Dephasing-induced vacuum-field Rabi splittings in microcavities with quantum wells

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We study the effects of dephasing on the vacuum-field Rabi splitting in semiconductor microcavities. We predict dephasing-induced resonances which can be probed by studying the absorption in modulated external fields. [S0163-1829(97)08831-0]

I. INTRODUCTION

In the recent past there has been considerable activity in the observation of the signatures of strong-interaction cavity physics[1–4] in solid-state systems.[5–7] Prototypes of vacuum-field Rabi splitting have been studied and experimentally observed in the context of excitons in quantum-well microcavities. Such studies have focused on cavities of various sizes ($\lambda/2$, λ, etc.) as well as systems with more than one quantum well. Efforts have been made to capture the physical implications both in frequency and in time domain. The observed phenomena can be well understood on the basis of a simple model where the excitons in quantum wells are modeled by simple harmonic oscillators. Thus, the semiconductor microcavities provide us with newer and technologically more promising systems to study the effects of cavity quantum electrodynamics (QED). However, there are effects in semiconductors which cannot be neglected and which can lead to qualitatively new observations. For example, dephasing processes have been known to be very prominent in systems involving excitonic resonances.[8] A natural question is what are the effects of dephasing in the context of semiconductor cavities. The simplest effect of dephasing is to increase the effective linewidth parameter in the expression for susceptibility leading to a broadening of the two components of the doublet. However, Bloembergen, Lotem, and Lynch[9] in a classic paper had predicted that the dephasing can lead to new resonances in the nonlinear response of a quantum system. Since then dephasing-induced or collision-induced resonances have been extensively studied. In this paper, we examine whether the dephasing can lead to more resonances[10] in the context of cavity QED in semiconductor quantum-well structures. In particular, we predict dephasing-induced resonances. We also show how these can be probed. We demonstrate that modulation spectroscopy is especially suited for this purpose.

The organization of the paper is as follows. In Sec. II we present the model and the numerical results. In Sec. III we derive the results used in numerical calculations and in Sec. IV we conclude the paper.

II. THE MODEL AND THE NUMERICAL RESULTS

Consider a system of excitons in a quantum well contained in a high-quality factor cavity. Let $b,b^\dagger$ $(a,a^\dagger)$ be the annihilation and creation operators for the excitons of frequency ω_0 (cavity mode of frequency ω_c) The cavity is driven by a coherent field of frequency ω_l. The Hamiltonian for the exciton-cavity system in a frame rotating with the frequency of the external field can be written as

$$H/\hbar=\delta a^\dagger a+\delta_0 b^\dagger b+g(a^\dagger b+ab^\dagger)-E(a+a^\dagger),$$
$$\delta=\omega_c-\omega_l,\quad \delta_0=\omega_0-\omega_l, \tag{1}$$

where E is the envelope function of the field driving the cavity mode and g is the exciton-photon coupling constant. Let the exciton (cavity mode) lifetime be $(2\gamma)^{-1}$ $[(2\kappa)^{-1}]$. The lifetime effects can be incorporated at the density-matrix level. Note that mean exciton amplitude will be given by

$$\langle\dot{b}\rangle=-i\delta_0\langle b\rangle-ig\langle a\rangle-\gamma\langle b\rangle, \tag{2}$$

and hence in the steady state

$$\langle b\rangle=(\gamma+i\delta_0)^{-1}(-ig)\langle a\rangle. \tag{3}$$

This result is equivalent to the standard harmonic-oscillator model of the susceptibility

$$\chi(\omega)=\frac{f/2\omega}{\omega_0-\omega-i\gamma}, \tag{4}$$

where f is the oscillator strength. At the level of Eq. (4) the effect of dephasing (specified by the relaxation parameter Γ) is easily accounted for by changing γ to $\gamma+\Gamma$. Dephasing can lead to additional complications since mean exciton amplitude decays at the enhanced rate $\gamma+\Gamma$; however, the decay rate of mean number of excitons $\langle b^\dagger b\rangle$ is not affected by dephasing effects. The effects of dephasing can be properly accounted for in the density-matrix framework. The dynamical equation for the density matrix ρ of the combined exciton-photon system is given by

$$\dot{\rho}=-i[H/\hbar,\rho]-\kappa(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a)$$
$$-\gamma(b^\dagger b\rho-2b\rho b^\dagger+\rho b^\dagger b)-\Gamma[b^\dagger b,[b^\dagger b,\rho]]. \tag{5}$$

Note the structure of the dephasing term in Eq. (5)—the Γ term will not appear in the equation for $\langle b^\dagger b\rangle$.

