# Resistance statistics in one-dimensional systems with correlated disorder

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We address the general problem of computing dc resistance fluctuations in one-dimensional Anderson models with spatially correlated disorder and discuss some examples of binary systems with Markovian correlations. As in the general case of uncorrelated disorder, we observe a growth of the relative resistance fluctuations $\langle \rho_N^2 \rangle / \langle \rho_N \rangle^2$ with the system length $N$. The largest sample-to-sample fluctuations are found in certain energy regions of quasipure systems with very low concentrations of defects, whereas constitutional entropy seems to rule the behavior of typical values of the resistance in different regions and no role appears to be played by the potential correlation length. We express the growth of relative fluctuations in terms of the entropy function characterizing different possible localization lengths of the wave function and observe convergence toward a universal lognormal distribution in the presence of an extended state. [S0163-1829(97)14617-3]

## I. INTRODUCTION

Dynamical properties of one-dimensional systems are of importance for understanding the behavior of many structures, like superlattices, multilayers, and linear polymers. It is quite well established that in the presence of disorder they possess localized eigenstates, and that quantum diffusion is absent under very general circumstances.[1–3] However, different behaviors are often found when certain rules or correlations are imposed on the disorder. For example, memory effects may lead to unexpected behaviors of the eigenstates localization lengths; e.g., they can be inversely proportional to the correlation length of the disorder in some regions of the spectrum of electronic[4] and harmonic[5] chains. Moreover, there are disordered heterostructures exhibiting properties similar to those of aperiodic ordered structures with same composition[6] or short-range correlation.[7] Other examples of the influence of disorder correlation on the dynamics of a system are the existence of a point spectrum of extended eigenstates in one-dimensional Schrödinger equations with correlated potential barriers or wells[1,8] and in other models,[9,10] where in some cases anomalous quantum diffusion may take place in spite of the disordered nature of the system.[9] As a matter of fact, spatial correlations of disorder lead in many cases to the appearance of peculiar and nonintuitive properties.

In the present work we investigate the statistical properties of electrical dc resistance in some one-dimensional Anderson models with Markov-correlated disorder. We consider the case of diagonal disorder, in which motion is described by the stationary equation

$$E\psi_n = \varepsilon_n \psi_n + \psi_{n-1} + \psi_{n-1}, \tag{1}$$

where $E$ is the system energy, $\varepsilon_n$ are site energies, and $\psi_n$ the wave function amplitude at site $n$.

The resistance of one-dimensional disordered electronic systems is a random variable, taking different values in different systems with identical statistical composition. Sample-to-sample fluctuations can be very large, and in the general case of uncorrelated disorder the variance of resistance distribution is found to grow faster than the mean value.[11] The four-probe resistance $\rho_N$ at $T=0$ for a system of $N$ sites and Fermi energy $E$ is given by the Landauer formula[12–14]

$$\rho_N = \frac{h}{q_e^2} \frac{R_N(E)}{T_N(E)}, \tag{2}$$

where $q_e$ is the charge of the carriers, $h$ the Planck constant, and $T_N(E)$ and $R_N(E)$ are the transmission and reflection coefficients. When $N$ is large $T_N \ll R_N$, and typical, i.e., most probable, values of resistance $\tilde{\rho}$ grow exponentially with the length of the system at a rate given by the Lyapunov characteristic exponent $\lambda$:[1,14]

$$\tilde{\rho}_N \simeq \frac{h}{q_e^2} e^{2N\lambda}. \tag{3}$$

The Lyapunov exponent $\lambda$ is a function of the energy $E$ yielding the most probable rate of exponential growth of particular solutions of Eq. (1): $\psi_N \approx \psi_0^{N\lambda}$. According to the Borland conjecture[1] $\lambda$ is equal to the inverse of the localization length $\xi$ of eigenstates $\phi$ of Eq. (1), having about the same energy as $\psi$, i.e., $\phi_N \approx \phi_0^{-N/\xi}$; $\lambda$ being not random, the logarithmic resistance turns out to be a self-averaging quantity:[13]

$$\lim_{N\to\infty} \frac{1}{N} \ln \rho_N = \lim_{N\to\infty} \frac{1}{N} \langle \ln \rho_N \rangle = 2\lambda,$$

where $\langle \cdots \rangle$ denotes the average over the disorder realizations.

In analogy with Eq. (3), the $q$th moment of the probability distribution of $\rho$ is related to the exponential growth rate of the $2q$th moment of wave functions. By defining

$$L(q) = \lim_{N\to\infty} \frac{1}{N} \ln \left\langle \left| \frac{\psi_N}{\psi_0} \right|^q \right\rangle, \tag{4}$$

it follows from Eq. (2)

 

$$\lim_{N\to\infty}\frac{1}{N}\ln\langle\rho_N^q\rangle = L(2q), \tag{5}$$

and fluctuations of $\rho_N$ grow asymptotically as

$$\langle\rho_N^q\rangle \simeq e^{NL(2q)}.$$

$L(q)$ are called generalized Lyapunov exponents and are connected to the local growth rates of solutions of Eq. (1) (Ref. 14) (see also the discussion in Sec. IV).

The purpose of this paper is to show how $L(q)$ can be computed in the case of spatially correlated disorder, and to discuss some physically relevant examples. To this aim, we make use of a method developed recently for computing the generalized Lyapunov exponents of Markov-correlated random matrices.[15] This method is illustrated in Sec. II, where the motion equation (1) is therefore recast in terms of transfer matrices. Section III is devoted to discussing the results from different models characterized by different correlation rules, focusing in particular on the growth rate of typical resistivity $\tilde{\rho}$, of first low moments $L(2q)$, and of relative fluctuations $\langle\rho_N^2\rangle/\langle\rho_N\rangle^2$, and investigating their behavior as functions of energy and composition. We consider for simplicity binary systems, examining in detail the three cases specified below.

(a) The *correlated alloy* (Sec. III A 1), in which the probability of occurrence of each type of atom at a given site depends on the type that occurred at the previous site and is different from zero.

