Weak disorder in the two-dimensional *XY* dipole ferromagnet

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The effects of random field and random anisotropy axis disorder on the two-dimensional *XY* dipole ferromagnet are studied. It is shown that the disorder leads to an instability of the ferromagnetic phase. The correlation function of the magnetization is calculated at low temperatures using the self-consistent harmonic approximation. In the random-field case, the correlation function obeys a power law as a function of the distance. For random axis disorder, a logarithmically slow decrease of the correlation function is found. [S0163-1829(97)02030-4]

I. INTRODUCTION

Recently a renewed interest of dipole effects in thin magnetic films has appeared. The dipole forces are inevitably present in any magnetic matter. Albeit weak, these long-range forces lead to new physics at large scales. In three-dimensional (3D) magnets, dipole forces produce domain structures and modify critical behavior.[1] The dipole effects in 2D magnetic films are even stronger. A remarkable property of the dipole forces is their ability to stabilize long-range order in 2D degenerate ferromagnets.[2,3] Recently, it was suggested that dipole stabilization of long-range order is also possible in 2D Heisenberg antiferromagnets.[4] The properties of the low-temperature phases in these systems are not yet fully understood. Recent theoretical studies[5–7] suggest an unusual behavior for 2D dipole ferromagnets. It is argued that at low temperatures, the properties of these systems are intermediate between the usual long-range order and critical point behavior.

An experimental test of these theoretical predictions is difficult since real magnetic films lack continuous symmetry. Even a weak in-plane anisotropy strongly modifies the properties of the 2D magnet. The only exception is the case of hexagonal anisotropy. It is expected that for a certain range of low temperatures such anisotropy is irrelevant and that the isotropic *XY* behavior is restored at large scales.[8,9] Another appropriate system is the ferroelectric smectic-C film. Unfortunately, the dipole force in such a system becomes relevant only at very large scales.[10]

During the past decade considerable advances in the growth of magnetic films with hexagonal symmetry have been achieved. Growing iron films on the (111) face of the fcc Ag was reported in Ref. 11. Recently, the Ru film has been grown on an hexagonal graphite substrate.[12] Thus, one can hope to observe the dipole stabilization of the long-range order experimentally in such films. These advances and also the recent observation of dipolar induced striped domain structure[13] in a 2D magnet have simulated theoretical investigations of the dipole effects in the 2D systems (e.g., Refs. 4–7).

There are many theoretical works devoted to the ideal case of the pure film. At the same time, the effect of disorder on the properties of the dipolar films has not been investigated. In the present work, the effect of weak disorder on the 2D *XY* ferromagnet with the dipole (and exchange) forces is studied at low temperatures. In the Heisenberg dipole magnet, the magnetization normal to the film has a finite correlation radius.[2,3] Hence, the results are also applicable to the Heisenberg case. Two types of disorder will be considered: random field and random anisotropy axis. From symmetry consideration, one can expect that for other types of locally correlated disorder, the system will belong to one of these two universality classes.

Despite a lot of effort, our present understanding of the random field and random anisotropy systems[14,15] is still incomplete. The standard perturbation methods are not applicable for these systems because of the multiplicity of energy minima in the systems.[16,17] To deal with problems including complicated energy landscapes, the technique of replica symmetry breaking was developed.[18,19] Originally, this technique was suggested for the mean-field problem.[20] Only simple non-mean-field toy models were completely studied with the aid of replica symmetry-breaking calculations.[21] However, recently suggested was the variational procedure regarding replica symmetry-breaking effects (the self-consistent harmonic approximation[22–28]). In contrast to other methods (e.g., renormalization group), this procedure takes into account peculiarities of complicated energy landscapes of disordered systems, and thus allows us to obtain reasonable results.[28] In particular, this approach seems to be effective for the pure exchange random-field *XY* model.[25–27]

Below we assume that in the low-temperature phase the effects of topological defects can be neglected. Recent numerical[29] and theoretical[30–33] studies support this assumption for disordered systems with short-range interactions in spatial dimensions $D>2$. As seen below, the behavior of the random axis dipole magnet can be interpreted as an effective increase of the dimension. This allows us to hope that for the random axis disorder the vortexless approximation is correct. In the random-field case, the dipole force turns out to be irrelevant. The role of vortices in the 2D short-range random-field *XY* model is not very well studied. Numerical work[29] suggests that the system has a finite correlation radius, and the results of the vortexless approximation[25–27] are valid only for not too large scales.

