PHYSICAL REVIEW B VOLUME 56, NUMBER 4 15 JULY 1997-II

Universal behavior of magnetoconductance due to weak localization in two dimensions

A. Zduniak,* M. I. Dyakonov,† and W. Knap
Groupe d'Etude des Semiconducteurs, Université Montepellier II, CNRS-URA 0357, 34095 Montpellier, France

(Received 21 March 1996; revised manuscript received 3 February 1997)

Magnetoconductance due to weak localization is studied experimentally for different semiconductor heterostructures. We observe that, when presented as a function of the appropriately normalized magnetic field, different samples show very similar high-field behavior. A theoretical description is developed that allows one to describe in a consistent way both the high- and low-field limits. The theory predicts universal magnetic field dependence ($B^{-1/2}$) of the conductivity correction for two-dimensional systems in the high-field limit. Low-field magnetoconductance depends strongly on spin and phase relaxation processes. Comparison of the theory with experiment confirms the universal behavior in high fields and allows one to estimate the spin and phase relaxation times. [S0163-1829(97)07024-0]

I. INTRODUCTION

Weak-field magnetoconductance is due to quantum corrections to the conductivity, $\Delta\sigma$, arising from interference of electron waves scattered along closed paths in opposite directions. This interference is destroyed by the magnetic field because of the phase shift between the corresponding amplitudes, which is equal to $2\pi\Phi/\Phi_0$, where $\Phi=BS$ is the magnetic flux through the area S of a closed path, $\Phi_0=\pi\hbar c/e$ is the elementary flux quantum. This idea is the basis of the theory of the weak localization magnetoconductance.[1]

To describe the magnetic field dependence of the quantum correction to conductivity three characteristic field values are usually introduced: B_φ, B_{so}, and B_{tr}, where

$$B_\varphi=\frac{\hbar c}{4eD\tau_\varphi},\quad B_{so}=\frac{\hbar c}{4eD\tau_s},\quad B_{tr}=\frac{\hbar c}{4eD\tau}. \tag{1}$$

Here $D=l^2/2\tau$ is the diffusion coefficient, $l=v_F\tau$ is the mean free path, v_F is the Fermi velocity, and τ, τ_s, and τ_φ are the elastic scattering, the spin relaxation, and the phase-breaking times, respectively. If

$$\tau_\varphi,\tau_s \gg \tau, \tag{2}$$

as usually is the case, we have $B_\varphi, B_{so} \ll B_{tr}$.

Since $S\sim Dt$ is the average area of a closed path passed by a diffusing electron during time t, the quantities B_φ, B_{so}, and B_{tr} define values of the magnetic field at which the flux through the area of a closed path, passed by a diffusing electron during the respective relaxation time, becomes comparable to Φ_0. The so-called ''transport field'' B_{tr} is actually defined as field at which $2\pi Bl^2=\Phi_0$.

When the magnetic field becomes greater than both B_φ and B_{so}, the phase-breaking and spin relaxation processes become irrelevant, since now only paths with the round-trip time $t\lesssim\Phi_0/BD$ contribute, and this time is less than τ_φ and τ_s. The contribution of longer paths is killed by the random phase difference introduced by the magnetic field.

A single parameter of the dimension of magnetic field being left, B_{tr}, the quantum correction to conductivity should depend on magnetic field only through the ratio B/B_{tr}.[1] Thus, for $B\gg B_\varphi, B_{so}$ the quantum correction to conductivity should be a universal function of $x=B/B_{tr}$. Until recently this function was known only for $x\ll 1$, i.e., in the magnetic field range where $B_\varphi, B_{so}\ll B\ll B_{tr}$, which exists if the inequality (2) is valid. In this range the contributing paths still involve many collisions and the diffusion approximation holds. The quantum correction to conductivity in this magnetic field range is well known[1] to be proportional to $\ln x$ [for the two-dimensional (2D) case]. For higher magnetic fields, when B becomes on the order of or greater than B_{tr}, paths contributing to the quantum correction have dimensions on the order of the mean free path l or less, so that the diffusion approximation breaks down. However, unless the value of $\ln(\tau_\varphi/\tau)$ is unrealistically high, a substantial part of the conductivity correction changes in this magnetic field range.

Kawabata[2] proposed a theory going beyond the diffusion approximation and not limited by the condition $B\ll B_{tr}$, as well as by the inequality (2). This theory is often used, with success, to fit the experimental data for $B\gtrsim B_{tr}$. Though Kawabata's expression for magnetoconductance is basically correct, it has the following drawbacks: (a) it is overcomplicated, since magnetoconductivity is given as the difference between two diverging sums, representing $\Delta\sigma(B)$ and $\Delta\sigma(0)$, and an unphysical cutoff is introduced to avoid this divergence; (b) it does not reveal the universal behavior at $B\gg B_\varphi, B_{so}$.[3]

It was recently pointed out in Refs. 4 and 5 that Kawabata's theory erroneously takes into account closed paths with one and two collisions, which have zero area and do not contribute to magnetoconductance, but give a seemingly diverging result for the quantum correction (in the weak localization limit $\lambda/l\to 0$, λ being the Fermi wavelength). The universal function of $x=B/B_{tr}$ describing the magnetic field dependence of $\Delta\sigma$ was found in Ref. 4. It was also shown that in the high-field limit ($x\gg 1$) the quantum correction $\Delta\sigma(B)$ decreases as $1/\sqrt{x}$, and that this behavior is determined by triangular closed paths with dimensions much smaller than the mean free path l. Actually these results are hidden in Kawabata's formulas and could be extracted from them after some manipulation.

