Influence of microwave fields on the electron tunneling through a quantum dot

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We consider a quantum dot coupled to two leads in the presence of external microwave (MW) fields which are applied to the dot and two leads, respectively. The amplitudes, phases, and frequencies of the MW fields are taken arbitrarily. Under the adiabatic approximation, the formula for the time-dependent current $j(t)$ and the average current $\langle j(t)\rangle$ are obtained. Some analytical results for several special cases are presented and studied numerically. When the MW field is applied only to one lead, the detailed structure of the main resonant peak in the average current vs gate voltage is in good agreement with the experiment of Kouwenhoven *et al.* [Phys. Rev. Lett. **73**, 3443 (1994)]. For the case with different frequencies of MW fields in different regions, some interesting effects occur, including multiple-photon-assisted tunneling, the combination of sidebands and the electron-photon pump effect, and the coherent effect. [S0163-1829(97)09028-0]

The experimental investigation of time-dependent effects of tunneling transport in nanostructures was started by Sollner *et al.*[1] Recently, time-dependent transport phenomena in mesoscopic systems have attracted more and more attention. The essential feature of mesoscopic physics is the phase coherence of the charge carriers. As pointed out by Jauho, Wingreen, and Meir,[2] for time-dependent processes external time-dependent perturbation affects the phase coherence different in different parts of the system. An energy scale $\hbar w$ in the time-dependent problem is introduced. Quite a few interesting effects have been observed, or are expected to occur, such as the ac response and transients in resonant-tunneling devices, single-electron pumps and turnstiles, interactions between ultrashort laser pulses and charge carriers, and so forth.

In the theoretical aspect, Tien and Gordon studied the effect of microwave (MW) radiation on superconducting tunneling devices as far back as the early 1960s.[3] Since then, different theoretical approaches have been developed, such as the time-dependent Schrödinger equation,[4–6] the transfer Hamiltonian,[7,8] the master equation,[9,10] the Wigner function,[11] and the nonequilibrium Green's-function method.[2,12–14] In contrast to the stationary transport theory, the theory of time-dependent processes in mesoscopic systems is still in its premature stage.

The motivation for this work was the recent experiments of Kouwenhoven *et al.*[15] in the study of photon-assisted tunneling through a quantum dot. They observed, in current-gate voltage curves, that a shoulder emerges on the left of the Coulomb peak and a negative current emerges on the right when the system is applied a MW field on one lead only. Kouwenhoven *et al.* explained this by using numerical simulations.[15,16] The system which we consider is a quantum dot coupled to two leads which are connected to the reservoirs. We assume that external MW fields are coupled to the three regions of the system (the left lead, the right lead, and the quantum dot), respectively. The Hamiltonian of the system can be written as

$$H(t)=\sum_{k\in L}\varepsilon_k(t)a_k^\dagger a_k+\sum_{p\in R}\varepsilon_p(t)b_p^\dagger b_p+\varepsilon_0(t)c^\dagger c+\left(\sum_k L_k a_k^\dagger c+\sum_p R_p b_p^\dagger c+\text{H.c.}\right), \quad (1)$$

where $a_k^\dagger$ (a_k), $b_p^\dagger$ (b_p), and $c^\dagger$ (c) are creation (annihilation) operators in the left lead, right lead, and dot, respectively. For simplicity, we only consider a single state in the quantum dot, and neglect intradot electron-electron Coulomb interaction. The fourth term denotes the tunneling part which is time dependent. Under the adiabatic approximation, the time-dependent MW field can be reflected in the single-electron energies $\varepsilon_\alpha(t)$ (here $\alpha=0$, k, and p represents the dot, the left lead, and right lead, respectively). We seperate $\varepsilon_{k/p}(t)$ into two parts: $\varepsilon_{k/p}(t)=\varepsilon_{k/p}+\Delta_{L/R}(t)$, where L and R correspond the left and right leads, respectively, and $\varepsilon_{k/p}$ is the time-independent single electron energies without the MW fields. $\Delta_{L/R}(t)$ is a time-dependent part from the external microwave fields, and we assumed it can be written as $\Delta_{L/R}(t)=\Delta_{L/R}\cos(\omega_{L/R}t+\phi_{L/R})$. Similarly for the dot energy $\varepsilon_0(t),\varepsilon_0(t)=\varepsilon_0+\Delta_0(t)$ and $\Delta_0(t)=\Delta_0\cos\omega_0 t$. The amplitudes, phases, and frequencies of the external MW fields in different region can be taken arbitrarily.