(b) The *chain with single insertions*, a model resembling the structure of some linear polymers in which one of the two kinds of sites can appear only individually (Sec. III B).

(c) The *random dimer model* (Sec. III C), introduced by Wu and co-workers,[9] where one of the two species can appear consecutively only an even number of times, and which may possess an extended state.

For comparison we also discuss some properties of the *uncorrelated binary alloy* (Sec. III A 2). In Sec. IV the behavior of resistance fluctuations is put in connection with the entropy function characterizing the probability density of different wave function growth rates, and the main findings are given in Sec. V.

In all the cases considered here it turns out that there is a value of concentration of the two species, which depends on the energy and which produces the largest growth rate of fluctuations. The very interesting point is that this value is different from the value which gives rise to the largest growth of resistance. There are regions of the spectrum where fluctuations grow faster while resistance grows slowly, if the abundance of nonconducting species is very low. In other regions of the spectrum both seem to reach their maximum growth for maximum compositional entropy. In none of the analyzed cases does a relation with the spatial correlation length of the potential emerge, and growth of the variance is always found to be larger than the growth of the average resistance, implying the divergence of relative fluctuations with the system size.

## II. GENERALIZED LYAPUNOV EXPONENTS OF WAVE FUNCTIONS IN CORRELATED RANDOM POTENTIALS

Techniques dealing with products of random matrices can be used in the present problem by rewriting the motion equation, Eq. (1), in the matrix form

$$\mathbf{u}_i = \mathbf{A}_i\mathbf{u}_{i-1},$$

where

$$\mathbf{u}_{i-1} = \begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix} \text{ and } \mathbf{A}_i = \begin{pmatrix}E-\varepsilon_i & -1\\ 1 & 0\end{pmatrix}, \tag{6}$$

where $i \geqslant 1$ and $\psi_0 = 0$. Exponential growth rates of moments of the wave function, Eq. (4), are then given by

$$L(q) = \lim_{N\to\infty}\frac{1}{N}\ln\left\langle\left|\frac{\mathbf{A}_N\mathbf{A}_{N-1}\cdots\mathbf{A}_2\mathbf{A}_1\mathbf{u}_0}{\mathbf{u}_0}\right|^q\right\rangle.$$

In Ref. 15 explicit expressions for $L(q)$ in the case of Markov-correlated random matrices have been derived. Such expressions are generalizations of the respective expressions for the uncorrelated case[14] based on the fact that, apart from exceptional cases, the product in the above equation is dominated for large $N$ by its largest eigenvalue in modulo. Thus $L(q)$ can be determined by computing $\lim_{N\to\infty} 1/N\ln\langle|\mathrm{Tr}\mathbf{X}_N|^q\rangle$, with $\mathbf{X}_N = \mathbf{A}_N\mathbf{A}_{N-1}\cdots\mathbf{A}_2\mathbf{A}_1$. Computation can be simplified by exploiting the identity (holding for any matrix $\mathbf{M}$ and positive integer $q$) $(\mathrm{Tr}\mathbf{M})^q = \mathrm{Tr}(\mathbf{M}^{\otimes q})$, where $\otimes$ indicates the direct product, and

$$\mathbf{M}^{\otimes q} = \underbrace{\mathbf{M}\otimes\mathbf{M}\otimes\cdots\otimes\mathbf{M}}_{q \text{ times}}.$$

Let us consider the case in which local energies $\varepsilon_i$ in Eq. (6) are chosen from a discrete set of $Q$ values. For a Markov-correlated process the probability of a given sequence $\{\varepsilon_i\}$ in a chain of length $N$ is

$$P_N(\{\varepsilon_i\}) = T(\varepsilon_{N,}\varepsilon_{N-1})T(\varepsilon_{N-1,}\varepsilon_{N-2})\cdots T(\varepsilon_{2,}\varepsilon_1)P_1(\varepsilon_1), \tag{7}$$

where $T(\varepsilon_{i,}\varepsilon_j) = T_{ij}$ is the stochastic matrix element giving the conditional probability of getting $\varepsilon_i$ after $\varepsilon_j$ and $P_1$ is the probability for the sequence to start with $\varepsilon_1$. Therefore, by denoting $\mathbf{B} = \mathbf{A}^{\otimes q}$, the average of $\mathbf{X}_N{}^{\otimes q}$ can be written as

$$\langle\mathbf{X}_N^{\otimes q}\rangle = \sum_{\{\varepsilon_i\}}\mathbf{B}(\varepsilon_N)T(\varepsilon_N,\varepsilon_{N-1})\mathbf{B}(\varepsilon_{N-1}) \times T(\varepsilon_{N-1},\varepsilon_{N-2})\cdots\mathbf{B}(\varepsilon_2)T(\varepsilon_2,\varepsilon_1)\mathbf{B}(\varepsilon_1)P_1(\varepsilon_1). \tag{8}$$

By introducing the $2^qQ\times 2^qQ$ matrix

$$Y(l,\varepsilon;l',\varepsilon') = B_{ll'}(\varepsilon)T(\varepsilon,\varepsilon'), \tag{9}$$

Eq. (8) turns into

$$\langle (X_N^{\otimes q})_{lm}\rangle = \sum_{\{\varepsilon_i\}} \sum_{\{l_i\}} Y(l,\varepsilon_N;l_{N-1},\varepsilon_{N-1}) \times Y(l_{N-1},\varepsilon_{N-1};l_{N-2},\varepsilon_{N-2})\cdots \times Y(l_2,\varepsilon_2;l_1,\varepsilon_1)B(\varepsilon_1)P_1(\varepsilon_1).$$

