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Single-electron tunneling at high temperature

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The electromagnetic environment in which a small tunnel junction circuit is embedded plays a crucial role in its transport properties. Although the theory of single-electron tunneling is well established, few analytical results are known. We use a real-time formulation to obtain new predictions for the high-temperature conductance of single- and double-junction systems in series with a resistor. We discuss the implications of our results for recently proposed metrological thermometry based on Coulomb blockade of single-electron tunneling. [S0163-1829(97)06928-2]

I. INTRODUCTION

For tunnel junctions with tunnel resistance much greater than the quantum of resistance $R_K = h/e^2 \approx 25.8\ \mathrm{k}\Omega$, the conduction electrons in the electrodes are well localized on either side of the junction. The conduction mechanism in these junctions consists of uncorrelated single-electron tunnel events that can be considered to occur instantaneously. This single-electron tunneling regime has been reviewed in Refs. 1–3. When such a tunnel event occurs, one electron charge flows through the circuit connected in series with the junction where it may excite electromagnetic modes. Thus, in principle, single-electron tunneling is an ''inelastic'' process in the sense that part of the electrostatic energy dissipated in the tunnel event is transferred to the electromagnetic environment and not to the quasiparticle degrees of freedom in the electrodes. Inelastic processes are revealed by nonlinearities in the current-voltage characteristic of the junction. The theory of single-electron tunneling has been worked out in the case of an arbitrary linear electromagnetic environment.[4,5] It relates the tunneling rates to the environment impedance through a series of integral transforms. Schematically, this treatment shows that inelastic processes are relevant only when the impedance of the environment is larger than R_K over a sufficiently large frequency range and that tunneling is elastic in all other circumstances. The latter situation prevails in most experiments since the typical impedance of connecting leads is always of the order of the vacuum impedance $Z_0 = \sqrt{\mu_0/\epsilon_0} \approx 377\ \Omega$ at microwave frequencies. Only few experiments have been conducted in the inelastic tunneling regime because the fabrication of a controlled high-impedance environment is difficult.[6–8] These experiments only checked the predictions of the theory at low temperatures,[9,10] where the reduction of the conductance due to the environment is most pronounced. At high-temperatures, analytical predictions and tests of the theory are presently lacking. In particular, it appeared recently that detailed understanding of this high temperature regime is important since linear arrays of small junctions have been proposed for metrological thermometry (see Ref. 11 for a review).

In this paper, we use the time-domain formulation of the theory of single-electron tunneling[12] to obtain high-temperature systematic expansions for the conductance of single- and double-junction systems connected to an Ohmic environment.

II. TRANSPORT PROPERTIES OF A SINGLE JUNCTION + ENVIRONMENT

In this section, we make the link between the initial energy-domain formulation of the theory of single-electron tunneling of Refs. 4, 5, and 9 and the time-domain formulation of Refs. 12 and 13. We give explicit formulas for the calculation of transport quantities in the time-domain formulation and discuss their advantages.

The generic circuit displaying Coulomb blockade of tunneling consists of a tunnel junction in series with an arbitrary external impedance $Z_{\mathrm{ext}}(\omega)$, as depicted in Fig. 1. The tunnel junction itself can be decomposed into the parallel combination of two functional elements: a capacitor characterized by its capacitance C and a pure tunnel element characterized by its tunnel resistance R_T assumed much larger than R_K. It is then apparent that the only relevant impedance $Z(\omega)$ for the tunnel process is the parallel combination of Z_{ext} and C. Furthermore, it is assumed that $R_T \gg Z_{\mathrm{ext}}(\omega)$, so that the voltage drop occurs across the junction, and the electrostatic energy change in a tunnel event is $\Delta E = eV$. The theory of single-electron tunneling relates the tunneling

