_k$  only; therefore  $\mathcal{G}(k, \epsilon) = \mathcal{G}(\epsilon_k, \epsilon)$ . It follows from Eq. (63) that  $\Gamma$  is independent of  $k$  (and so is the level shift). Therefore,  $n(k, \epsilon) = n(\epsilon_k, \epsilon)$ .

Equation (63) is a self-consistent equation for the level width. Integrating the right-hand side of the equation, we find that

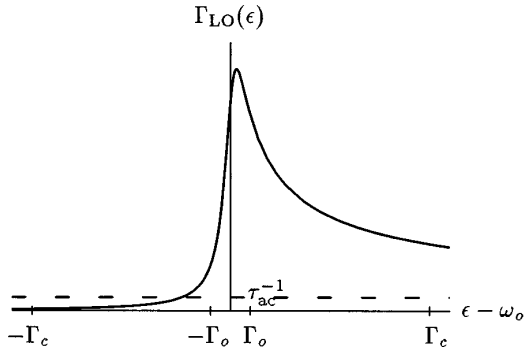


FIG. 4. The behavior of the electron level width due to optical phonons, at the vicinity of the threshold for LO-phonon emission, when the electron level width at the bottom of the subband is considered.

$$\Gamma_{LO}(\epsilon) = \alpha^* \omega_o^{3/2} \left\{ \left[ \epsilon - \omega_o - \Delta\epsilon(\epsilon - \omega_o) + \frac{i}{2} \Gamma(\epsilon - \omega_o) \right]^{-1/2} + \left[ \epsilon - \omega_o - \Delta\epsilon(\epsilon - \omega_o) - \frac{i}{2} \Gamma(\epsilon - \omega_o) \right]^{-1/2} \right\}. \quad (66)$$

When  $\epsilon - \omega_o \ll \omega_o$  the arguments of  $\Gamma(\epsilon - \omega_o)$  and  $\Delta\epsilon(\epsilon - \omega_o)$  are close to the bottom of the band. In this case the level shift can be neglected since it is just a small renormalization. The level width close to the bottom of the band  $\Gamma_o$ , is due to scattering with acoustic phonons and due to thermal recombination. The latter contribution to  $\Gamma_o$  can be neglected, while the acoustic-phonon contribution can be considered as a constant (though not the same as the width due to acoustic phonons at the threshold). We assume that  $\Gamma_o$  is small compared to all other energy scales of the problem. Thus, close to the threshold,

$$\Gamma_{LO}(\epsilon) = \alpha^* \omega_o^{3/2} \left\{ \left[ \epsilon - \omega_o + \frac{i}{2} \Gamma_o \right]^{-1/2} + \left[ \epsilon - \omega_o - \frac{i}{2} \Gamma_o \right]^{-1/2} \right\}. \quad (67)$$

The behavior of  $\Gamma_{LO}(\epsilon)$  when  $\epsilon$  is close to  $\omega_o$  is shown in Fig. 4.

From expression (67), we see that above the threshold for  $\epsilon - \omega_o \gg \Gamma_o/2$ ,  $\Gamma(\epsilon)$  can be written as

$$\Gamma(\epsilon) = \Gamma_{LO}(\epsilon) = 2 \alpha^* \omega_o \left( \frac{\omega_o}{\epsilon - \omega_o} \right)^{1/2} = \left( \frac{\Gamma_c^3}{\epsilon - \omega_o} \right)^{1/2}. \quad (68)$$

Comparing expression (68) to expression (41), it is evident that above the threshold  $\Gamma(\epsilon)$  is equal to  $\Gamma(\epsilon_k)$  when the on-shell energy  $\epsilon_k$  is replaced by the off-shell energy  $\epsilon$ . Since  $\Gamma_o$  is the smallest energy scale in the problem, expression (68) can be used for the electron level width above the threshold.

