Piezoelectric effects in quasicrystals

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(Received 20 December 1996)

Piezoelectric effects in quasicrystals induced by both phonon and phason fields are investigated. Some characteristic features are predicted. Nonvanishing piezoelectric coefficients are deduced for two-dimensional quasicrystals with crystallographically allowable and forbidden symmetries as well as three-dimensional icosahedral and cubic quasicrystals. Our results show that (1) while there may be no piezoelectric effect in quasicrystals induced by phonon fields, the effect may yet be nonzero due to phason fields, and (2) the piezoelectric effect may be due (a) only to phonon fields, (b) only to phason fields, or (c) to both. [S0163-1829(97)01129-6]

I. INTRODUCTION

Since the discovery of a quasicrystal (QC) in an Al-Mn alloy in 1984, significant progress has been made concerning the structural and static properties. However, the knowledge and understanding of the physical properties beyond those are still limited. Some investigations on elasticity have been made by a number of authors.[1–4] Fujiwara, Laissardiere, and Yamamoto[5] have discussed the electronic structure and electron transport of quasicrystals. Brandmüller and Claus[6] have calculated the piezoelectricity tensor which is restricted to that part associated with the phonon degree of freedom. Obviously, a full description of piezoelectric effects in QC's requires one to consider both phonon and phason fields. Yang and co-workers[7,8] have recently studied the thermodynamics of equilibrium properties of QC's, which is realized by extending related formulation for ordinary crystals[9] to the case of QC's.

On the other hand, the extensive usage of piezoelectric crystals has made a great change in this world.[9,10] It is natural to ask what is the piezoelectric behavior of a quasicrystal. Although it is still difficult to investigate this property experimentally because of the small size of the samples for most of the quasicrystals and the difficulty to measure effects pertaining to the phason field, we think it is worthwhile to propose a theoretical insight into it. This is the purpose of this paper. We will use group theory to explore the piezoelectricity of QC's including the contribution from both phonon and phason fields. With the help of this method we discuss piezoelectric effects in all two-dimensional (2D) quasicrystals with crystallographically allowable and forbidden symmetries[11] and three-dimensional (3D) icosahedral and cubic quasicrystals. It is found that the piezoelectric behavior of QC's is more complicated than that of ordinary crystals because of the presence of the phason field. The nonvanishing piezoelectric constants are tabulated for all 2D and 3D QC's in Tables I–III. The following section is devoted to deducing the nonvanishing piezoelectric components of QC's by means of thermodynamic and group-theoretical methods. Conclusions are given in Sec. III.

II. NONVANISHING PIEZOELECTRIC CONSTANTS

A. Thermodynamic consideration

It is well known that a prominent feature that distinguishes QC's from ordinary crystals is the appearance of the phason degree of freedom in hydrodynamics for QC's, which leads to two kinds of strain fields, the phonon strain

$$E_{ij}=\frac{1}{2}\,(\partial_j u_i+\partial_i u_j), \tag{1}$$

and the phason strain

$$W_{ij}=\partial_j w_i$$

as well as two kinds of stress fields: the phonon stress T_{ij} and the phason stress H_{ij}.[3,4] In this case, the first and second laws of thermodynamics take the following form:

$$dU=T_{ij}dE_{ij}+H_{ij}dW_{ij}+F_i dD_i+\theta dS. \tag{2}$$

The Gibbs free energy is

$$G=U-T_{ij}E_{ij}-H_{ij}W_{ij}-F_iD_i-\theta S. \tag{3}$$

Combining it with Eq. (2) we have

$$dG=-E_{ij}dT_{ij}-W_{ij}dH_{ij}-D_idF_i-Sd\theta, \tag{4}$$

where F is the electric field, D is the electric displacement, θ is the temperature, and S is the entropy. In isothermal conditions, the electric displacement caused by stresses (direct piezoelectric effect) obeys

$$\begin{aligned} dD_i&=-\left(\frac{\partial^2 G}{\partial F_i\partial T_{jk}}\right)_\theta dT_{jk}-\left(\frac{\partial^2 G}{\partial F_i\partial H_{jk}}\right)_\theta dH_{jk}\\ &=d^{(1)}_{ijk}dT_{jk}+d^{(2)}_{ijk}dH_{jk}. \end{aligned} \tag{5}$$