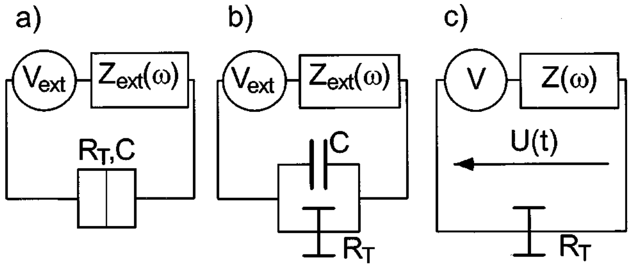

FIG. 1. (a) The generic circuit displaying single-electron tunneling in the presence of an arbitrary environment consists of a tunnel junction in series with an arbitrary impedance $Z_{\mathrm{ext}}(\omega)$, and biased with a voltage source V_{ext}. (b) The tunnel junction is separated into two functional elements: a capacitor with capacitance C, and a pure tunnel element represented by the double T symbol. This description of the system can be transformed to (c), using Thévenin's theorem. The only relevant impedance for the tunneling process is $Z(\omega)$, the parallel combination of C and $Z_{\mathrm{ext}}(\omega)$.

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rates through the junction to the equilibrium correlation function $J(t)=\langle[\varphi(t)-\varphi(0)]\varphi(0)\rangle$ of the phase $\varphi(t)=\int_{-\infty}^{t}dt' e[U(t')-V]/\hbar$ across the total impedance $Z(\omega)$ at temperature T. More precisely, the tunneling rate $\Gamma(\Delta E)$ through the junction, for a global change in the electrostatic energy ΔE, is given by the convolution product:

$$\Gamma(\Delta E)=\frac{1}{e^2R_T}\int_{-\infty}^{+\infty}\gamma(\epsilon)P(\epsilon-\Delta E)d\epsilon=\frac{1}{e^2R_T}\gamma*P(\Delta E), \tag{1}$$

where ‘‘*’’ denotes convolution, $\gamma(E)=E/(1-e^{-\beta E})$, with $\beta=1/k_BT$, and where the function $P(E)$ denotes the Fourier transform of $\exp J(t)$:

$$P(E)=\int_{-\infty}^{+\infty}\exp\left[J(t)+\frac{iEt}{\hbar}\right]\frac{dt}{2\pi\hbar}. \tag{2}$$

The interpretation of Eq. (1) is the following: $\gamma(E)/R_Te^2$ is the probability per unit time that a tunnel event converts an energy E into quasiparticle excitations in the electrodes. On the other hand, $P(E)$ is the probability that the electromagnetic environment absorbs an energy E during the tunneling process. The convolution product structure reflects the fact that all possible partitions of the available energy into electromagnetic excitations (‘‘photons’’) and quasiparticle excitations contribute to the tunneling rate. Finally, the phase correlation function $J(t)$ is itself related to the dissipative part of the impedance $Z(\omega)$ by the quantum fluctuation-dissipation theorem:

$$J(t)=2\int_{-\infty}^{+\infty}\frac{\mathrm{Re}Z(\omega)}{R_K}\,\frac{e^{-i\omega t}-1}{1-e^{-\beta\hbar\omega}}\,\frac{d\omega}{\omega}. \tag{3}$$

Although the above theory formally solves the problem of calculating the rate $\Gamma(\Delta E)$, the multiple embedding of integral equations hinders analytical calculations and complicates numerical evaluations. Indeed, the explicit dependence of the rate on the impedance has only been obtained for the low-energy, low-impedance regime.[10] In other cases, one must resort to numerical evaluations that are mainly performed using fast Fourier transform. An alternate numerical approach circumvents the Fourier transformations (2) and (3): the function $P(E)$ is obtained by solving the inhomogeneous integral Minnhagen equation,[15] which directly connects $P(E)$ to the impedance $Z(\omega)$.

An alternate formulation of the theory was proposed in Ref. 12. In this formulation the rate $\Gamma(E)$ is directly related to the phase correlation function $J(t)$, without going through the determination of $P(E)$. This approach is *a priori* attractive because $J(t)$ is more amenable to analytical calculations than $P(E)$. The link between the two formulations is made by noticing that any convolution product $\gamma*f(E)$ with a function $f(E)$ such as the one in Eq. (1) can be expressed by the time domain integral:

$$\gamma*f(E)=\int_{-\infty}^{+\infty}\tilde{\gamma}(t)\tilde{f}(t)e^{iEt/\hbar}\frac{dt}{2\pi\hbar}, \tag{4}$$

where $\tilde{f}(t)$ and $\tilde{\gamma}(t)$ are the respective inverse Fourier transform of f and γ. The expression of $\tilde{\gamma}(t)$ is[12]

$$\tilde{\gamma}(t)=i\pi\hbar^2\frac{d}{dt}\delta(t)-\frac{\pi^2}{\beta^2}\mathrm{csch}^2\frac{\pi t}{\hbar\beta}.$$