Contrary to the Boltzmann description, the electron-LO-phonon scattering contributes to the electron level width be-

low the threshold as well. When  $\omega_o - \epsilon \gg \Gamma_o$  the electron width due to optical phonons decays like  $|\epsilon - \omega_o|^{-3/2}$ , much faster than the decay above the threshold (see Fig. 4). The contribution of the acoustic phonons to the electron width below the threshold  $\tau_{ac}^{-1}$ , will be negligible compared to  $\Gamma_{LO}$  for  $\omega_o - \epsilon \ll (\Gamma_o \tau_{ac})^{2/3} \Gamma_c/2$ . Since  $\Gamma_o \tau_{ac} \ll 1$  and  $\Gamma_c \tau_{ac} \gg 1$ , the right-hand side of the inequality above is much smaller than  $\Gamma_c$ , and much larger than  $\Gamma_o$  [see the crossing point of  $\Gamma_{LO}(\epsilon)$  and  $\tau_{ac}^{-1}$  in Fig. 4].

The QKE that is obtained by applying the generalization rules to the BE for the luminescence photons, Eq. (51), is

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \left[ \frac{1}{2} + N(f, n, \nu') \right] \delta \left[ \gamma(f, n, \nu') | \nu' - \nu_{f,n} - \Delta \nu(f, n, \nu') \right] \right\} \\ & = \delta \left[ \gamma(f, n, \nu') | \nu' - \nu_{f,n} - \Delta \nu(f, n, \nu') \right] \\ & \quad \times \int \frac{dk}{2\pi} \int d\epsilon \delta \left[ \Gamma(\epsilon) | \epsilon - \epsilon_k - \Delta \epsilon(\epsilon) \right] | M_n^{\text{lum}}(k_o) |^2 \\ & \quad \times 2\pi \delta(\nu' - \epsilon - \epsilon) n(\epsilon_k, \epsilon), \end{aligned} \quad (69)$$

where  $\nu'$  is the photon off-shell energy,  $N$  is the photon occupation function, and  $\gamma$  and  $\Delta \nu$  are the photon energy level width and shift, respectively. All the photon functions are written for  $m=0$ .

By applying the generalization rules to the right-hand side of the full BE, Eq. (43), one can find the photon level width

$$\begin{aligned} \gamma(n, \nu') & = - \int \frac{dk}{2\pi} \int d\epsilon \delta \left[ \Gamma(\epsilon) | \epsilon - \epsilon_k - \Delta \epsilon(\epsilon) \right] \\ & \quad \times | M_n^{\text{lum}}(k_o) |^2 2\pi \delta(\nu' - \epsilon - \epsilon) n(\epsilon_k, \epsilon). \end{aligned} \quad (70)$$

Note that  $\gamma < 0$  corresponds to photon generation. It is evident from expression (70) that  $\gamma$  does not depend on  $f$ , and therefore neither does  $\Delta \nu$ .

Using Eq. (70), Eq. (69) can be written as

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \left[ \frac{1}{2} + N(f, n, \nu') \right] \delta \left[ \gamma(n, \nu') | \nu' - \nu_{f,n} - \Delta \nu(n, \nu') \right] \right\} \\ & = - \gamma(n, \nu') \delta \left[ \gamma(n, \nu') | \nu' - \nu_{f,n} - \Delta \nu(n, \nu') \right]. \end{aligned} \quad (71)$$

The right-hand side of Eq. (71) is the photon-generating term, the term from which the luminescence spectral distribution will be obtained.

As in the classical case, the luminescence source can be characterized by the spectral dependence of  $E(\nu') d\nu'$ , that is obtained by multiplying the right-hand side of Eq. (71) by  $\nu' d\nu'/L$  and summing over  $f$  and  $n$

$$\begin{aligned} E(\nu') d\nu' & = - \frac{\nu'}{L} \sum_n \sum_f \gamma(n, \nu') \\ & \quad \times \delta \left[ \gamma(n, \nu') | \nu' - \nu_{f,n} - \Delta \nu(n, \nu') \right] d\nu'. \end{aligned} \quad (72)$$

The main contribution to  $E(\nu')$  comes from  $f$  and  $n$ , such that  $\nu_{f,n}$  is close to  $\nu'$ .