Apparently, no systematic experimental investigation of magnetoconductivity in the limit $B\gg B_{tr}$ has been done so mfar. Some results can be found in Ref. 5. The universal behavior of experimental data for different samples was pointed out for the first time in Ref. 6. However, in the absence of a theory valid both for $B \gg B_{tr}$ and $B \ll B_{tr}$ these experimental results could not be adequately interpreted. Some experimental studies of the quantum correction $\Delta\sigma(B)$ in a 2D electron gas were devoted to the determination of the absolute value of the correction $\Delta\sigma(0)$ (see Ref. 7). Experimentally $\Delta\sigma(0)$ is determined by the difference between the values of conductivity for $B \gg B_{tr}$ and $B=0$. In Ref. 7 experimental results were interpreted in the frame of the conventional diffusion approximation theory.

Precise experimental determination of $\Delta\sigma(B)$ is difficult because very often the condition $B \gg B_{tr}$ is fulfilled for fields when another magnetotransport phenomenon, the Shubnikov–de Haas oscillations, starts to dominate and the small weak localization correction cannot be extracted from conductivity measurements any more. Choosing a particular sample carrier density and mobility is necessary to overcome this difficulty.

The purpose of this work is to study experimentally the magnetoconductance of 2D electrons in a wide magnetic field range, covering both the low-field ($B \ll B_{tr}$) and the high-field ($B \gg B_{tr}$) limits, and to compare the data with the theoretical predictions. We find that when presented as a function of normalized magnetic field ($x=B/B_{tr}$), different samples, indeed, show universal high-field behavior. We also show that the experimental results in the whole magnetic field range ($B \gg B_{tr}$ and $B \ll B_{tr}$) can be interpreted in the frame of the same theory.

II. THEORETICAL BACKGROUND

It is convenient to use the expression for the quantum correction to the conductivity derived in Ref. 8 (see also Ref. 9), which may be written as

$$\Delta\sigma = -\frac{e^2}{2\pi^2\hbar}F, \quad F = 2\pi l^2 W(0), \tag{3}$$

where $W(\boldsymbol{r})d^2r$ is the probability of finding an electron in the area d^2r around the point $\boldsymbol{r}$ after an arbitrary number of collisions, provided the electron started at $\boldsymbol{r}=0$. Thus $W(0)$ is the total probability for an electron to return to the origin. In the presence of magnetic field, $W(0)$ should be understood as the quasiprobability of return, in which all paths are weighted with a phase factor determined by the magnetic flux through the path area.[9]

After Eq. (3) is established, one may consider the essentially classical problem of finding $W(0)$. The quantum mechanics enters only through the phase factor $\exp(2\pi iBS/\Phi_0)$, which should be attached to each path in the presence of magnetic field.

It was shown in Refs. 4 and 5 that the quasiprobability $W(\boldsymbol{r})$ is given by a sum over the number of collisions N:

$$W(r) = \sum_{N=3}^{\infty} W^{(N)}(\boldsymbol{r}), \tag{4}$$

where $W^{(N)}$ is the contribution of paths with N collisions; the terms with $N=1,2$, corresponding to paths with one and two collisions, that have zero area, should be neglected. The relation between the functions $W^{(N+1)}(\boldsymbol{r})$ and $W^{(N)}(\boldsymbol{r})$ is given by the equation

$$W^{(N+1)}(\boldsymbol{r}) = \int d^2r'\; P(\boldsymbol{r}-\boldsymbol{r}')\exp\left[i\frac{e}{\hbar c}\mathbf{B}(\boldsymbol{r}\times\boldsymbol{r}')\right]W^{(N)}(\boldsymbol{r}'), \tag{5}$$

where

$$W^{(1)}(\boldsymbol{r}) = P(r) = \frac{\exp(-r/l)}{2\pi r l}. \tag{6}$$

The quantity $P(r)d^2r$ gives the probability for an electron starting from the origin to experience the first collision around point $\boldsymbol{r}$.

Thus $W(\boldsymbol{r})$ satisfies the equation[4,5]

$$W(\boldsymbol{r}) = \int d^2r'\; P(\boldsymbol{r}-\boldsymbol{r}')\exp\left[i\frac{e}{\hbar c}\mathbf{B}(\boldsymbol{r}\times\boldsymbol{r}')\right]W(\boldsymbol{r}') + W^{(3)}(\boldsymbol{r}). \tag{7}$$

Equation (7) with $W^{(1)}(\boldsymbol{r})=P(r)$, instead of $W^{(3)}(\boldsymbol{r})$, in the right-hand side was first derived by Kawabata[2] (see also Ref. 9). The difference is due to our neglecting paths with one and two collisions, as explained in Refs. 4 and 5 and above.

The solution of Eq. (7) by the method first used by Kawabata[2] gives the universal functional dependence of $\Delta\sigma(B)$ on $x=B/B_{tr}$:[4]

$$\Delta\sigma(B) = -\frac{e^2}{2\pi^2\hbar}F(x), \quad F(x) = x\sum_{n=0}^{\infty}\frac{P_n^3}{1-P_n}, \tag{8}$$

$$P_n = s\int_0^{\infty} dt\; \exp(-st-t^2/2)L_n(t^2), \tag{9}$$

where $s=(2/x)^{1/2}$ and L_n is the n-th Laguerre polynomial.

For the low-field limit ($x \ll 1$) one obtains the result of the diffusion approximation[1]

$$F(x) = 1.96 - \ln(x), \tag{10}$$

while for the high-field limit ($x \gg 1$) Eq. (8) gives[4]

$$F(x) = 7.74/\sqrt{x}. \tag{11}$$

The function $F(x)$ diverges logarithimically as x goes to zero. This happens because Eqs. (7)–(9) do not take into account the phase-breaking and spin-relaxation processes, thus the results are valid for $B \gg B_\varphi, B_{so}$. $F(x)$ is in fact a sum of terms $P_n^3, P_n^4, P_n^5, \ldots$, which give separate contributions of closed paths with 3,4,5, . . . collisions. It may be seen that in the high-field limit only triangular paths contribute, which could be anticipated since the contribution of more complex paths with larger areas is destroyed by the high magnetic field. Moreover, at $B \gg B_{tr}$, the magnetic flux through the area of a typical triangle greatly exceeds Φ_0. Thus, at these high fields only very small triangles (with dimensions much less than l) contribute. It was shown in Ref. 4 that the asymptotic $B^{-1/2}$ law for $x \gg 1$ is related to the area distribution function for small triangles with $S \ll l^2$, which was found to be proportional to $S^{1/2}$.