Assuming that $\dot{\tilde{f}}(t)=(d/dt)\tilde{f}(t)$ is continuous at $t=0$, Eq. (4) becomes

$$\begin{aligned}\gamma*f(E)=\dot{\tilde{f}}(0)\left(\frac{1}{\beta}+\frac{E}{2}\right)&\\ -\frac{i\hbar}{2}\dot{\tilde{f}}(0)-\frac{\pi}{2\beta}\int_{-\infty}^{+\infty}\frac{dt}{\hbar\beta}\{\tilde{f}(t)e^{iEt/\hbar}&\\ -\tilde{f}(0)\}\mathrm{csch}^2\frac{\pi t}{\hbar\beta}.&\end{aligned} \tag{5}$$

The tunneling rate through the tunnel element is determined by applying this result to Eq. (1) with $f(\Delta E)=P(\Delta E)$. One gets, using symmetry properties of $J(t)$ (Ref. 9),

$$\begin{aligned}\Gamma(E)=\frac{1}{e^2R_T}\Bigg\{\frac{1}{\beta}+\frac{E-E_C}{2}-\frac{\pi}{\beta}\int_0^{+\infty}\frac{dt}{\hbar\beta}\{\mathrm{Re}\exp[J(t)&\\ +iEt/\hbar]-1\}\mathrm{csch}^2\frac{\pi t}{\hbar\beta}\Bigg\},&\end{aligned} \tag{6}$$

where $E_C=i\hbar\dot{J}(0)$ is the charging energy of the junction in its environment [see Eq. (66) of Ref. 9]. The current $I(V)=e[\Gamma(eV)-\Gamma(-eV)]$ flowing through the junction in response to an applied voltage V is readily obtained from the above result, as well as the differential conductance, which is written as

$$\begin{aligned}G(V)=\frac{dI(V)}{dV}=\frac{1}{R_T}\Bigg[1+2\int_0^{+\infty}\frac{dt}{\hbar\beta}\,\frac{\pi t}{\hbar\beta}\mathrm{Im}e^{J(t)}&\\ \times\cos\frac{eVt}{\hbar}\mathrm{csch}^2\frac{\pi t}{\hbar\beta}\Bigg].&\end{aligned} \tag{7}$$

The above time-domain formulas coincide with Kubo formulas for the composite system formed by the electrodes and the electromagnetic environment. The functions $\tilde{\gamma}(t)$ and $\exp J(t)$ are real-time representations of retarded Green functions for the electrons and for the charge translation operator $e^{i\varphi}$. These Green functions, evaluated with the unperturbed density matrix in the absence of tunneling, can only describe weak-tunneling situations. Strong-tunneling corrections could be incorporated using better approximations of the Green functions. Finally, the time-domain formulas given above for the rate and the conductance require one less integration stage than the equivalent energy-domain formulas, and are thus well suited for making numerical evaluations, as already noticed in Ref. 12. We now apply the time-domain formulation to derive analytical results for single- and double-junction systems placed in an Ohmic environment.

III. CONDUCTANCE OF A SINGLE JUNCTION PLACED IN AN OHMIC ENVIRONMENT

We assume here that $Z_{\mathrm{ext}}(\omega)=R$ so that the real part of the impedance seen by the pure tunnel element is $\mathrm{Re}Z(\omega)=R/[1+(RC\omega)^2]$. The integration (3) leading to