It is convenient to write  $\gamma(n, \nu')$  in the form

$$\gamma(n, \nu') = - \frac{1}{\pi R^2 |J_1(\kappa_n R)|^2} \mathcal{C}\xi(\nu'), \quad (73)$$

where  $\mathcal{C}$  is given by Eq. (56), and

$$\begin{aligned} \xi(\nu') &= \int \frac{dk}{2\pi} \int d\epsilon \delta[\Gamma(\epsilon)|\epsilon - \epsilon_k - \Delta\epsilon(\epsilon)] \\ &\quad \times 2\pi\delta(\nu' - \epsilon - \epsilon)n(\epsilon_k, \epsilon). \end{aligned} \quad (74)$$

When the electrons' energy is taken to be on shell—that is, when the electron-smearred  $\delta$  function is reduced to a singular  $\delta$  function— $\xi(\nu')$  is reduced to expression (57). In writing  $\gamma$  in the form (73), the normalization volume has been separated from the rest of the function—that is, independent of the index  $n$ —and can therefore be taken out of the summation in  $E(\nu')$ .

Taking the normalization volume to infinity the sums over  $f$  and  $n$  are transformed into integrals. From Eq. (73) it is evident that  $\gamma|_{R \rightarrow \infty} \rightarrow 0$ . This is due to the fact that the photons are emitted into an “infinite” space, while their interaction with the electrons is confined to the finite volume of the wire. In this case  $\gamma$  and  $\Delta\nu$  are negligible compared to the other widths, and the smeared photon  $\delta$  function that appears in the expression for  $E(\nu')$  becomes a singular  $\delta$  function, giving

$$\begin{aligned} E(\nu') &= \mathcal{C}\nu' \xi(\nu') \int_{-\infty}^{\infty} \frac{df}{2\pi} \int_0^{\infty} \frac{\kappa}{2\pi} \frac{d\kappa}{2\pi} \delta(\nu' - \nu_{f,n}) \\ &= \mathcal{C}\nu' D(\nu') \xi(\nu'). \end{aligned} \quad (75)$$

Since  $\nu' D(\nu')$  is a smooth function it is clear at this point that in order to understand the dependence of  $E(\nu')$  on  $\nu'$ , the behavior of  $\xi(\nu')$  should be analyzed.

Performing the integration over  $\epsilon$  in Eq. (74), and substituting the expression we obtained for  $n(\epsilon_k, \epsilon)$ , we obtain

$$\begin{aligned} \xi(\nu') &= (2m_e)^{1/2} \frac{2\pi |M^{\text{exc}}(\bar{\nu})|^2 D(\bar{\nu})}{\Gamma(\nu' - \epsilon)} \\ &\quad \times \int_0^{\infty} \frac{d\epsilon_k}{\epsilon_k^{1/2}} \delta(\Gamma(\nu' - \epsilon)|\nu' - \epsilon - \epsilon_k) \\ &\quad \times \langle N(\nu' - \epsilon + \epsilon_k + \epsilon_g) \rangle. \end{aligned} \quad (76)$$

Since  $\epsilon_k - \omega_o \ll \omega_o$ , we substitute  $\omega_o$  for  $\epsilon_k$  in the one-dimensional density of states. This integral contains in fact two Lorentzians. The first is of width  $\Gamma(\nu' - \epsilon)$ : The electron level width at energy  $\epsilon = \nu' - \epsilon$ . The second Lorentzian is of width  $\Delta\nu_o/\eta$ : A width that is proportional to the spectral width of the exciting photon source. Since the excitation is such that  $\epsilon_k$  is close to  $\omega_o$ , and all the energy scales that can characterize the level width are much smaller than  $\omega_o$ , the integrand goes to zero when  $\epsilon_k$  approaches zero from above. Thus the lower boundary of the integral can be taken to minus infinity. As a result the spectral distribution of the luminescence is given by the function