The aim of the article consists in calculating the correlation function of the magnetization for random field and random anisotropy axis disorder with the self-consistent harmonic approximation. In both cases, the disorder leads to the destruction of the ferromagnetic phase. In the random-field case, the correlation function of the magnetization obeys the same power law as for the random-field magnet with the only exchange forces.[25–27] In the random axis case, the correlation function is a logarithmically slow decreasing function of the distance.

The results of this work are valid at scales where unbound vortices are absent. The role of topological defects deserves further attention.

II. THE MODEL

Consider the system with the following Hamiltonian:

$$H=-\sum_{\langle ij\rangle} K\mathbf{m}_i\mathbf{m}_j+\mu_B^2\sum_{i,j}\frac{\mathbf{m}_i\mathbf{m}_j-3(\mathbf{m}_i\mathbf{n}_{ij})(\mathbf{m}_j\mathbf{n}_{ij})}{|\mathbf{r}_i-\mathbf{r}_j|^3}+V_{\mathrm{imp}}, \tag{1}$$

where $\Sigma_{\langle ij\rangle}$ denotes summation over neighboring sites $\mathbf{r}_i$ of the lattice with spins $\mathbf{m}_i$; $\mathbf{n}_{ij}=(\mathbf{r}_i-\mathbf{r}_j)/|\mathbf{r}_i-\mathbf{r}_j|$; V_{imp} is the disorder energy. The spin vectors $\mathbf{m}_i$ lie in the plane of the film. For the random-field disorder

$$V_{\mathrm{imp}}=-\sum_i \mathbf{h}_i\mathbf{m}_i\,;\overline{h_i^\alpha h_j^\beta}\sim\delta_{\alpha\beta}\delta_{ij}\,;\alpha,\beta=x,y. \tag{2}$$

For the random axis disorder

$$V_{\mathrm{imp}}=-\sum_i (\nu_i\mathbf{m}_i)^2;\overline{\nu_i^\alpha\nu_j^\beta}\sim\delta_{\alpha\beta}\delta_{ij}\,;\alpha,\beta=x,y. \tag{3}$$

The dipolar interaction of neighboring spins is assumed to be small in comparison with their exchange interactions. Thus, the ground state of the pure system is ferromagnetic.

Consider the continuous limit of the Hamiltonian (1). In-plane anisotropy related to the discrete symmetry of the lattice will not be considered further. In the low-temperature limit, one can neglect the fluctuations of the absolute value of the local magnetization (assuming that vortices are irrelevant). Hence, the Hamiltonian can be expressed in terms of the angle $\phi(\mathbf{r})$ between the magnetization vector and some fixed spatial direction. For the types of disorder considered, the continuous Hamiltonian is

$$H=\int d^2r\frac{J}{2}(\nabla\phi)^2+\int\int d^2rd^2r'\frac{g}{|\mathbf{r}-\mathbf{r}'|^3}[\cos(\phi(\mathbf{r})-\phi(\mathbf{r}'))-3\cos(\phi(\mathbf{r})-\theta(\mathbf{r}-\mathbf{r}'))\cos(\phi(\mathbf{r}')-\theta(\mathbf{r}-\mathbf{r}'))]$$
$$+\int[h_x(\mathbf{r})\cos(p\phi(\mathbf{r}))+h_y(\mathbf{r})\sin(p\phi(\mathbf{r}))]d^2r, \tag{4}$$

where $\theta(\mathbf{r}-\mathbf{r}')$ is the angle between the vector $(\mathbf{r}-\mathbf{r}')$ and the fixed direction; the parameter p equals $p=1$ for the random-field disorder and $p=2$ for the random axis disorder; $h_x(\mathbf{r})$ and $h_y(\mathbf{r})$ are random fields. We assume that these fields are Gaussian and δ correlated, i.e.,

$$\overline{h_\alpha(\mathbf{r})h_\beta(\mathbf{r}')}=\Delta\,\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}');\alpha,\beta=x,y. \tag{5}$$

After disorder averaging, with the aid of the replica method, the effective Hamiltonian takes the form

$$H_R=\sum_a\int d^2r\frac{J}{2}(\nabla\phi^a)^2+\int\int d^2rd^2r'\frac{g}{|\mathbf{r}-\mathbf{r}'|^3}\sum_a[\cos(\phi^a(\mathbf{r})-\phi^a(\mathbf{r}'))-3\cos(\phi^a(\mathbf{r})-\theta(\mathbf{r}-\mathbf{r}'))\cos(\phi^a(\mathbf{r}')-\theta(\mathbf{r}-\mathbf{r}'))]$$
$$-\frac{\Delta}{2T}\int\sum_{a,b}\cos[p(\phi^a(\mathbf{r})-\phi^b(\mathbf{r}))]d^2r, \tag{6}$$

where a,b are the replica indices, T is the temperature.

