Electronic states of a two-dimensional electron system in a lateral superlattice and a perpendicular magnetic field

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(Received 25 July 1997)

The electronic state of a two-dimensional electron system (2DES) in the presence of a perpendicular uniform magnetic field and a lateral superlattice (LS) is investigated theoretically. A comparative study is made between a LS induced by a spatial electrostatic potential modulation (referred to as a PMLS) and that induced by a spatial magnetic-field modulation (referred to as a MMLS). By utilizing a finite-temperature self-consistent Hartree-Fock approximation scheme, the dependence of the electronic state on different system parameters (e.g., the modulation period, the modulation strength, the effective electron-electron interaction strength, the averaged electron density, and the system temperature) is studied in detail. The inclusion of exchange effect is found to bring qualitative changes to the electronic state of a PMLS, leading generally to a nonuniform spin splitting, and consequently the behavior of the electronic state becomes similar to that of a MMLS. The Landau-level coupling is taken into account, and is found to introduce some interesting features not observed before. It is also found that, even in the regime of intermediate modulation strength, the density dependence of the spin splitting of energy levels, either for a PMLS or a MMLS, can be qualitatively understood within the picture of a 2DES in a perpendicular magnetic field with the modulation viewed as a perturbation. [S0163-1829(97)02248-0]

I. INTRODUCTION

The motion of an electron in a perpendicular uniform magnetic field and a periodic potential is a nontrivial problem, and has been intensively studied for decades,[1] although the experimental exploration of such systems was hindered by the requirement of an extremely high magnetic field. However, the recent advance in submicron technology has made it possible to fabricate microstructures in which one can reach the interested regime even with a moderate magnetic field.[2] Because of this, there has been a considerable amount of experimental and theoretical studies on the transport and optical properties of a two-dimensional electron system (2DES) in a perpendicular uniform magnetic field and a lateral superlattice (or lateral modulation), and some fascinating phenomena are observed.[2,3]

A lateral superlattice (LS) can be realized by imposing on a 2DES a spatial electrostatic potential modulation (this type of LS will be referred to as a PMLS) or a spatial magnetic field modulation (this type of LS will be referred to as a MMLS).[2] Both a PMLS and a MMLS have been experimentally realized and intensively studied in the literature.[2–4] In this paper, we wish to report a comparative study of the electronic state of a 2DES in a PMLS and in a MMLS, with one-dimensional lateral modulations. In calculating the electronic state, a finite temperature self-consistent Hartree-Fock approximation scheme[4,5] is employed, allowing us to investigate the dependence of the electronic state on various system parameters, i.e., the modulation period, the modulation strength, the effective electron-electron interaction strength, the averaged electron density, and the system temperature.

In this paper, we focus on the electronic state, more precisely on the energy-level structure, because it is the basis of understanding the transport and optical properties of a PMLS and a MMLS. As we aim at obtaining a qualitative and overall picture of the electronic state of a PMLS and a MMLS, we have restricted ourselves to the self-consistent Hartree-Fock approximation for simplicity, where the exchange effect is included, but the correlation effect is ignored.[4,5] This approximation has been widely employed in the literature, and is believed to be reliable when the filling of Landau levels is not too small.[4,6] The present work improves some previous theoretical studies of a MMLS, including those of ourselves, as the exchange effect is now included, and the properties of a PMLS and a MMLS are investigated comparatively. Also, this work is complementary to some previous studies of the PMLS, for example, Ref.4, as the coupling between Landau levels is considered. Some interesting features are found in the energy levels arising from the Landau-level coupling, which to our knowledge have been previously unnoticed.

This paper is organized as follows: in Sec. II, we introduce the procedure of calculating the electronic state, where the finite temperature Green's function method is used within the self-consistent Hartree-Fock approximation. Section III contains our numerical results and a discussion. Finally, in Sec. IV, a summary is provided.