$$\begin{aligned} \xi(\nu') &= \left( \frac{2m_e}{\omega_o} \right)^{1/2} \frac{2\pi |M^{\text{exc}}(\bar{\nu})|^2 D(\bar{\nu})}{\Gamma(\nu' - \epsilon)} I \left( \frac{c}{\bar{\nu}} \right)^3 \\ &\quad \times \int_{-\infty}^{\infty} d\epsilon_k \left( \frac{\Gamma(\nu' - \epsilon)/2\pi}{(\nu' - \epsilon - \epsilon_k)^2 + \Gamma(\nu' - \epsilon)^2/4} \right) \\ &\quad \times \left( \frac{\Delta\nu_o/2\pi}{[(\nu' - \epsilon + \eta\epsilon_k + \epsilon_g) - \nu_o]^2 + \Delta\nu_o^2/4} \right). \end{aligned} \quad (77)$$

The region of validity of the BE is evident from Eq. (77). In order to obtain the result of the BE, Eq. (57), one has to replace the first Lorentzian with a  $\delta$  function. This can be done only when its width is smaller than that of the second Lorentzian, i.e.,  $\Gamma(\nu' - \epsilon) \ll \Delta\nu_o/\eta$ . It follows from this inequality that the Boltzmann description of the luminescence spectra is correct only far from the threshold, when  $\nu' - \bar{\nu}' \gg \eta^2 \Gamma_c^3 / \Delta\nu_o^2 \equiv \nu_c$ . The quantum interval  $\nu_c$  differs from the naive estimate  $\Gamma_c$  (see the discussion at the end of Sec. III A): It is larger for “narrow”-band excitation ( $\Delta\nu_o \ll \Gamma_c$ ), and smaller for “wide”-band excitation ( $\Delta\nu_o \gg \Gamma_c$ ).

For an excitation that is mostly of frequencies above  $\bar{\nu} + \nu_c$ , the spectral dependence of the energy of the emitted photons will behave according to the predictions of the BE (see Figs. 2 and 3). However, as long as the excitation is above the threshold, very close to the threshold  $\xi(\nu')$  will increase linearly with  $\nu'$ , and not as a square root; see Eq. (59).

Non-Boltzmann behavior is obtained when  $\Gamma(\nu' - \epsilon) \gg \Delta\nu_o/\eta$ —that is, when  $\nu' - \bar{\nu}' \ll \nu_c$ —and the second Lorentzian can be treated as a  $\delta$  function. In this case one obtains

$$\begin{aligned} \xi(\nu') &\propto \frac{\eta}{2\pi(1+\eta)^2} \left[ \left( \nu' - \bar{\nu}' - \frac{\tilde{\nu}_o}{1+\eta} \right)^2 \right. \\ &\quad \left. + \frac{\eta^2}{4(1+\eta)^2} \frac{\Gamma_c^3}{\nu' - \bar{\nu}'} \right]^{-1}. \end{aligned} \quad (78)$$

Here we bring two specific examples of an extreme non-Boltzmann behavior of  $\xi(\nu')$ . In both examples the excitation is centered within the quantum interval, i.e.,  $\tilde{\nu}_o \ll \nu_c$ , and “narrow” band, i.e.,  $\Delta\nu_o \ll \Gamma_c$ . Due to the latter inequality  $\nu_c \gg \Gamma_c$ .

In the first case the detuning is large,  $\tilde{\nu}_o \gg \Gamma_c$ ; therefore the energy width of an electron excited by the central excitation frequency,  $\epsilon_k = \omega_o + \tilde{\nu}_o/(1+\eta)$ —that is, of the order of  $(\Gamma_c^3/\tilde{\nu}_o)^{1/2}$ —is small compared to its distance from the threshold. This is true for most of the electrons. In such a case the electron states are “well defined,” but the prediction of the BE for the electron distribution is wrong, since its width  $\Delta\nu_o/(1+\eta)$  is much smaller than the width of the states. As a result, it follows from Eq. (78) that the luminescence spectra is symmetric and centered at the classical position  $\bar{\nu}' + \tilde{\nu}'_o$ , with  $\tilde{\nu}'_o = \tilde{\nu}_o/(1+\eta)$ , but its width is given by the quantum-mechanical width of the electron states (see Fig. 5),