For the converse piezoelectric effect

$$\begin{aligned} dE_{ij}&=-\left(\frac{\partial^2 G}{\partial T_{ij}\partial F_k}\right)_\theta dF_k=\underline{d}^{(1)}_{ijk}dF_k,\\ dW_{ij}&=-\left(\frac{\partial^2 G}{\partial H_{ij}\partial F_k}\right)_\theta dF_k=\underline{d}^{(2)}_{ijk}dF_k. \end{aligned} \tag{6}$$

The coefficients $d^{(1)}_{ijk}$, $d^{(2)}_{ijk}$, $\underline{d}^{(1)}_{ijk}$, $\underline{d}^{(2)}_{ijk}$ are called piezoelectric constants, which are the tensors of rank 3. The components can be divided into two types denoted by superscripts (1) and (2), which are associated with the phonon and phason fields,

TABLE I. Piezoelectric constants for 2D QC's with crystallographically allowable symmetries. In this table the indices jk in the phason strain W_{jk} are arranged in the order of 11, 22, 23, 12, 13, and 21.

Point groups	Piezoelectric constants $d^{(1)}$	$d^{(2)}$
1	$\begin{pmatrix} d^{(1)}_{11} & d^{(1)}_{12} & d^{(1)}_{13} & d^{(1)}_{14} & d^{(1)}_{15} & d^{(1)}_{16} \\ d^{(1)}_{21} & d^{(1)}_{22} & d^{(1)}_{23} & d^{(1)}_{24} & d^{(1)}_{25} & d^{(1)}_{26} \\ d^{(1)}_{31} & d^{(1)}_{32} & d^{(1)}_{33} & d^{(1)}_{34} & d^{(1)}_{35} & d^{(1)}_{36} \end{pmatrix}_{18}$	$\begin{pmatrix} d^{(2)}_{111} & d^{(2)}_{122} & d^{(2)}_{123} & d^{(2)}_{112} & d^{(2)}_{113} & d^{(2)}_{121} \\ d^{(2)}_{211} & d^{(3)}_{222} & d^{(2)}_{223} & d^{(2)}_{212} & d^{(2)}_{213} & d^{(2)}_{221} \\ d^{(2)}_{311} & d^{(2)}_{322} & d^{(2)}_{323} & d^{(2)}_{312} & d^{(2)}_{313} & d^{(2)}_{321} \end{pmatrix}_{18}$
2 $(2 \parallel x_2)$	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & 0 & d^{(1)}_{16} \\ d^{(1)}_{21} & d^{(1)}_{22} & d^{(1)}_{23} & 0 & d^{(1)}_{25} & 0 \\ 0 & 0 & 0 & d^{(1)}_{34} & 0 & d^{(1)}_{36} \end{pmatrix}_{8}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & d^{(2)}_{112} & 0 & d^{(2)}_{121} \\ d^{(2)}_{211} & d^{(2)}_{222} & 0 & 0 & d^{(2)}_{213} & 0 \\ 0 & 0 & d^{(2)}_{323} & d^{(2)}_{312} & 0 & d^{(2)}_{321} \end{pmatrix}_{9}$
2 $(2 \parallel x_3)$	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & d^{(1)}_{24} & d^{(1)}_{25} & 0 \\ d^{(1)}_{31} & d^{(1)}_{32} & d^{(1)}_{33} & 0 & 0 & d^{(1)}_{36} \end{pmatrix}_{8}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & d^{(2)}_{223} & 0 & d^{(2)}_{213} & 0 \\ d^{(2)}_{311} & d^{(2)}_{322} & 0 & d^{(2)}_{312} & 0 & d^{(2)}_{321} \end{pmatrix}_{8}$
m $(m \perp x_2)$	$\begin{pmatrix} d^{(1)}_{11} & d^{(1)}_{12} & d^{(1)}_{13} & 0 & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & d^{(1)}_{24} & 0 & d^{(1)}_{26} \\ d^{(1)}_{31} & d^{(1)}_{32} & d^{(1)}_{33} & 0 & d^{(1)}_{35} & 0 \end{pmatrix}_{10}$	$\begin{pmatrix} d^{(2)}_{111} & d^{(2)}_{122} & 0 & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & d^{(2)}_{223} & d^{(2)}_{212} & 0 & d^{(2)}_{221} \\ d^{(2)}_{311} & d^{(2)}_{322} & 0 & 0 & d^{(3)}_{313} & 0 \end{pmatrix}_{9}$