In the wide-band limit, i.e., the linewidths $\Gamma^\alpha(\varepsilon)\equiv 2\pi\rho_\alpha(\varepsilon)\alpha(\varepsilon)\alpha^*(\varepsilon)$ [$\rho_\alpha(\alpha)$ is the density of states of the lead, and $\alpha=L$ and R] are energy-independent constants, and we can use the general time-dependent current formula derived by Wingreen, Jauho, and Meir[14,2] to the system under consideration, and obtain the time-dependent current $j_L(t)$ and the average current $\langle j_L(t)\rangle$ as [the current $j_L(t)$ is from the left lead into the dot]

$$j_L(t)=-\frac{e}{\hbar}\int\frac{d\varepsilon}{2\pi}\Gamma^L\left\{\sum_{\alpha=L,R}f_\alpha(\varepsilon)\Gamma^\alpha|A_\alpha(\varepsilon,t)|^2+f_L(\varepsilon)\text{Im}A_L(\varepsilon,t)\right\}, \quad (2)$$

$$\langle j(t)\rangle = -\frac{2e}{\hbar}\frac{\Gamma^L\Gamma^R}{\Gamma^L+\Gamma^R}\int\frac{d\varepsilon}{2\pi}\{f_L(\varepsilon)\mathrm{Im}\langle A_L(\varepsilon,t)\rangle - f_R(\varepsilon)\mathrm{Im}\langle A_R(\varepsilon,t)\rangle\}, \quad (3)$$

in which $f_{L/R}(\varepsilon)$ is the Fermi distribution function of electrons in left and/or right lead, and $A_\alpha(\varepsilon,t)$ is defined as

$$A_\alpha(\varepsilon,t) = \int dt_1 G^r_{00}(tt_1)\exp\left(i\varepsilon(t-t_1) - i\int_t^{t_1} dt_2\Delta_\alpha(t_2)\right), \quad (4)$$

where $\alpha = L,\ R$ and $G^r_{00}(tt_1) \equiv -i\theta(t-t_1)\langle\{c(t),c^\dagger(t_1)\}\rangle$,

Using $\Delta_0(t)=\Delta_0\cos\omega_0 t$, and $\Delta_\alpha(t)=\Delta_\alpha\cos(\omega_\alpha t+\phi_\alpha)$, $(\alpha = L,R)$, we obtain

$$A_\alpha(\varepsilon t) = \exp\left\{-i\frac{\Delta_0}{\omega_0}\sin\omega_0 t + i\frac{\Delta_\alpha}{\omega_\alpha}\sin(\omega_\alpha t+\phi_\alpha)\right\} \times\sum_{k,n} J_k\left(\frac{\Delta_0}{\omega_0}\right)J_n\left(\frac{\Delta_\alpha}{\omega_\alpha}\right)e^{-in\phi_\alpha} \times\frac{e^{t(ik\omega_0 - in\omega_\alpha)}}{\varepsilon-\varepsilon_0+i\dfrac{\Gamma}{2}+n\omega_\alpha-k\omega_0}. \quad (5)$$

Substituting Eq. (5) into Eqs. (2) and (3), we can evaluate the time-dependent current $j_L(t)$ and the average current when the system (the quantum dot, the left lead, and the right lead) is applied the external MW fields with arbitrary amplitudes, phases, and frequencies.