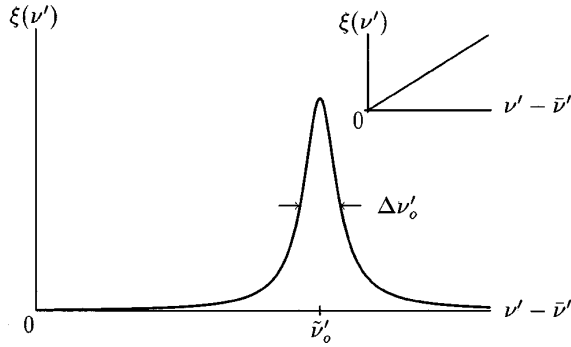


FIG. 5. The behavior of  $\xi(\nu')$  as a function of  $\nu'$  for  $\nu_c \gg \tilde{\nu}_o \gg \Gamma_c \gg \Delta\nu_o$  ( $\eta = \frac{1}{3}$ ,  $\tilde{\nu}_o = 15\Delta\nu_o$ , and  $\Gamma_c = 5\Delta\nu_o$ ), where  $\tilde{\nu}'_o = \tilde{\nu}_o/(1+\eta)$  and  $\Delta\nu'_o = \eta(\Gamma_c^3/\tilde{\nu}_o)^{1/2}/(1+\eta)$ . An enlargement of its behavior for  $\nu' - \bar{\nu}' \ll (\eta^2/4)\Gamma_c^3/\tilde{\nu}_o^2$  is shown in the inset.

$$\Delta\nu'_o = \frac{\eta}{1+\eta} \left[ \frac{\Gamma_c^3}{\tilde{\nu}_o/(1+\eta)} \right]^{1/2}.$$

In the second case the detuning is small,  $\tilde{\nu}_o \ll \Gamma_c$ , and the electrons are excited to “badly defined” states, since the broadening of these states is larger than their distance from the threshold. As a result the luminescence spectral distribution differs greatly from that predicted by the BE. One finds from Eq. (78) that (see Fig. 6)

$$\xi(\nu') \propto \frac{2}{\pi\eta\Gamma_c^3} (\nu' - \bar{\nu}') \quad \text{when } \nu' - \bar{\nu}' \ll \Gamma_c$$

and

$$\xi(\nu') \propto \frac{\eta}{2\pi(1+\eta)^2} \frac{1}{(\nu' - \bar{\nu}')^2} \quad \text{when } \nu_c \gg \nu' - \bar{\nu}' \gg \Gamma_c.$$

In conclusion, let us note that the hole dispersion plays an essential role in determining the quantum behavior of the luminescence spectra. The width of the second Lorentzian in Eq. (77),  $\Delta\nu_o/\eta$ , is not equal to the width of the classical luminescence line  $\Delta\nu_o/(1+\eta)$ , and the integral is not a simple convolution of the classical luminescence profile with the spectral function of an electron state. One can see from Eq. (77) that for a flat hole band ( $\eta=0$ ) the second Lorentzian does not depend on the integration variable, and the first Lorentzian is integrated to 1, unexpectedly restoring the BE result.

The luminescence due to electrons that were excited *below* the threshold ( $\epsilon < \omega_o$ ) by an excitation centered *above* the threshold ( $\tilde{\nu}_o > 0$ ) is not obtained by the simple exchange of  $\Gamma_{LO}$  by  $\tau_{ac}^{-1}$ , as in the classical case. As long as  $\nu' - \bar{\nu}' < (\Gamma_o \tau_{ac})^{2/3} \Gamma_c/2$ , one should take  $\Gamma = \Gamma_{LO}$  from expression (67) since  $\Gamma_{LO}(\nu' - \epsilon) > \tau_{ac}^{-1}$ ; therefore, although the electrons are below the threshold, it is the optical-phonon contribution to the electron width that is dominant.