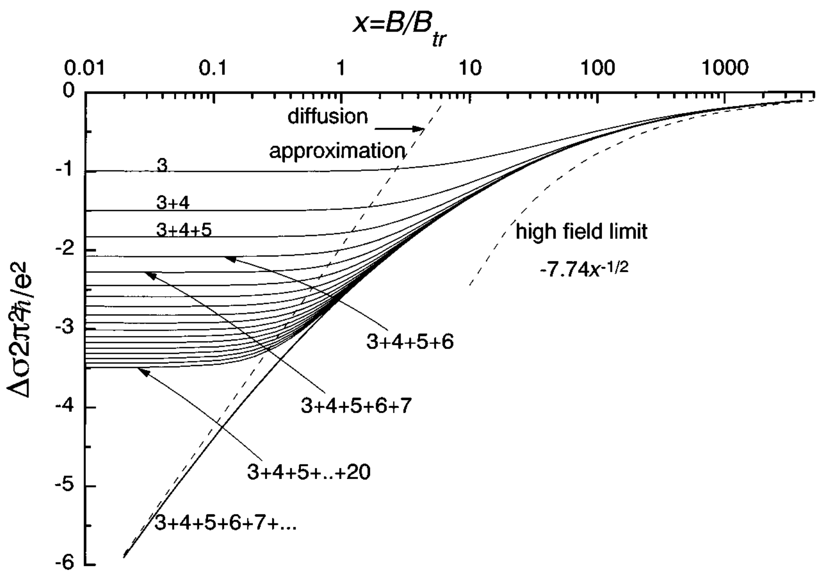

FIG. 1. Calculated magnetic field dependence of the quantum correction $\Delta\sigma$ for $\tau_\varphi=\tau_s=\infty$ (solid line) together with low- and high-field asymptotics (dashed lines) (Ref. 4). The contribution of all paths involving an arbitrary number of collisions $N\leqslant N_{\max}$ $N_{\max}=3,4\ldots,20$, are also presented.

Figure 1 presents results of numerical calculations of the quantum correction $\Delta\sigma(B)$, as given by Eqs. (8) and (9) together with the low- and high-field asymptotics.[4] In addition, the calculated contributions of all paths involving an arbitrary number of collisions $N\leqslant N_{\max}$, for $N_{\max}=3,4,\ldots,20$ are presented. One can see that with increasing magnetic field the contribution of shorter and shorter paths is destroyed until finally only triangular paths remain, which determine the high-field behavior. At $B=0$ the contribution of a path with N collisions to the quantity F in Eqs. (3) and (8) is $1/(N-2)$.

It is easy to take into account the phase-breaking and spin-relaxation processes and thus to obtain a complete formula describing the quantum correction to conductivity not only in the high-field limit ($B\gg B_\varphi, B_{\mathrm{so}}$), as Eqs. (8) and (9), but also in the low-field limit ($B\lesssim B_\varphi, B_{\mathrm{so}}$). If, for the moment, we consider phase-breaking processes only, an additional factor $\exp(-|\mathbf{r}-\mathbf{r}'|/l_\varphi)$ should be introduced into the integrand of Eqs. (5) and (7), where $l_\varphi=v_F\tau_\varphi=l\tau_\varphi/\tau$. This leads to the following modification of the expression for P_n in Eq. (9):

$$P_n=s\int_0^\infty dt\ \exp[-s(1+\beta_\varphi)t-t^2/2]L_n(t^2), \qquad (12)$$

where $\beta_\varphi=\tau/\tau_\varphi=B_\varphi/B_{\mathrm{tr}}$. We denote by $F(x,\beta_\varphi)$ the function defined as $F(x)$ in Eq. (8), but with P_n modified according to Eq. (12). Thus, if spin relaxation is ignored,

$$\Delta\sigma(B)=-\frac{e^2}{2\pi^2\hbar}F(x,\beta_\varphi). \qquad (13)$$

The calculated field dependencies of $\Delta\sigma(B)$, for different values of the parameter β_φ, are presented in Fig. 2. There is an obvious resemblance of the curve family in Fig. 2 to the one in Fig. 1, which is natural since the phase-breaking processes limit the number of collisions for paths contributing to quantum interference by $N_{\max}\sim\tau_\varphi/\tau$. The smaller the value of β_φ, the earlier the universal curve for $\beta_\varphi=0$ is reached. One can also see from Fig. 2 that for very small values of β_φ the change of $\Delta\sigma$ in the range of validity of the diffusion approximation ($B\ll B_{\mathrm{tr}}$) may be comparable to, or even less