Because the process in Eq. (7) is supposed to converge to a stationary state for $N\to\infty$, in this limit the quantity between brackets should not depend on the initial matrix $\mathbf{B}(\varepsilon_1)P_1(\varepsilon_1)$. We thus chose $P_1(\varepsilon_1)\equiv\Sigma_{\varepsilon_0}T(\varepsilon_1,\varepsilon_0)$ and rewrite the above equation as

$$\langle (X_N^{\otimes q})_{lm}\rangle = \sum_{\varepsilon_N} \sum_{\varepsilon_0} Y^N(l,\varepsilon_N,;m,\varepsilon_0),$$

so that for even $q$

$$\langle |\mathrm{Tr}\mathbf{X}_N|^q\rangle = \langle |\mathrm{Tr}[\mathbf{X}_N^{\otimes q}]|\rangle = \langle \mathrm{Tr}[\mathbf{X}_N{}^{\otimes q}]\rangle = \mathrm{Tr}\langle \mathbf{X}_N{}^{\otimes q}\rangle = \sum_l \langle (X_N^{\otimes q})_{ll}\rangle = \mathrm{Tr}\mathbf{Y}^N.$$

For large $N$ the rightmost expression is dominated by the largest eigenvalue in the modulus of $\mathbf{Y}$, $y(q)$,

$$\langle |\mathrm{Tr}[\mathbf{X}_N{}^{\otimes q}]|\rangle \simeq a|y(q)|^N,$$

where $a$ is a constant independent of $N$, and generalized Lyapunov exponents are given by

$$L(q)=\ln|y(q)|. \tag{10}$$

This expression holds for $q$ even or for any positive integer $q$ whenever $Y_{ij}>0 \ \ \forall i,j$.

## III. RESISTANCE FLUCTUATIONS IN CORRELATED RANDOM BINARY SYSTEMS

In this section we discuss the behavior of the dc resistance and of its relative fluctuations in some different cases in which the site energies $\varepsilon_i$, Eq. (1), are allowed to assume two values $\varepsilon_a$ and $\varepsilon_b$. We consider first the case of a *correlated alloy* (Sec. III A) where sites of energy $\varepsilon_i$ may follow sites of energy $\varepsilon_j$ with a finite probability $T_{ij}$. A comparison between the behavior of this system and the one of the *uncorrelated alloy* will put into a major evidence the effects of correlation. We then discuss a model that we have called the *chain with single insertions*, in which two consecutive sites of energy $\varepsilon_b$ are not allowed, i.e., $T_{bb}=0$ (Sec. III B). This model is inspired to the basical structure of some linear polymer in which two consecutive units of the same kind do not occur. An example of such a system is doped emeraldine,[16] in which two consecutive units of iminequinone are never found in practice. The chain with single insertions is also in some relation with the other case that we consider, the *random dimer model* (Sec. III C). Here one of the two species can appear only in pairs,[9] giving rise to an extended state which is responsible for conduction and, according to numerical simulation, to quantum diffusion. This model has been used to represent protonated polyaniline,[9] a good conducting polymer obtained from doped polyaniline by protonic addition.[16]

Computation of $L(q)$, Eq. (10), has been done by selecting $y(q)$ through iterated applications of $\mathbf{Y}$ to an arbitrary initial vector. Besides computing the first low $L(2q)$'s we have considered the quantity

$$\beta(E)=L(4)-2L(2)=\lim_{N\to\infty}\frac{1}{N}\ln\frac{\langle\rho_N^2\rangle}{\langle\rho_N\rangle^2}. \tag{11}$$

Abrahams and Stephen[11] have shown that in the general case of uncorrelated disorder this quantity is positive, implying fluctuations of resistance growing faster than its average value for increasing $N$. The Lyapunov exponent $\lambda$, which yields the growth of the typical resistance $\tilde{\rho}$, Eq. (3), has been also computed by a Monte Carlo procedure, and the dependence of $\lambda$ and $\beta$ on the transition probabilities $T_{ij}$ has been investigated in the energy ranges relevant for the considered models.

### A. Random alloys

#### *1. Correlated alloy*

The stochastic matrix for this model is given by

$$\mathbf{T}=\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix},$$

$T_{ab}$ representing the probability of finding a site of energy $\varepsilon_a$ after a site of energy $\varepsilon_b$. Note that besides the usual normalization of the conditional probabilities, $\Sigma_i T_{ij}=1$, the above matrix also satisfies $\Sigma_j T_{ij}=1$, implying for this case the same average concentration for the two types of sites: $n_i=0.5$; $\mathbf{Y}$ in Eq. (9) takes the form

$$\mathbf{Y}=\begin{pmatrix} p\mathbf{A}_a^{\otimes q} & (1-p)\mathbf{A}_a^{\otimes q} \\ (1-p)\mathbf{A}_b^{\otimes q} & p\mathbf{A}_b^{\otimes q} \end{pmatrix},$$

where $\mathbf{A}_a=\mathbf{A}(\varepsilon_a)$ and $\mathbf{A}_b=\mathbf{A}(\varepsilon_b)$.

A spatial correlation function for the potential can be defined as $\langle\varepsilon_n\varepsilon_m\rangle$, from which results the spatial correlation length

$$l^{-1}=\lim_{|n-m|\to\infty}\frac{\ln|\langle\varepsilon_n\varepsilon_m\rangle|}{|n-m|}=-\ln|1-2p|. \tag{12}$$

Notice that when $p=1/2$ the correlation length vanishes, and the largest eigenvalue of $\mathbf{Y}$ is the same as that of the matrix $(\mathbf{A}_a^{\otimes q}+\mathbf{A}_b^{\otimes q})/2$ (see Appendix A), which corresponds to the uncorrelated case.[14] When $p\to 1$ the system shows very long homogeneous sequences with the same $\varepsilon_i$, whereas when $p\to 0$ it resembles a periodic system with a few defects. In both limits, $l\to\infty$.

The correlated random alloy has been investigated by several authors,[4,5] showing, among other results, that the localization length $\xi$ defined as the inverse of the Lyapunov exponent is not generally proportional to the spatial correlation length of the potential.