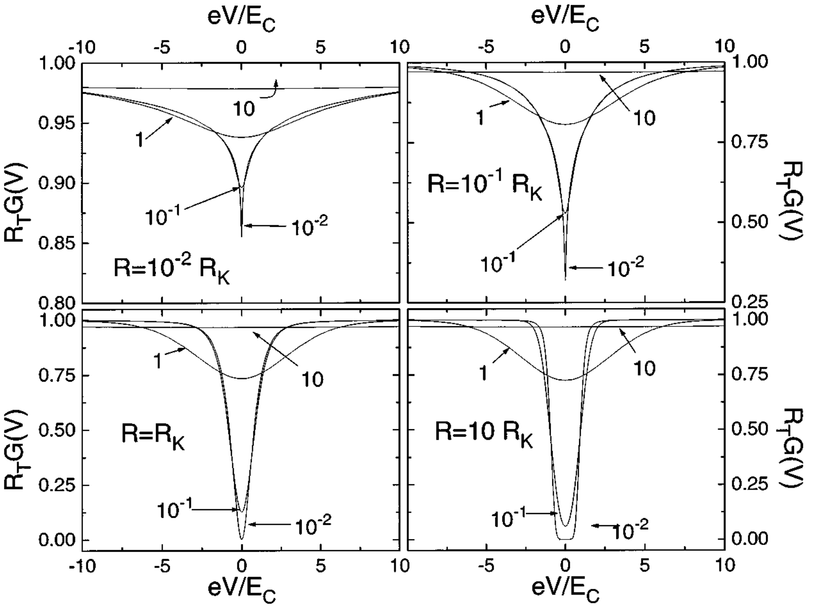

FIG. 2. Differential conductance of a weakly conducting tunnel junction in series with a resistor, for different values of the resistance R and different temperatures. The curves are labeled by the ratio k_BT/E_C. Coulomb blockade of tunneling manifests itself as a dip at $V=0$. For a given resistance, the dip gets deeper as the temperature is reduced, and for a given temperature, the dip gets deeper when R increases (note the enlarged vertical scale of the top panels). For resistances $R \gg R_K$, a finite gap develops at low temperature (bottom right). For low resistances, a singularity develops at $V=0$, and the conductance eventually reaches 0 at $T=0$ (Ref. 10). See also Fig. 3.

$J(t)$ can be performed exactly:[16]

$$J(t)=\frac{\pi R}{R_K}\left\{[1-\exp(-\omega_C|t|)]\left(\cot\frac{\beta\hbar\omega_C}{2}-i\right)-\frac{2|t|}{\hbar\beta}+2\sum_{n=1}^{+\infty}\frac{\omega_C^2[1-\exp(-\omega_n|t|)]}{2\pi n(\omega_n^2-\omega_c^2)}\right\}, \tag{8}$$

where $\omega_n=2\pi n/\hbar\beta$ are the Matsubara frequencies and $\omega_c=1/RC$ is the cutoff frequency of $Z(\omega)$. This phase correlation function is identical to the correlation function for the position of a quantum particle coupled to an Ohmic bath. The infinite sum can further be expressed using special functions:[17]

$$2\sum_{n=1}^{+\infty}\frac{\omega_C^2[1-\exp(-\omega_n t)]}{2\pi n(\omega_n^2-\omega_c^2)}$$

$$=-\frac{1}{\pi}\left[2\gamma+\Psi(-x)+\Psi(x)+2\ln(1-y)+\frac{y}{1+x}\times{}_2F_1(1,1+x,2+x,y)+\frac{y}{1-x}\,{}_2F_1(1,1-x,2-x,y)\right],$$

where γ is here Euler's constant, Ψ is the logarithmic derivative of the Gamma function, ${}_2F_1$ is the hypergeometric function, $y=\exp(-2\pi t/\hbar\beta)$, and $x=\beta E_C R_K/2\pi^2 R$. Since $J(t)$ is known exactly for this impedance, the calculations of Γ, $I(V)$, or $G(V)$, only require one to evaluate a single integral. In Fig. 2, we used Eq. (7) to evaluate the differential conductance $G(V)$, for different values of the resistance R and temperature T. The dip at $V=0$ is called the Coulomb zero bias anomaly. A finite Coulomb gap is recovered at low temperature only for $R \gg R_K$. In Fig. 3, we plot the zero-voltage conductance of the system for different temperatures and different values of the resistance R. For the sake of comparison, we also plot the predictions obtained using the approximate expression of $P(E)$ obtained in Ref. 10, which is valid at low energies and for low impedances. We see, as expected, that both calculations agree at low temperature and that the temperature range in which the approximation is valid increases as R is decreased. Although $G(0)$ vanishes at $T=0$ for any resistance, this effect is unobservable in practice for small resistances since one then finds $G(0) \propto T^{2R/R_K}$ when $T\to 0$.