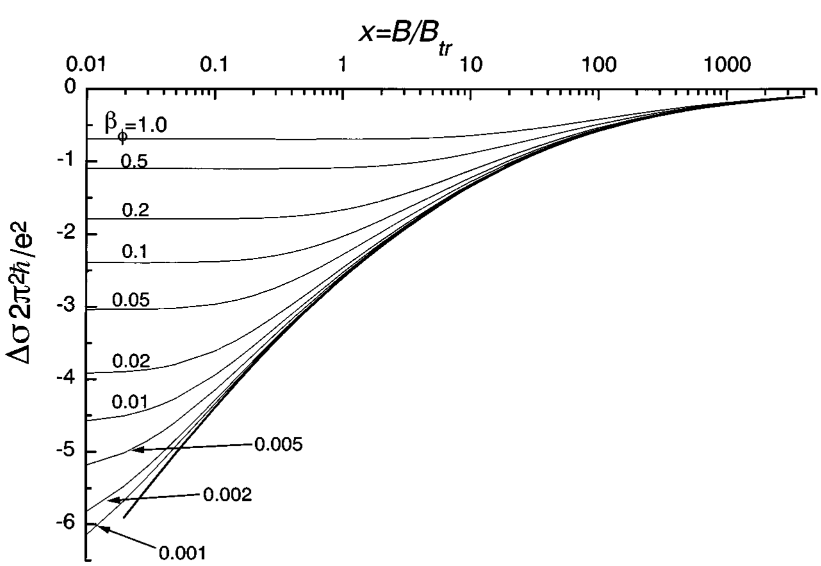

FIG. 2. Calculated magnetic field dependencies of $\Delta\sigma$ for different values of $\beta_\varphi=\tau/\tau_\varphi$. Spin relaxation is not taken into account ($\tau_s=\infty$).

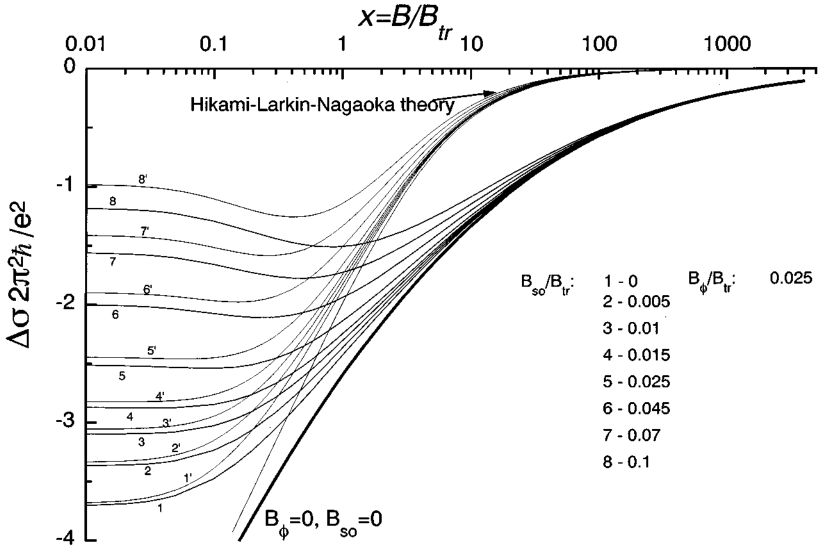

FIG. 3. Calculated magnetic field dependencies of $\Delta\sigma$ for $\beta_\varphi = 0.025$ and different values of B_{so}/B_{tr}. Curves 1–8 were calculated using Eq. (15), curves $1'$–$8'$ were calculated by using the HLN formula for the same values of parameters.

than, its change beyond this range. Thus, even for $\beta_\varphi = 0.001$, the variation of $\Delta\sigma$ in the range $B < 0.5 B_{tr}$ is less than half of its total variation. The reason for this is that the relevant parameter is not τ_φ/τ, but rather $\ln(\tau_\varphi/\tau)$. In practice this quantity is never very large. The zero-field value of the function $F(x,\beta_\varphi)$ in Eq. (13) is

$$F(0,\beta_\varphi) = \ln\left(\frac{1+\beta_\varphi}{\beta_\varphi}\right) = \ln\left(\frac{\tau_\varphi}{\tau}+1\right), \tag{14}$$

which, for $\tau_\varphi/\tau \gg 1$ is close to the value $\ln(\tau_\varphi/\tau)$ derived in the diffusion approximation.

We now introduce spin relaxation. This may be done by a simple generalization of the Hikami-Larkin-Nagaoka (HLN) results.[10] As it is well known (see Refs. 1 and 10) the quantum correction may be split into a positive singlet part, which is not influenced by spin relaxation (in the absence of magnetic impurities), and the negative triplet part, which depends on the spin relaxation rate. In the 2D case, when the fluctuating magnetic field responsible for spin relaxation lies in the 2D plane, the two in-plane spin components relax with the same time constant, τ_s, while the normal-to-the-plane spin component is destroyed two times faster. These considerations lead to the following formula, which accounts for the phase-breaking and spin-relaxation processes and is applicable for arbitrary magnetic fields (limited only by the condition $\omega_c \tau \ll 1$, where ω_c is the cyclotron frequency):

$$\Delta\sigma(B) = -\frac{e^2}{2\pi^2\hbar}\left[F(x,\beta_{s1}) + \frac{1}{2}F(x,\beta_{s2}) - \frac{1}{2}F(x,\beta_\varphi)\right], \tag{15}$$

where

$$\beta_{s1} = \frac{B_\varphi + B_{so}}{B_{tr}}, \quad \beta_{s2} = \frac{B_\varphi + 2B_{so}}{B_{tr}}, \quad \beta_\varphi = \frac{B_\varphi}{B_{tr}}. \tag{16}$$

The quantities B_φ, B_{so}, and B_{tr} are defined by Eq. (1), where τ_s should be understood as the relaxation time for the in-plane spin components.[11,12] For $x = B/B_{tr} \ll 1$ Eq. (15) reduces to the HLN formula:

$$\Delta\sigma(B) = -\frac{e^2}{2\pi^2\hbar}\left[\Psi\left(\frac{1}{2}+\frac{1}{x}\right) - \Psi\left(\frac{1}{2}+\frac{\beta_{s1}}{x}\right) + \frac{1}{2}\Psi\left(\frac{1}{2}+\frac{\beta_\varphi}{x}\right) - \frac{1}{2}\Psi\left(\frac{1}{2}+\frac{\beta_{s2}}{x}\right)\right], \tag{17}$$

where Ψ is the di-gamma function. For large x the universal curve given by Eqs. (8) and (9) is recovered.