m $(m \perp x_3)$	$\begin{pmatrix} d^{(1)}_{11} & d^{(1)}_{12} & d^{(1)}_{13} & 0 & 0 & d^{(1)}_{16} \\ d^{(1)}_{21} & d^{(1)}_{22} & d^{(1)}_{23} & 0 & 0 & d^{(1)}_{26} \\ 0 & 0 & 0 & d^{(1)}_{34} & d^{(1)}_{35} & 0 \end{pmatrix}_{10}$	$\begin{pmatrix} d^{(2)}_{111} & d^{(2)}_{122} & 0 & d^{(2)}_{112} & 0 & d^{(3)}_{121} \\ d^{(2)}_{211} & d^{(2)}_{222} & 0 & d^{(2)}_{212} & 0 & d^{(2)}_{221} \\ 0 & 0 & d^{(2)}_{323} & 0 & d^{(2)}_{313} & 0 \end{pmatrix}_{10}$
222	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d^{(1)}_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d^{(1)}_{36} \end{pmatrix}_{3}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d^{(2)}_{213} & 0 \\ 0 & 0 & 0 & d^{(2)}_{312} & 0 & d^{(2)}_{321} \end{pmatrix}_{4}$
$mm2$ $(2 \parallel x_3)$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & d^{(1)}_{24} & 0 & 0 \\ d^{(1)}_{31} & d^{(1)}_{32} & d^{(1)}_{33} & 0 & 0 & 0 \end{pmatrix}_{5}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & d^{(2)}_{223} & 0 & 0 & 0 \\ d^{(2)}_{311} & d^{(2)}_{322} & 0 & 0 & 0 & 0 \end{pmatrix}_{4}$
$2mm$ $(2 \parallel x_1)$	$\begin{pmatrix} d^{(1)}_{11} & d^{(1)}_{12} & d^{(1)}_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d^{(1)}_{26} \\ 0 & 0 & 0 & 0 & d^{(1)}_{35} & 0 \end{pmatrix}_{5}$	$\begin{pmatrix} d^{(2)}_{111} & d^{(2)}_{122} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d^{(2)}_{212} & 0 & d^{(2)}_{221} \\ 0 & 0 & 0 & 0 & d^{(2)}_{313} & 0 \end{pmatrix}_{5}$
4	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & d^{(1)}_{15} & -d^{(1)}_{14} & 0 \\ d^{(1)}_{31} & d^{(1)}_{31} & d^{(1)}_{33} & 0 & 0 & 0 \end{pmatrix}_{4}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & d^{(2)}_{113} & 0 & -d^{(2)}_{123} & 0 \\ d^{(2)}_{311} & d^{(2)}_{311} & 0 & d^{(2)}_{312} & 0 & -d^{(2)}_{312} \end{pmatrix}_{4}$
$\overline{4}$	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & -d^{(1)}_{15} & d^{(1)}_{14} & 0 \\ d^{(1)}_{31} & -d^{(1)}_{31} & 0 & 0 & 0 & d^{(1)}_{36} \end{pmatrix}_{4}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & -d^{(2)}_{113} & 0 & d^{(2)}_{123} & 0 \\ d^{(2)}_{311} & -d^{(2)}_{311} & 0 & d^{(2)}_{312} & 0 & d^{(2)}_{312} \end{pmatrix}_{4}$
422	$\begin{pmatrix} 0 & 0 & 0 & d^{(1)}_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d^{(1)}_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{1}$	$\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d^{(2)}_{123} & 0 \\ 0 & 0 & 0 & d^{(2)}_{312} & 0 & -d^{(2)}_{312} \end{pmatrix}_{2}$
$4mm$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d^{(1)}_{15} & 0 \\ 0 & 0 & 0 & d^{(1)}_{15} & 0 & 0 \\ d^{(1)}_{31} & d^{(1)}_{31} & d^{(1)}_{33} & 0 & 0 & 0 \end{pmatrix}_{3}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & d^{(2)}_{113} & 0 & 0 & 0 \\ d^{(2)}_{311} & d^{(2)}_{311} & 0 & 0 & 0 & 0 \end{pmatrix}_{2}$

TABLE I. (*Continued*).

