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Localization and delocalization of light under oblique incidence

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It is found experimentally that light is localized for a disordered binary multilayer, which is periodic on average and is composed of cryolite and antimonite, under oblique incidence. The localization length of p-polarized light increases with the incident angle, while it is almost a constant for s-polarized light, when the incident angle varies between 0° and 70°. This effect can be attributed to the vector nature of light, i.e., the reflection of p waves on the multilayer interfaces becomes small when the incident angle approaches Brewster's angle. The corresponding localization lengths are calculated by the transfer matrix method. Our measurements agree with the simulation of the transfer matrix method, when the self-averaging effect plays a role. [S0163-1829(97)02221-2]

I. INTRODUCTION

It has been demonstrated that disorder can induce localization of classical waves in a one-dimensional system.[1–12] The corresponding localization length ξ for such layered media can be measured and calculated, where ξ, as usual, is defined as $\xi = -L/\langle \ln T \rangle$. Here L is the total thickness of the system, T is the transmittance of light and $\langle \cdots \rangle$ represents an average over the statistical distribution of the disordered system. In this aspect the similarity of light localization to other classical wave localization seems to be apparent. However, light has an inherent vector nature that is quite different from scalar waves such as the acoustic wave. This vector nature should become apparent in light localization related phenomena. By using long-wavelength theory and the Monte Carlo method Sipe *et al.* have predicted that the delocalization effect, i.e., the divergence of the localization length of p-polarized waves, can occur for a disordered layered media under certain conditions.[13] The physical reason is that when the incident angle is varied the reflection and transmission on the multilayer interfaces are different for p and s waves. Therefore, the interference between these reflections and transmissions are also different for p and s waves. This leads to a divergence of the localization length for p waves. Specifically, if the incidence angle equals Brewster's angle, the p wave can pass completely through the multilayer without reflection. This is rather an interesting effect because there is no such analogy for scalar waves. Until now there has not been any experimental evidence, though the exponential decay of light with the wavelengths inside the band gap for the regular multilayers is well known.

In our experiment on the measurements of light transmission in a disordered binary multilayer composed of cryolite and antimonite we find that, at wavelengths either inside or outside the corresponding photonic band gap the exponential relationship between transmittance and multilayer thickness exists under oblique incidence. In addition, the localization length increases with incident angle for p-polarized waves, while it remains nearly unchanged for s-polarized waves, as the incident angle varies from 0° to 70°. The corresponding localization length is also calculated by the optical transfer matrix method.[14] Based on both measurement and simulation it is believed that this effect may be attributed to the delocalization process, which is related to the light polarization. The main experimental and calculated results are reported in this paper.

II. EXPERIMENT

The experimental setup is as follows. The multilayer is placed vertically on a stage, which can be horizontally rotated. Thus, the incidence angle of the light beam is precisely adjustable. We use a Model 3900s Ti: Sapphire cw laser pumped by a Model 171 Ar^+ laser as a light source. Its output is about 100 mW, horizontally polarized at 790 and 880 nm. The laser beam passes horizontally through the multilayer. In such an arrangement the plane of polarization is in the plane of incidence, corresponding to a p wave. For an incident s wave, a pair of special broad band reflectors[15] is used to change the polarization of the laser beam from the horizontal to the vertical direction. In the measurement the laser beam goes first through a Glan-Thomson prism that improves the degree of linear polarization of the beam to better than 10^4. This is rather important, because at larger angles of incidence the transmission intensities of the p and s waves can differ by 10^5. Then the beam is incident on the multilayer. The transmitted light finally enters a box, which blocks all scattered light coming from various elements, and inside the box is a detector, a Model PD300 NOVA laser power monitor (Ophir Optronics Ltd.). The highest sensitivity of this detector is 0.1 nW and the lowest transmitted intensity in our experiment is several nW, so that the measurement may be regarded as reliable. For each experimental run the input laser intensity is calibrated.