The numerically calculated magnetic field dependencies of $\Delta\sigma$ for $B_\varphi/B_{tr} = 0.025$ and different values of the parameter B_{so}/B_{tr} are presented in Fig. 3. For comparison the results following from the HLN formula for same values of the parameters are given. Large values of x are, of course, beyond the range of validity of the HLN theory. Within its range of validity ($x \ll 1$), the corresponding curves are close to each other, and are more so the less the value of B_{so}/B_{tr}. The difference in zero field ($x=0$) is due to the difference between the exact value $F(0,\beta) = \ln(1+1/\beta)$ and the value $\ln(1/\beta)$, which follows from the HLN formula. One can see that even for rather small values of $B_\varphi = 0.025 B_{tr}$ (curves 8, $8'$) the HLN formula does not describe the ''antilocalization'' minimum accurately enough. This, again, is related to the fact that the relevant parameters are not just τ_φ/τ, and τ_s/τ but rather $\ln(\tau_\varphi/\tau)$ and $\ln(\tau_s/\tau)$.

Some remarks are in order, concerning the notion of universality for the high-field behavior of the quantum correction $\Delta\sigma$. It is universal in the sense that for $B \gg B_\varphi, B_{so}$ and *for a given elastic scattering mechanism* $\Delta\sigma$ depends on magnetic field only through the parameter $x = B/B_{tr}$. For the case of isotropic scattering (short-range impurity potential) this dependence was calculated in Ref. 4, and it is for this case that the formulas given above were derived. For $x \ll 1$, when the diffusion approximation is valid, the dependence $\Delta\sigma \sim \ln x$ is always the same, whatever is the elastic scattering mechanism, since the diffusion process may be characterized by a single parameter D, or equivalently, B_{tr}. However for $x \gtrsim 1$, when the contributing paths have linear dimensions on the order of l or less, this is no longer the case, because now the geometry and area distribution of small closed paths are strongly correlated to the angular dependence of the scattering cross section. Thus, the shape of the $\Delta\sigma(x)$ curve for $x \gtrsim 1$ should depend on the nature of the elastic scattering and this dependence could, in principle, give information about the scattering mechanism.

In the region of universal behavior the magnetoconductance is temperature independent, since the only relevant parameter, τ, does not depend on temperature for the case of impurity scattering of degenerate electrons.

III. EXPERIMENT

Pseudomorphic $Al_{0.32}Ga_{0.68}As/In_{0.15}Ga_{0.85}As/GaAs$ quantum wells are studied. They were grown by the molecular beam epitaxy technique. The two-dimensional electron gas is formed in the 13-nm-thick InGaAs layer. All samples are δ doped with Si (doping density $N_\delta = 25\times 10^{12}\ \mathrm{cm}^{-2}$). They have $Al_xGa_{1-x}As$ spacer of thickness 2–6 nm. The samples are Hall bars with geometry precisely defined by photolithography and the etching process. They have a length of 1.0 mm and a width of 0.1 mm, and two current and four voltage probes. The distance between voltage probes is 0.3 mm. The samples are characterized by independent luminescence, high-field transport, and cyclotron emission experiments.[13]

Reproducible and stable weak magnetic fields, necessary for measurements of the weak localization magnetoconductance are realized in the following way. We use a system of two superconducting coils (80 kG/80 kG) placed in the same cryostat. The constant field in one of the coils (sample coil) is compensated by the slowly tuned spread field of the other one. Typically the constant magnetic field in the sample coil is of the order of 50 G. The compensating coil field is tuned in the range from 0 to 80 kG. The magnetic field applied to the sample is determined on the basis of measurements of the Hall voltages induced on the sample and known coil coefficients. This way we can obtain in the sample space magnetic fields from about -600 to $+600$ G. Small sample dimensions and the geometry of the coils ensure a practically uniform magnetic field.

To measure the conductivity as a function of applied magnetic field the standard direct current method is used. A constant current is passed through the sample and voltages between conductivity and Hall probes are measured. Special care is taken to avoid any thermocouple voltage effects. A high precision voltmeter allowing one to measure nV changes of mV signals is used to measure the conductivity voltage. To avoid mechanical and temperature instabilities, the sample is not directly immersed in the liquid helium but is enclosed in the vacuum tight sample holder and cooled by helium exchange gas under 50-mbar pressure. The constant current applied to the sample does not exceed 10 μA, so that there is no appreciable sample heating. More details about the experimental system are given in Ref. 14.

After slow cooling (to 4.2 K) in the dark the samples have carrier concentration N_s varying from 1.05×10^{12} to $1.6\times 10^{12}\ \mathrm{cm}^{-2}$. Illumination by an infrared emitting diode allows us to increase persistently the carrier concentration by about 20% of its initial value for each sample. This behavior of the samples under infrared illumination is caused by metastable properties of *DX* Si centers present in the $Al_xGa_{1-x}Ga$ layer.[13] After each illumination Shubnikov–de Haas and zero field conductivity measurements are performed to establish the value of N_s and the mobility μ. In all investigated samples only the lowest electric subband is populated.

TABLE I. Parameters of the samples.