#### IV. CONCLUSIONS

We have presented a recipe that allows one to generalize the BE to a QKE in spatially homogeneous situations. The

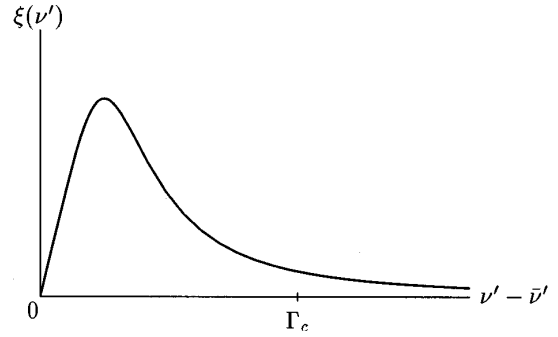


FIG. 6. The behavior of  $\xi(\nu')$  as a function of  $\nu'$  for  $\nu_c \gg \Gamma_c \gg \tilde{\nu}_o, \Delta\nu_o$  ( $\eta = \frac{1}{3}$ ,  $\tilde{\nu}_o = 2\Delta\nu_o$ , and  $\Gamma_c = 15\Delta\nu_o$ ).

QKE obtained by employing this recipe is the same as that obtained by the Keldysh-Green function technique in the self-consistent Born approximation. The advantage of this method for writing the QKE is that it provides a physical understanding of the terms in the QKE, and it allows one to neglect those terms that were negligible in the Boltzmann description. This is due to the fact that the equations are written for quantities that are similar to the quantities described by the Boltzmann equation.

Spatially homogeneous situations are realized in many optical experiments in which the samples are thin and only weakly absorbing. In this case it can be assumed that the incident optical fields are not attenuated within the sample, and that the density of excited carriers is homogeneous, and so is the luminescence source.

We considered the specific example of hot luminescence from a QWR. The recipe described above was used in order to generalize the set of BE's that describe the problem to a set of QKE's. Solving these equations, we were able to describe the luminescence spectral distribution.

We have shown that there is a domain of luminescence frequencies that correspond to a domain of photoexcited electron energies, for which the quantum description of the luminescence spectral distribution leads to a different behavior than that given by the Boltzmann description. This quantum domain could not be easily guessed from level width considerations. Two other nontrivial conclusions were obtained. The first is the role played by the hole mass in the definition of the quantum domain. When the hole dispersion relation is flat the quantum domain shrinks to zero, and the results of the Boltzmann description are retrieved. The second nontrivial conclusion is that there is an energy domain below the threshold for LO-phonon emission, in which the LO-phonon contribution to the electron level width is dominant.

#### ACKNOWLEDGMENTS

One of the authors (Y.L.) is indebted to P. Wölfle for drawing his attention to the importance of QKE's in optical phenomena. The research was supported by the Israel Academy of Sciences.