	Piezoelectric constants	
Point groups	$d^{(1)}$	$d^{(2)}$
$\overline{4}2m$ $(2\parallel x_1)$	$\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36}^{(1)} \end{pmatrix}_2$	$\begin{pmatrix} 0 & 0 & d_{123}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 \\ 0 & 0 & 0 & d_{312}^{(2)} & 0 & d_{312}^{(2)} \end{pmatrix}_2$
3	$\begin{pmatrix} d_{11}^{(1)} & -d_{11}^{(1)} & 0 & d_{14}^{(1)} & d_{15}^{(1)} & -2d_{22}^{(1)} \\ -d_{22}^{(1)} & d_{22}^{(1)} & 0 & d_{15}^{(1)} & -d_{14}^{(1)} & -2d_{11}^{(1)} \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_6$	$\begin{pmatrix} d_{111}^{(2)} & -d_{111}^{(2)} & d_{123}^{(2)} & -d_{222}^{(2)} & d_{113}^{(2)} & -d_{222}^{(2)} \\ -d_{222}^{(2)} & d_{222}^{(2)} & d_{113}^{(2)} & -d_{111}^{(2)} & -d_{123}^{(2)} & -d_{111}^{(2)} \\ d_{311}^{(3)} & d_{311}^{(2)} & 0 & d_{312}^{(2)} & 0 & -d_{312}^{(2)} \end{pmatrix}_6$
32 $(2\parallel x_1)$	$\begin{pmatrix} d_{11}^{(1)} & -d_{11}^{(1)} & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14}^{(1)} & -2d_{11}^{(1)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_2$	$\begin{pmatrix} d_{111}^{(2)} & -d_{111}^{(2)} & d_{123}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_{111}^{(2)} & -d_{123}^{(2)} & -d_{111}^{(2)} \\ 0 & 0 & 0 & d_{312}^{(2)} & 0 & -d_{312}^{(2)} \end{pmatrix}_3$
$3m$ $(m\perp x_1)$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15}^{(1)} & -2d_{22}^{(1)} \\ -d_{22}^{(1)} & d_{22}^{(1)} & 0 & d_{15}^{(1)} & 0 & 0 \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_4$	$\begin{pmatrix} 0 & 0 & 0 & -d_{222}^{(2)} & d_{113}^{(2)} & -d_{222}^{(2)} \\ -d_{222}^{(2)} & d_{222}^{(2)} & d_{113}^{(2)} & 0 & 0 & 0 \\ d_{311}^{(2)} & d_{311}^{(2)} & 0 & 0 & 0 & 0 \end{pmatrix}_3$
6	$\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & d_{15}^{(1)} & 0 \\ 0 & 0 & 0 & d_{15}^{(1)} & -d_{14}^{(1)} & 0 \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_4$	$\begin{pmatrix} 0 & 0 & d_{123}^{(2)} & 0 & d_{113}^{(2)} & 0 \\ 0 & 0 & d_{113}^{(2)} & 0 & -d_{123}^{(2)} & 0 \\ d_{311}^{(2)} & d_{311}^{(2)} & 0 & d_{312}^{(2)} & 0 & -d_{312}^{(2)} \end{pmatrix}_4$
$\overline{6}$	$\begin{pmatrix} d_{11}^{(1)} & -d_{11}^{(1)} & 0 & 0 & 0 & -2d_{22}^{(1)} \\ -d_{22}^{(1)} & d_{22}^{(1)} & 0 & 0 & 0 & -2d_{11}^{(1)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_2$	$\begin{pmatrix} d_{111}^{(2)} & -d_{111}^{(2)} & 0 & -d_{222}^{(2)} & 0 & -d_{222}^{(2)} \\ -d_{222}^{(2)} & d_{222}^{(2)} & 0 & -d_{111}^{(2)} & 0 & -d_{111}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_2$
622	$\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$	$\begin{pmatrix} 0 & 0 & d_{123}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{123}^{(2)} & 0 \\ 0 & 0 & 0 & d_{312}^{(2)} & 0 & -d_{312}^{(2)} \end{pmatrix}_2$
$6mm$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15}^{(1)} & 0 \\ 0 & 0 & 0 & d_{15}^{(1)} & 0 & 0 \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{113}^{(2)} & 0 \\ 0 & 0 & d_{113}^{(2)} & 0 & 0 & 0 \\ d_{311}^{(2)} & d_{311}^{(2)} & 0 & 0 & 0 & 0 \end{pmatrix}_2$
$\overline{6}m2$ $(m\perp x_1)$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -2d_{22}^{(1)} \\ -d_{22}^{(1)} & d_{22}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$	$\begin{pmatrix} 0 & 0 & 0 & -d_{222}^{(2)} & 0 & -d_{222}^{(2)} \\ -d_{222}^{(2)} & d_{222}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$
In other cases		$d^{(1)}=d^{(2)}=0$