II. CALCULATION OF THE ELECTRONIC STATE

Let we consider a 2DES in the presence of a uniform magnetic field and a LS. The system is located in the xy plane, and the magnetic field is applied perpendicularly in the z direction. The Hamiltonian of the system can be written as

$$H=\sum_{\sigma}\frac{1}{2m_b}\int d\mathbf{r}\psi_\sigma^\dagger(\mathbf{r})\left(\mathbf{p}+\frac{e\mathbf{A}(\mathbf{r})}{c}\right)^2\psi_\sigma(\mathbf{r})$$
$$+\sum_{\sigma\sigma'}\tfrac{1}{2}\int d\mathbf{r}\,d\mathbf{r}'\,\psi_{\sigma'}^\dagger(\mathbf{r}')\psi_\sigma^\dagger(\mathbf{r})v(\mathbf{r}-\mathbf{r}')\psi_\sigma(\mathbf{r})\psi_{\sigma'}(\mathbf{r}')$$
$$+\sum_{\sigma}\int d\mathbf{r}\;\psi_\sigma^\dagger(\mathbf{r})[g^*\mu_B B(\mathbf{r})\sigma_z+U(\mathbf{r})]\psi_\sigma(\mathbf{r}), \qquad (1)$$

where $\mathbf{A}(\mathbf{r})=\mathbf{A}_0+\mathbf{A}_1$ is the vector potential, $v(\mathbf{r})$ the Coulomb interaction potential, and $B(\mathbf{r})$ the total magnetic field in the z direction. g^* is the effective g factor, m_b the electron band mass, and $\mu_B=e\hbar/2m_ec$ with m_e the electron mass in vacuum. The vector potential $\mathbf{A}_0$ is chosen as $\mathbf{A}_0(\mathbf{r})=(0,B_0x,0)$ which leads to a uniform magnetic field B_0 in the z direction. A PMLS is assumed to be induced by a lateral modulation potential $U(\mathbf{r})=U_0\cos(2\pi x/a)$, which is a periodic function along the x direction, and $\mathbf{A}_1$ is set to zero. A MMLS is assumed to be induced by an additional vector potential $\mathbf{A}_1(\mathbf{r})=[0,-(a/2\pi)B_1\cos(2\pi x/a),0]$, which leads to a lateral modulation magnetic field $B_1\sin(2\pi x/a)$ along the x direction, and $U(\mathbf{r})=0$ is assumed. It is clear that the system is translational invariant in the y direction.

In this paper, we will calculate the electronic state of a 2DES in a PMLS and in a MMLS via the evaluation of finite temperature Green's function in the self-consistent Hartree-Fock approximation.[4,5] The exchange effect is taken into account, but the correlation effect is ignored for simplicity.[4] The calculation procedure is briefly discussed below for completeness, although the method has been used by others as well.[3,4] Within the self-consistent Hartree-Fock approximation, the finite temperature Green's function of the system satisfies the following Dyson's equation:

$$G_\sigma(x,x',y-y',i\omega_n)$$
$$=G_{\sigma0}(x,x',y-y',i\omega_n)$$
$$+\frac{1}{\beta\hbar^2}\sum_{\sigma',\omega_1}\int dx_1dx_2dy_1dy_2G_{\sigma0}(x,x_1,y-y_1,i\omega_n)$$
$$\times v(x_1-x_2,y_1-y_2)G_{\sigma'}(x_2,x_2,y_2-y_2,i\omega_1)$$
$$\times G_\sigma(x_1,x',y_1-y',i\omega_n)$$
$$-\frac{1}{\beta\hbar^2}\sum_{\omega_1}\int dx_1dx_2dy_1dy_2G_{\sigma0}(x,x_1,y-y_1,i\omega_n)$$
$$\times v(x_1-x_2,y_1-y_2)G_\sigma(x_1,x_2,y_1-y_2,i\omega_1)$$
$$\times G_\sigma(x_2,x',y_2-y',i\omega_n). \qquad (2)$$