In the following we study the dependence of the average current $\langle j(t)\rangle$ on the parameters of the external MW fields. Several interesting phenomena emerge while the parameters change. Some of them have been observed in the experiment, e.g., the electron-photon pump.[15] We shall compare our calculated result with the experiment. Here we only consider the zero-temperature situation. Since the temperature in the experiment is usually lower 100 mK,[15] $k_BT/\hbar\omega \approx 0.1$ (quantum regime). Thus this consideration is quite reasonable. In fact, in the quantum regime, the finite temperature does not qualitatively change $j(t)$ and $\langle j\rangle$, but only makes current peak broader and lower.

(1) *The case with same phase and frequence of MW fields*: First let us consider the case in which the external MW field in each region (the quantum dot, the left leads and the right lead) has the same phase, $\phi_L=\phi_R=0$, and the same frequency, $\omega_0=\omega_R=\omega_L\equiv\omega$. In this case the $\langle j_L(t)\rangle$ reduces to

$$\langle j(t)\rangle = \frac{e}{\hbar}\Gamma^L\Gamma^R\int\frac{d\varepsilon}{2\pi}\sum_k\frac{1}{(\varepsilon-\varepsilon_0-kw)^2+\Gamma^2/4} \times\left\{f_L(\varepsilon)J_k^2\left(\frac{\Delta_0-\Delta_L}{\omega}\right) - f_R(\varepsilon)J_k^2\left(\frac{\Delta_0-\Delta_R}{\omega}\right)\right\}, \quad (6)$$

where $\Gamma\equiv\gamma^L+\Gamma^R$. In the following, we shall discuss two special applications of the average current formula, Eq. (6).

(a) *$\Delta_0=\Delta_L=0$, $\Delta_R\neq 0$ (asymmetrical case)*. Here the external MW field is applied only in the right lead. Now Eq. (6) can be separated into two terms ($T=0$ K):

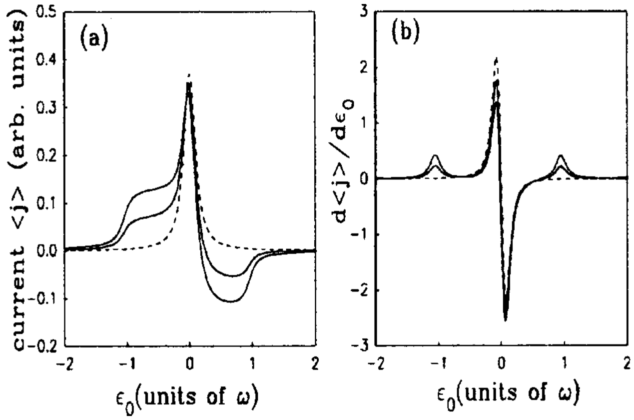

FIG. 1. (a) The average current $\langle j\rangle$ vs ε_0 for a MW field applied only on the right lead. Two solid curves correspond to $\Delta_R/\omega = 0.5$ and 0.7. The dotted curve is the main resonant peak without MW field. The parameters are $\Gamma=0.2\omega$ and $\mu_l=-\mu_r=0.05\omega$. (b) The derivatives $\partial\langle j\rangle/\partial\varepsilon_0$ of the curves shown in (a).

$$\langle j\rangle = \frac{e}{\hbar}\Gamma^L\Gamma^R\int_{\mu_r}^{\mu_l}\frac{d\varepsilon}{2\pi}\frac{1}{(\varepsilon-\varepsilon_0)^2+\Gamma^2/4} - \frac{e}{\hbar}\Gamma^L\Gamma^R \times\sum_k\int_0^{\mu_r-\varepsilon_0}\frac{d\varepsilon}{2\pi}\frac{1}{(\varepsilon-kw)^2+\Gamma^2/4}\left\{J_k^2\left(\frac{\Delta_R}{\omega}\right)-\delta_{k0}\right\}. \quad (7)$$

In Eq. (7) we already changed the lower limit of the integral in the second term from $-\infty$ to 0. In fact, by using the residual theorem one can easily prove that the integral from $-\infty$ to 0 is zero. The first term in Eq. (7) is the average current without an external MW field. This term comes from the dc source-drain voltage $v_{sd}=\mu_l-\mu_r$. Only the second term in Eq. (7) represents the effect from the MW field, and depends on the amplitude Δ_R and the frequency ω. One can clearly see that the external MW field causes a change of the current which is independent of the voltage v_{sd}.