- <sup>1</sup>L. V. Keldysh, Zh. Éksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
- <sup>2</sup>E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Course of Theoretical Physics Vol. 10 (Pergamon, Oxford, 1981).
- <sup>3</sup>J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).
- <sup>4</sup>S. Schmitt-Rink, D. S. Chemla, and H. Haug, Phys. Rev. B **37**, 941 (1988).
- <sup>5</sup>M. Lindberg and S. W. Koch, Phys. Rev. B **38**, 3342 (1988).
- <sup>6</sup>A. V. Kuznetsov, Phys. Rev. B **44**, 8721 (1991).
- <sup>7</sup>Y. Levinson, Zh. Éksp. Teor. Fiz. **65**, 331 (1973) [Sov. Phys. JETP **38**, 162 (1974)].
- <sup>8</sup>S. A. Bulgadaev, B. I. Kaplan, and Y. Levinson, Zh. Éksp. Teor. Fiz. **70**, 1550 (1976) [Sov. Phys. JETP **43**, 808 (1976)].
- <sup>9</sup>Y. Levinson, Phys. Rev. B **51**, 16 898 (1995).
- <sup>10</sup>M. Potemski *et al.*, Phys. Rev. Lett. **66**, 2239 (1991);.
- <sup>11</sup>S. M. Badalian, U. Rössler, and M. Potemski, J. Phys., Condens. Matter **5**, 6719 (1993).
- <sup>12</sup>L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
- <sup>13</sup>Y. Levinson, Zh. Éksp. Teor. Fiz. **57**, 660 (1969) [Sov. Phys. JETP **30**, 362 (1970)].
- <sup>14</sup>A. V. Kuznetsov, Phys. Rev. B **44**, 13381 (1991).
- <sup>15</sup>K. E. Sayed, L. Bányai, and H. Haug, Phys. Rev. B **50**, 1541 (1994).
- <sup>16</sup>L. Bányai *et al.*, Phys. Rev. Lett. **75**, 2188 (1995).
- <sup>17</sup>S. Bar-Ad *et al.*, Phys. Rev. Lett. **77**, 3177 (1996).
- <sup>18</sup>S. Tarucha, T. Honda, and T. Saku, Solid State Commun. **94**, 413 (1995).
- <sup>19</sup>T. Honda, S. Tarucha, T. Saku, and Y. Tokura, Jpn. J. Appl. Phys. **34**, L72 (1995).
- <sup>20</sup>A. Yacoby *et al.*, Phys. Rev. Lett. **77**, 4612 (1996).
- <sup>21</sup>A. Yacoby *et al.*, Solid State Commun. **101**, 77 (1997).
- <sup>22</sup>J. M. Calleja *et al.*, Solid State Commun. **79**, 911 (1991).
- <sup>23</sup>E. Kapon, D. M. Hwang, and R. Bhat, Phys. Rev. Lett. **63**, 430 (1989).
- <sup>24</sup>M. Walther *et al.*, Appl. Phys. Lett. **60**, 521 (1992).
- <sup>25</sup>J. F. Ryan *et al.*, Phys. Rev. B **53**, R4225 (1996).
- <sup>26</sup>B. Y.-K. Hu and S. D. Sarma, Phys. Rev. B **48**, 5469 (1993).
- <sup>27</sup>A. Brataas, A. G. Mal'shukov, V. Gudmundsson, and K.A. Chao, J. Phys. Condens. Matter **8**, L325 (1996).
- <sup>28</sup>F. Rossi and E. Molinari, Phys. Rev. Lett. **76**, 3642 (1996).
- <sup>29</sup>T. Ogawa and T. Takagahara, Phys. Rev. B **43**, 14325 (1991).
- <sup>30</sup>T. Ogawa and T. Takagahara, Phys. Rev. B **44**, 8138 (1991).
- <sup>31</sup>S. Benner and H. Haug, Phys. Rev. B **47**, 15750 (1993).
- <sup>32</sup>P. A. Knipp and T. L. Reinecke, Phys. Rev. B **48**, 5700 (1993).
- <sup>33</sup>J. R. Senna and S. D. Sarma, Phys. Rev. Lett. **70**, 2593 (1993).
- <sup>34</sup>J. R. Senna and S. D. Sarma, Phys. Rev. B **48**, 4552 (1993).
- <sup>35</sup>L. Rota, F. Rossi, P. Lugli, and E. Molinari, Phys. Rev. B **52**, 5183 (1995).
- <sup>36</sup>A. Sa'ar *et al.*, Phys. Rev. B **54**, 2675 (1996).
- <sup>37</sup>M. Suhrke, S. Wilke, and R. Keiper, J. Phys. Condens. Matter **2**, 6743 (1990).
- <sup>38</sup>V. F. Gantmakher and Y. Levinson, *Carrier Scattering in Metals and Semiconductors*, Modern Problems in Condensed Matter Sciences Vol. 19 (North-Holland, Amsterdam, 1987).
- <sup>39</sup>G. D. Mahan, *Many Particle Physics* (Plenum, New York, 1990).