The effect of bare modulations (U and $\mathbf{A}_1$) has been absorbed into $G_{\sigma0}(x,x',y-y',i\omega_n)$, the unperturbed Green's function. The exact Green's function depends on $y-y'$ only, as H is y-translational invariant. This property should be retained in the self-consistent Hartree-Fock approximation. After the frequency summation over ω_1 and a Fourier transform over $y-y'$, the Dyson's equation becomes

$$G_\sigma(x,x',k,i\omega_n)$$
$$=G_{\sigma0}(x,x',k,i\omega_n)$$
$$+\sum_{\sigma'}\frac{1}{\hbar}\int dx_1dx_2dy_2G_{\sigma0}(x,x_1,k,i\omega_n)v(x_1-x_2,y_2)$$
$$\times G_{\sigma'}(x_2,x_2,y_2-y_2,0)G_\sigma(x_1,x',k,i\omega_n)$$
$$-\frac{1}{\hbar}\int dx_1dx_2dy_2G_{\sigma0}(x,x_1,k,i\omega_n)v(x_1-x_2,y_2)$$
$$\times G_\sigma(x_1,x_2,y_2,0)e^{iky_2}G_\sigma(x_2,x',k,i\omega_n). \qquad (3)$$

Next, let us expand $G_\sigma(x,x',k,i\omega_n)$ as

$$G_\sigma(x,x',k,i\omega_n)=\sum_{\alpha,\beta}[G_\sigma(k,i\omega_n)]_{\alpha,\beta}\varphi_\alpha(x)\varphi_\beta^*(x'), \qquad (4)$$

with respect to a completed set $\{\varphi_\alpha(x)\}$. Then the Dyson's equation can be cast into a matrix form

$$G_\sigma(k,i\omega_n)=G_{\sigma0}(k,i\omega_n)+G_{\sigma0}(k,i\omega_n)V_1G_\sigma(k,i\omega_n)$$
$$+G_{\sigma0}(k,i\omega_n)V_2G_\sigma(k,i\omega_n), \qquad (5)$$

where the matrix elements of V_1 and V_2 are given by

$$[V_1]_{\alpha,\beta}=\sum_{\sigma'}\frac{1}{\hbar}\int dx_1dx_2dy_2\varphi_\alpha^*(x_1)\varphi_\beta(x_1)$$
$$\times v(x_1-x_2,y_2)G_{\sigma'}(x_2,x_2,y_2-y_2,0), \qquad (6)$$

$$[V_2]_{\alpha,\beta}=-\frac{1}{\hbar}\int dx_1dx_2dy_2\varphi_\alpha^*(x_1)\varphi_\beta(x_2)$$
$$\times v(x_1-x_2,y_2)G_\sigma(x_1,x_2,y_2,0)e^{iky_2}. \qquad (7)$$

The exact Green's function satisfies $G_\sigma^*(x,x',y-y',\tau)=G_\sigma(x',x,y'-y,\tau)$. As the self-consistent Hartree-Fock approximation should retain this property, one can verify that matrixes V_1 and V_2 should be Hermitian.

The Dyson's equation can be rewritten as

$$G_\sigma^{-1}(k,i\omega_n)=G_{\sigma0}^{-1}(k,i\omega_n)-V_1-V_2. \qquad (8)$$

With an appropriate base set, $G_{\sigma0}^{-1}(k,i\omega_n)$ can be written as $i\omega_n-(E_{\sigma0}(k)-\mu)/\hbar$, with $E_{\sigma0}(k)$ a diagonal matrix whose diagonal elements are the energy levels of the 2DES in the presence of magnetic field and bare modulations. μ is the chemical potential determined by the given averaged electron density. Similarly, one may choose a base set so that $E_{\sigma0}/\hbar+V_1+V_2$ becomes diagonal, and one has $G_\sigma^{-1}(k,i\omega_n)=i\omega_n-(E_\sigma(k)-\mu)/\hbar$. Thus, by self-consistently diagonalizing a Hermitian matrix $E_{\sigma0}+\hbar(V_1+V_2)$, one obtains self-consistent energy levels $E_\sigma(k)$, as matrixes V_1 and V_2 also depend on $E_\sigma(k)$. The wave function can be obtained at the same time. The diagonal elements of matrix $E_\sigma(k)$ will be denoted as $E_{n\sigma}(k)$.