	N_s (cm^{-2})	μ (cm^2/V s)	B_{tr} (G)	τ (ps)	τ_s (ps)	τ_φ (ps)	$x_{\max}$
*S*1	1.85×10^{-12}	14 300	32	0.6	19	21	22
S2	1.10×10^{-12}	37 000	7.9	1.5	29	59	34
*S*3	1.15×10^{-12}	41 000	6.2	1.7	27	77	39
*D*1					90τ	250τ	

To calculate the B_{tr} parameter we used Eq. (1). For the case of a degenerate 2D electron gas the diffusion constant can be expressed as

$$D = \frac{1}{2}\left(\frac{\hbar k_f}{m}\right)^2 \tau, \tag{18}$$

where m is the effective mass and $k_f = \sqrt{2\pi N_s}$ is the Fermi wave vector. Thus, we can rewrite Eq. (1) in the form

$$B_{\mathrm{tr}} = \frac{ec}{4\pi\hbar N_s \mu^2}. \tag{19}$$

The B_{tr} parameter derived from Eq. (19) is in the range from 6 to 50 G for our samples. Mobility, carrier density, and B_{tr} for a few samples, discussed in more detail in the following paragraph, are listed in Table I.

Figures 4(a)–4(d) show an example of direct experimental records of Hall and conductivity voltages as functions of magnetic field for one of our samples. Scales of vertical and horizontal axes of Figs. 4(a)–4(d) are subsequently magnified by a factor of 10 in order to show the low-field behavior of conductivity. One can see how, by magnifying the scales by three orders of magnitude one comes from the Shubnikov–de Haas oscillations and the quantum Hall effect [Fig. 4(a)] to the subtle features [Fig. 4(d)] related to the antilocalization effects (see Refs. 15 and 16). Figures 4(b) and 4(c) show weak localization behavior, i.e., the conductance increases with increasing magnetic field.

Two-dimensional conductivity $\sigma(B)$ was calculated on the basis of measured voltage, current, and known sample geometry. In order to compare with theory the results were presented as $\sigma(B)-\sigma(0)$ in units $(e^2/2\pi^2\hbar)$ versus the parameter $x = B/B_{\mathrm{tr}}$. In Fig. 5 we show results for one of the samples. Scales in Fig. 5 were chosen in order to make the weak localization effect visible. One can see that positive magnetoconductivity is followed by Shubnikov–de Haas oscillations. For x higher than 300, Shubnikov–de Haas oscillations dominate and weak localization correction to conductivity cannot be extracted from the measurements any more. Generally Shubnikov–de Haas oscillations start at fields for which $\mu B/c$ is around 1. On the other hand the theoretical model presented above is valid for $\mu B/c$ much smaller than 1. Therefore to avoid the influence of Shubnikov–de Haas oscillations in the region of the validity of theory we limited the comparison between the theory and experiment to the fields for which $\mu B/c$ is smaller than 0.1. This condition imposes the maximal value of x for each sample $x_{\max}$

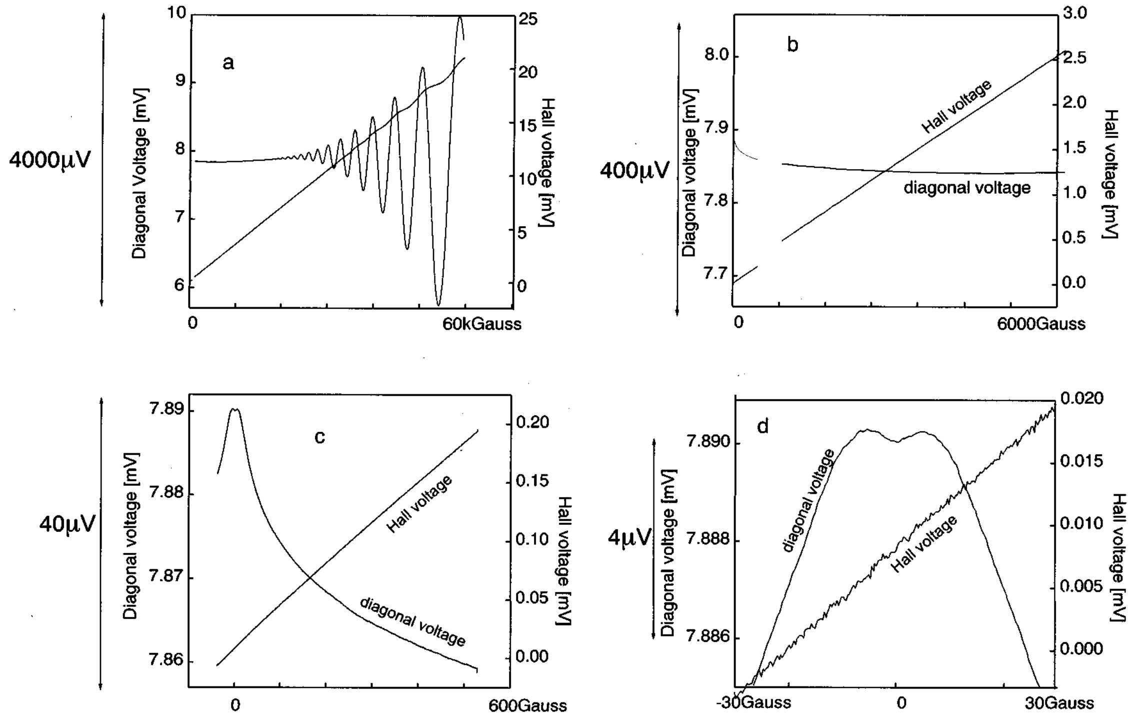

FIG. 4. Example of magnetoresistance measurements seen in different scales (a) Shubnikov–de Haas oscillations and Hall voltage, (b) and (c) weak localization phenomena and Hall voltage, (d) ''antilocalization minimum'' and Hall voltage. The gap in the experimental curves at magnetic field ~0.05 T is related to the change of the experimental system design for measurements at very low magnetic field.