In Fig. 1, $2\lambda=2\xi^{-1}$ is plotted for different $p$ as a function of $E$ for $\varepsilon_a=-\varepsilon_b=\varepsilon=0.5$. The band spectrum of the pure $\varepsilon_i$ system is given by $\varepsilon_i-2\leq E\leq\varepsilon_i+2$ whereas, with this choice of parameters, the spectrum of the pure periodic system is $\varepsilon^2\leq E^2\leq 4+\varepsilon$ $(0.5\leq|E|\leq\sqrt{4.5})$. When $E$ is close to or in the gap of the periodic system obtained for $p=0$,

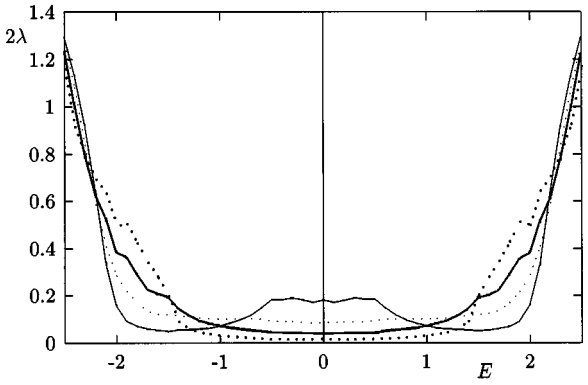


FIG. 1. Exponential growth rate of the typical resistance $2\lambda$ for the correlated random alloy (Sec. III A) at different values of correlation: $p=0.2$ (thin solid line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

λ is maximum at small values of $p$ and decreases for increasing $p$. On the contrary, when $E$ approaches the band edge of the periodic system ($|E|>1.5$), λ shows a maximum at high values of $p$, i.e., when long sequences of pure $\varepsilon_i$ chains are present in the system, and decreases as $p$ decreases. This behavior is independent of the potential correlation length and can be understood if one notes that at a given energy the largest contributions to the Lyapunov exponent, i.e., to the localization of the wave function, are given by sequences of sites that are ''out of the band'' at that energy, i.e., sequences whose characteristic energy band (obtained in the infinite length limit) does not contain the considered energy. Therefore, when the system is very similar to a periodic one the wave function is more localized for energies near or in the gap, while for $|E|>2-|\varepsilon_i|$ one of the two types of site is out of its own band, and maximum localization is obtained when long sequences of consecutive identical sites are generated, i.e., for $p$ close to 1.

Figure 2 shows that the first low moments of resistance grow with the system size in the same qualitative way as λ, but from Fig. 3 it can be seen that relative fluctuations, Eq. (11), behave in a quite different way, $\beta$ exhibiting a very different dependence on $p$ and $E$. When $E$ is in the gap of

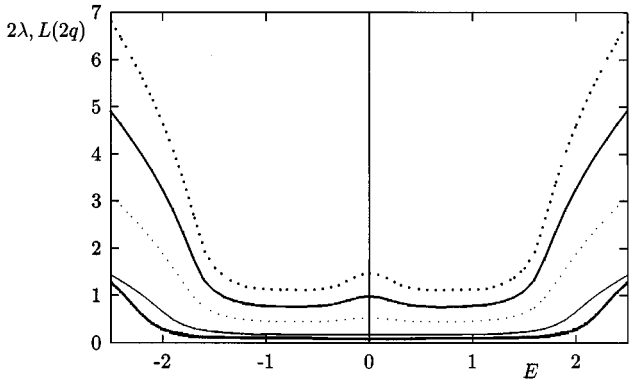


FIG. 2. Cumulants $L(2q)$ and double of the Lyapunov exponent $2\lambda$ for the correlated alloy at $p=0.4$: $q=1$ (thin line), $q=2$ (dotted line), $q=3$ (thick line), $q=4$ (bullets), and $2\lambda$ (bold line).

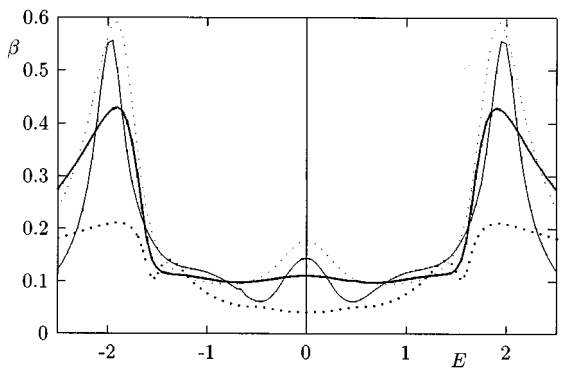


FIG. 3. Exponential growth rate of relative fluctuations $\beta$ in the correlated alloy at different correlations: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

the pure periodic system, $-0.5\leqslant E\leqslant 0.5$, the smallest values of $\beta$ are obtained when more sequences ''in the band'' are present (that is, for $p$ close to unity) in analogy with λ and $L(q)$. But the largest values are not attained for $p$ tending to zero, as one could expect. Depending on the energy, there is some intermediate value of $p$ which maximizes fluctuations. The same happens between the band edge of the periodic system and that of a pure $\varepsilon_i$ system ($1.5<|E|<\approx 2.1$). In these regions a maximum $\beta$ is reached for $p\approx 0.4$. Such a behavior could suggest for these energy regions a proportionality between the relative fluctuations of resistance and the potential correlation length, with maximum fluctuations corresponding to about the minimum correlation. For the particular model in consideration, the inverse correlation length as a function of $p$, Eq. (12), is approximately proportional to the entropy funtion measuring the degree of correlated disorder,

$$H=-\Sigma_j n_j \Sigma_i T_{ij}\ln T_{ij}\,, \tag{13}$$

which in the present case reads

$$H=-p\ln(p)-(1-p)\ln(1-p).$$

Both $l^{-1}$ and $H$ are convex functions of $p$, with a maximum at $p=0.5$, and vanish at $p=0$ and $p=1$. Nevertheless, such a common behavior is not general. In the cases discussed below these quantitities will be found to behave in different ways, showing that at least at some energy values resistance fluctuations seem rather to be related to the entropy than to the potential correlation length. One more thing to note is that $\beta$ is always positive. Since the same property is observed in all the cases considered in the following, we conclude that a faster growth of variance with respect to the average value when the system size increases can be likely considered a general property of correlated system, in analogy to the case of uncorrelated potentials.[11]