The uniform magnetic field B_0 introduces an energy scale $\hbar\omega_c=\hbar eB_0/m_bc$ and a length scale $1/\alpha=(\hbar/m_b\omega_c)^{1/2}$. It can be shown that the system under investigation is characterized by the following parameters: (1) the modulation period $p=a\alpha$; (2) the modulation strength u_0, defined as

$u_0 = U_0/\hbar\omega_c$ for a PMLS, and defined as $u_0 = (B_1/B_0)(p/2\pi)$ for a MMLS; (3) the effective electron-electron interaction strength $v_e = e^2\alpha/\epsilon_0\hbar\omega_c$, with ϵ_0 the effective background dielectric constant (we consider here an ideal 2DES for simplicity, and the inclusion of subband effect is straightforward); (4) the dimensionless averaged electron density $\nu = 2\pi n_e/\alpha^2$, with n_e the averaged electron density of the 2DES; (5) the system temperature $\beta = \hbar\omega_c/k_BT$; and (6) the effective spin splitting $g_0 = g^*\mu_B B_0/\hbar\omega_c$. Note that, the parameter u_0 above is defined such that the electronic state of a PMLS and a MMLS could be directly compared in the perturbation regime ($|u_0| \ll 1$).[3] Note also that, as $E_{\sigma 0}$, V_1, and V_2 are periodic functions of k with periodicity $k_p = a\alpha^2$, E_σ is also a periodic function of k with the same periodicity k_p.

III. RESULTS AND DISCUSSIONS

In this section, our numerical results and their discussions are presented. The system parameters (p, u_0, v_e, ν, and β) are chosen such that they take reasonable values and are in a presently accessible regime (for example, for a 2DES in a GaAs-based PMLS and MMLS). However, in order for the spin splitting to be clearly observable in the figures, the effective spin splitting $g_0 = 0.1$ is assumed throughout. In the following, we also limit ourselves to the regime of $\nu > 1$, where the self-consistent Hartree-Fock approximation is believed to be more appropriate.[6]

In most previous studies, the modulation potential or field is treated as a perturbation, and the coupling between different Landau levels is neglected.[3,4] However, in this paper we take the Landau-level coupling into account. The well-known Landau levels[5] in the Landau gauge are used as the base set for the expansion described in Sec. II, and typically 20 Landau levels are included. The inclusion of more Landau levels is found to have a negligible effect on the lowest several energy levels, but demands a much larger computational capacity. Note that because of the introduction of modulation, originally flat Landau levels now become Landau bands with nonzero width.

An iteration method is used in the self-consistent diagonalization of the matrix $E_{\sigma 0} + \hbar(V_1 + V_2)$. With each iteration, the eigenstate of the previous iteration is used as an input, and the newly generated eigenstate is combined with the input to form an input for the next iteration. This input differs from the old one by a controlled, but not necessarily small, correction. Convergency is assumed to be achieved if the difference of energy levels between two consecutive iterations is smaller than a prescribed number, and this behavior continues for the next several iterations. In our numerical results present below, the maximum relative difference in energy between the last several iterations before exiting the iteration loop is typically smaller than 10^{-6}, much smaller than the features in each curve shown in the figures. This indicates that the features observed in the following figures are free from possible numerical artifacts.