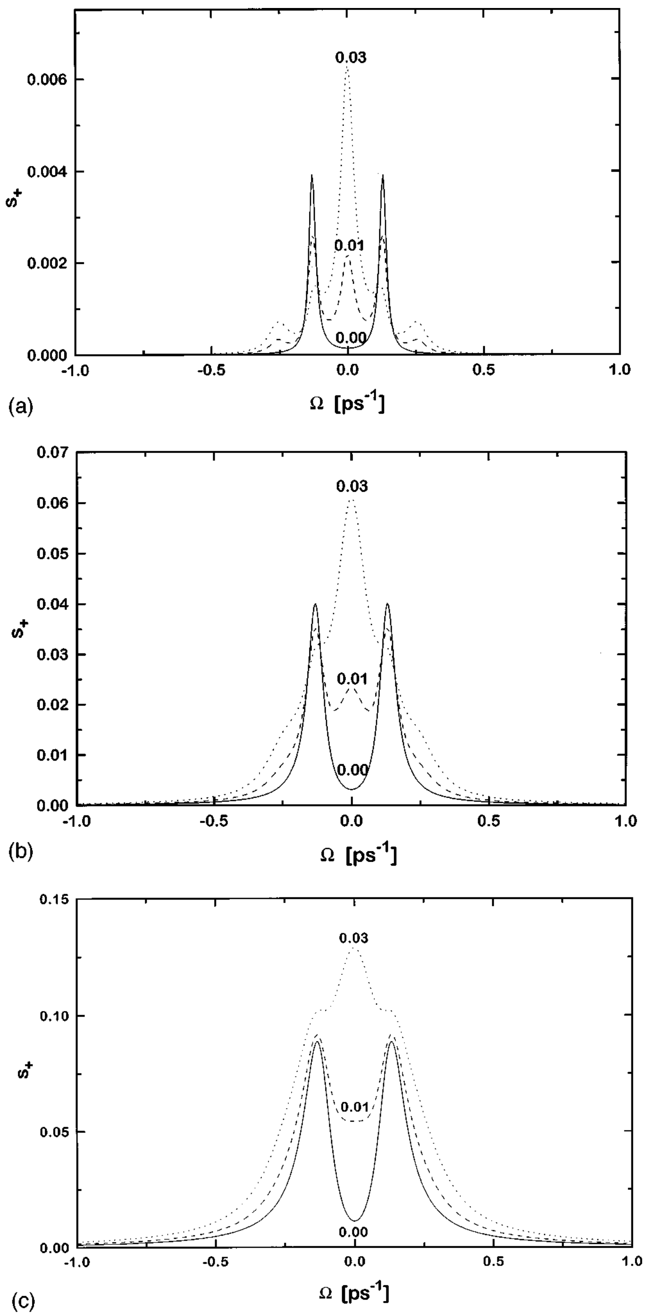

FIG. 1. Normalized cosine component of the signal s_+ as a function of the modulation frequency for (a) $\kappa=0.01\ \mathrm{ps}^{-1}$, (b) $\kappa=0.05\ \mathrm{ps}^{-1}$, and (c) $\kappa=0.10\ \mathrm{ps}^{-1}$. The various curves are labeled by the corresponding values of Γ. Other parameters are as follows: $g=0.13\ \mathrm{ps}^{-1}$, $\gamma=0.02\ \mathrm{ps}^{-1}$, and $\delta=\delta_0=0.0$.

The following question now arises: what is the signature of Γ in a strongly coupled exciton-photon system? One simple consequence of changing γ to $\gamma+\Gamma$ in Eq. (4) is to broaden the vacuum-field Rabi split resonances. However, much more remarkable effects arise in modulation spectroscopy. Consider for simplicity the case when $\omega_0=\omega_c=\omega_l$. We assume that the external field is weakly modulated at Ω, i.e., we replace E by

$$E\rightarrow E(1+2m\cos\Omega t), \tag{6}$$

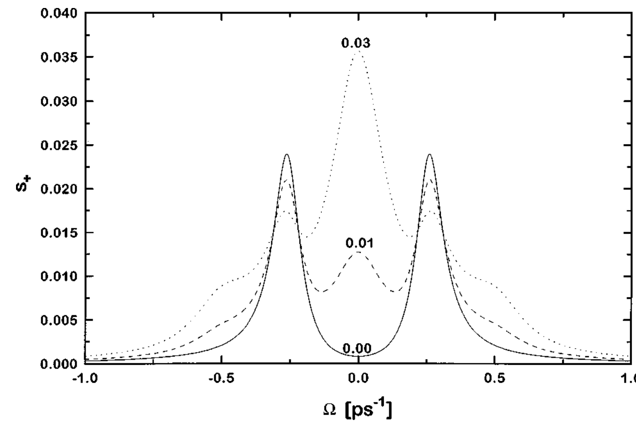

FIG. 2. Normalized cosine component of the signal S_+ as a function of the modulation frequency for $g=0.26\ \mathrm{ps}^{-1}$, $\kappa=0.10\ \mathrm{ps}^{-1}$, $\gamma=0.02\ \mathrm{ps}^{-1}$, and $\delta=\delta_0=0.0$. The curves are labeled by the values of Γ.