#### *2. Uncorrelated alloy*

It can be interesting to compare the present model with the case of an uncorrelated alloy. Figure 4 shows the resistance growth rate for a system in which $\varepsilon_a$ ($\varepsilon_b$) occurs with unconditional probability $p$ $(1-p)$. As expected the Lyapunov exponent is larger when the presence of the sites

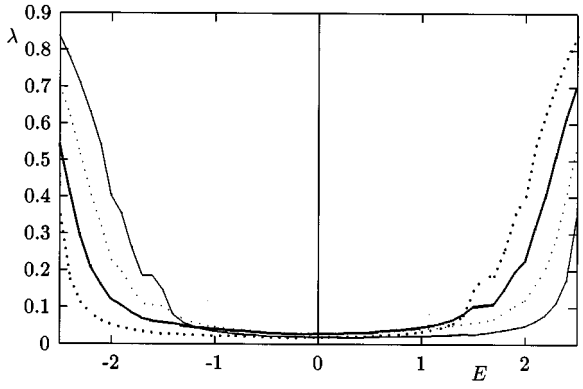


FIG. 4. Lyapunov exponent λ for the uncorrelated alloy (Sec. III A) at different concentrations $p$ of the $a$ species: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

''out of the band'' is larger. However, it is just in this situation that fluctuations attain their minimum (Fig. 5). In the region $1.5<E<2.5$ $(-2.5<E<-1.5)$, which is out of the band of $\varepsilon_a$ $(\varepsilon_b)$, fluctuations grow faster when there are few sites of energy $\varepsilon_a$ $(\varepsilon_b)$. Thus we come to the unexpected result that in an almost pure system with a low defect concentration, fluctuations of resistance depend very heavily on the position and on the actual number of these defects. On the other hand, intuition would suggest that maximum fluctuations should correspond to the maximum entropy of the system $S=-\Sigma_i p_i \ln p_i$, which is attained at $p=0.5$. Actually it can be seen that this happens only at energies within the band gap of the limit periodic system (Fig. 5). Similar features are found in the correlated systems discussed below, suggesting that they could be common to those disordered chains composed of two fundamental units, when one of the two is present at low values of concentration and the energy is out of its characteristic band. In the absence of correlation such units are to be identified with sites of the two species $a$ and $b$. When correlation is introduced, the role of basic elements in the system is played by consecutive sequences of identical sites or of alternate sites, namely, consecutive $a$, $b$, or $ab$ sequences. From this point of view, while the uncorrelated alloy and the other systems discussed in the following are formed by only two of such units, the correlated

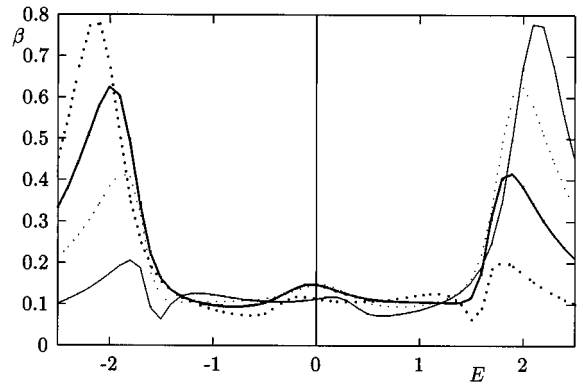


FIG. 5. $\beta$ for uncorrelated alloy at different concentrations: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

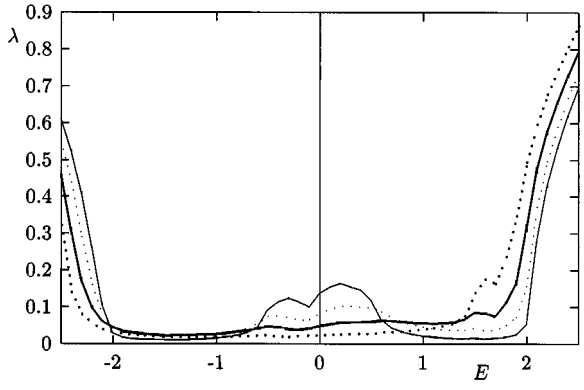


FIG. 6. λ for the chain with single insertion at different value of correlation (Sec. III B): $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

alloy has the structure of a ternary system since all the above three types of sequence appear in it.

### B. Single insertion chain

The introduction of Markovian correlations makes it possible to construct a variety of very different models. One of these is inspired to the structure of disordered doped polynaniline (emeraldine), a linear polymer in which quinoid rings are randomly placed in a chain of benzoid rings, and that is an important intermediate stage in the realization of conducting polyaniline.[16] However, we are not interested here in a faithful modeling of such polymers which would need the introduction of different hopping terms in Eq. (1).

Correlation in this case is described by the matrix

$$\mathbf{T}=\begin{pmatrix} p & 1 \\ 1-p & 0 \end{pmatrix},$$

implying, as desired, that a site of kind $a$ may be followed by another site of the same kind with probability $p$, and that a site of kind $b$ is never followed by a site of the same kind. The average concentration of the two species is $n_a=1/(2-p)$ and $n_b=(1-p)/(2-p)$; as a curiosity it can be note that maximum compositional entropy of the system is obtained for $p=p_m=(\sqrt{5}-1)/2$, the golden mean.