We now apply the time-domain formulation to determine the high-temperature expansion of the zero-voltage conductance. This expansion is systematically obtained by injecting a small-time high-temperature expansion of $\exp J(t)$ into Eq. (7). Up to the second order in βE_C, one gets

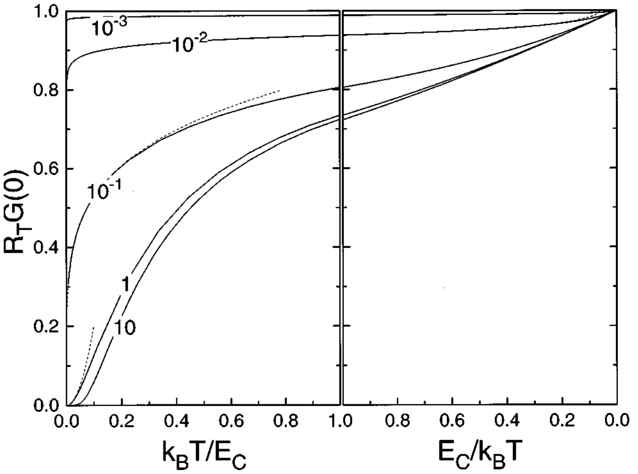

FIG. 3. Zero-voltage conductance of a weakly conducting tunnel junction in series with a resistance R, as a function of temperature, for different values of the resistance. The curves are labeled by the value of R/R_K. The right panel completes the coverage of the whole temperature range by taking not the temperature as the horizontal coordinate, but its inverse. The predictions of Eq. (7) using Eq. (8) (full lines) are compared with those made using the approximate $P(E)$ of Ref. 10 (dashes). In the case $R/R_K=10$, the approximation deviates already at very low temperature and was not plotted here, for the sake of clarity.

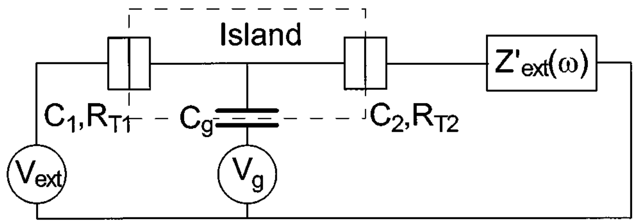

FIG. 4. Single-electron transistor with an arbitrary electromagnetic environment described by the impedance $Z'_{\mathrm{ext}}(\omega)$.

$$R_T G(0) = 1 - \frac{\beta E_C}{3} + (\beta E_C)^2 \left(\frac{1}{15} + \frac{3\zeta(3) R_K}{2\pi^4 R} \right) + O((\beta E_C)^3), \tag{9}$$

where ζ is the Riemann zeta function. Determination of higher orders requires numerical evaluations of definite integrals. This expansion is valid in the inverse temperature β range defined by $(R_K/R)(\beta E_C)^2 \ll 1$; this range vanishes in the limit $R \to 0$. In this limit, one directly finds $R_T G(0) = 1$ by setting $J(t) = 0$ in Eq. (7). Thus, $G(0)$ is a nonanalytic function of R, at $\beta = R = 0$. Recently, using the path-integral formalism, Wang, Göppert, and Grabert[18] calculated the high-temperature expansion of the conductance of a tunnel junction of arbitrary tunnel resistance R_T in series with an arbitrary resistance R. In the weak-tunneling limit $R_T \gg R, R_K$, their result coincides with Eq. (9).

We now address the effect of the electromagnetic environment on a double-junction system, in which Coulomb blockade is already present in the absence of any series impedance.