The dipole force leads to the lowering of the symmetry in comparison with systems with short-range interactions. In presence of dipolar interactions, there is no symmetry with respect to the rotations in either spin or coordinate spaces. In the pure system, this is not an important point since simultaneous spin and coordinate rotations still remain symmetries of the magnet, and generation of new relevant operators is not expected.[5] However, in the impure system, the disorder leads to the loss of symmetry with respect to transformations of the spatial coordinates. Hence, for the dipole system the spin rotational symmetry is also lost. The only symmetry which is not broken by the dipole forces is the spin inversion symmetry. Thus, there are two different universality classes: the class including systems with the spin inversion symmetry and the class without such a symmetry.

The symmetry group of the disorder averaged effective Hamiltonian (6) includes (for any integer p) the following elements:

(1) the rotations of all replicas of the magnetization vector $(\cos\phi^a,\sin\phi^a)$ and the spatial coordinates on the same angle;

(2) the axis symmetries simultaneously in spin and coordinate spaces;

(3) the simultaneous change of the sign of all spin vector replicas: $\phi^a\to\phi^a+\pi$.

For $p=2$ (and for any even p) there is also the symmetry with respect to changing the sign of any one (or several) spin replica. This means that there are two different universality classes in the problem with arbitrary order p of anisotropy: odd p (the random field class) and even p (the random axis class). The disorder which conserves symmetry with respect to the spin rotations, i.e., random temperature disorder, belongs to the random axis class.

III. SELF-CONSISTENT HARMONIC APPROXIMATION

The procedure used for the calculations consists in obtaining an extremum of the variational free energy

$$F_{\mathrm{VAR}}=F_0+\langle H_R-H_0\rangle_0 \tag{7}$$

with respect to the quadratic trial Hamiltonian

$$H_0=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\sum_{ab} G_{ab}^{-1}(\mathbf{q})\phi^a(\mathbf{q})\phi^b(-\mathbf{q}). \tag{8}$$

In Eq. (7) the Hamiltonian H_R is determined from Eq. (6), $F_0=-T\ln\mathrm{Tr}\exp(-H_0/T)$ is the free energy corresponding to the trial Hamiltonian, $\langle\ \ldots\rangle_0$ denotes Gibbsian averaging with the Hamiltonian H_0. For the sake of simplicity, we assume that the temperature $T=1$. The low-temperature limit then corresponds to the case of large J in the Hamiltonian (4).

To average the Hamiltonian (6) it is convenient to rewrite its second term in the form

$$E_{\mathrm{dip}}=-\frac{1}{2}\int\int d^2r\,d^2r'\frac{g}{|\mathbf{r}-\mathbf{r}'|^3}\sum_a[\cos(\phi^a(\mathbf{r})-\phi^a(\mathbf{r}'))+3\cos(\phi^a(\mathbf{r})+\phi^a(\mathbf{r}'))\cos(2\theta(\mathbf{r}-\mathbf{r}'))$$
$$+3\sin(\phi^a(\mathbf{r})+\phi^a(\mathbf{r}'))\sin(2\theta(\mathbf{r}-\mathbf{r}'))]. \tag{9}$$

The Gaussian average of the third term in Eq. (9) is zero since this term is odd in ϕ. The second term in Eq. (9) also gives zero after averaging. To show this, it suffices to note that for the self-consistent solution $G_{ab}(\mathbf{q})$, which will be found below, the integral $\int G_{aa}(\mathbf{q})[d^2q/(2\pi)^2]$ diverges in the infrared limit.

One can find the variational free energy per volume

$$F_{\mathrm{VAR}}=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\left[Jq^2\sum_a G_{aa}(\mathbf{q})-\ln\det\hat{G}_{ab}(\mathbf{q})\right]-\frac{\Delta}{2}\sum_{a\neq b}\exp\left(-\frac{p^2}{2}B_{ab}\right)-\frac{g}{2}\int\frac{d^2x}{|\mathbf{x}|^3}\sum_a\exp\left[-\frac{1}{2}\langle(\phi_a(\mathbf{x})-\phi_a(\mathbf{0}))^2\rangle_0\right], \tag{10}$$

where an irrelevant constant term is omitted and

$$B_{ab}=\langle(\phi_a(\mathbf{x})-\phi_b(\mathbf{x}))^2\rangle_0=\int\frac{d^2q}{(2\pi)^2}[G_{aa}(\mathbf{q})+G_{bb}(\mathbf{q})-G_{ab}(\mathbf{q})-G_{ba}(\mathbf{q})]. \tag{11}$$