The disordered binary multilayers used here are the same as those used in our previous experiment.[7] They consist of both cryolite and antimonite layers and are periodic on average. The disorder is produced by choosing the thickness of the antimonite layers randomly from a flat distribution around an average value of $\langle b \rangle = d/4n_b = 57$ nm, while the thickness of all the cryolite layers are the same and $\langle a \rangle = d/4n_a = 122$ nm. Here $n_a = 1.33$ and $n_b = 2.83$ are the corresponding refractive indices and $d = 650$ nm. The degree of disorder for the multilayer is defined as $D = [\Sigma_i (b_i - \langle b \rangle)^2/M]^{1/2}/l_{ab}$, and b_i, M, and $l_{ab} = \langle a \rangle + \langle b \rangle$ are, respectively, the real thickness, total number of antimonite lay-

TABLE I. The designed thickness of the antimonite layers, in units of nm, and their sequences. Bi, Ci, and Di are the thicknesses of the ith layer for the multilayers with $M=10$, 15, and 21, respectively. In the cases of $M=15$ and 21, the layer thicknesses are symmetrically distributed with respect to the central layer, the eighth and the eleventh layer, respectively, ie., $C1=C15, C2=C14, \ldots, C7=C9$ and $D1=D21, D2=D20, \ldots, D10=D12$. Therefore, the values of $Ci(i>8)$ and $Di(i>11)$ are omitted in the table. The first antimonite layer of each multilayer is in direct contact with the surface of the optical glass substrate, which has a refractive index of 1.52.

i	1	2	3	4	5	6	7	8	9	10	11
Bi	23	46	80	87	92	87	80	46	23	9	
Ci	16	20	27	69	79	83	90	94			
Di	14	16	18	37	48	75	80	85	88	92	95

ers, and the average multilayer period. In order to show the relationship between the transmission and the total thickness of the multilayer, three kinds of disordered multilayers are prepared. These have the same $D=0.16$ and $l_{ab}=179$ nm, but different total lengths $L=10$, 15, and 21 in units of l_{ab} (i.e., $M=10$, 15, and 21), respectively. The designed thickness and sequence of the antimonite layers for these multilayers are listed in Table I.

According to the definition of the light localization length, $\xi=-L/\langle \ln T \rangle$, ξ can be obtained by measuring the transmission of light passing through the disordered multilayers with various L and then by the linear regression of different pairs of L and $\ln T$. For the corresponding regular quarter wave stack ($\lambda=650$ nm) the first band gap is located around 526 to 849 nm. In the case of a regular binary multilayer, within the gap light decays exponentially with the total thickness, and the transmittance is very low because of perfect Bragg reflection. However, outside the gap there is no exponential relationship between transmittance and the total thickness. To see the dependence of the transmittance of the disordered multilayer on the angle of incidence both inside and outside the gap, i.e., to confirm if the light is still localized, the experiments are performed at two wavelengths, namely, 790 nm (inside the gap) and 880 nm (outside the gap).

III. EXPONENTIAL DECAY OF LIGHT

The transmittances of the s and p wave, T_s and T_p, are measured for the multilayer with $M=10$, 15, and 21 and incident angle of $\theta=0°$ to 70°. Figure 1(a) shows the variation of T_s and T_p with θ for the $M=21$ multilayer at two wavelengths, 790 and 880 nm, while in Fig. 1(b) the variation for the $M=15$ multilayer is given. One finds that T_p increases by five to six orders of magnitude for both multilayers when θ varies from 0° to 70°. On the contrary, the changes of T_s are very small, within one order of magnitude. The exponential decay has been verified for the disordered multilayer at normal incidence in the previous experiment, so the present measurements of T_s imply that for s waves such an exponential relationship between T_s and M exists too under oblique incidence. We further examine T_p. The dependencies of T_p on M at 790 and 880 nm are measured. The results show that at small θ, 0° and 10°, the measured T_p coincides with the exponential relationship quite well. But when θ goes up, a marked deviation of T_p in the $M=10$ multilayer is observed, while that in the $M=15$ and 21 multilayers still follow an exponential decay. As θ becomes larger, the deviation becomes more pronounced. This behavior occurs at both wavelengths. In addition, at 790 nm the exponential variation of T_p with $M=15$ and 20 holds for all incident angles between 0° and 70°. However, at 880 nm this relationship fails at θ larger than 45°. Such a kind of devia-

