High-field miniband transport in semiconductor superlattices in parallel electric and magnetic fields

P. Kleinert
Paul-Drude-Institut für Festkörperelektronik, Hausvogteiplatz 5-7, 10117 Berlin, Germany

V. V. Bryksin
Physical Technical Institute, Politekhnicheskaya 26, 194021 St. Petersburg, Russia
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We present a rigorous quantum-mechanical description of the miniband transport in semiconductor superlattices under the mutual influence of high electric and magnetic fields aligned parallel to the growth direction. Strong current oscillations appear due to Landau and Wannier-Stark quantization of the electronic states and due to the scattering on polar-optical phonons. The combined influence of an electric and magnetic field on magnetophonon and electrophonon resonances is treated by a lateral electron distribution function, which is the solution of a quantum-kinetic equation. The lateral electron heating due to the field dependent coupling between longitudinal and transverse degrees of freedom via the distribution function is essential to understand miniband transport properties of superlattices. Intracollisional field effects are taken into account. Strong magnetic fields lead to a pronounced enhancement of Wannier-Stark current oscillations. The experimentally detected crossover in the temperature dependence of the current is reproduced. The influence of an electric field on magnetophonon resonances is investigated.
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I. INTRODUCTION

Two quite different transport regimes have been discriminated in semiconductor superlattices (SL's). At low electric fields, i.e., when $\hbar\Omega = e\mathcal{E}d \ll \Delta$ (where Ω is the Bloch frequency, $\mathcal{E}$ the electric field, d the SL period, and Δ the miniband width) the states are extended, and the current increases linearly with increasing field. This is the so-called mobility or miniband transport regime. If, on the other hand, the Bloch frequency is larger than some characteristic collision broadening ($\Omega > 1/\tau$), electrons approach the minizone boundary, where the effective mass is negative, and eventually undergo Bragg diffraction leading to electric-field-induced localized states. The spatial overlap between these localized Stark-ladder states decreases with increasing field and with it the transition probability. In this so-called hopping regime, the band conduction model breaks down, and transport is due to scattering mediated carrier transitions between Wannier-Stark ladder states. Negative differential conductivity (NDC) may occur. Recently, the existence of Bloch oscillations in SL's has been demonstrated by electro-optic experiments[1,2] and by a direct observation of the submillimeter wave emission from spatial carrier oscillations.[3] Neglecting Wannier-Stark localization and interminiband transitions such Bloch oscillations have been studied by solving the Boltzmann transport equation in the relaxation time approximation.[4]

Another interesting subject is the magnetotransport in SL's that manifests such fundamental phenomena as quantum interference and electron localization. It is the aim of the present paper to consider miniband transport under the mutual influence of both an electric ($\mathcal{E}$) and a magnetic ($\mathcal{H}$) field aligned parallel to the growth axis of the SL. This gives rise to a complete quantization of the energy spectrum and a reduction in the dimensionality due to the magnetic-field-induced quantization of the in-plane motion. In this case the Hamiltonian of the system decouples into two separable parts: One refers to the motion along the SL axis under the influence of the SL potential and the electric field, and the other one describes the in-plane (x,y) motion in the presence of the magnetic field. The carrier motion is characterized by two characteristic frequencies (the Bloch frequency Ω and the cyclotron frequency $\omega_c = e\mathcal{H}/m^*c$) so that phase locking may occur, if their ratio is a rational number.[5] Due to the superimposed periodic motions in the transverse plane and along the SL axis, the current exhibits pronounced structures that depend on the ratio between the two frequencies ω_c and Ω. Combined Stark-cyclotron or Stark-cyclotron-phonon resonances are predicted to occur under the condition $n\omega_c = l\Omega$ or $n\omega_c = |l\Omega \pm \omega_0|$, respectively, where ω_0 is the optic-phonon frequency and n,l are integer numbers.

There are only few experimental and theoretical studies dealing with this subject. Noguchi *et al.*[6] treated transport in the mobility regime under high magnetic fields transverse to the interfaces and detected clear magnetophonon resonances. These resonances are magnified in SL's, because the phonon scattering rate is enhanced due to the peaked structure of the density of states. On the other hand, they observed a suppression of optical-phonon scattering when the magnetic field is strong enough ($\Delta < \hbar\omega_c$) to create real forbidden gaps. These effects have been studied theoretically on the basis of the balance equation method.[7]

At high electric fields, when hopping is the dominant transport mechanism, Stark-cyclotron resonances have been reported in an experimental work.[8] Their theoretical analysis is presented in Refs. 9 and 10. From the temperature dependence of the current it has been concluded that acoustic-phonon-assisted (quasielastic) scattering is not dominant.[8] Combined Stark-cyclotron resonances have also been identified in photocurrent measurements.[11] As the magnetic field was increased, the intensity of the Stark transitions was enhanced and conversely, Landau transitions became more pronounced with increasing electric field. It has been pointed out that disorder or Coulomb interaction may cause a coupling of the lateral and longitudinal electron motion, which leads to anticrossings.[11]

Only few theoretical papers treated galvanomagnetic effects in SL's in the hopping transport regime[9,10,12] by employing the Wannier-Stark representation of the Hamiltonian. The interest of the authors focused on the field dependence of the scattering matrix element. The electron distribution was treated by the mean energy gain method,[13,14] which is claimed to be a fairly good approximation if the intrawell relaxation is much faster than interwell hopping. In this approximation the coupling between the longitudinal and transverse carrier motion via the field dependent distribution function is not adequately treated. However, the consideration of this coupling is essential to understand the nonseparable influence of electric and magnetic fields on the miniband transport. Furthermore, the determination of the nonequilibrium distribution function and the related current density is a serious theoretical problem that has to be solved. On the one hand it has been pointed out that the electric field reduces the coupling between quantum wells and tends to localize the electronic wave functions. On the other hand, the electric field leads also to a field-dependent energy transfer to the lateral degrees of freedom, which has not been treated in most previous theoretical works. The importance of this lateral electron heating, making the high-field miniband transport effectively a three-dimensional problem, has been stressed by Gerhardts.[15] The coupling between the hopping transport along the SL axis and the in-plane motion mainly determines the temperature dependence of the miniband transport in a SL. Other than in narrow band semiconductors, where the lateral momentum distribution function $n(\boldsymbol{k}_\perp)$ is nearly constant,[16] a strong temperature dependence of the current has been observed in SL's (Ref. 17). From miniband transport measurements (at $\mathcal{H}=0$) two different temperature regimes have been discriminated.[17] At low temperatures and sufficiently high electric fields electrons diffuse through the crystal by hopping processes and the current increases with increasing temperature. Above a characteristic temperature the eigenfunctions extend over many SL periods and the current starts to decrease with increasing temperature. A theoretical study of the measured temperature characteristics requires a consistent treatment of field effects in both transport regimes and a calculation of the lateral electron distribution function from a kinetic equation. With respect to the mutual influence of parallel magnetic and electric fields the knowledge of the nonequilibrium distribution function is necessary in order to understand, why a magnetic field affects transitions between Stark ladder states, and why an electric field changes magnetophonon resonances.