In this paper, our results for the PMLS case sometimes resemble those of Ref. 4. However, one important difference should be noted: in our paper, the variable k of function $E_{n\sigma}(k)$ varies in the interval $[0,k_p]$, with $k_p = a\alpha^2$ defined in Sec. II, but in Ref. 4, k was limited to a smaller interval

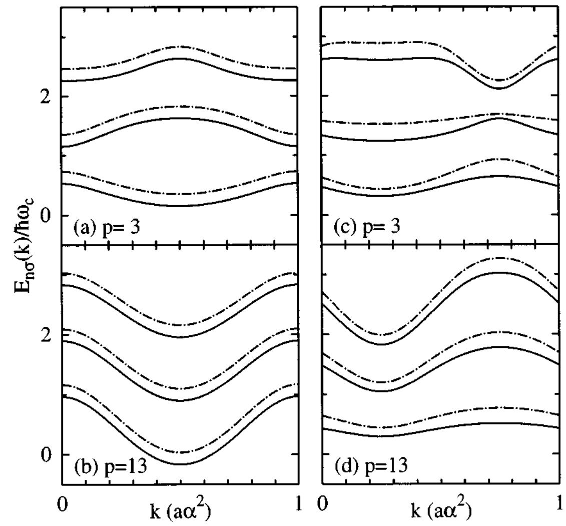

FIG. 1. The first six energy levels of a 2DES in a PMLS [(a) and (b)] and in a MMLS [(c) and (d)] for two typical values of p, without the electron-electron interaction. $u_0 = 0.6$. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively.

$[0,k_p/2]$, as $E_{n\sigma}(k)$ was assumed to be symmetrical about $k = k_p/2$. In this paper, the same model modulation potential is used as that in Ref. 4. However, our results show that this k symmetry is no longer there, because of the inclusion of the Landau-level coupling.

In Fig. 1, the lowest six energy levels of a 2DES in a PMLS [panels (a) and (b)] and in a MMLS [panels (c) and (d)] are shown for two values of the modulation period p without electron-electron interaction. u_0 takes a value of 0.6, in the regime of intermediate modulation strength. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. It can be clearly seen in Fig. 1 that, if one identifies $k\alpha^{-2}$ as a point along the x axis (without modulation, k/α^2 is the cyclotron center of a Landau state), $E_{n\sigma}(k)$ closely resembles the modulation profile, when the modulation period a is much larger than the magnetic length α^{-1}, i.e., $p \gg 1$. This behavior was also observed in previous studies,[1–4] and the underlying physics for this behavior is simple: when $p \gg 1$, the electron wave function (i.e., the cyclotron orbit) of low-lying states is not strongly modified by the modulation.

By comparing panels (a) and (b), one observes that, for a PMLS with a shorter modulation period ($p = 3$), the variation of energy levels (the bandwidth) is smaller than that for a PMLS with a longer modulation period ($p = 13$). This is a general behavior, and it is because the effective modulation strength, determined by the intra- and inter-Landau-level matrix elements, decreases as p decreases[3,4] (for a fixed u_0). The MMLS has a stronger inter-Landau level coupling,[3] therefore this p dependence is less obvious. In this paper, we will study the electronic state for typical values of p. The commensurability problem (i.e., the detailed p dependence), which was the main focus in previous studies,[2,3] will not be considered.

Comparing the cases of a PMLS and a MMLS in Fig. 1, one observes that, for a PMLS, the spin splitting defined as $\Delta E_n(k) = E_{n\uparrow}(k) - E_{n\downarrow}(k)$ is independent of k. For a

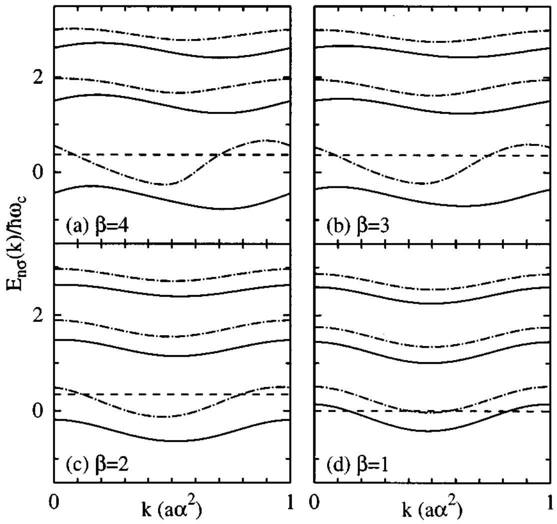

FIG. 2. The β dependence of the energy levels for a PMLS, with the exchange effect included. $p=13$, $u_0=0.6$, $v_e=0.8$, and $\nu=1.6$. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. The dashed line indicates the position of the chemical potential.