$=c/(10\mu B_{\rm tr})$. We mention here that the so-called quantum lifetime τ_q, which defines the amplitude of Shubnikov–de Haas oscillations, for the studied $In_xGa_{1-x}As$ quantum wells was found to be about 10 times less than τ.[17] This means that we compare our experiments with the theory in magnetic fields for which $\omega_c\tau_q<0.01$, where $\omega_c=eB/mc$. Using Eq. (19) one obtains

$$x_{\rm max}=\frac{4\pi\hbar N_s\mu}{10ec}. \quad (20)$$

One can see that high values of $x_{\rm max}$ can be obtained for samples with high mobility and carrier density. However, when increasing carrier density one should avoid the population of the second electron subband. For heterojunctions this limits the carrier density to about 0.6×10^{12} cm^{-2}. Values of $x_{\rm max}$ obtained for $Ga_xIn_{1-x}As$ quantum wells used in our experiments varied between 20 and 40 (see Table I).

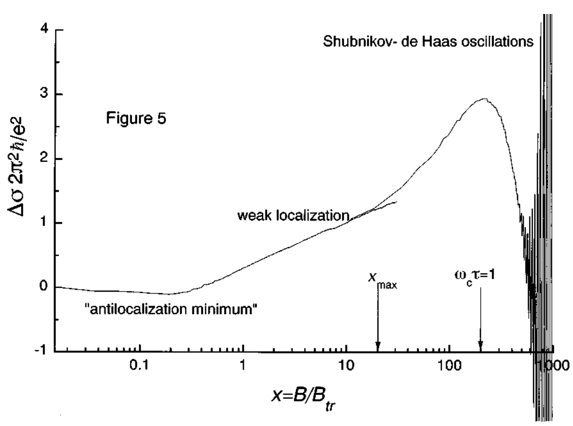

FIG. 5. Magnetoconductivity of one of the samples vs the parameter x.

IV. RESULTS AND DISCUSSION

In Fig. 6(a) we show the measured magnetoconductivity presented as a function of normalized magnetic field ($x=B/B_{\rm tr}$). Our results are presented together with previous measurements done on heterojunctions by Dresselhaus[6] (dashed lines). The universality manifests itself by the fact that different curves are almost parallel for $x=B/B_{\rm tr}$ greater than 1.

We fitted the experimental data, using Eq. (15) and calculating numerically the function $F(x,\beta)$ defined by Eqs. (8) and (12). We also used the recursion relation for P_n, derived in Ref. 2. Direct calculation becomes more and more difficult for small values of x, since to get reasonable accuracy, values of P_n for very high n are needed. The asymptotic formula for P_n at $n\gg1$ (see Refs. 2 and 5) is

$$P_n\cong[(1+\beta)^2+(2n+1)x]^{-1/2}. \quad (21)$$

Values of P_n were calculated using Eq. (12) for increasing n, until the difference between these values and the ones given by Eq. (21) became less than 0.1%. For higher n Eq. (21) was used.

Four examples of the fits are shown in Fig. 6(b). Values of the fit parameters are given in Table I. One can see from Fig. 6 that both low-field and high-field behavior is well reproduced. We can notice that for $x>1$ experimental data follow similar functional dependence approaching the theo-

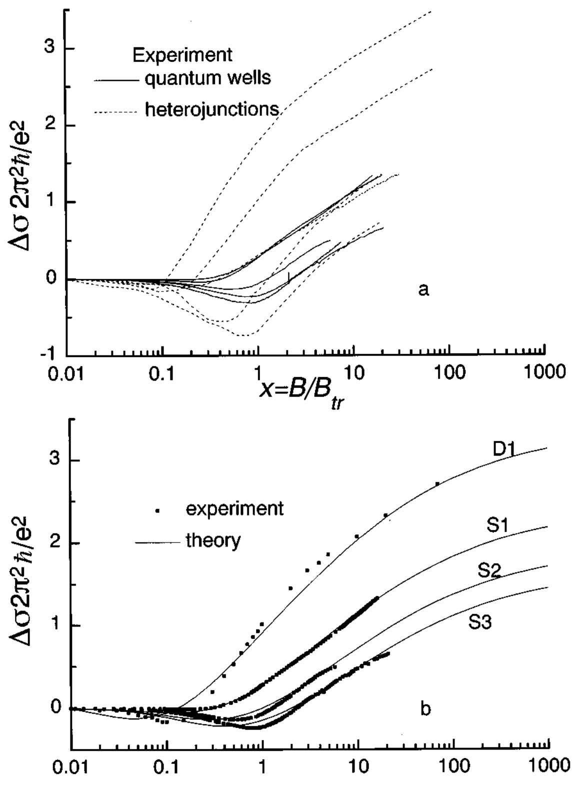

FIG. 6. (a) Magnetoconductivity of $Ga_xIn_{1-x}As$ quantum wells (straight lines) and heterojunctions (Ref. 6) (dashed lines) presented as a function of normalized magnetic field. (b) Examples of fits by formula (15) for whole range of magnetic field.

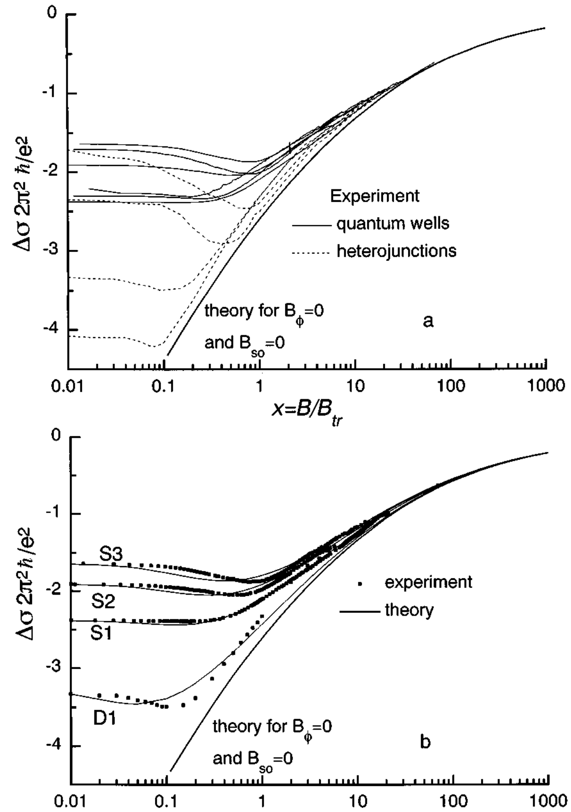

FIG. 7. (a) Same as in Fig. 6(a) but presented in another way. Universality manifests itself by the fact that experimental curves (solid and dashed lines) follow similar functional dependence approaching the theoretically predicted behavior (heavy line). (b) Examples of fits by Eq. (15).