In this paper we present a rigorous quantum theory of the high-field miniband transport in SL's. Intracollisional field effects are included and the nonequilibrium distribution function is calculated from a suitable quantum-kinetic equation.

II. BASIC THEORY

The mobility transport regime ($\Omega\tau \ll 1$) was thoroughly investigated within the semiclassical Boltzmann transport model.[18,19] In the opposite case $\Omega\tau \gg 1$, when Wannier-Stark localization (WSL) prevails, the Boltzmann approach becomes inadequate and must be replaced by a quantum-transport theory, which allows the determination of the nonequilibrium distribution function. In high electric fields the distribution function of electrons differs strikingly from its equilibrium form and has to be calculated from a kinetic equation. If constant electric and magnetic fields are applied to the SL structure the Laplace transformed distribution function $f(\boldsymbol{k},s)$ is the solution of the equation (see, e.g., Refs. 20,21)

$$\frac{e\vec{\mathcal{E}}}{\hbar}\nabla_{\boldsymbol{k}} f(\boldsymbol{k},s) = \sum_{\boldsymbol{k}'} \Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}|s) f(\boldsymbol{k}',s), \tag{1}$$

where the operator $\Delta\hat{W}$ encloses both scattering in and scattering out contributions:

$$\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}|s) = \hat{W}(\boldsymbol{k}',\boldsymbol{k}|s) - \delta_{\boldsymbol{k}',\boldsymbol{k}} \sum_{\boldsymbol{k}''} \hat{W}(\boldsymbol{k},\boldsymbol{k}''|s). \tag{2}$$

$\hat{W}$ exhibits a characteristic dependence on the electric and magnetic fields and was calculated in Ref. 22. At the presence of a magnetic field the scattering probability is an operator defined by

$$\hat{W}(\boldsymbol{k}',\boldsymbol{k}|s) = W(\boldsymbol{k}',\boldsymbol{k}|s) + \frac{i}{\hbar}\delta_{\boldsymbol{k}\boldsymbol{k}'}\{\varepsilon[\boldsymbol{k}' + \boldsymbol{A}(i\nabla_{\boldsymbol{k}'})] - \varepsilon[\boldsymbol{k} - \boldsymbol{A}(i\nabla_{\boldsymbol{k}})]\}. \tag{3}$$

Intracollisional field effects due to both electric and magnetic fields are included. This expression allows the simultaneous treatment of arbitrarily strong electric and magnetic fields. In Eq. (3) $\varepsilon(\boldsymbol{k})$ is the superlattice dispersion relation, $\boldsymbol{A}$ the vector potential of the magnetic field in the symmetric gauge, and W the transition rate, which depends on the electric and magnetic fields and which plays a fundamental role in the transport. Without any scattering ($\Omega\tau \to \infty$) the wave vector component along the field direction changes with time t according to $k_z(t) = k_0 + e\mathcal{E}t/\hbar$, which results in an oscillatory drift velocity ($v_z = \partial\varepsilon/\partial\hbar k_z$). The frequency of these oscillations in $\boldsymbol{k}$ space is $2\pi e\mathcal{E}/\hbar G_0$, where G_0 is the shortest vector of the reciprocal lattice along the $\mathcal{E}$ direction. In the one-band case this periodic motion is quantized and leads to the Wannier-Stark ladder. The time averaged current vanishes unless scattering induced transitions between Stark levels occur. Therefore, the identification of the main scattering processes is crucial for the calculation of the current. Here we focus on optical-phonon scattering described by the Fröhlich Hamiltonian,

$$H_{e\text{-ph}} = \frac{1}{\sqrt{2N}} \sum_{\boldsymbol{k},\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}}[\gamma_{\boldsymbol{q}} b_{\boldsymbol{q}}^\dagger a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}-\boldsymbol{q}} + \text{H.c.}], \tag{4}$$

where $\omega_{\boldsymbol{q}}$ is the phonon frequency, $\gamma_{\boldsymbol{q}}$ the electron-phonon coupling constant, and $a_{\boldsymbol{k}}$ ($b_{\boldsymbol{k}}$) are the electron (phonon) field operators. The envisaged nonperturbative inclusion of external electric and magnetic fields requires a due treatment of intracollisional field effects, i.e., a consideration of field dependent scattering rates. Using diagrammatic techniques such a scattering rate was obtained in[22,23]

$$W(\boldsymbol{k}',\boldsymbol{k}|s)=2\,\mathrm{Re}\int_0^\infty dt\; e^{-st}\sum_{\boldsymbol{q}}\;\omega_{\boldsymbol{q}}^2|\gamma_{\boldsymbol{q}}|^2[(N_{\boldsymbol{q}}+1)e^{-i\omega_q t}+N_{\boldsymbol{q}}e^{i\omega_q t}]\exp\left\{\frac{i}{\hbar}\int_{-t/2}^{t/2}d\tau\{\varepsilon[\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{F}\tau+\boldsymbol{A}(i\nabla_k)]-\varepsilon[\boldsymbol{k}-\boldsymbol{F}\tau-\boldsymbol{A}(i\nabla_k)]\}\right\}\delta_{\boldsymbol{k}',\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{F}t}, \tag{5}$$

where s is an adiabatic parameter (the Laplace transformed time variable), $\boldsymbol{F}=e\vec{\mathcal{E}}/\hbar$ and $N_{\boldsymbol{q}}=1/[\exp(\hbar\omega_{\boldsymbol{q}}/k_BT)-1]$ is the phonon distribution function. Equation (5) takes into account intracollisional field effects due to both electric and magnetic fields. The Houston representation (5) of the scattering probability W has been derived to the lowest order in the electron-phonon coupling and for low carrier concentrations, where correlation effects can be neglected.

As mentioned above the motion of electrons perpendicular to the layers is strictly periodic when scattering is neglected so that periodic boundary conditions can be imposed along the k_z direction[16]

$$f(\boldsymbol{k}_\perp,k_z+\Lambda_z,s)=f(\boldsymbol{k}_\perp,k_z,s), \tag{6}$$

where Λ_z is the reciprocal lattice vector. This periodicity condition can be used to simplify the equation for $f(\boldsymbol{k})$. To this end we introduce a lateral distribution function $n(\boldsymbol{k}_\perp,s)=\Sigma_{k_z}f(\boldsymbol{k},s)$, which allows a reformulation of the kinetic equation (1) in such a way that the boundary condition (6) is automatically fulfilled. The simplification results from the fact that already one degree of freedom (namely, k_z) has been eliminated and that spherical symmetry applies to the remaining part $n(\boldsymbol{k}_\perp,s)$ of the distribution function. $n(\boldsymbol{k}_\perp,s)$ describes the heating of the lateral electron motion and strongly deviates from its equilibrium expression if high electric fields are applied. To derive an equation for $n(\boldsymbol{k}_\perp,s)$ we make the ansatz

$$f(\boldsymbol{k},s)=n(\boldsymbol{k}_\perp,s)+\sum_{\boldsymbol{k}'}\hat{F}(\boldsymbol{k}',\boldsymbol{k}|s)f(\boldsymbol{k}',s)+\frac{\hbar k_z}{e\mathcal{E}}\sum_{\boldsymbol{k}'}\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)f(\boldsymbol{k}',s), \tag{7}$$

with

$$\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)=\sum_{k_z}\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}|s). \tag{8}$$