Figure 1(a) shows a numerical calculation of the average current $\langle j\rangle$ vs ε_0. We have chosen $e=\hbar=1$ units, $\Gamma^L=\Gamma^R=\Gamma/2$ (symmetrical barriers), and all the energies are measured in ω. The value of v_{sd} is taken as $v_{sd}=\mu_l-\mu_r=0.1\omega$. Figure 1(b) is the derivative of the average current shown in Fig. 1(a) with respect to ε_0. In comparison to the experiment by Kouwenhoven *et al.*,[15] one can see that the following aspects are in good agreement: (1) There is a shoulder on the left side of the main resonant peak, and a negative current on the right side in Fig. 1(a), corresponding to the electron-photon pump. Two photon-assisted-tunneling (PAT) peaks appear in Fig. 1(b). (2) With increasing Δ_R, the shoulder becomes higher, and the magnitude of the negative current larger. But the location of the peaks of PAT is independent of the microwave field amplitude Δ_R. (3) The distance between the peak of PAT [in Fig. 1(b)] and the main resonant peak is almost unchanged, and equals ω, i.e., a photon's energy. Here we should point out, since we neglected the intradot Coulomb interaction in our model, that we cannot obtain a series of Coulomb oscillation peaks. Instead, we can only obtain a single peak, but with a detailed structure.

(b) *$\Delta_0=0$, $\Delta_R=\Delta_L\equiv\Delta\neq 0$ (symmetric case).* In this case two leads couple the external MW fields with the same amplitude and frequency, but no MW field is applied on the dot. From Eq. (6), one has

$$\langle j\rangle=\frac{e}{\hbar}\,\Gamma^L\Gamma^R\sum_k\; j_k^2\left(\frac{\Delta}{\omega}\right)\int_{\mu_r-\varepsilon_0}^{\mu_l-\varepsilon_0}\frac{d\varepsilon}{2\pi}\,\frac{1}{(\varepsilon-kw)^2+\Gamma^2/4}. \tag{8}$$

If $\Gamma<\omega$, the main resonant peak is split into a set of peaks (which is not shown here). The location of the peaks is at $[(\mu_l-\mu_r)/2]+k\omega$. The height of the kth peak is determined by the Bessel function $J_k^2(\Delta/\omega)$. It is easy to prove that the summation of the heights of all peaks is equal to the height of the original main resonant peak. This phenomenon can be explained by whether the tunneling electron can absorb or emit one or several photons. If $\Gamma>\omega$, these peaks are broadened and overlap with each other (which is not shown here), and the external MW fields do not emerge from quantum action.

(2) *The case with different phases of MW fields:* Now we consider the case in which the external MW field in each region has the same frequency ($\omega_0=\omega_L=\omega_R\equiv\omega$), but the phase and the amplitude may be different. In general, MW fields have five independent parameters: Δ_0, Δ_L, Δ_R, ϕ_L, and ϕ_R. It is easy to prove that the situation under consideration is equivalent to the case with only two independent parameters. $\Delta_L'=\sqrt{\Delta_L^2+\Delta_0^2-2\Delta_0\Delta_L\cos\phi_L}$ and $\Delta_R'=\sqrt{\Delta_R^2+\Delta_0^2-2\Delta_0\Delta_R\cos\phi_R}$, with the other three parameters fixed: $\Delta_0'=0$ and $\phi_L'=\phi_R'=0$.