The behavior of the Lyapunov exponent, shown in Fig. 6, can be understood in this case with the same arguments used for the correlated and uncorrelated alloys discussed above: when the system is similar to a periodic one with defects, that is, at low values of $p$, maximum typical resistance growth is obtained out of the band of the pure periodic system. It is also obtained when the system resembles a pure $a$ type system, i.e., for $p$ close to 1, at energies $E>\varepsilon_a+2$ (Fig. 7), that is, out of the band of the pure $a$ system. On the other hand, $\beta$ is found to have a maximum at $p\approx p_m$ in the whole relevant energy range, with the exception of the region $E<\approx-\sqrt{4+\epsilon}$. This region belongs only to the spectrum of the pure $a$ system, and possesses maximum fluctuations at very low concentration of $ab$ sequences $(p\to 1)$. On the other hand, low concentrations of $a$ sequences $(p\to 0)$ give rise to relevant fluctuations for $E>2+\epsilon_a=1.5$, namely,

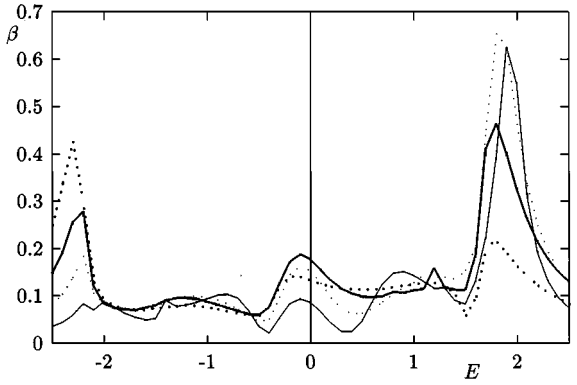


FIG. 7. Growth rate $\beta$ for the single insertion chain: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (thick line), and $p=0.8$ (bullets).

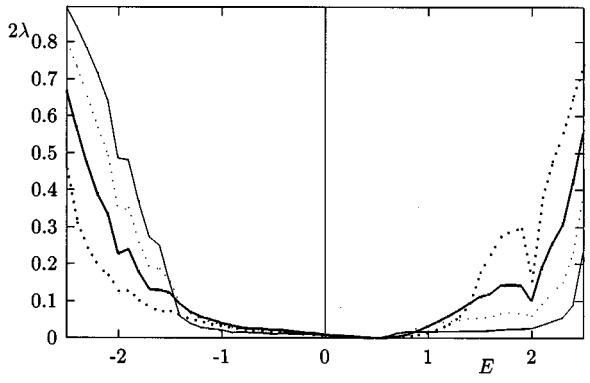


FIG. 8. Lyapunov exponent λ for the random dimer model (Sec. III C) for different values of correlation: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (bold line), and $p=0.8$ (bullets).

out of the characteristic band of $a$ but within the characteristic band of the periodic system. Thus from this point of view we are in a situation very similar to that of the uncorrelated alloy, where changes in the position of a few defects ($ab$ pairs in the present case) or statistical fluctuations of their concentration can sensibly modify the value of resistance.

Finally, this case yields strong evidence for the absence of a relation among the typical growth rate λ, its fluctuations $\beta$, and the potential correlation length $l$. Indeed, in the present case the potential spatial correlation length $l$ and the generalized entropy $H$ behave in a completely different way as functions of $p$:

$$H=-\frac{p}{2-p}\ln p-\frac{1-p}{2-p}\ln(1-p)$$

is a convex function with a maximum at $p=p_m$, and

$$l=-\frac{1}{\ln(1-p)}$$

is monotone with limit values $l=0$ for $p=1$ and $l=\infty$ for $p=0$.

### C. Random dimer model

This model has been investigated quite extensively in the last years[9,10] because, at variance with uncorrelated disordered one-dimensional systems, it has been found to support quantum diffusion. It has also been used[9] for describing the structure of some conducting conjugate polymers, like protonated emeraldine.[16] In this model one of the energies, say, $\varepsilon_b$, can occur only in pairs. When $|\varepsilon_a-\varepsilon_b|\leqslant 2$ (that is, when $\varepsilon_b$ is in the band of the pure $\varepsilon_a$ system) it possesses one extended state at the energy $E_0=\varepsilon_b$ that, together with a set of states very close in energy, is responsible for long-range electronic transport.

We can write the stochastic matrix describing this model as

$$\mathbf{T}=\begin{pmatrix} p & 0 & p \\ 1-p & 0 & 1-p \\ 0 & 1 & 0 \end{pmatrix}.$$

In fact, the probability of getting a site of energy $\varepsilon_a$ is nonzero only if it occurs after another $\varepsilon_a$ or after an $\varepsilon_b$ pair. Consequently we can distinguish three different kinds of sites. Sites of type 1, which possess energy $\varepsilon_a$ and which may appear after another site of the same type, or after an integer number of $\varepsilon_b$ pairs; sites of type 2, with energy $\varepsilon_b$, which may appear after sites of type 1 or after an integer number of $\varepsilon_b$ pairs; and sites of type 3, which have the same energy as type 2 but always occur after one such site, in order to complete the dimer. On the whole, the relative concentration of sites with energy $\varepsilon_a$ and $\varepsilon_b$ is $n_a=p/(2-p)$ and $n_b=2(1-p)/(2-p)$, respectively, and according to Eq. (9):

$$\mathbf{Y}=\begin{pmatrix} p\mathbf{A}_a^{\otimes q} & \mathbf{0} & p\mathbf{A}_a^{\otimes q} \\ (1-p)\mathbf{A}_b^{\otimes q} & \mathbf{0} & (1-p)\mathbf{A}_b^{\otimes q} \\ \mathbf{0} & \mathbf{A}_b^{\otimes q} & \mathbf{0} \end{pmatrix}.$$

Numerical resources required for computation of the maximum eigenvalue of the above matrix are sensibly reduced by the observation (Appendix B) that $\mathbf{Y}$ possesses the same eigenvalues as

$$\begin{pmatrix} p\mathbf{A}_a^{\otimes q} & \mathbf{A}_b^{\otimes q} \\ (1-p)\mathbf{A}_b^{\otimes q} & \mathbf{0} \end{pmatrix}$$

(the remaining eigenvalues being equal to zero).

The existence of the extended state is signaled by a vanishing Lyapunov exponent λ at $E_0=\varepsilon_b$ (Fig. 8). This happens independently of the probability $p$, which, on the other hand, rules the behavior of λ in the whole energy range. As in the previous cases the dependence of λ on $p$ can be understood in terms of the probability of the occurrence of ''in-band'' and ''out-of-band'' sequences of sites and is not ruled by the potential correlation length which has the same dependence on $p$ as in the single insertion chain:

$$l=-\frac{1}{\ln(1-p)};$$

also in this case maximum entropy is obtained for $p=p_m=[\sqrt{(5)}-1]/2$.