Owing to Eqs. (1) and (7) the newly introduced operator $\hat{F}$ must satisfy the equation

$$\frac{\partial\hat{F}(\boldsymbol{k}',\boldsymbol{k}|s)}{\partial k_z}=\frac{\hbar}{e\mathcal{E}}[\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}|s)-\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)]. \tag{9}$$

It is seen from the ansatz (7) that the boundary condition (6) is only fulfilled when the last term on the right hand side of Eq. (7) vanishes:

$$\sum_{\boldsymbol{k}'}\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)f(\boldsymbol{k}',s)=0. \tag{10}$$

This equation can be used to derive an integral equation for the lateral distribution function $n(\boldsymbol{k}_\perp,s)$ (cf. Ref. 16). For this purpose a new operator $\hat{F}_1$ is defined by

$$f(\boldsymbol{k},s)=n(\boldsymbol{k}_\perp,s)+\sum_{\boldsymbol{k}'}\hat{F}_1(\boldsymbol{k}',\boldsymbol{k}|s)n(\boldsymbol{k}'_\perp,s), \tag{11}$$

which we compare with Eq. (7) [under the condition (10)] and obtain

$$\hat{F}_1(\boldsymbol{k}',\boldsymbol{k}|s)=\hat{F}(\boldsymbol{k}',\boldsymbol{k}|s)+\sum_{\boldsymbol{k}''}\hat{F}_1(\boldsymbol{k}',\boldsymbol{k}''|s)\hat{F}(\boldsymbol{k}'',\boldsymbol{k}|s). \tag{12}$$

To derive an equation for $n(\boldsymbol{k}_\perp)$ we insert Eq. (11) into Eq. (10) and arrive at an expression that can be cast into the form

$$\sum_{\boldsymbol{k}'}\Delta\hat{W}_1(\boldsymbol{k}',\boldsymbol{k}_\perp|s)n(\boldsymbol{k}'_\perp,s)=0, \tag{13}$$

where $\hat{W}_1$ itself satisfies the integral equation

$$\begin{aligned}\Delta\hat{W}_1(\boldsymbol{k}',\boldsymbol{k}_\perp|s)&=\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)+\sum_{\boldsymbol{k}''}\hat{F}_1(\boldsymbol{k}',\boldsymbol{k}''|s)\times\Delta\hat{W}(\boldsymbol{k}'',\boldsymbol{k}_\perp|s)\\&=\Delta\hat{W}(\boldsymbol{k}',\boldsymbol{k}_\perp|s)+\sum_{\boldsymbol{k}''}\hat{F}(\boldsymbol{k}',\boldsymbol{k}''|s)\times\Delta\hat{W}_1(\boldsymbol{k}'',\boldsymbol{k}_\perp|s).\end{aligned} \tag{14}$$

Now we insert Eq. (9) into the k_z derivative of Eq. (12) and compare the result with the difference $\hat{W}_1(\boldsymbol{k}',\boldsymbol{k})-\hat{W}_1(\boldsymbol{k}',\boldsymbol{k}_\perp)$ expressed by using Eq. (14). We obtain

$$\frac{\partial\hat{F}_1(\boldsymbol{k}',\boldsymbol{k}|s)}{\partial k_z}=\frac{\hbar}{e\mathcal{E}}[\Delta\hat{W}_1(\boldsymbol{k}',\boldsymbol{k}|s)-\Delta\hat{W}_1(\boldsymbol{k}',\boldsymbol{k}_\perp|s)], \tag{15}$$

which completely agrees with Eq. (9). From Eqs. (9) and (15) we conclude that an iteration of Eq. (14) results in an asymptotic expansion of $\hat{W}_1$ with respect to the electric field. In the hopping regime, when $\Omega\tau\gg 1$, it is a reasonable approximation to retain only the first term in this expansion so that we can identify the quantity $\Delta\hat{W}_1$ with $\Delta\hat{W}$ in Eq. (13). Physically it is obvious that for the considered field alignments the lateral distribution function $n(\boldsymbol{k}_\perp,s)$ does not depend on the angle but only on $|\boldsymbol{k}_\perp|$. This will be confirmed by the calculation of the scattering probability W in the next section. In this case the second term on the right hand side of Eq. (3) does not give any contribution and we obtain from Eq. (13)

$$\sum_{k_z,k'}[W(\boldsymbol{k}',\boldsymbol{k}|s)n(\boldsymbol{k}'_\perp,s)-W(\boldsymbol{k},\boldsymbol{k}'|s)n(\boldsymbol{k}_\perp,s)]=0. \tag{16}$$

This is a convenient final form of the quantum-kinetic equation because already one degree of freedom (namely, k_z) has been eliminated. Together with the normalization condition for $n(\boldsymbol{k}_\perp)$ the linear integral equation (16) has a unique solution. As we will show in Sec. IV the current density can likewise be calculated from this special distribution function.

III. SCATTERING ON POLAR-OPTICAL PHONONS

We will focus on the particular case of scattering on polar-optical phonons with a narrow phonon bandwidth so that resonant-type current anomalies are not essentially smoothed out by an integration over the phonon dispersion. There are many different models for the electron-phonon interaction in SL's, which do not agree with each other and are, therefore, the subject of some controversial discussions. A due consideration of the electric-field dependent overlap of wave functions localized in neighboring wells via the form factor is inevitable to decide whether optical-phonon mediated current anomalies should occur in experiments or not. On the basis of the dielectric continuum model Shon and Nazareno[10] obtained very small form factors and related current contributions resulting from LO-phonon scattering. This indicates that a realistic description of the scattering on polar-optical phonons in SL's is necessary to derive reliable quantitative results. However, to keep our presentation transparent and simple we do not go into a detailed analysis of optical-phonon modes in the layers and rely on the simple bulk phonon model, which already reproduces the main qualitative features of the electron-phonon interaction in a SL. In this case the $\boldsymbol{q}$ sum in Eq. (5) is expressed by an integral ($\Sigma_{\boldsymbol{q}}\ldots\to(a/2\pi)^3\int d^3\boldsymbol{q}\ldots$). The exponential term in Eq. (5) is periodic in q_z ($q_z\to q_z+2\pi/d$) and depends on $\boldsymbol{q}_\perp$ only via $\varepsilon(\boldsymbol{k}_\perp+\boldsymbol{q}_\perp)$ so that the characteristic transverse electron and phonon momenta are of the same order of magnitude. Denoting the exponential term in Eq. (5) by $g(\boldsymbol{q})$ and the remaining $\boldsymbol{q}$-dependent factor by $|\gamma_{\boldsymbol{q}}|^2h(\omega_{\boldsymbol{q}})$ the right hand side of Eq. (5) has the following structure:

$$W=\left(\frac{a}{2\pi}\right)^3\int d^3\boldsymbol{q}|\gamma_{\boldsymbol{q}}|^2h(\omega_{\boldsymbol{q}})g(\boldsymbol{q}). \tag{17}$$