IV. INFLUENCE OF THE ELECTROMAGNETIC ENVIRONMENT ON THE CONDUCTANCE OF A SINGLE-ELECTRON TRANSISTOR

The single-electron transistor (SET), whose circuit is shown in Fig. 4, was first operated by Fulton and Dolan.[19] It consists of two tunnel junctions with tunnel resistances large compared to R_K, connected in series with an arbitrary impedance. Effects of the electromagnetic environment on the SET have been reported in Ref. 20. The theory we use here neglects the possibility of simultaneous tunnel events (''cotunneling''[21,22]): when tunneling occurs through a given junction, the other junction is treated as a passive capacitor contributing to the electromagnetic environment of the former junction. The charging energy $E_C = e^2/2(C_1 + C_2 + C_g)$ is now the electrostatic energy of one electron on the island. In the case where the capacitances C_1 and C_2 of the two junctions are equal, the zero-voltage conductance of the SET is given by[12,23,24]

$$\frac{G(n_g)}{G_0} = \sum_{n=-\infty}^{+\infty} p(n) \frac{\beta}{2} [\gamma * P(E_{n+1} - E_n) + \gamma * P(E_{n-1} - E_n)]. \tag{10}$$

Here, $n_g = C_g V_g / e$ is the dimensionless gate charge, $E_n = E_C (n - n_g)^2$ is the electrostatic energy with n excess electrons on the island, $p(n) = \exp(-\beta E_n) / \Sigma_{m=-\infty}^{+\infty} \exp(-\beta E_m)$ is the probability of this configuration, and $G_0 = 1/(R_{T1} + R_{T2})$ is the series conductance. In the case when the two capacitances are unequal, the electromagnetic environment seen by the two tunnel elements differ and the expression is slightly more complicated. The above expression for the conductance can be used directly, with the convolution products evaluated using Eq. (6). At high temperature, however, many charge states of the island are populated and an alternate formulation is more convenient. First, using Poisson's resummation formula, we transform Eq. (10) into a ratio of Fourier series that converge rapidly at high temperature:

$$\frac{G(n_g)}{G_0} = \frac{h_0 + 2 \sum_{k=1}^{+\infty} \cos(2\pi k n_g) h_k}{1 + 2 \sum_{k=1}^{+\infty} \cos(2\pi k n_g) \exp(-\pi^2 k^2 / \beta E_C)}, \tag{11}$$

where

$$h_k = \frac{\beta^2}{\sqrt{4\pi\beta E_C}} \left(\gamma * P * \left\{ \exp - \frac{\beta E^2}{4 E_C} \cos \frac{\pi k E}{E_C} \right\} \right) (-E_C).$$

Applying Eq. (5) yields

$$h_k = \exp\left(- \frac{\pi^2 k^2}{\beta E_C} \right) [1 - \beta E_C - I_k],$$

with

$$I_k = \pi \int_0^{+\infty} \frac{dt}{\hbar\beta} \left\{ \mathrm{Re} \exp\left(J(t) - \frac{E_C t^2}{\beta \hbar^2} - \frac{i E_C t}{\hbar} \right) \cosh\left(\frac{2\pi k t}{\hbar\beta} \right) - 1 \right\} \mathrm{csch}^2 \frac{\pi t}{\hbar\beta},$$

so that finally

$$\frac{G(n_g)}{G_0} = 1 - \beta E_C - \frac{I_0 + 2 \sum_{k=1}^{+\infty} \cos(2\pi k n_g) \exp(-\pi^2 k^2/\beta E_C) I_k}{1 + 2 \sum_{k=1}^{+\infty} \cos(2\pi k n_g) \exp(-\pi^2 k^2/\beta E_C)}. \tag{12}$$

This expression for the conductance shows that the gate charge modulation of the SET is exponentially suppressed at temperatures such that $\beta E_C < 1$. In this regime, the conductance is

$$\frac{G}{G_0}\approx 1-\beta E_C-I_0=1-\beta E_C-\pi\int_0^{+\infty}\frac{dt}{\hbar\beta}\left\{\mathrm{Re}\exp\left(J(t)-\frac{E_C t^2}{\beta\hbar^2}-\frac{iE_C t}{\hbar}\right)-1\right\}\mathrm{csch}^2\frac{\pi t}{\hbar\beta}. \tag{13}$$