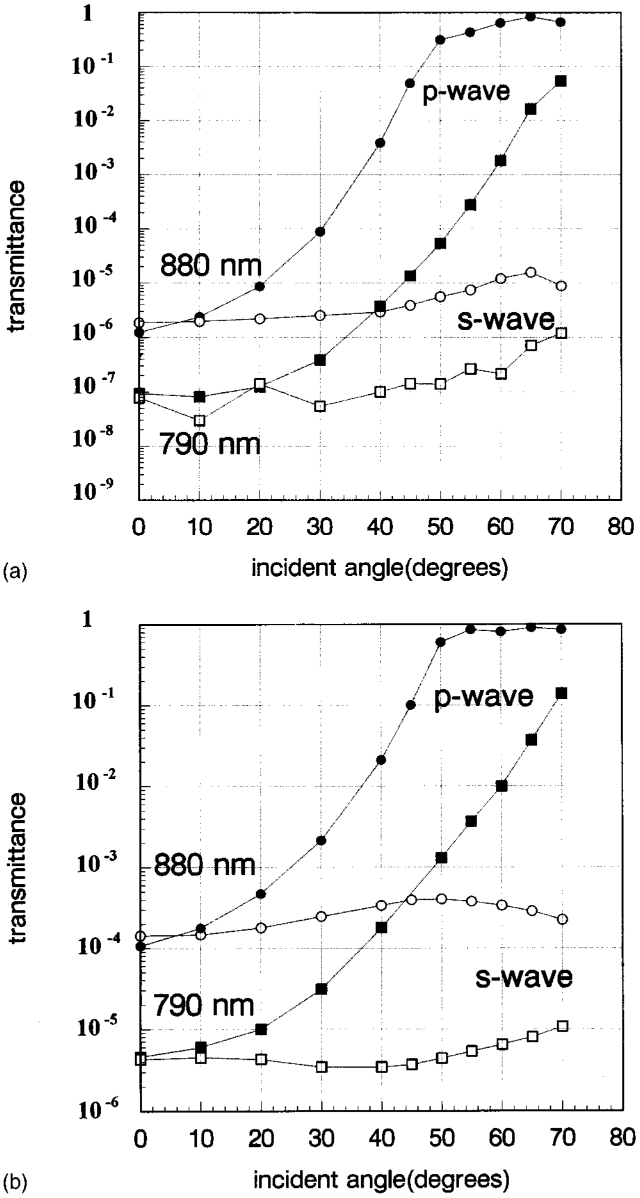

FIG. 1. The variation of measured transmittances of p and s waves with incident angle at 790 and 880 nm for two multilayers of (a) 21 periods (b) 15 periods.

tion may be caused by the longer real localization length and limited total multilayer thickness. From this experiment we know that the localization length ξ_p increases with θ. In the experiment we measure only the transmittance of the multilayer with a particular layer thickness distribution and the averaging over different sequences of layers is not taken into account. So, in our case a self-averaging effect should play an essential role in the exponential decay of light. The previous study has pointed out that this self-averaging can be effective only if the total thickness of the multilayer is much greater than the corresponding localization length, otherwise the error may be remarkable. At normal incidence ξ_p is only $1\sim2\ l_{ab}$ and the total thickness of the multilayer is 10, 15, and 21 l_{ab}, respectively, thus $L\gg\xi_p$ is well satisfied. However, as θ increases ξ_p becomes large. The self-averaging effect may fail, first, for the thinnest multilayer of $M=10$, and then even for the thicker multilayer. The measured results for $\theta=50°$ and 55° show that there the exponential decay is destroyed even for the multilayers with $M=15$ and 21. Based on our experiment and the above discussions it is believed that, as long as the total multilayer thickness is much longer than the corresponding localization length, i.e., the self-averaging effect does work, then, the exponential relationship between T_p and L is still valid in the case of oblique incidence.