We neglect the weak $\boldsymbol{q}_\perp$ dependence in $\omega_{\boldsymbol{q}}$ and $\gamma_{\boldsymbol{q}}$ and consider the Fourier representation of $g(\boldsymbol{q})$ with respect to q_z:

$$\begin{aligned}W&=\left(\frac{a}{2\pi}\right)^3\int d^2\boldsymbol{q}_\perp\int_{-\infty}^{\infty}dq_z|\gamma_{q_z}|^2h(\omega_{q_z})\sum_{l=-\infty}^{\infty}g_l(\boldsymbol{q}_\perp)e^{ilq_zd/2\pi}\\&=\left(\frac{a}{2\pi}\right)^3\int d^2\boldsymbol{q}_\perp\int_0^{2\pi/d}dq_z\\&\quad\times\sum_{n=-\infty}^{\infty}|\gamma_{q_z+2\pi n/d}|^2h(\omega_{q_z+2\pi n/d})\sum_{l=-\infty}^{\infty}g_l(\boldsymbol{q}_\perp)e^{ilq_zd/2\pi}.\end{aligned} \tag{18}$$

Here $g_l(\boldsymbol{q}_\perp)$ are the Fourier coefficients of $g(\boldsymbol{q})$. In the restricted q_z interval ($0\leqslant q_z\leqslant 2\pi/d$) the factor $|\gamma_{q_z+2\pi n/d}|^2h(\omega_{q_z+2\pi n/d})$ depends only weakly on q_z, so that the $l=0$ term dominates the l sum in Eq. (18), and we obtain

$$W\cong\left(\frac{a}{2\pi}\right)^3\int d^3\boldsymbol{q}|\gamma_{q_z}|^2h(\omega_{q_z})g_{l=0}(\boldsymbol{q}_\perp). \tag{19}$$

Making use of the equation for the Fourier coefficient $g_{l=0}$,

$$g_{l=0}(\boldsymbol{q}_\perp)=\frac{d}{2\pi}\int_0^{2\pi/d}dq_zg(\boldsymbol{q}_\perp,q_z), \tag{20}$$

and considering only dispersionless optical phonons ($\omega_{q_z}\to\omega_0$), we arrive at the following expression for the transition probability:

$$\begin{aligned}W(\boldsymbol{k}',\boldsymbol{k}|s)&=2\Gamma\omega_0^2\,\mathrm{Re}\int_0^\infty dt\,e^{-st}[(N_0+1)e^{-i\omega_0t}+N_0e^{i\omega_0t}]\\&\quad\times\int d^2\boldsymbol{q}_\perp\int_0^{2\pi/d}dq_z\exp\Bigg(\frac{i}{\hbar}\int_{-t/2}^{t/2}d\tau\{\varepsilon[\boldsymbol{k}+\boldsymbol{q}\\&\quad-\boldsymbol{F}\tau+\boldsymbol{A}(i\nabla_{\boldsymbol{k}})]-\varepsilon[\boldsymbol{k}-\boldsymbol{F}\tau-\boldsymbol{A}(i\nabla_{\boldsymbol{k}})]\}\Bigg)\\&\quad\times\delta(\boldsymbol{k}'-\boldsymbol{k}-\boldsymbol{q}+\boldsymbol{F}t),\end{aligned} \tag{21}$$

where

$$\Gamma=\frac{a}{2\pi}\int_0^{2\pi/a}dq_z|\gamma_{\boldsymbol{q}_\perp=\boldsymbol{0},q_z}|^2 \tag{22}$$

is an average of the electron-phonon coupling constant along the field direction. The replacement of the scattering matrix element by an averaged screened coupling constant is only a crude approximation which allows us here, however, to derive a number of analytical results. As the q integration has been extended over the entire momentum space, the lattice constant a enters this average and not the SL period d.

The motion along the SL axis is not affected by the magnetic field and is characterized by the tight-binding dispersion relation,

$$\varepsilon(k_z)=\frac{\Delta}{2}[1-\cos(k_zd)]. \tag{23}$$

The $\boldsymbol{k}_\perp$ and k_z dependencies separate from each other so that the τ integral is easily calculated in Eq. (21). We obtain

$$W(\boldsymbol{k}',\boldsymbol{k}|s)=2\Gamma\omega_0^2\ \mathrm{Re}\int_0^\infty dt\ e^{-st}[(N_0+1)e^{-i\omega_0 t}+N_0 e^{i\omega_0 t}]\exp\left\{i\frac{\Delta}{\hbar\Omega}(\cos k_z' d-\cos k_z d)\times\sin\left(\frac{\Omega t}{2}\right)\right\}\int \boldsymbol{d}^2\boldsymbol{q}_\perp F(\boldsymbol{k}_\perp,\boldsymbol{k}_\perp',\boldsymbol{q}_\perp|t). \quad (24)$$

The Green's function F is defined by

$$F(\boldsymbol{k}_\perp,\boldsymbol{k}_\perp',\boldsymbol{q}_\perp|t)=\exp\left\{\frac{it}{\hbar}\{\varepsilon[\boldsymbol{k}_\perp+\boldsymbol{q}_\perp+\boldsymbol{A}(i\nabla_{k_\perp})]-\varepsilon[\boldsymbol{k}_\perp-\boldsymbol{A}(i\nabla_{k_\perp})]\}\right\}\delta_{\boldsymbol{k}_\perp',\boldsymbol{k}_\perp+\boldsymbol{q}_\perp}. \quad (25)$$

Its Laplace transformed components satisfy the differential equation

$$\left\{s-\frac{i}{\hbar}\{\varepsilon[\boldsymbol{k}_\perp+\boldsymbol{q}_\perp+\boldsymbol{A}(i\nabla_{k_\perp})]-\varepsilon[\boldsymbol{k}_\perp-\boldsymbol{A}(i\nabla_{k_\perp})]\}\right\}\times F(\boldsymbol{k}_\perp,\boldsymbol{k}_\perp',\boldsymbol{q}_\perp|s)=\delta_{\boldsymbol{k}_\perp',\boldsymbol{k}_\perp+\boldsymbol{q}_\perp}, \quad (26)$$

which we solve by making use of the ansatz

$$F(\boldsymbol{k}_\perp,\boldsymbol{k}_\perp',\boldsymbol{q}_\perp|s)=\sum_{\lambda\lambda'} a_{\lambda\lambda'}(\boldsymbol{q}_\perp|s)\psi_\lambda^*(\boldsymbol{k}_\perp)\psi_{\lambda'}(\boldsymbol{k'}_\perp), \quad (27)$$

with normalized orthogonal wave functions ψ_λ, which are solutions of the eigenvalue problem

$$\varepsilon[\boldsymbol{k}+\boldsymbol{A}(i\nabla_k)]\psi_\lambda(\boldsymbol{k}_\perp)=E_\lambda\psi_\lambda(\boldsymbol{k}_\perp). \quad (28)$$