retically predicted universal behavior. On the other hand, for values of the parameters β obtained from the fits for our samples, the difference between all theoretical curves for $x > 10$ is small and comparable with the experimental accuracy (see Fig. 3). Therefore in order to demonstrate clearly the universal behavior in high field, we present all experimental results in another way in Fig. 7(a). In this figure every experimental curve was shifted vertically by some constant value. For each curve the shift was chosen in such a way that for $x = x_{max}$ experimental data coincide with the theoretical curve.

One can see from Fig. 7(a) that for $x > 1$ all experimental curves approach the theoretical curve for $B_\varphi = B_s = 0$ in a similar way. It is worth noting that from Fig. 7(a) one can estimate the absolute value of the quantum correction to conductivity $\Delta\sigma(0)$. Precise experimental determination of $\Delta\sigma(0)$ is difficult because it requires measurement of $\Delta\sigma(B)$ for x in the range from 100 to 1000 [see Fig. 7(a)]. As discussed earlier for such high x another magnetotransport phenomenon—the Shubnikov–de Haas oscillations—dominates (see Fig. 5) and the weak localization correction $\Delta\sigma$ cannot be extracted from conductivity measurements any more. The absolute value of the correction $\Delta\sigma(0)$ estimated for our samples changed in the range from 1.6 to 2.3 while for Dresselhaus results—in the range from 1.7 to 4.1—in the units $(2\pi^2\hbar/e^2)$. Generally higher values of $\Delta\sigma(0)$ are obtained for samples with longer phase relaxation times.

Figure 7(b) shows examples of the fits of experimental data by Eq. (15). The fitting parameters are, of course, the same as those in Fig. 6(b). Theoretical curves reproduce correctly the values $\Delta\sigma(0)$ and of the $\Delta\sigma$ versus magnetic field dependence. We would like to stress that for all cases values of $\Delta\sigma(0)$ depend not only on momentum and phase relaxation rates but also on the spin relaxation rate. The fitted values of the ratios τ_φ/τ and τ_s/τ are given in the table. We found that for our samples τ_φ/τ changes from 35 to 50 and τ_s/τ in the range from 15 to 35. Since τ can be determined from zero field mobility and known effective mass (Ref. 12) we calculated that the values of τ_φ change in the range 35–50 ps and τ_s in the range 15–35 ps.

The functional dependence of $\Delta\sigma$ on magnetic field was calculated for the simple case of isotropic scattering. In the absence of a theory valid for an arbitrary scattering mechanism, we have compared our data with the existing theoretical results for isotropic scattering, although a certain contribution of small-angle scattering by ionized impurities in the δ-doped layer should be present. We did not find any appreciable disagreement between our data and the theory. This may indicate that in our pseudomorphic samples short range isotropic alloy scattering is the dominating scattering mechanism.

The theoretical curves reproduce quite well the general shape of the experimental curves both in the low- and high-field regions. However, the description of the region of the antilocalization minimum, where spin relaxation effects are important, is not very accurate. The reason for this is that the effect of spin relaxation on quantum correction to conductivity in GaAs heterostructures cannot be accurately described by a single parameter τ_s.[18] More precise theoretical and experimental studies of spin effects in weak localization were presented in Ref. 16. It was shown that anisotropic spin splitting of the conduction band should be taken into account to describe accurately the antilocalization minimum. Consideration of these subtle spin effects is beyond the scope of this work. We would like to stress, however, that the presented theory allows for an overall description of both the low- and high-field regions.

In summary, we have studied weak localization corrections to the magnetoconductivity of a 2D electron gas. A theory describing both the low field, where closed paths with many collisions are involved, and the high-field region, where paths with only a few collisions are important, has been presented. The theoretically predicted universal behavior of magnetoconductance in the high-field limit has been experimentally confirmed.

Note added in proof. We come to know that the theory of weak localization magnetoresistance beyond the diffusion approximation was developed by Gasparyan and Zuzin.[19] They showed that at $B \geqslant B_{\mathrm{tr}}$ an additional contribution to magnetoconductivity, due to so-called ''hat diagrams,'' exists. The physical meaning of this contribution was recently explained in Ref. 20.

The correction due to hat diagrams does not change the overall behavior of $\Delta\sigma(B)$. In the magnetic field range experimentally investigated in this paper the correction only slightly modifies $\Delta\sigma(B)$ as given by Eq. (15), the difference not exceeding 10%. We have checked that the values of the spin and phase relaxation times (parameters τ_ϕ and τ_s) derived by using the theory of Refs. 19 and 20 differ from those obtained above by less than 5%, which is within experimental error. Thus all the conclusions of this work remain valid.

ACKNOWLEDGMENTS

One of the authors (M.D.) acknowledges partial support of this work by the Russian Foundation for Fundamental Research (Grant No. 96-02-17896) and by the Swedish Royal Academy of Sciences (Grant No. 1240).

*Permanent address: Institute of Experimental Physics, Warsaw University, Poland.

†Permanent address: A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

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