respectively. From the symmetry property $E_{ij}=E_{ji}$ provided by their definition (1) we have

$$d_{ijk}^{(1)}=d_{ikj}^{(1)}, \quad \underline{d}_{ijk}^{(1)}=\underline{d}_{jik}^{(1)}. \tag{7}$$

Furthermore, by comparing Eqs. (5) and (6) we have

$$d_{ijk}^{(1)}=\underline{d}_{jki}^{(1)}, \quad d_{ijk}^{(2)}=\underline{d}_{jki}^{(2)}. \tag{8}$$

Consequently, the number of the independent piezoelectric constants $d_{ijk}^{(1)}$ and $\underline{d}_{ijk}^{(1)}$ is 18. The number of independent coefficients $d_{ijk}^{(2)}$ and $\underline{d}_{ijk}^{(2)}$ is 27 for 3D QC's and 18 for 2D QC's, since for 2D QC's W_{ij} with $i=1,2$ have only six components W_{11}, W_{22}, W_{23}, W_{12}, W_{13}, and W_{21}. However, additional restrictions arise from the point-group symmetry inherent in the quasicrystal considered and generally lead to a reduction in the number of independent piezoelectric constants. This is discussed in the following.

B. Number of independent piezoelectric constants

According to the higher-dimensional description of QC's a quasicrystal structure can be generated by projecting a higher-dimensional lattice (V) onto the physical space (V_E) where $V=V_E+V_I$ with V_I being the complementary space. Consequently, a vector in V_E transforms under the vector representation (Γ_A) of the symmetry group of the structure considered, whereas a vector in V_I transforms under another irreducible representation (Γ_B). Once the trans-

TABLE II. Piezoelectric constants for 2D QC's with noncrystalline symmetries. In this table the indices jk in the phason strain W_{jk} are arranged in the order of 11, 22, 23, 12, 13, and 21.

$5,10,8,12$

$$d^{(1)}=\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & d_{15}^{(1)} & 0 \\ 0 & 0 & 0 & d_{15}^{(1)} & -d_{14}^{(1)} & 0 \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_4$$

$5m,10mm,8mm,12mm$

$$d^{(1)}=\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15}^{(1)} & 0 \\ 0 & 0 & 0 & d_{15}^{(1)} & 0 & 0 \\ d_{31}^{(1)} & d_{31}^{(1)} & d_{33}^{(1)} & 0 & 0 & 0 \end{pmatrix}_3$$

$52,1022,822,1222$

$$d^{(1)}=\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$$

$5,\overline{10}$

$$d^{(2)}=\begin{pmatrix} d_{111}^{(2)} & -d_{111}^{(2)} & 0 & d_{112}^{(2)} & 0 & d_{112}^{(2)} \\ d_{112}^{(2)} & -d_{112}^{(2)} & 0 & -d_{111}^{(2)} & 0 & -d_{111}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_3$$

$52,\overline{10}2m(\text{with } 2\perp x_1)$

$$d^{(2)}=\begin{pmatrix} d_{111}^{(2)} & -d_{111}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_{111}^{(2)} & 0 & -d_{111}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$$

$5m,\overline{10}m2(\text{with } m\perp x_1)$

$$d^{(2)}=\begin{pmatrix} 0 & 0 & 0 & d_{112}^{(2)}0 & 0 & d_{112}^{(2)} \\ d_{112}^{(2)}0 & -d_{112}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_1$$

$\overline{8},\overline{12}$

$$d^{(2)}=\begin{pmatrix} 0 & 0 & d_{123}^{(2)} & 0 & d_{113}^{(2)} & 0 \\ 0 & 0 & -d_{113}^{(2)} & 0 & d_{123}^{(2)} & 0 \\ d_{311}^{(2)} & -d_{311}^{(2)} & 0 & d_{312}^{(2)} & 0 & d_{312}^{(2)} \end{pmatrix}_4$$