Inserting the ansatz (27) into Eq. (26), the coefficients $a_{\lambda\lambda'}$ are easily calculated. From an inverse Laplace transformation we obtain

$$F(\boldsymbol{k}_\perp,\boldsymbol{k}_\perp',\boldsymbol{q}_\perp|t)=\sum_{\lambda\lambda'}\exp\left\{\frac{it}{\hbar}(E_{\lambda'}-E_\lambda)\right\}\psi_\lambda^*(\boldsymbol{k}_\perp)\psi_{\lambda'}(\boldsymbol{k}_\perp')\times\int \boldsymbol{d}^2\boldsymbol{k}_\perp''\psi_\lambda(\boldsymbol{k}_\perp''-\boldsymbol{q}_\perp)\psi_{\lambda'}^*(\boldsymbol{k}_\perp''). \quad (29)$$

In the symmetric gauge $\{\boldsymbol{A}(\boldsymbol{r})=e[\vec{\mathcal{H}}\times\boldsymbol{r}]/2\hbar c\}$ Eq. (28) has the solution

$$E_\lambda\equiv E_{m,n}=\hbar\omega_c\left(n+\frac{m+|m|+1}{2}\right), \quad (30)$$

$$\psi_{n,m}(k,\phi)=2\sqrt{\pi}\frac{\sqrt{(n+1)\dots(n+|m|)}}{|m|!}\times(2l_B^2)^{(|m|+1)/2}k^{|m|}e^{-(kl_B)^2}\frac{n!|m|!}{(n+|m|)!}\times L_n^{|m|}(2(kl_B)^2)e^{-im\phi}, \quad (31)$$

with $l_B=\sqrt{\hbar/m^*\omega_c}$ being the magnetic length and $n=0,1,2,\dots,\ m=0,\pm1,\pm2,\dots$ is the discrete set of quantum numbers. It is straightforward to calculate the Green's function (29) by making use of the equations

$$\int \boldsymbol{d}^2\boldsymbol{k}_\perp\psi_{nm}(\boldsymbol{k}_\perp)=\frac{(-1)^n}{\sqrt{2\pi l_B^2}}\delta_{m,0}, \quad (32)$$

and

$$\psi_{n0}(k_\perp)=2\sqrt{2\pi l_B^2}e^{-(k_\perp l_B)^2}L_n(2(k_\perp l_B)^2). \quad (33)$$

Inserting this result into Eq. (24) we obtain the final expression for the transition rate

$$W(\boldsymbol{k}',\boldsymbol{k}|s)=8\Gamma\omega_0^2 e^{-(k_\perp l_B)^2-(k_\perp' l_B)^2}\times\mathrm{Re}\int_0^\infty dt\ e^{-st}[(N_0+1)e^{-i\omega_0 t}+N_0e^{i\omega_0 t}]\times\exp\left\{i\frac{\Delta}{\hbar\Omega}(\cos k_z'd-\cos k_z d)\sin\left(\frac{\Omega t}{2}\right)\right\}\times\sum_{nn'=0}^\infty(-1)^{n-n'}e^{i(n'-n)\omega_c t}\times L_n(2(k_\perp l_B)^2)L_{n'}(2(k_\perp' l_B)^2). \quad (34)$$

Equation (34) shows that the scattering is isotropic in the $\boldsymbol{k}_\perp$ plane and does not introduce any angular dependence of the distribution function. Therefore, it is possible to expand $n(\boldsymbol{k}_\perp)$ in orthogonal Laguerre polynomials,

$$n(\boldsymbol{k}_\perp)=2\exp(-(k_\perp l_B)^2)\sum_{n=0}^\infty(-1)^n L_n(2(k_\perp l_B)^2)N_n. \quad (35)$$

The kinetic equation (16) is used to determine the unknown coefficients N_n. The imposed normalization of the lateral electron distribution function

$$\frac{a^2}{(2\pi)^2}\int \boldsymbol{d}^2\boldsymbol{k}_\perp n(\boldsymbol{k}_\perp)=1 \quad (36)$$

is converted into the condition $\Sigma_{n=0}^\infty\tilde{N}_n=1$, where $\tilde{N}_n=(a^2/2\pi l_B^2)N_n$.

Now we are in the position to derive an explicit form of the kinetic equation (16). To this end Eq. (34) is inserted into Eq. (16). The k_z,k_z' integrals are elementary and lead to the function

$$G(t)=J_0^2\left(\frac{\Delta}{\hbar\Omega}\sin\frac{\Omega t}{2}\right)=\sum_{l=-\infty}^\infty f_l\left(\frac{\Delta}{\hbar\Omega}\right)e^{-il\Omega t}, \quad (37)$$

which is periodic $[G(t+2\pi/\Omega)=G(t)]$ and may, therefore, be Fourier transformed as indicated. The Fourier coefficients f_l depend on the electric field and are given by

$$f_l\left(\frac{\Delta}{\hbar\Omega}\right)=\frac{1}{\pi}\int_0^\pi dt\cos(lt)J_0^2\left(\frac{\Delta}{\hbar\Omega}\sin\frac{t}{2}\right)=\frac{1}{\pi}\int_0^\pi dx\ J_l^2\left(\frac{\Delta}{\hbar\Omega}\sin x\right). \quad (38)$$

Physically, the Fourier transformation (37) simply means a switching to the Stark-ladder representation, which is appropriate in the hopping transport regime. In this representation all states are completely discrete so that the expression for the current and the kinetic equation exhibit a singular structure. In this case meaningful results can only be obtained if one reintroduces the neglected phonon dispersion. This would allow a consistent microscopic consideration of the hopping transport in this field configuration. Here, however, we prefer a phenomenological approach and identify the variable s of the Laplace transformation in Eq. (34) with some effective scattering rate $1/\tau$, which leads to a collisional broadening of Landau and Stark levels. Putting everything together we obtain from Eqs. (16), (34), and (37) the following set of homogeneous linear equations for $\tilde{N}_n$:

$$\sum_{n'=0}^{\infty}\sum_{l=-\infty}^{\infty} f_l\left(\frac{\Delta}{\hbar\Omega}\right)\left[\frac{1}{\delta^2+[(n'-n)\gamma-l\kappa-1]^2}+\frac{1}{\delta^2+[(n'-n)\gamma-l\kappa+1]^2}\right]\times\left\{\exp\left[\frac{\beta}{2}[(n'-n)\gamma-l\kappa]\right]\tilde{N}_{n'}-\exp\left[-\frac{\beta}{2}[(n'-n)\gamma-l\kappa]\right]\tilde{N}_n\right\}=0. \tag{39}$$