As in the single-junction case, a high-temperature expansion of this conductance can be obtained by injecting a short-time high-temperature expansion for $J(t)$ in this expression. For the sake of the comparison with the case of the single junction, we will now further assume that the impedance $Z'_{\mathrm{ext}}(\omega)$ is an Ohmic resistance R', and that $C_g \ll C_1 = C_2 = C/2$. The environment seen from each tunnel element is then described by

$$\mathrm{Re}Z'(\omega)=\frac{R'/4}{1+[(R'/4)C\omega]^2},$$

which is the same quantity as appears in the case of the single junction connected to a resistor R, provided that $R'=4R$. Thus, in the present case, $J(t)$ is also given by Eq. (8), substituting R with $R'/4$.

In Fig. 5, we plot calculations of the maximum and minimum conductance of the SET for different values of the resistance connected in series with it, for all temperatures. The results, which reproduce known low-temperature results,[12,24] show in particular that even a small environmental resistance causes, at all temperatures, a noticeable reduction from the conductance of a SET with zero-impedance environment. At high temperature Eq. (13) can be expanded in the same way as done for the single junction:

$$\frac{G}{G_0}=1-\frac{2}{3}\beta E_C+(\beta E_C)^2\left[\frac{1}{60}+0.286\cdots\frac{R_K}{R'}+0.135\cdots\left(\frac{R_K}{R'}\right)^2\right]+O((\beta E_C)^3).$$

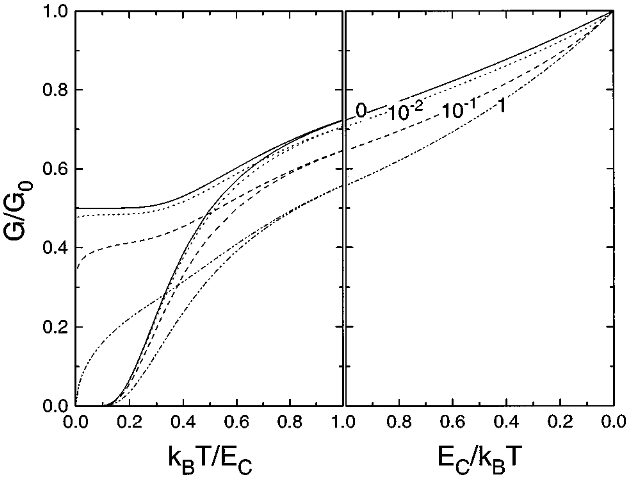

FIG. 5. Zero-voltage conductance of a weakly conducting SET in series with a resistance R', as a function of temperature, for different values of the resistance. The curves are labeled by the value of R'/R_K. As in Fig. 3, both panels encompass the complete temperature range. The gate charge modulation of the SET is simply rendered here by plotting the maximum and minimum conductance of the SET in the left panel.

In the case where $R'=0$, the expansion can also be performed, and one finds

$$\frac{G}{G_0}=1-\frac{1}{3}\beta E_C+\frac{1}{15}(\beta E_C)^2+O((\beta E_C)^3). \tag{14}$$

Note the change in the βE_C coefficient, clearly showing that G is a nonanalytic function of R', at $\beta=R'=0$, as previously seen for the single junction. The latter expansion (14) is identical to the expansion (9) where the limit $R\rightarrow+\infty$ is taken. This is not a mere coincidence since one can indeed prove a rigorous equality between the limit of Eq. (7) when $R\rightarrow+\infty$ and the nonmodulating part h_0 of Eq. (11) when $R'=0$. Thus, at high temperature, the conductance of a single junction in series with a large resistance $R \gg R_K$ coincides with the conductance of a symmetric SET with same series tunnel resistance and charging energy, but with no resistance in series.