Variation of Eq. (10) over G_{ab} gives an equation for the replica correlation function

$$G_{ab}^{-1}(\mathbf{q})=Jq^2\delta_{ab}-\sigma_{ab}+g\,\delta_{ab}\int\frac{d^2x}{|\mathbf{x}|^3}[1-\exp(i\mathbf{qx})]\exp\left[-\int\frac{d^2q}{(2\pi)^2}[1-\exp(i\mathbf{qx})]G_{aa}(\mathbf{q})\right], \tag{12}$$

where

$$\sigma_{ab}|_{a\neq b}=\Delta p^2\exp\left[-\frac{p^2}{2}B_{ab}\right];\quad \sum_b \sigma_{ab}=0. \tag{13}$$

Below the solution of Eq. (12) is found in the form of the replica nonsymmetric Parisi matrix.[18,19] Such a structure of the solution reflects a complicated energy landscape of the system.[28]

IV. RANDOM FIELD

In the random-field case ($p=1$), it is easy to solve Eq. (12). The solution is the same, up to a small correction, as the one found for the system without the dipole force.[25] To verify this, one can use the result[25] for the correlation function

$$\langle\mathbf{m}(\mathbf{R})\mathbf{m}(\mathbf{0})\rangle=\langle\cos(\phi^a(\mathbf{R})-\phi^a(\mathbf{0}))\rangle\sim\frac{1}{R^s};\quad s\geqslant 2 \tag{14}$$

at large R. Substituting the correlation function (14) into the right-hand side (rhs) of Eq. (12), one sees that the dipole contribution to the rhs

$$g\,\delta_{ab}\int\frac{d^2x}{|\mathbf{x}|^3}(1-\exp(i\mathbf{qx}))\langle\cos(\phi^a(\mathbf{x})-\phi^a(\mathbf{0}))\rangle=gO(q^2). \tag{15}$$

Since g is small, the dipole contribution is small in comparison with the exchange term $Jq^2\delta_{ab}$ at any q. Hence, the solution[25] and the correlation function (14) remains valid in our problem.

We see that the dipole force is irrelevant for such a type of disorder. One can estimate the scaling dimension of the dipole energy as $\Delta_{\mathrm{dip}}=1-s$. This scaling dimension is negative in agreement with our conclusion.

The exponent s is nonuniversal and depends both on the temperature and the disorder.[25,27] For low temperatures and sufficiently weak disorder $s=2$.[27]

V. RANDOM ANISOTROPY

In this case one needs to rewrite replica matrices with the aid of the Parisi parametrization.[18,19] The matrix $G_{ab}^{-1}(\mathbf{q})$ is parametrized by the diagonal element $G_{aa}^{-1}(\mathbf{q})=T(\mathbf{q})-\tilde{\sigma}$ and the function $-\sigma(y)$, $0\leqslant y\leqslant 1$, where

$$T(\mathbf{q})=Jq^2+g\int\frac{d^2x}{|\mathbf{x}|^3}[1-\exp(i\mathbf{qx})]\exp\left[-\int\frac{d^2q}{(2\pi)^2}\times[1-\exp(i\mathbf{qx})]G_{aa}(\mathbf{q})\right],\tag{16}$$

$\sigma(y)$ and $\tilde{\sigma}=\int_0^1\sigma(y)dy$ correspond to the parametrization of the matrix σ_{ab} Eq. (13). Using the formulas of inversion of the Parisi matrices,[23] one finds for the parametrization $[G_{aa}(\mathbf{q}),b(\mathbf{q};y)]$ of the matrix $G_{ab}(\mathbf{q})$

$$G_{aa}(\mathbf{q})=\frac{1}{T(\mathbf{q})}\left[1+\int_0^1\frac{dy\,\delta(y)}{y^2(\delta(y)+T(\mathbf{q}))}\right];\tag{17}$$

$$G_{aa}(\mathbf{q})-b(\mathbf{q};x)=\frac{1}{x[\delta(x)+T(\mathbf{q})]}-\int_x^1\frac{dy}{y^2[\delta(y)+T(\mathbf{q})]},\tag{18}$$

where

$$\delta(y)=\int_0^y dz\,\sigma'(z)z.\tag{19}$$

Equation (17) is written in the form corresponding to the case of $\sigma(0)=0$. This condition can be justified *a posteriori*.