Here $\delta=1/\omega_0\tau$ is the broadening parameter, $\gamma=\omega_c/\omega_0$ ($\kappa=\Omega/\omega_0$) the parameter of the magnetic (electric) field, and $\beta=\hbar\omega_0/k_BT$. In deriving Eq. (39) the smooth $\boldsymbol{k}_\perp$ dependence of the scattering out term has been neglected.[24] Adding the normalization condition $\Sigma_{n=0}^{\infty}\tilde{N}_n=1$, Eq. (39) gives us an inhomogeneous set of linear equations for $\tilde{N}_n$, which has a unique solution. In the limit of vanishing magnetic fields ($\gamma\to 0$) the sum over n' in Eq. (39) can be converted into an integral so that the spectrum becomes continuous and one can chose $\delta\to +0$. Another convenient form of Eq. (39) is obtained by making use of the substitution $\tilde{N}_n\to\nu_n$ with $\tilde{N}_n=\nu_n\exp(-\beta\gamma n)$:

$$\sum_{n'=0}^{\infty}\sum_{l=-\infty}^{\infty} f_l\left(\frac{\Delta}{\hbar\Omega}\right)\left[\frac{1}{\delta^2+[(n'-n)\gamma-l\kappa-1]^2}+\frac{1}{\delta^2+[(n'-n)\gamma-l\kappa+1]^2}\right]e^{-\beta(n+n')/2}\times\{e^{-\beta l\kappa/2}\nu_{n'}-e^{\beta l\kappa/2}\nu_n\}=0. \tag{40}$$

In the limit of high electric fields and narrow bandwidths ($\Delta/\hbar\Omega\ll 1$), only the $f_{l=0}$ component survives in Eq. (40) and one gets the solution $\nu_n=$const. Exploiting the normalization condition we obtain

$$\tilde{N}_n=2\sinh\left(\frac{\beta\gamma}{2}\right)e^{-\beta\gamma(n+1/2)}. \tag{41}$$

This is the thermal equilibrium distribution function, which has been used in Ref. 9 to study resonance phenomena in the hopping transport regime. In general, the lateral distribution function strongly deviates from Eq. (41) and has to be calculated from Eq. (39) or Eq. (40).

IV. CALCULATION OF THE CURRENT

The current density is calculated from the electron distribution function via

$$\boldsymbol{j}=-\frac{e}{\hbar V}\sum_{\boldsymbol{k}(1\mathrm{BZ})}\varepsilon(\boldsymbol{k})\nabla_{\boldsymbol{k}}f(\boldsymbol{k}), \tag{42}$$

where V is the crystal volume. This expression has been obtained by an integration by parts and allows a straightforward use of the kinetic equation (1). In our context it is expedient to use another form, which expresses the current density by the lateral distribution function directly. This is accomplished by inserting Eq. (11) into (42) and considering Eq. (15) and Eq. (13). Finally, we arrive at a representation for the current

$$j_z=-\frac{n}{\mathcal{E}}\sum_{\boldsymbol{kk}'}\Delta\hat{W}_1(\boldsymbol{k}',\boldsymbol{k})n(\boldsymbol{k}'_\perp)\varepsilon(\boldsymbol{k}), \tag{43}$$

which has already the asymptotic form ($j_z\sim 1/\mathcal{E}$) relevant in the NDC regime. To the leading order in the asymptotic expansion of the current with respect to the electric field, we identify the renormalized scattering rate $\hat{W}_1$ with $\hat{W}$ and consider again Eq. (13), which yields

$$j_z=\frac{n}{\mathcal{E}}\frac{\Omega_0^2}{(2\pi)^6}\int \boldsymbol{d}^3\boldsymbol{k}\int \boldsymbol{d}^3\boldsymbol{k}'\,\hat{W}(\boldsymbol{k}',\boldsymbol{k})n(\boldsymbol{k}'_\perp)[\varepsilon(\boldsymbol{k}')-\varepsilon(\boldsymbol{k})]. \tag{44}$$

The second term on the right hand side of Eq. (3) is proportional to $\delta_{\boldsymbol{k},\boldsymbol{k}'}$ so that it does not give any contribution to the current density (44). An explicit form for the current density j_z can be derived from Eq. (44) by inserting Eq. (34) and Eq. (35) into Eq. (44) and by calculating the k_z, k_z' integrals, which again results in the factor $G(t)$ [cf. Eq. (37)]. We obtain

$$\frac{j_z}{j_0}=\frac{\omega_c}{\pi}\sum_{nn'=0}^{\infty}\tilde{N}_{n'}\,\mathrm{Re}\int_0^\infty dt\; e^{-st+i(n'-n)\omega_c t}\times[e^{\beta}e^{-i\omega_0 t}-e^{i\omega_0 t}]G(t), \tag{45}$$

with

$$j_0=\frac{n}{E}\frac{m^*\omega_0^3a^2\Gamma}{e^{\beta}-1} \tag{46}$$

being a reference current density.

In the quasiclassical limit, when the hopping length is much smaller than the localization length ($\hbar\Omega\ll\Delta$), there is only a weak field dependence in $G(t)$ which can be neglected:

$$G(t)\approx J_0^2\left(\frac{\Delta t}{2\hbar}\right). \tag{47}$$

In this quasiclassical limit the intracollisional field effect does not play any significant role and the lateral distribution function is given by Eq. (41). Then the t integral in Eq. (45) is elementary and we obtain

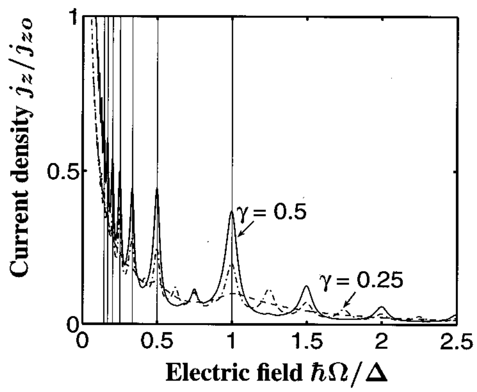

FIG. 1. Current density j_z/j_{zo} as a function of the electric-field parameter $\hbar\Omega/\Delta$ for $\alpha=1$, $\beta=10$, $\delta=0.05$, and different magnetic-field parameters $\gamma=\omega_c/\omega_0$. The dashed, dash-dotted, and solid lines are calculated from Eqs. (39) and (50) with $\gamma=0$, 0.25, and 0.5, respectively. Peaks due to Wannier-Stark transitions at $\hbar\Omega/\Delta=1/n$ are marked by vertical lines. The reference current density $j_{zo}=enm^*\omega_0^3\Gamma a^2 d/\Delta$ has been introduced.