MMLS, this is no longer the case, as there is a spatially varying magnetic field. This shows an important difference between the PMLS and MMLS when the exchange effect is not included: the spin splitting is nonuniform for a MMLS. Note that this nonuniform spin splitting can be clearly observed for a MMLS with either a short or a long modulation period. Earlier studies[2,3] show that this nonuniform spin splitting can even survive the inclusion of the self-consistent Hartree potential (i.e., the mean electrostatic potential). However, we will demonstrate that, if the exchange effect is taken into account, the spin splitting of a PMLS with a large p also becomes nonuniform, thus this difference between the PMLS and MMLS will no longer be obvious.

Before we study the exchange effect, it should be pointed out that the exchange effect becomes important only if the Fermi surface of the system is sharp (clear) enough.[5,6] Thus, the system temperature must be sufficiently low, i.e., $\beta>1$. This is demonstrated in Fig. 2, where the temperature (β) dependence of energy levels is shown for a PMLS, with the exchange effect included. Other system parameters are $p=13$, $u_0=0.6$, $v_e=0.8$, and $\nu=1.6$. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. The dashed line indicates the position of the chemical potential. At a lower temperature [see panel (a) with $\beta=4$], the spin splitting is obviously nonuniform and larger than the bare spin splitting (compare with Fig. 1). As the temperature increases (β decreases), the spin splitting becomes almost uniform (see panel (d) with $\beta=1$) and smaller. The case of a MMLS is similar to that of a PMLS; therefore, it is not shown explicitly. When the modulation period is short, the temperature effect in either a PMLS or a MMLS is quite similar to what is shown in Fig. 2: the spin splitting reduces toward its bare value as β decreases, so it is not shown. In order to study the exchange effect unambiguously, in the following, we will focus on the regime of low temperatures.

In Fig. 2, there is an interesting feature not observed in previous studies, to our knowledge.[3,4] At a higher temperature (a smaller β), $E_{n\sigma}(k)$ versus k is almost symmetrical about $k_p/2$, but, as β increases, this is no longer the case. One observes that the spin splitting becomes non-uniform, and the narrowing of spin splitting in the k interval $[0,k_p/2]$ is accompanied by the widening of the spin splitting when k lies in the other half interval $[k_p/2,k_p]$ [see the lowest two energy levels in panel (a) of Fig. 2]. We attribute this breaking of the k symmetry to the inclusion of Landau-level coupling, because without the Landau-level coupling, one can show that $E_{n\sigma}(k)$ is indeed symmetrical about $k_p/2$.[4] It must be pointed out that the occurrence of the nonuniform spin splitting is also determined by the averaged electron density and the modulation period. Because an uneven k distribution of electrons is more likely when $p\gg1$ (recalling that k/α^2 is the center of cyclotron orbit, therefore the k distribution is closely related to the x-distribution of electrons), the nonuniform spin splitting is easier to see when the modulation period is large.