Let us introduce an auxiliary variable

$$g[\delta(y)]=\int\frac{d^2q}{(2\pi)^2}\frac{1}{T(\mathbf{q})+\delta(y)}.\tag{20}$$

The matrix σ_{ab} can be expressed via $g[\delta]$ as follows

$$\sigma(y)=\Delta p^2\exp\left[-\frac{p^2}{2}B(y)\right],\tag{21}$$

where

$$B(y)=2\left(\frac{1}{y}g[\delta(y)]-\int_y^1\frac{dz}{z^2}g[\delta(z)]\right).\tag{22}$$

Differentiating Eqs. (19), (21), and (22) with respect to y, and excluding $\sigma(y)$ and $B(y)$, one finds

$$\frac{d}{d\delta}\left(\frac{dg[\delta(y)]}{d\delta}\right)^{-1}=-\frac{p^2}{y}.\tag{23}$$

Equations (16), (17), (20), and (23) constitute a closed system to be solved for $p=2$.

We search for $T(\mathbf{q})$ Eq. (16) in the form of

$$T(q)=qf\left(\ln\frac{1}{q}\right),\tag{24}$$

where $f(\ln(1/q))$ is a slow varying function of q, $f(\ln(1/q))\to 0$ as $q\to 0$, and the ultraviolet cutoff is set equal to 1. The scales of interest are $q\ll 1$, where $f(\ln(1/q))\ll 1$. Calculation of $g[\delta]$ Eq. (20) gives

$$g[\delta]=\frac{1}{2\pi}\left[\int\frac{dq}{f(\ln(1/q))}-\delta\int\frac{dq}{f(\ln(1/q))[qf(\ln(1/q))+\delta]}\right]\approx\text{const}-\frac{\delta}{2\pi}\int^{\ln(1/\delta)}\frac{dz}{f^2(z)};\quad(\delta\ll 1).\tag{25}$$

Then, one obtains from Eq. (23)

$$y[\delta]=\frac{p^2}{2\pi}\delta f^2\left(\ln\frac{1}{\delta}\right)\left[\int^{\ln(1/\delta)}\frac{dz}{f^2(z)}\right]^2.\tag{26}$$

Expressing δ as a function of y produces

$$\delta[y]=\frac{2\pi y}{p^2f^2(\ln(1/y))}\left[\int^{\ln(1/y)}\frac{dz}{f^2(z)}\right]^{-2}.\tag{27}$$

Substitution of $\delta[y]$ into Eq. (17) yields

$$G_{aa}(q)=\frac{1}{qf(\ln(1/q))}\left[1+\int_0^1\frac{dm}{m^2}\frac{(2\pi/p^2)m}{(2\pi/p^2)m+qf(\ln(1/q))f^2(\ln(1/m))\left\{\int^{\ln(1/m)}[dz/f^2(z)]\right\}^2}\right].\tag{28}$$

With change of the variable $y=p^2/2\pi m$, one can rewrite Eq. (28) in the form

$$G_{aa}(q)=\frac{1}{qf(\ln(1/q))}\left[1+\frac{2\pi}{p^2qf(\ln(1/q))}I\left(\frac{1}{qf(\ln(1/q))}\right)\right],\tag{29}$$

where

$$I(b)=\int_{p^2/2\pi}^{\infty}\frac{dy}{yf^2(\ln y)\{\int^{\ln y}[dz/f^2(z)]\}^2+b}.\tag{30}$$

The main contribution to $I(b)$ at large b comes from the region

$$y\gg\frac{b}{f^2(\ln b)\{\int^{\ln b}[dz/f^2(z)]\}^2}=y_c.\tag{31}$$

This allows us to calculate the integral (30)

$$I(b)=\int_{y_c}^{\infty}\frac{dy}{yf^2(\ln y)\{\int^{\ln y}[dz/f^2(z)]\}^2}=\left.\frac{1}{\int^{\ln y}[dz/f^2(z)]}\right|_{\infty}^{y_c}.\tag{32}$$

Since $f(z)\to 0$ as $z\to\infty$, the integral $\int^{\infty}[dz/f^2(z)]$ diverges. Thus, after substitution of Eq. (32) into Eq. (29), one finds (at small q)

$$G_{aa}(q)\approx\frac{2\pi}{p^2q^2f^2(\ln(1/q))}\left[\int^{\ln(1/q)}\frac{dz}{f^2(z)}\right]^{-1}. \tag{33}$$