In Fig. 6, we plotted on the same graph the conductance of a single junction in series with a resistance R and the conductance of a SET in series with a resistance R', for

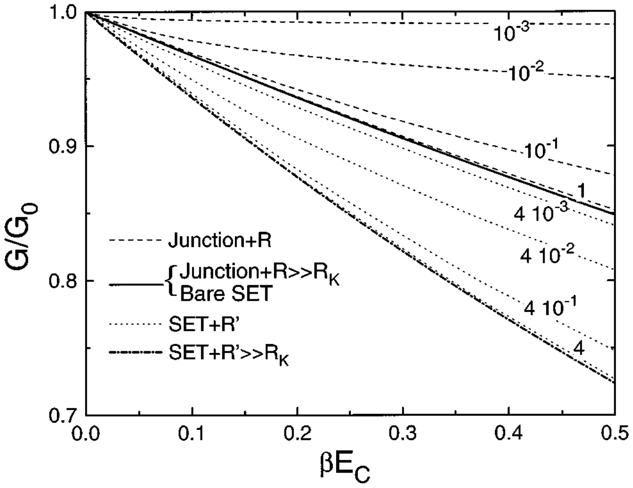

FIG. 6. Normalized high-temperature zero-voltage conductance of a single junction ($G_0=1/R_T$) in series with a resistance R, and of a SET $[G_0=1/(R_{T2}+R_{T1})]$ in series with a resistance R', as a function of the dimensionless inverse temperature βE_C. The curves are labeled by the value of R/R_K or R'/R_K. In this temperature domain the SET has no gate charge modulation. At finite temperature, the conductance is reduced even by a small resistance in series. Note also that for small resistances the relative reduction with respect to the $R=R'=0$ value depends weakly on temperature and is the same for the junction and for the SET, provided $R'=4R$ in order to take into account the capacitive division occurring in the SET (see text). For large values of the resistances (but still $R,R' \ll 1/G_0$), the effect saturates for both the junction and the SET. At high temperature, an asymmetric junction + resistance system ($R_T \gg R$) is found equivalent to a symmetric SET with same series tunnel resistance and charging energy, but with no resistance in series.

different values of the resistances and as a function of temperature, in the regime where there is no modulation with the gate voltage of the SET ($\beta E_C < 1$). For resistances R and R' providing the same dissipation (i.e., with $R' = 4R$), the relative reduction of conductance with respect to the $R = R' = 0$ case is nearly the same for the single- and double-junction system, and depends weakly on temperature for small resistances $R, R' \ll R_K$. This relative reduction of conductance is of the order of 1%, for environmental impedances of the order of the vacuum impedance Z_0, which are typical for nanofabricated circuits at high frequencies. For larger values of the resistances (but still smaller than the tunnel resistance), the conductance reaches an asymptotic curve in both the case of the single junction and of the SET. In the case of the single junction, the conductance reaches that of a bare SET, as shown above.

We now discuss the implications of our results for the proposal of Pekola *et al.* to use tunnel junction arrays for metrological thermometry.[14] These authors have shown that the voltage half-width half-depth of the Coulomb zero-bias anomaly of a bare SET is $V_{1/2} \simeq 5.44\ k_B T/e$ for $k_B T \gg E_C$, thus providing a *primary thermometer*. This thermometer is restricted to the temperature range in which the conductance is reduced by at most a few percent. Our results indicate that typical environmental impedances yield large corrections to the reduction of the zero-voltage conductance in this temperature range. For instance, at $\beta E_C = 0.1$, the conductance of a bare SET is reduced by 3.3%. An additional series resistance of $R' = 400\ \Omega \approx Z_0$ yields an extra reduction of 0.65%, and certainly also affects the voltage width of the Coulomb zero-bias anomaly. Practical realization of a SET-based primary thermometer thus requires one to place the SET in a specially designed low-impedance electromagnetic environment.

V. CONCLUSION

Using the real-time formalism of single-electron tunneling, we have shown how single-electron charging effects vanish at high temperature in the case of single and double junctions placed in an electromagnetic environment. We have found that moderate environmental resistances appreciably modify Coulomb blockade, even at high temperature. In the case of resistances $R \ll R_K$, the relative reduction of conductance depends weakly on temperature up to $k_B T \approx E_C \sqrt{R_K/R}$. This effect is of particular importance for metrological thermometry based on Coulomb blockade of single-electron tunneling.

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