(3) *The case with different frequencies of MW fields:* For simplicity, we assume $\phi_L=\phi_R=0$. By taking $\phi_L=\phi_R=0$ in Eqs. (3) and (5), the average current $\langle j\rangle$ can be obtained straightforwardly. In this case, some interesting effects emerge. In the following we present three examples.

(a) *Multiple-photon-assisted tunneling:* Considering the case with $\omega_L=\omega_0\equiv\omega$ and $\Delta_L=\Delta_0\equiv\Delta$, where ω/ω_R is not a simple integer ratio, we set the source-drain voltage $v_{\rm sd}=\mu_l-\mu_r=0$. In this case the tunneling through the left barrier will be independent of the MW fields, and only the tunneling through the right barrier is related to MW fields. The average current $\langle j\rangle$ becomes

$$\langle j\rangle=\frac{e}{\hbar}\,\Gamma^L\Gamma^R\int_0^{-\varepsilon_0}\frac{d\varepsilon}{2\pi}\left\{\frac{1}{\varepsilon^2+\Gamma^2/4}-\sum_{k,n}\frac{J_k^2\left(\frac{\Delta}{\omega}\right)J_n^2\left(\frac{\Delta_R}{\omega sR}\right)}{(\varepsilon+n\omega_R-k\omega)^2+\Gamma^2/4}\right\}. \tag{9}$$

The photon-assisted tunneling is displayed in steps in the $\langle j\rangle$ vs ε_0 curves, corresponding to peaks in the derivative of the current, $\partial\langle j\rangle/\partial\varepsilon_0$, as shown in Fig. 2. The location of the peaks is not only at $k\omega_R$ and $k\omega$ but at $k\omega_R\pm n\omega$. This means that the electron in the tunneling can emit or absorb photons with different frequencies in a more complicated way. For example, the electron may absorb a ω_R photon and a ω photon, or absorb a ω_R photon and emit a ω photon, and so forth. Here the situation is more complicated than the conventional multiple-photon-assisted tunneling. Notice that if the ω/ω_R is the simple integer ratio, then the coherent effect will play an important role [see (c) below].

(b) *The combination of sidebands and electron-photon pump effects:* Now let us consider $\Delta_L=0$, where ω_0/ω_R is not a simple integer ratio. Then $\langle j\rangle$ becomes

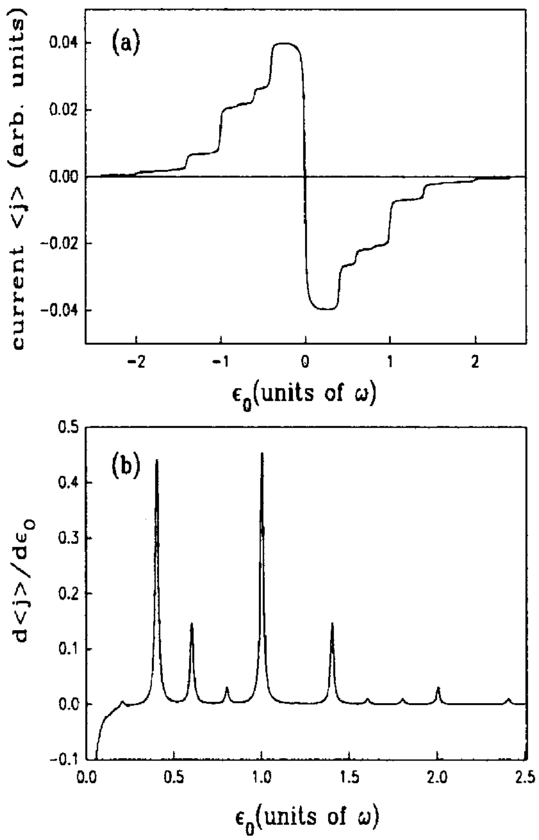

FIG. 2. (a) $\langle j\rangle$ vs ε_0 for MW fields with different frequencies, and $\Gamma=0.02\omega_R$, $\mu_l=\mu_r=0$, $\omega_L=\omega_0=0.41\omega_R$, and $\Delta_L/\omega_L=\Delta_0/\omega_0=\Delta_R/\omega_R=1$. (b) $\partial\langle j\rangle/\partial\varepsilon_0$ of the curves shown in (a).