With respect to the growth of fluctuations the same considerations made in the previous cases hold, with larger values corresponding to large compositional entropies at energies belonging to the common band of the two constituent species, and to low concentration of one of the two species at energies not belonging to the band of the corresponding pure system. Moreover, as we have shown in a previous paper,[15] this model exhibits universal fluctuations of the finite-size localization lengths when the energy is close to the one of extended state. When $E$ approaches $E_0$ the probability distribution for different growth rates $\gamma$ in finite sequences tends to a log normal whose mean and variance depend on a single parameter, the typical localization length:

$$P(\gamma)=\sqrt{\frac{\xi}{2\pi N}}\exp\left\{-\frac{(\gamma-N/\xi)^2}{2N/\xi}\right\}. \tag{14}$$

This implies in turn a log-normal statistics for resistance, since the ratio between reflection and transmission coefficients in Eq. (2) is exponetially dominated by the Lyapunov exponent for large $N$. Thus the localization length is also the only parameter ruling the behavior of resistance in this asymptotic limit. Convergence toward such a distribution seems to be characteristic of localization-delocalization transitions, and has been observed both in one-dimensional systems at low values of disorder and in higher-dimensional systems close to the mobility edge.[17]

## IV. FLUCTUATIONS AND WAVE FUNCTION GROWTH RATE PROBABILITIES

It has been observed in the previous section that the growth of resistance fluctuations can be very large at energies out of the characteristic band of the pure system formed by one of the two basic units composing the alloy. Remarkably, the largest fluctuations in this situation are not obtained at high concentrations of this unit or for large values of the compositional entropy, but when such a unit is very scarce. Thus, the physical picture is that in a quasipure system with a low concentration of defects (i.e., of units for which the considered energy is out of the related energy band) the actual value of resistance can change very much from sample to sample, due to a different position of the defects or to their different concentration, which is sample independent only in the thermodynamic limit. On the other hand, in alloys with comparable concentrations of the two basic sequences, self-averaging appears to become more effective, making ordering and the actual concentration in the chain less important, with maximum fluctuations corresponding to about maximum values of the compositional entropy. As recalled in Sec. I the growth of resistance is connected with the growth of solutions of Eq. (1). It is therefore possible to relate resistance fluctuations to the probability distribution of different growth rates in finite systems.

The problem of different growth rates of the wave function in different realizations of a disordered chain has been addressed by Paladin and Vulpiani[18] by introducing an entropy function describing the limit probability distribution for growth rates different from the Lyapunov exponent $\lambda$. By grouping together the sequences which lead to a growth rate within a range $d\gamma$ around a value $\gamma$, the moments of the wave function, Eq. (4), are expressed by

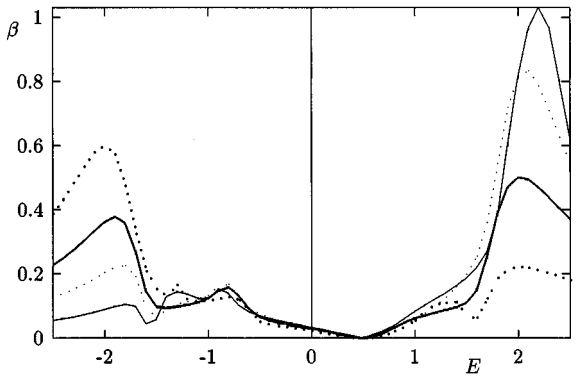


FIG. 9. Fluctuation exponential growth rate $\beta$ for the random dimer model: $p=0.2$ (thin line), $p=0.4$ (dotted line), $p=0.6$ (bold line), and $p=0.8$ (bullets).

$$\langle|\psi_N|^q\rangle\approx\int d\gamma P_N(\gamma)e^{Nq\gamma},$$

where $P_N(\gamma)$ is the probability density for an exponential growth rate $\gamma$ in a chain of $N$ sites. By assuming $P_N(\gamma)$ to vanish exponentially for $N\to\infty$, the entropy $S(\gamma)\geq 0$ is defined by

$$P_N(\gamma)\propto e^{-NS(\gamma)}.$$

For $\gamma=\lambda$, $S=0$, and a saddle point approximation for the integral yields

$$L(q)=\lim_{N\to\infty}\frac{1}{N}\ln\left\langle\left|\frac{\psi_N}{\psi_0}\right|^q\right\rangle=\underbrace{\max}_{\text{over }\gamma}[q\gamma-S(\gamma)], \tag{15}$$

generating the Legendre transformation

$$\frac{dL(q)}{dq}=\gamma^*$$

and

$$L(q)=q\gamma^*-S(\gamma^*);$$

$\gamma^*=\gamma^*(q)$ is the value of $\gamma$ that maximizes Eq. (15) for each given $q$, yielding the most relevant contribution to the integral for large $N$.

The entropy $S(\gamma^*)$ can be obtained from $L(q)$ by inverting the above relations:

$$S(\gamma^*)=q\frac{dL(q)}{dq}-L(q). \tag{16}$$

It is easily seen that $\beta$ of Eq. (11) (see Fig. 9) can be expressed as an integral of $S$, establishing a relation between the exponential growth rate of relative fluctuations of resistance and the wave function probability distribution:

$$\frac{\langle \rho_N^2 \rangle}{\langle \rho_N \rangle^2} \simeq \exp\left[ 4N \int_2^4 \frac{S(\gamma^*)}{q^2} dq \right]. \tag{17}$$

In the Gaussian case $S$ takes the form $S(\gamma) = (\gamma - N\lambda)^2/(2N\sigma^2)$, and

$$\frac{\langle \rho_N^2 \rangle}{\langle \rho_N \rangle^2} \simeq e^{4N\sigma^2}.$$