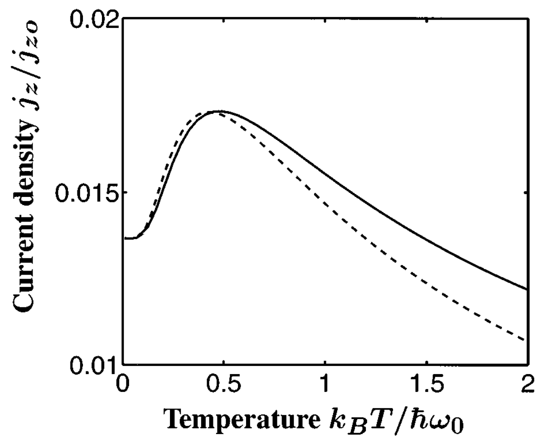

FIG. 2. Temperature dependence of the current density calculated from Eqs. (39) and (50) (solid line) for $\alpha=0.2$, $\gamma=0.5$, $\hbar\Omega/\Delta=1.5$, and $\delta=0.05$. The dashed line is obtained from Eq. (48) by using the equilibrium distribution function (41).

$$\frac{j_z}{j_0}=\frac{4\gamma}{\alpha\pi^2}e^{-\beta\gamma/2}\sinh\frac{\beta\gamma}{2}\sum_{nn'=0}^{\infty}e^{-\beta\gamma n'}\{H[(n'-n)\gamma-1]e^{\beta}-H[(n'-n)\gamma+1]\},\tag{48}$$

with

$$H(x)=K(\sqrt{1-(x/\alpha)^2})\Theta(\alpha-|x|).\tag{49}$$

Here K is the complete elliptic integral of the first kind and $\alpha=\Delta/\hbar\omega_0$ a bandwidth parameter. In the quasiclassical case the band transport dominates and there are no resonances associated with the Wannier-Stark ladder.

Beyond the field interval, where the semiclassical treatment applies, the electric field dependence of $G(t)$ has to be retained. In this general case one might exploit Eq. (37) and obtains from Eq. (45),

$$\frac{j_z}{j_0}=\frac{\gamma\delta}{\pi}\sum_{l=-\infty}^{\infty}f_l\left(\frac{\Delta}{\hbar\Omega}\right)\sum_{nn'=0}^{\infty}\tilde{N}_{n'}\left[\frac{e^{\beta}}{\delta^2+[(n'-n)\gamma-l\kappa-1]^2}-\frac{1}{\delta^2+[(n'-n)\gamma-l\kappa+1]^2}\right].\tag{50}$$

This equation together with Eq. (39) [or Eq. (40)] are the main theoretical results of our paper.

Figure 1 shows the electric field dependence of the current density calculated from Eqs. (39) and (50) for $\alpha=1$, $\beta=10$, and $\delta=0.05$. The current density is expressed in units of $j_{zo}=enm^*\omega_0^3\Gamma a^2 d/\Delta$. Without any magnetic field ($\gamma=0$, dashed line) there are only smooth oscillations due to transitions between Wannier-Stark levels. These oscillatory current anomalies in the NDC regime are smooth due to the finite collisional broadening parameter ($\delta=0.05$). Suppressed local current maxima appear at $\hbar\Omega/\Delta=1/(\alpha l)$ with l being a positive integer. The positions of these field strengths are marked by vertical lines in Fig. 1. Application of a magnetic field leads to a pronounced enhancement of these Stark transitions as shown in Fig. 1 by the solid and dash-dotted lines. The magnetic-field mediated increase of the current around the Stark ladder is related to a dimensionality reduction and the corresponding peaked density of states, which results from the quantization of the in-plane motion. Indeed, it has been shown[25] that oscillatory current anomalies in two-dimensional SL's are much more pronounced than in the three-dimensional counterparts. This dimensionality effect has been identified in experiments,[11] too. The combined effect of the electric-field-induced localization and the magnetic field related dimensionality reduction gives rise to a longitudinal magnetoresistance, which is negative over a substantial range of magnetic fields. In addition to the enhanced Wannier-Stark transitions, Fig. 1 exhibits also peaks due to magnetophonon resonances (all structures above $\hbar\Omega/\Delta=1$ and between the Stark ladder). The intensity of these peaks is modulated by the electric field.

The temperature dependence of the current density, as calculated from Eqs. (39) and (50), is shown in Fig. 2 by the solid line (where the parameters $\alpha=0.2$, $\gamma=0.5$, $\hbar\Omega/\Delta=1.5$, and $\delta=0.05$ have been used). At low temperatures the current density decreases exponentially with decreasing temperature. This clearly designates the hopping transport regime. At $T=0$ the current remains nonzero, which is a consequence of the nonanalytic field dependence of the current density [$j_z\sim\mathcal{E}^{3/2}\exp(-C/\mathcal{E})$]. Such a dependence is also known from the Franz-Keldysh effect, and is due to the energy supply by the electric field. With increasing temperature the current reaches a maximum followed by a second temperature regime, where the carrier transport is due to more or less extended states so that the current decreases with increasing temperature. In this band transport regime the semiclassical Boltzmann transport model becomes applicable. Both temperature regimes were observed in experiments.[17] In Fig. 2 the dashed line has been calculated from Eq. (48). This approximation, which is formally equivalent to the mean energy gain method, leads to an overestimate of the band transport character at high temperatures. With an increasing bandwidth parameter α Eq. (41) completely fails to approximate the lateral distribution function

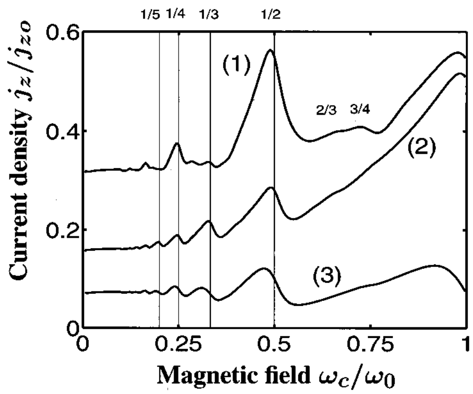

FIG. 3. Current density j_z/j_{zo} as a function of the magnetic-field parameter ω_c/ω_0 for $\beta=1$ and $\delta=0.1$. For curves (1), (2), and (3) the bandwidth and field parameters are ($\alpha=1$, $\hbar\Omega/\Delta=0.5$), ($\alpha=1$, $\hbar\Omega/\Delta=1$), and ($\alpha=0.5$, $\hbar\Omega/\Delta=1$), respectively. The main peak positions at $\gamma=\omega_c/\omega_0=1/n$ are indicated by thin vertical lines. $j_{zo}=enm^*\omega_0^3\Gamma a^2 d/\Delta$ is a suitable reference current density.

and the current-temperature dependence calculated from Eq. (50) together with Eq. (39) deviates even qualitatively from results obtained from Eq. (48).

The magnetic-field dependence of the current density is shown in Fig. 3 for $\beta=1$ and $\delta=0.1$. Curves (1), (2), and (3) where calculated with ($\alpha=1$, $\hbar\Omega/\Delta=0.5$), ($\alpha=1$, $\hbar\Omega/\Delta=1$), and ($\alpha=0.5$, $\hbar\Omega/\Delta=1$), respectively. Electric-field modified, broad magneto-phonon resonances appear at magnetic fields satisfying the condition $n\gamma=l\kappa\pm1$, where n,l are integers. The main peaks are due to transitions between different Landau levels associated with the $l=0$ Stark ladder state. Their position is marked by vertical lines. A comparison between curves (1) and (2) shows that in the considered NDC region the current density is lower at higher electric fields. A decrease of the current is also observed when the miniband width decreases [(2)→(3)]. This is due to the bandwidth dependence of the current. With increasing electric field the peak at $\omega_c=\omega_0$ is enhanced, similar to the enhancement of Wannier-Stark oscillations by a magnetic field. However, the peak at $\omega_c=\omega_0/2$ behaves quite differently and decreases with increasing electric field. Therefore, we conclude that the effects exerted by the electric field are not independent of the magnetic field. The mutual influence of the electric and magnetic fields on the current density described by the lateral distribution function is not separable. Curve (1) exhibits additional structures at $\gamma=3/4$ ($n=2,l=1$) and 2/3 ($n=3,l=2$), which result from transitions between different Stark levels. A reduction of the bandwidth parameter α leads to a suppression of the current at high magnetic fields. This is shown by curve (3), where for $\gamma=1$ the miniband width Δ is smaller than the cyclotron energy $\hbar\omega_c$ so that forbidden gaps appear, which leads to a gradual reduction of the optical-phonon scattering until at $\omega_0+\Delta<\omega_c$ it is completely forbidden. This observation is in accordance with other theoretical work[7] and experiments.[6] The oscillation amplitudes of the current density increase with increasing temperature (not shown in Fig. 3) because the LO-phonon scattering is stronger when the temperature becomes higher.