This allows us to calculate the correlation function of the angles $\phi^a(\mathbf{x})$:

$$\begin{aligned}\frac{1}{2}\langle(\phi^a(\mathbf{x})-\phi^a(\mathbf{0}))^2\rangle &= \int\frac{d^2q}{(2\pi)^2}(1-\exp(i\mathbf{qx}))G_{aa}(\mathbf{q})\\ &= -\frac{1}{p^2}\int(1-J_0(qx))\frac{d}{dq}\ln\int^{\ln(1/q)}\frac{dz}{f^2(z)}dq\\ &\approx\frac{1}{p^2}\ln\int^{\ln x}\frac{dz}{f^2(z)},\end{aligned} \tag{34}$$

where $J_0(qx)$ is a Bessel function. Substituting the correlation function (34) into Eq. (16), one obtains a closed equation for $f(\ln(1/q))$:

$$qf\left(\ln\frac{1}{q}\right)=\text{const}\int\frac{d^2x}{|\mathbf{x}|^3}(1-\exp(i\mathbf{qx}))\left[\int^{\ln x}\frac{dz}{f^2(z)}\right]^{-1/p^2+o(1)}. \tag{35}$$

The slow x dependence of the expression in the square brackets allows us to reduce Eq. (35) to the form

$$f\left(\ln\frac{1}{q}\right)=\text{const}\left[\int^{\ln(1/q)}\frac{dz}{f^2(z)}\right]^{-1/p^2+o(1)}. \tag{36}$$

The asymptotic solution of this equation at small q can be found in the form of a power law as,

$$f\left(\ln\frac{1}{q}\right)\sim\left(\ln\frac{1}{q}\right)^{-1/(p^2-2)}=\left(\ln\frac{1}{q}\right)^{-1/2}. \tag{37}$$

One can now calculate the correlation function of the magnetization. With the accuracy up to the double logarithmic correction

$$\langle\mathbf{m}(\mathbf{R})\mathbf{m}(\mathbf{0})\rangle=\langle\cos(\phi_a(\mathbf{R})-\phi_a(\mathbf{0}))\rangle\sim\frac{1}{\sqrt{\ln R}}. \tag{38}$$

The replica Green function, $G_{ab}(\mathbf{q})$, contains more information than simply the correlation function (38). One can extract from G_{ab} the partition function for the quantity $(\phi(\mathbf{R})-\phi(\mathbf{0}))$. The effective potential corresponding to this partition function is organized hierarchically.[24] The structure of the effective potential determines the dynamics of the system. Proceeding as in Ref. 34, one can estimate the typical height of the highest potential barriers (that is the barriers corresponding to the lowest hierarchy level) as $U(R)\sim R$.

VI. CONCLUSION

In conclusion, we find that there is no long-range order in the impure magnetic film. This result agrees with the Imry-Ma-type arguments.[35] If the ferromagnetic order is destroyed in a region of size l, then the loss in dipole energy is $\Delta E_d\sim l^{2D-3}$, where $D=2$ is the spatial dimension. At the same time, the gain in disorder energy is $\Delta E_i\sim l^{D/2}$. Since in the 2D case $\Delta E_i\sim\Delta E_d$ one can expect the typical lower critical dimension behavior, that is the absence of the long-range order.

The effect of the dipole forces on the random-field Ising model (RFIM) has been studied in Ref. 36. The Imry-Ma-type arguments show that in this system the dipole forces do not shift the lower critical dimension. However, as well as in the XY model, these forces damp fluctuations in the RFIM. In particular the roughness exponent decreases.[36]

Since we have worked with the continuous Hamiltonian (4) the effect of the vortices has been neglected. This approximation is reasonable if the correlation function varies slowly with distance. Hence, one expects that for the random axis disorder, vortices are irrelevant. The effects of the vortices on the random-field magnet are deferred for later investigations.

The behavior (38) of the random axis dipole ferromagnet for not too large a system can be interpreted as the existence of the ferromagnetic long-range order. In the absence of dipole forces the correlation function of the impure XY model obeys a power law for any spatial dimension $D<4$.[25–27] On the other hand, in the ''clean'' 2D XY ferromagnet, the dipole forces stabilize the long-range order.[2,3] Thus, one can conclude that the ability of the dipole force to stabilize the long-range order persists in impure systems—but only if the disorder does not break the spin inversion symmetry.

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