$$\langle j\rangle=\frac{e}{\hbar}\,\Gamma^L\Gamma^R\left\{\int_0^{\mu_1-\varepsilon_0}\frac{d\varepsilon}{2\pi}\sum_k\frac{J_k^2\left(\frac{\Delta_0}{\omega_0}\right)}{(\varepsilon-k\omega)^2+\Gamma^2/4}-\int_0^{\mu_r-\varepsilon_0}\frac{d\varepsilon}{2\pi}\sum_{k,n}\frac{J_k^2\left(\frac{\Delta_0}{\omega_0}\right)J_n^2\left(\frac{\Delta_R}{\omega_R}\right)}{(\varepsilon+n\omega_R-k\omega)^2+\Gamma^2/4}\right\}. \tag{10}$$

In Fig. 3 we present the curves of $\langle j\rangle$ vs ε_0 with a very small bias voltage, $v_{\rm sd}=\mu_l-\mu_r=0.05\omega_0$, showing a set of peaks at the location of $k\omega_0$. A shoulder emerges on the left of each current peak and a negative current on the right, provided $\Delta_R/\omega_R\neq0$. Moreover the distance between the shoulder and its main peak depends only on ω_R, and the height of the shoulder and the magnitude of the negative current depend only on Δ_R/ω_R. We explained this as a combination of two effects: the the sideband and the electron-photon pump effects. In fact the MW field applied on the quantum dot causes the sidebands, and the MW field applied on the right lead in on asymmetric way induces the electron-photon pump effect. The point here is that the two frequencies ω_0/ω_R must not be simple integer ratios; otherwise this effect will be suppressed by the coherent effect.

(c) *Coherent effect:* Finally we consider the case with $\omega_R=\omega_L$ and $\delta_R=\Delta_L$, but the ratio of $\omega_R:\omega_0$ is a simple integer ratio. Then $\langle j\rangle$ becomes

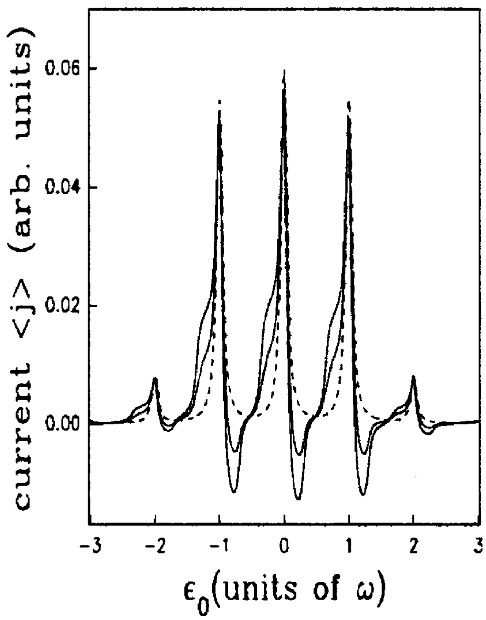

FIG. 3. $\langle j\rangle$ vs ε_0 in small bias voltage. $\Delta_R/\omega_R=0$ corresponds to the dotted curve. $\Delta_R/\omega_R=0.7$ and 0.5 correspond to two solid curves, respectively. $\Gamma=0.1\omega_0$, $\mu_l=-\mu_r=0.025\omega_0$, $\omega_R=0.33\omega_0$, and $\Delta_0/\omega_0=1.4$.