V. SUMMARY

In summary we have presented a quantum-kinetic description of the interplay between Landau and Stark quantization in the high-field miniband transport of SL's. High electric and magnetic fields aligned parallel to the SL axis lead to a complete localization of all electronic states. Hopping transport is only possible when electrons are scattered on impurities or phonons. Current maxima are observed at combined magnetophonon and electro-phonon resonances, which appear at $n\omega_c=l\Omega\pm\omega_0$. Due to the dimensionality reduction by the magnetic field the transport becomes effectively one dimensional, which leads to a pronounced enhancement of Wannier-Stark current oscillations. The amplitudes of magnetophonon and electrophonon resonances show a quite different temperature behavior. Magnetophonon resonance peaks are most pronounced at high temperatures, when optical-phonon scattering is strong. Contrary, the observation of electrophonon resonances requires low lattice temperatures. Other theoretical papers[7,9,10,12] arrived at similar conclusions but neglected the intracollisional field effects and/or the lateral electron heating described by the electron distribution function. However, a due consideration of these effects is inevitable in order to understand, why the magnetic field influences the electric-field driven electron motion along the SL axis and conversely, why an electric field affects the magnetophonon resonances. This coupling between the longitudinal and transverse degrees of freedom is described by a lateral distribution function, which is the solution of the quantum-kinetic equation (39). As a result the electric and magnetic-field effects as well as their temperature dependence are not separable. Three main peculiarities of the miniband transport have been attributed to a coupling between the longitudinal and in-plane motion described by $n(\boldsymbol{k}_\perp)$: (i) the magnetic-field mediated enhancement of Wannier-Stark oscillations due to the dimensionality reduction and the associated peak structure of the density of states, (ii) a decrease of the current due to a suppression of optical-phonon scattering once $\hbar\omega_c$ becomes larger than the miniband width Δ so that real gaps appear in the energy spectrum of the lateral electron motion, (iii) a crossover in the temperature dependence of the current as observed in experiments.

We presented a general framework for a quantum-transport theory in SL's at high electric and magnetic fields in which intracollisional field effects are accounted for. The electron distribution function is calculated from a quantum-kinetic equation. It has been pointed out that a consequent microscopic approach requires the consideration of the phonon dispersion, which removes the singular structure of the current.

Stark-cyclotron resonance effects have been measured[8] in the superlattice transport assisted by quasielastic scattering. At low temperatures the elastic scattering is mainly due to ionized background impurities and layer fluctuations.[8] To analyze these experimental results elastic scattering mechanisms have to be incorporated into our approach. This is straightforward and will be done in the near future.

[1] J. Feldmann, K. Leo, J. Shah, D. A. B. Miller, J. E. Cunningham, T. Meier, G. von Plessen, A. Schulze, P. Thomas, and S. Schmitt-Rink, Phys. Rev. B **46**, 7252 (1992).
[2] T. Dekorsy, P. Leisching, K. Köhler, and H. Kurz, Phys. Rev. B **50**, 8106 (1994).
[3] C. Waschke, H. G. Roskos, R. Schwedler, K. Leo, H. Kurz, and K. Köhler, Phys. Rev. Lett. **70**, 3319 (1993).
[4] H. N. Nazareno and R. A. Masut, Solid State Commun. **101**, 819 (1997).
[5] F. Claro, M. Pacheco, and Z. Barticevic, Phys. Rev. Lett. **64**, 3058 (1990).
[6] H. Noguchi, H. Sakaki, T. Takamasu, and N. Miura, Phys. Rev. B **45**, 12 148 (1992).
[7] W. M. Shu and X. L. Lei, Phys. Rev. B **50**, 17 378 (1994).
[8] L. Canali, M. Lazzarino, L. Sorba, and F. Beltram, Phys. Rev. Lett. **76**, 3618 (1996).
[9] N. H. Shon and H. N. Nazareno, Phys. Rev. B **53**, 7937 (1996).
[10] N. H. Shon and H. N. Nazareno, Phys. Rev. B **55**, 6712 (1997).
[11] A. Alexandrou, E. E. Mendez, and J. M. Hong, Phys. Rev. B **44**, 1934 (1991).
[12] Y. Lyanda-Geller and J. P. Leburton, Phys. Rev. B **52**, 2779 (1995).
[13] R. Tsu and G. Döhler, Phys. Rev. B **12**, 680 (1975).
[14] D. Calecki, J. F. Palmier, and A. Chomette, J. Phys. C **17**, 5017 (1984).
[15] R. R. Gerhardts, Phys. Rev. B **48**, 9178 (1993).
[16] V. V. Bryksin and Y. A. Firsov, Zh. Éksp. Teor. Fiz. **61**, 2373 (1971) [Sov. Phys. JETP **34**, 1272 (1971)].
[17] H. T. Grahn, K. von Klitzing, K. Ploog, and G. H. Döhler, Phys. Rev. B **43**, 12 094 (1991).
[18] A. Y. Shik, Fiz. Tekh. Poluprovodn. **7**, 261 (1973) [Sov. Phys. Semicond. **7**, 187 (1973)].
[19] R. A. Suris and B. S. Shchamkhalova, Fiz. Tekh. Poluprovodn. **18**, 1178 (1984) [Sov. Phys. Semicond. **18**, 738 (1984)].
[20] L. Reggiani, *Hot-Electron Transport in Semiconductors* (Springer-Verlag, Berlin, 1985).
[21] H. Haug and A. P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer-Verlag, Berlin, 1996).
[22] V. V. Bryksin and Y. A. Firsov, Fiz. Tverd. Tela (Leningrad) **15**, 3235 (1973) [Sov. Phys. Solid State **15**, 2158 (1973)].
[23] V. V. Bryksin, V. C. Woloschin, and A. W. Rajtzev, Fiz. Tverd. Tela (Leningrad) **22**, 3076 (1980) [Sov. Phys. Solid State **22**, 1796 (1980)].
[24] B. Movaghar, Semicond. Sci. Technol. **2**, 185 (1987).
[25] P. Kleinert and V. V. Bryksin, Superlatt. Microstruct. (to be published).