IV. LOCALIZATION LENGTH: MEASURED AND CALCULATED

The measured values of ξ_p and ξ_s are derived from the exponential dependence of T on M. At the same time they are calculated by the optical transfer matrix method. The comparisons between measurement and calculation at 790 and 880 nm are given in Figs. 2(a) and 2(b), respectively. For an s wave, when θ varies from 0° to 70°, ξ_s is almost unchanged at the wavelengths of either inside or outside the gap and the measured values fit very well with the calculated ones. For a p wave, however, two experimental phenomena can be identified. First, the divergence of ξ_p appears, i.e., a steep rise in ξ_p can be seen in a certain angle range, namely $\sim65°$ for the transmission of 790 nm and $\sim45°$ for that of 880 nm. Second, the coincidence of measurement with calculation is good when ξ_p is small, while the error becomes large as $\xi_p>3\,l_{ab}$. Moreover, a longer ξ_p may lead to a larger error. This disagreement may be attributed to the fact that the self-averaging effect does not work due to too short a total thickness of the multilayer, as we have described in the last paragraph. In our case, because of the higher refractive contrast between two coating materials, the angle at which light with p polarization passes completely through the binary multilayers cannot be approached [see Eq. (9) of Ref. 13]. What we present here are the phenomena when the angle of incidence is getting close to this angle. Even so, our experiment gives a verification of the prediction of Sipe *et al.*[13]

Furthermore, our measurement can provide a rough estimate of how many layers are needed practically for getting a reliable experimental localization length. The thickest multilayer we used is 21 l_{ab} and the error starts to occur at $\xi_p>3\,l_{ab}$, so the minimal ratio of total thickness to localization length should be greater than 7.

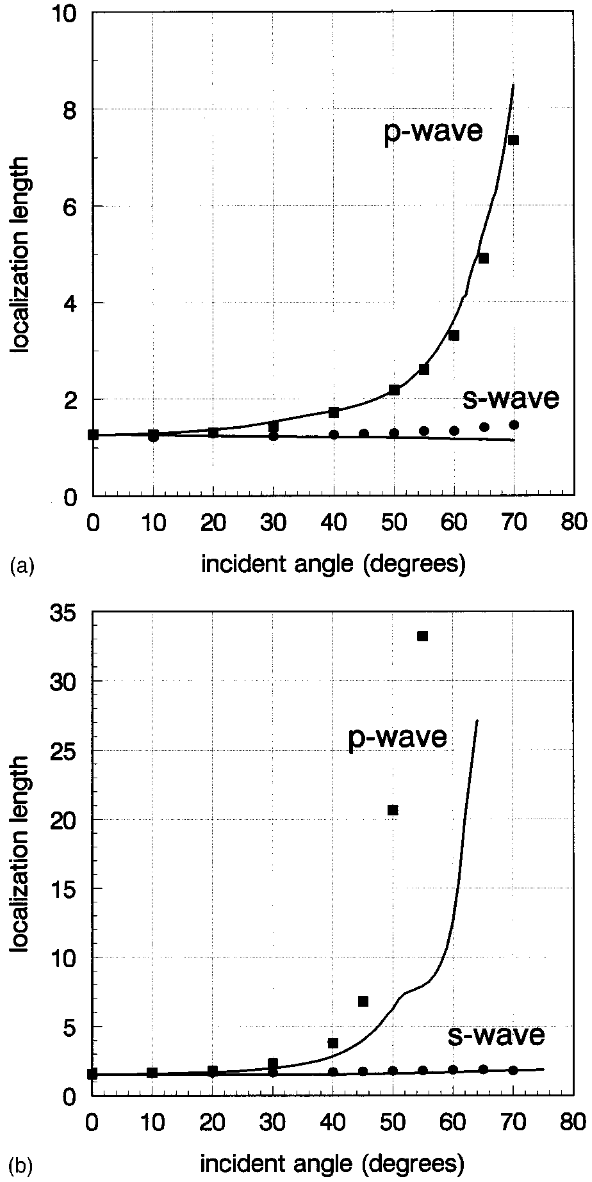

FIG. 2. The localization length, in units of l_{ab}, as a function of incident angle for p and s waves at (a) 790 nm (b) 880 nm. Solid line: calculated curve.

V. CONCLUSION

A polarization related change of light localization length for a random one-dimensional system has been experimentally demonstrated. The localization length increases with the incident angle of the p-polarized light, while it remains a constant for s waves. It originates physically from the different properties of reflection and transmission in the interfaces of a random multilayer for the two types of polarized waves. Therefore, this effect can be observed only for the localization of a vector wave. The optical transfer matrix calculations are in reasonably good agreement with the measurements.

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[1] See, for example, *Scattering and Localization of Classical Waves in Random Media*, edited by P. Sheng (World Scientific, Singapore, 1990); *Photonic Band Gap and Localization*, edited by C. M. Soukoulis (Plenum, New York, 1993).

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