$\overline{8}2m,\overline{12}2m(\text{with } 2\parallel x_1)$

$$d^{(2)}=\begin{pmatrix} 0 & 0 & d_{123}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 \\ 0 & 0 & 0 & d_{312}^{(2)} & 0 & d_{312}^{(3)} \end{pmatrix}_2$$

$\overline{8}m2,\overline{12}m2(\text{with } m\perp x_1)$

$$d^{(2)}=\begin{pmatrix} 0 & 0 & 0 & 0 & d_{113}^{(2)} & 0 \\ 0 & 0 & -d_{113}^{(2)} & 0 & 0 & 0 \\ d_{311}^{(2)} & -d_{311}^{(2)} & 0 & 0 & 0 & 0 \end{pmatrix}_2$$

In other cases
Either $d^{(1)}=0$
or $d^{(2)}=0$
or $d^{(1)}=d^{(2)}=0$

TABLE III. Piezoelectric constants for 3D QC's. In this table the indices jk in the phason strain W_{jk} are arranged in the order of 11, 22, 33, 23, 31, 12, 32, 13, and 21.

	Piezoelectric constants	
Point groups	$d^{(1)}$	$d^{(3)}$
23	$\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{14}^{(1)} \end{pmatrix}_1$	$\begin{pmatrix} 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{132}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{132}^{(2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{132}^{(3)} \end{pmatrix}_2$
$\overline{43}m$	$\begin{pmatrix} 0 & 0 & 0 & d_{14}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{14}^{(1)} \end{pmatrix}_1$	$\begin{pmatrix} 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{123}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{123}^{(2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & d_{123}^{(3)} \end{pmatrix}_1$
432	$d^{(1)}=0$	$\begin{pmatrix} 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & -d_{123}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & -d_{123}^{(2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{123}^{(2)} & 0 & 0 & -d_{123}^{(2)} \end{pmatrix}_1$
$m\overline{3},m\overline{3}m$ $235,m\overline{3}\,\overline{5}$		$d^{(1)}=d^{(2)}=0$

formation properties of these vectors are specified, the physical property tensor of any rank can be determined by group representation theory.

To illustrate this procedure, consider the example of octagonal quasicrystals with $S_8(\bar{8})$ symmetry. This point group is a cyclic group, which has eight elements, and hence eight irreducible representations.[12] All are one-dimensional (1D). Among them one is the identity representation (Γ_1), one is the alternate representation (Γ_2), and the six other representations are decomposed into three pairs $(\Gamma_3,\Gamma_4,\Gamma_5)$. Two representations of each pair are conjugate to each other. In this case, $\Gamma_A=\Gamma_2+\Gamma_3$ and $\Gamma_B=\Gamma_4$. Therefore, the components $d^{(1)}_{ijk}$ $(\underline{d}^{(1)}_{ijk})$ transform under

$$(\Gamma_2+\Gamma_3)\times[(\Gamma_2+\Gamma_3)\times(\Gamma_2+\Gamma_3)]_s=4\Gamma_2+4\Gamma_3+\Gamma_4+2\Gamma_5, \quad (9)$$

where $[\cdots]_s$ is the symmetric part of the direct product. The components $d^{(2)}_{ijk}$ $(\underline{d}^{(2)}_{ijk})$ transform under

$$(\Gamma_2+\Gamma_3)\times[(\Gamma_2+\Gamma_3)\times\Gamma_4]=4\Gamma_1+\Gamma_3+4\Gamma_4+2\Gamma_5. \quad (10)$$

As it is well known, the number of nonvanishing independent components of a physical property tensor is just the number of the identity representations which are contained in the direct product. Since there is no identity representation in Eq. (9), $d^{(1)}_{ijk}=0$. This means that there is no piezoelectric effect caused by the phonon field. From Eq. (10) it follows that there are four independent components for $d^{(2)}_{ijk}$. The number of independent piezoelectric constants for each point group considered in this paper is listed in Tables I–III as a subscript attached to each matrix.