in Eq. (1). For weak modulation the mean cavity mode excitation can be written as

$$\langle a^{\dagger}a\rangle=S_0+mS_+\cos\Omega t+mS_-\sin\Omega t, \tag{7}$$

where S_0 is the excitation in the absence of modulation. The $S_\pm$ give the two quadratures in modulation spectroscopy. Let $\widetilde{S}_+$ be the value of S_+ in the absence of the quantum well for $\Omega=0$. We thus study the in-phase quadrature normalized to $\widetilde{S}_+$ (i.e., $s_+=S_+/\widetilde{S}_+$) as a function of Ω for various parameters of the quantum well and the cavity. We describe the evaluation of s_+ in the next section.

We now present the numerical results. For all our calculations we have chosen the external drive frequency to be resonant with the cavity/exciton frequency, i.e., we assume $\delta=\delta_0=0$. The parameter values for calculation were taken from Jacobson *et al.*[7] and they were as follows: $\gamma=0.02\ \mathrm{ps}^{-1}$, $\kappa=0.1\ \mathrm{ps}^{-1}$, $g=0.13\ \mathrm{ps}^{-1}$. We have also produced results with other values of κ and g. In order to observe the effect of dephasing in each of these sets we used three different values of Γ, namely, $\Gamma=0.0$, 0.01, and 0.03 ps^{-1}. The results for s_+ for $g=0.13\ \mathrm{ps}^{-1}$ and $\kappa=0.01$, 0.05, and 0.1 ps^{-1} are shown in Figs. 1(a), 1(b), and 1(c),

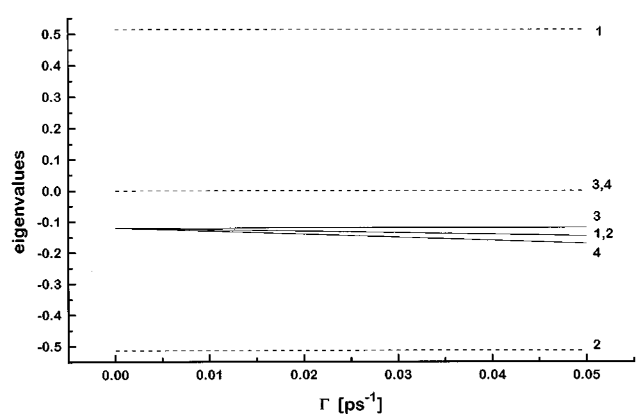

FIG. 3. Real (solid curves) and imaginary (dashed curves) parts of the eigenvalues of A as functions of the dephasing parameter Γ. Other parameters are as in Fig. 2. The various eigenvalues are correspondingly numbered.

respectively. It is clear from Fig. 1(a) that for $\Gamma=0.0$ one observes the standard vacuum-field Rabi splitting with the cosine component peaks around $\pm g$. For nonzero Γ there is a qualitative change: the peaks at $\pm g$ broaden and additional peaks appear at $\Omega=0$ and close to $\Omega\approx\pm 2g$ [see dashed and dotted curves in Fig. 1(a)]. The mathematical origin of these peaks can be traced to the eigenvalues of the matrix A defined by Eq. (9). A comparison of Figs. 1(a), 1(b), and 1(c) reveals that with an increase in the value of κ the resolution of the additional peaks can become a problem. However, due to the robust character of the peak at $\Omega=0$, it may still be resolvable even with decay rate $\kappa=0.1\ \mathrm{ps}^{-1}$ [see the dotted curve in Fig. 1(c)]. The curves for S_- (not shown) exhibit dispersionlike structures at $\Omega=0$, $\pm g$, $\pm 2g$. It may be noted that oscillations near $\pm 2g$ in S_- can be quite helpful in recognizing the additional resonances. We next present the results for $\kappa=0.1\ \mathrm{ps}^{-1}$ and $g=0.26\ \mathrm{ps}^{-1}$ in Fig. 2. It is clear from Figs. 2 and 1(a) that a twofold increase in g leads to much better resolution. It is thus clear from the above results that though the cavity of Yamamoto and co-workers does not adequately meet the requirements for observing and resolving the additional dephasing-induced resonances, a modification like reduction of decay rate or an increase in the coupling coefficient (by means of, say, reduction of the cavity volume) can lead to a direct observation of such resonances.