## V. SUMMARY

We computed and discussed the dc resistance fluctuations in some one-dimensional Anderson models with correlated random potentials. We considered in particular the asymptotic growth rates of the first low moments and found that the variance grows faster than the average resistance when increasing the system length, as in the case of uncorrelated disorder.[11] In all the examined models first to fourth moments of the resistance distribution have been observed to behave qualitatively in the same way as the typical value, but the growth of relative fluctuations $\langle \rho_N^2 \rangle / \langle \rho_N \rangle^2$ has been shown to be possibly very large also when typical and average resistance grow slowly. In particular, at energies out of the characteristic band of the system made of only one kind of atom, we observe the fastest growth when such a kind is present in a small quantity, showing that the resistance of different realizations of slightly doped systems with same statistical composition is very sensible to the actual position and concentration fluctuations of few defects. This is caused by the different values taken by the localization lengths of the eigenstates in different realizations or system portions, and can be quantitatively related to the entropy function characterizing the corresponding probability distribution. No evidence is found of a dependence of statistical quantities on the potential correlation length, but in some limited range of energy a qualitative relation with compositional entropy can be observed.

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## APPENDIX A: THE CORRELATED RANDOM ALLOY

The stochastic matrix $\mathbf{T}$ is in this case

$$\mathbf{T} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

and $\mathbf{Y}$ is

$$\mathbf{Y} = \begin{pmatrix} p\mathbf{B}_a & (1-p)\mathbf{B}_a \\ (1-p)\mathbf{B}_b & p\mathbf{B}_b \end{pmatrix},$$

where $\mathbf{B}_a = [\mathbf{A}(\varepsilon_a)]^{\otimes q}$ and $\mathbf{B}_b = [\mathbf{A}(\varepsilon_b)]^{\otimes q}$. Notice that when $p = 1/2$ the correlation is lost and the largest eigenvalue of $\mathbf{Y}$ is the same as that of the matrix $(\mathbf{B}_a + \mathbf{B}_b)/2$ which is the correct expression for the uncorrelated case.[14] Indeed, when $p = 1/2$, the following identities hold:

$$\frac{1}{2}\begin{pmatrix} \mathbf{B}_a & \mathbf{B}_a \\ \mathbf{B}_b & \mathbf{B}_b \end{pmatrix}\begin{pmatrix} \mathbf{U} \\ -\mathbf{U} \end{pmatrix} = 0,$$

where $\mathbf{U}$ is a column matrix and

$$(\mathbf{V} \quad \mathbf{V})\frac{1}{2}\begin{pmatrix} \mathbf{B}_a & \mathbf{B}_a \\ \mathbf{B}_b & \mathbf{B}_b \end{pmatrix} = [\tfrac{1}{2}\mathbf{V}(\mathbf{B}_a + \mathbf{B}_b) \quad \tfrac{1}{2}\mathbf{V}(\mathbf{B}_a + \mathbf{B}_b)]$$
$$= \lambda(\mathbf{V} \quad \mathbf{V}),$$

where $\mathbf{V}$ is a row matrix or

$$\tfrac{1}{2}\mathbf{V}(\mathbf{B}_a + \mathbf{B}_b) = \lambda \mathbf{V},$$

from which follows that each eigenvalue of $(\mathbf{B}_a + \mathbf{B}_b)/2$ is an eigenvalue of $\mathbf{Y}$. The remaining eigenvalues are equal to zero.

## APPENDIX B: THE RANDOM DIMER MODEL

In this case the stochastic matrix is given by

$$\mathbf{T} = \begin{pmatrix} p & 0 & p \\ 1-p & 0 & 1-p \\ 0 & 1 & 0 \end{pmatrix},$$

so that

$$\mathbf{Y} = \begin{pmatrix} p\mathbf{B}_a & \mathbf{0} & p\mathbf{B}_a \\ (1-p)\mathbf{B}_b & \mathbf{0} & (1-p)\mathbf{B}_b \\ \mathbf{0} & \mathbf{B}_b & \mathbf{0} \end{pmatrix}.$$

Consider the identity

$$\begin{pmatrix} p\mathbf{B}_a & \mathbf{0} & p\mathbf{B}_a \\ (1-p)\mathbf{B}_b & \mathbf{0} & (1-p)\mathbf{B}_b \\ \mathbf{0} & \mathbf{B}_b & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{U} \\ \mathbf{0} \\ -\mathbf{U} \end{pmatrix} = 0,$$

where $\mathbf{U}$ is a column matrix and

$$(\mathbf{V} \quad \mathbf{W} \quad \mathbf{V})\begin{pmatrix} p\mathbf{B}_a & \mathbf{0} & p\mathbf{B}_a \\ (1-p)\mathbf{B}_b & \mathbf{0} & (1-p)\mathbf{B}_b \\ \mathbf{0} & \mathbf{B}_b & \mathbf{0} \end{pmatrix}$$
$$= [p\mathbf{V}\mathbf{B}_a + (1-p)\mathbf{W}\mathbf{B}_b \quad \mathbf{W}\mathbf{B}_b \quad p\mathbf{V}\mathbf{B}_a + (1-p)\mathbf{W}\mathbf{B}_b]$$
$$= \lambda(\mathbf{V} \quad \mathbf{W} \quad \mathbf{V}).$$

This is equivalent to

$$(\mathbf{V} \quad \mathbf{W})\begin{pmatrix} p\mathbf{B}_a & \mathbf{B}_b \\ (1-p)\mathbf{B}_b & \mathbf{0} \end{pmatrix} = \lambda(\mathbf{V} \quad \mathbf{W}),$$

where $\mathbf{V}$ and $\mathbf{W}$ are row matrices. We can conclude that each eigenvalue of the matrix

$$\begin{pmatrix} p\mathbf{B}_a & \mathbf{B}_b \\ (1-p)\mathbf{B}_b & \mathbf{0} \end{pmatrix}$$

is an eigenvalue of $\mathbf{Y}$, the remaining eigenvalues being equal to zero.

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