$$\langle j\rangle=\frac{e\Gamma^L\Gamma^R}{\hbar}\int_{\mu_r-\varepsilon_0}^{\mu_l-\varepsilon_0}\frac{d\varepsilon}{2\pi}\left[\sum_{k,k',n,n'}J_k\left(\frac{\Delta_0}{\omega_0}\right)J_n\left(\frac{\Delta_R}{\omega_R}\right)\right.$$
$$\left.\times J_{k'}\left(\frac{\Delta_0}{\omega_0}\right)J_{n'}\left(\frac{\Delta_R}{\omega_R}\right)\frac{\delta_{(k-k')\omega_0,(n-n')\omega_R}}{(\varepsilon+n\omega_R-k\omega_0)^2+\dfrac{\Gamma^2}{4}}\right]. \tag{11}$$

For concreteness, taking $\omega_0=2\omega_R$, then coherent terms appear in the formula of $\langle j\rangle$. These terms make some of the peaks higher, and others lower, but the summation of the heights of all peaks is constant. Figure 4(a) shows $\langle j\rangle$ vs ε_0 for $\omega_0=2\omega_R$, which has a strong interference. For comparison, $\langle j\rangle$ vs ε_0 with $\omega=2.01\omega_R$ is also presented in Fig. 4(b), without interference. Physically, each current peak comes from contributions of the electrons tunneling through different channels. For example, the current peak at $\varepsilon_0=0.5\omega_0$ is contributed by electrons from the left lead tunneling to the right lead in many different ways: either tunneling by absorbing an ω_R photon, or by absorbing an ω_0 photon and emitting an ω_R photon, etc. Since the electron tunneling through different channels may have different phases, the coherent effect happens.

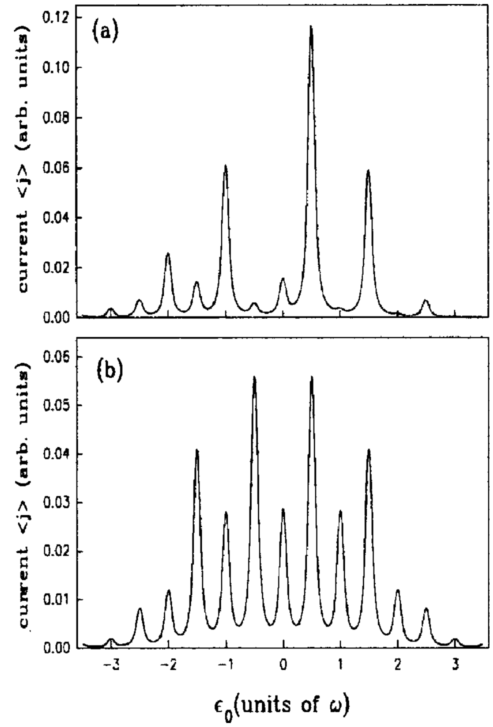

FIG. 4. (a) $\langle j\rangle$ vs ε_0 with $\omega_0=2\omega_R$, showing the strong coherent effect. (b) $\langle j\rangle$ vs ε_0 with $\omega_0=2.01\omega_R$ (no coherent effect is visible) for comparison. $\Gamma=0.1\omega_0$, $\mu_l=-\mu_r=0.05\omega_0$, and $\Delta_0/\omega_0=\Delta_R/\omega_R=\Delta_L/\omega_L=1.6$.

In Fig. 4(a), the current peak at $\varepsilon_0=0.5$ is enhanced twice as much as the current peak in Fig. 4(b), which is the simple summation of the contributions from different channels.

In summary, we studied electron tunneling through a quantum dot coupled to two leads in the presence of external MW fields. Various cases of MW fields have been considered. Formulas for the time-dependent current $j(t)$ and the average current $\langle j(t)\rangle$ are obtained. When the MW field is applied on only one lead (the asymmetric case), the detailed structure of the main resonant peak of the average current vs the gate voltage is in good agreement with the experiment of Kouwenhoven *et al.* For the case with different frequencies of MW fields, some interesting effects are theoretically expected to occur, including multiple-photon-assisted tunneling in a more complicated way, a combination of sidebands and electron-pump effects, and the coherent effect. These have to be verified by the future experiments.

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