C. Matrix form of the piezoelectric constants

The determination of explicit forms for these independent components is much more complicated than counting their number. From Eqs. (5) and (6) we can see that the transformation properties of $d^{(2)}_{ijk}$ $(\underline{d}^{(2)}_{jki})$ follow directly from those for $D_i(F_i)$ and $H_{jk}(W_{jk})$. If we find the precise components of D_i and W_{jk} that transform under the same constituent representations we can construct all the invariants formed by their combinations, and then establish the independent components $(d^{(2)}_{ijk})$. Using the same method given in Ref. 12, we find that $W_{11}-W_{22}$, $W_{21}+W_{12}$, and D_3 transform under the same representation (Γ_2) giving two invariants

$$D_3(W_{11}-W_{22}),\quad D_3(W_{21}+W_{12}). \quad (11)$$

Thus, we obtain the nonvanishing components

$$d^{(2)}_{311}=-d^{(2)}_{322},\quad d^{(2)}_{321}=d^{(2)}_{312}. \quad (12)$$

Similarly, (D_1,D_2) and (W_{23},W_{13}) transform under the same representation (Γ_3) giving two other invariants

$$D_1W_{23}+D_2W_{13},\quad D_1W_{13}-D_2W_{23}. \quad (13)$$

Then the corresponding nonvanishing components are

$$d^{(2)}_{123}=d^{(2)}_{213},\quad d^{(2)}_{113}=-d^{(2)}_{223}. \quad (14)$$

The piezoelectric constant tensor for point group $S_8(\bar{8})$ can be also written in the matrix form

$$d^{(2)}=\begin{pmatrix} 0 & 0 & d^{(2)}_{123} & 0 & d^{(2)}_{113} & 0 \\ 0 & 0 & -d^{(2)}_{113} & 0 & d^{(2)}_{123} & 0 \\ d^{(2)}_{311} & -d^{(2)}_{311} & 0 & d^{(2)}_{312} & 0 & d^{(2)}_{312} \end{pmatrix}_4, \quad (15)$$

where the subscript 4 stands for the number of independent components. The double indices labeling the phason strains are arranged in the order of 11, 22, 23, 12, 13, 21. Using this method we can determine the matrix forms of the piezoelectric properties for all QC's. All the results for 2D and 3D QC's are given in Tables I–III. Most of the piezoelectric constants $d^{(1)}$ for 2D QC's with crystallographically allowable symmetries listed in Table I have already been given in Refs. 9 and 13. The forms of the piezoelectric tensors for some crystallographically forbidden symmetries can also be found in Refs. 6 and 14. These results coincide with our $d^{(1)}$ given in this paper.

III. CONCLUSION

Our results show that (1) 2D QC's with crystallographically allowable symmetries can be divided into two classes according to their piezoelectric properties. One consists of those QC's with central symmetry, which have no piezoelectric effects. The other consists of those QC's without central symmetry, which have piezoelectric effects induced by both phonon and phason fields.

(2) 2D QC's with crystallographically forbidden symmetries can be divided into four classes. The first class consists of those QC's with central symmetry $(\bar{5},\bar{5}m,N/m,\ N/mmm$ with $N=10,8,12)$, which have no piezoelectric effects. The second class consists of those QC's with fivefold symmetry $(5,52,5m)$, which have piezoelectric effects induced by both phonon and phason fields. The third class consists of those QC's with evenfold rotoinversion symmetry $(\bar{N},\bar{N}m2$ with $N=10,8,12)$, which have only a piezoelectric effect induced by the phason field. The fourth class consist of those QC's with evenfold proper rotation symmetries $(N,N22,\ Nmm$ with $N=10,8,12)$, which have only a piezoelectric effect induced by the phonon field.

(3) 3D QC's can be divided into three classes. The first class $(m\bar{3},m\bar{3}m,\ \underline{235},m\bar{3}\bar{5})$ has no piezoelectric effects. The second class $(23,\bar{4}3m)$ has piezoelectric effects induced by both phonon and phason fields. The third class (432) has only piezoelectric effect induced by the phason field.

From the above characteristics it follows that we can separate the influence of the phonon and phason fields on piezoelectric effects in some QC's. An important consequence may be that experiments on piezoelectric effects in QC's of different classes can provide information about the phonon and phason fields, respectively. If such experiments could be performed, one could work backwards and use the results given here to extract information about the phonon and phason fields. This would help in further understanding the physics of QC's.

Finally we would like to say something about our two papers,[7,8] in which the piezoelectric behaviors of some QC's were discussed and where a mistake slipped in. The third class ($d^{(1)}=0$, $d^{(2)}\neq 0$) mentioned in item (2) was left out. This has been corrected in the present paper.

ACKNOWLEDGMENT

This project was supported by the National Natural Science Foundation of China.

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