III. THE MATHEMATICAL FORMULATION OF MODULATION SPECTROSCOPY

The basic result given by Eq. (7) is to be obtained from the solution of Eq. (5). For this purpose we derive equations for the mean values $\langle a\rangle$, $\langle b\rangle$, $\langle a^\dagger a\rangle$, $\langle b^\dagger b\rangle$, etc. These mean value equations can be written in the form

$$\dot{\Phi}=D\Phi+iE(t)Z, \tag{8}$$

$$\dot{\Psi}=A\Psi+iE(t)B\Phi, \tag{9}$$

where

$$\Phi=(\langle a\rangle,\langle b\rangle,\langle a^\dagger\rangle,\langle b^\dagger\rangle)^t, \tag{10}$$

$$\Psi=(\langle a^\dagger a\rangle,\langle b^\dagger b\rangle,\langle a^\dagger b\rangle,\langle ab^\dagger\rangle)^t, \tag{11}$$

$$Z=(-1,0,1,0)^t. \tag{12}$$

D, A, and B in Eqs. (8) and (9) are 4×4 matrices given by the following expressions:

$$D=\begin{pmatrix}\overline{D} & 0\\ 0 & \overline{D^*}\end{pmatrix},\quad \overline{D}=\begin{pmatrix}-\kappa-i\delta & -ig\\ -ig & -(\gamma+\Gamma)-i\delta_0\end{pmatrix}, \tag{13}$$

$$A=\begin{pmatrix}-2\kappa & 0 & -ig & ig\\ 0 & -2\gamma & ig & -ig\\ -ig & ig & -(\kappa+\Gamma+\gamma)-i(\delta_0-\delta) & 0\\ ig & -ig & 0 & -(\kappa+\Gamma+\gamma)+i(\delta_0-\delta)\end{pmatrix}, \tag{14}$$

$$B=\begin{pmatrix}1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}. \tag{15}$$

Recall that the external drive $E(t)$ is modulated at frequency Ω with modulation depth m, as per Eq. (6). Treating m to be much smaller than unity one can write down the expressions for the solutions for Φ and Ψ, retaining terms only up to first order in m,

$$\Phi=\Phi_0+m\Phi_+e^{-i\Omega t}+m\Phi_-e^{i\Omega t}, \tag{16}$$

$$\Psi=\Psi_0+m\Psi_+e^{-i\Omega t}+m\Psi_-e^{i\Omega t}. \tag{17}$$

It is then straightforward to write down the solutions for various orders of Φ and Ψ,

$$\Phi_0=-iE_0\tilde{\Phi}_0,\quad \tilde{\Phi}_0=(D)^{-1}Z, \tag{18}$$

$$\Phi_\pm=-iE_0\tilde{\Phi}_\pm,\quad \tilde{\Phi}_\pm=(D\pm i\Omega I)^{-1}Z, \tag{19}$$

$$\Psi_0=-E_0^2\tilde{\Psi}_0,\quad \tilde{\Psi}_0=A^{-1}B\tilde{\Phi}_0, \tag{20}$$

$$\Psi_\pm=-E_0^2\tilde{\Psi}_\pm,\quad \tilde{\Psi}_\pm=(A+i\Omega I)^{-1}B(\tilde{\Phi}_0+\tilde{\Phi}_\pm). \tag{21}$$

In Eqs. (19) and (21) I is the unit matrix. In what follows, we monitor the mean photon number $\langle a^\dagger a\rangle$ as function of the modulation frequency Ω. In terms of the components of $\Psi_0,\Psi_\pm$, $\langle a^\dagger a\rangle$ can be written as follows:

$$\begin{aligned}\langle a^\dagger a\rangle&=\Psi_{01}+m(\Psi_{+1}+\Psi_{-1})\cos\Omega t\\ &\quad -im(\Psi_{+1}-\Psi_{-1})\sin\Omega t\\ &=S_0+S_+\cos\Omega t+S_-\sin\Omega t,\end{aligned} \tag{22}$$

where the numerical subscript ''1'' represent the first component of the corresponding column matrices. Thus modulated fluorescence can be obtained from the solution of Eqs. (8) and (9). This is what has been done to obtain the numerical results reported in Sec. II. The locations and widths of various resonances are determined by the eigenvalues of the matrix A. We show in Fig. 3 the distribution of the eigenvalues of A. Note that the imaginary (real) part of the eigenvalues determine the location (width) of the additional resonances. Thus, it is clear from Fig. 3 that two of the eigenvalues are purely real and they determine the width of the central lobe, whereas the other two are complex conjugates characterizing the side lobes close to $\pm 2g$. Note also that an increasing Γ leads to an overall broadening of all the lobes, while their locations are insensitive to any change in Γ.

IV. CONCLUSIONS

We have examined the effects of dephasing on vacuum-field Rabi splittings in semiconductor microcavities. We show that besides broadening of vacuum-field Rabi splitting, one can also get dephasing-induced resonances. We further demonstrate that modulation spectroscopy could be one of the useful techniques to study these dephasing-induced resonances. We have presented numerical and analytical results to support these conclusions.

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