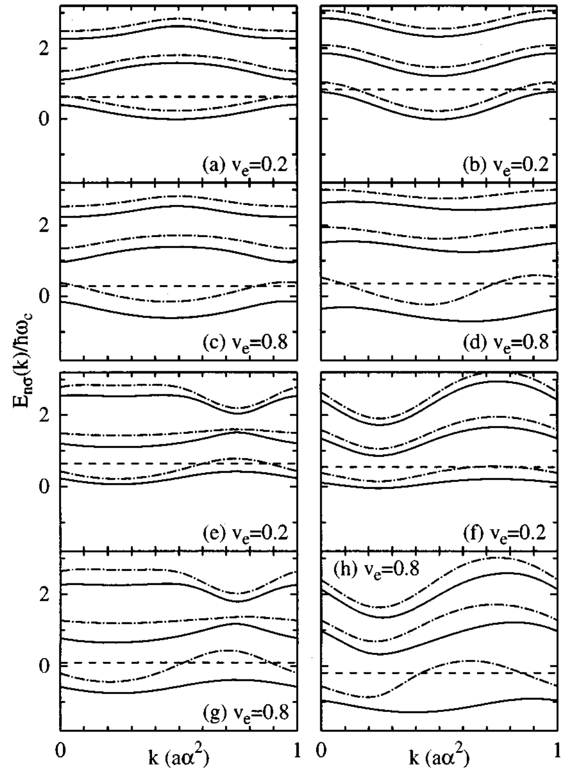

FIG. 3. The v_e dependence of the energy levels. The upper (lower) four panels are for a PMLS (MMLS), respectively. $u_0=0.6$, $\beta=3$, and $\nu=1.6$. In the left (right) four panels, $p=3$ ($p=13$), respectively.

Next, let us study the dependence of energy levels on the effective electron-electron interaction strength v_e, as v_e can be tuned by varying the strength of uniform component of the magnetic field. The lowest several energy levels are shown in Fig. 3 for two values of v_e. The upper four panels are for a PMLS, while the lower four panels are for a MMLS, respectively. Other system parameters are $u_0=0.6$, $\beta=3$, and $\nu=1.6$. In the left (right) four panels, $p=3$ ($p=13$), corresponding to the case of a short (long) modulation period, respectively. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. The dashed line indicates the position of the chemical potential.

When v_e is small, the spin splitting is small as expected [see panels (a), (b), (e), and (f)]. For the PMLS, the spin splitting is almost uniform, with either a short or long modulation period. In contrast, for the MMLS, the spin splitting is nonuniform even for a small value of v_e, as discussed above. Because the self-consistent Hartree-Fock approximation is a perturbation theory in nature,[5,6] and it should be more suitable and reliable for small values of v_e, the largest value of v_e studied in this paper is limited to be smaller than 1. For a larger, value of v_e, the iteration process is found to take more time to converge as one intuitively expected. In the case of $v_e=0.8$, for the PMLS, one observes that the spin splitting becomes larger as expected, but the spin splitting also becomes more nonuniform (the bandwidth increases) as the modulation period increases from $p=3$ to 13. For the MMLS, the spin splitting increases as v_e increases. However, the uniform to nonuniform behavior change of spin splitting (versus p) found for the PMLS case is less obvious. This is due to the presence of the spatially varying magnetic field.

There is a number of reasons why the spin splitting is almost uniform in a short-period PMLS. First, the effective modulation strength of a PMLS with a short modulation period is smaller than that with a long modulation period, as mentioned above. Second, when p is small, it is more difficult for the k distribution, or equivalently the x distribution, of electrons becomes uneven, because there is an energy penalty arising from the self-consistent Hartree potential. The calculation carried out for other small values of p (not shown here) corroborates these arguments. Note that the exchange effect is still observable in a short-period PMLS [see panel (c) of Fig. 3], which manifests itself as an overall increase of the spin splitting.

It is well known that for a 2DES without modulation, the exchange effect could introduce large corrections to the Landau levels.[5,6] Figure 3 shows that this picture is still qualitatively correct. Here an intermediate value of $u_0=0.6$ is chosen for the modulation strength, so that the exchange effect could be unambiguously identified. In contrast to a uniform system, the modulation studied in this paper introduces a self-consistent Hartree potential,[5,6] which is included via the self-consistent Hartree-Fock approximation. It is clear from Fig. 3 that, either for the PMLS or MMLS, the variation of a fully occupied energy level (bandwidth) is smaller than that of a partially occupied energy level. This is because the self-consistent Hartree potential becomes more important for a fully occupied energy level. The main effect of this self-consistent Hartree potential is to flatten the variation of the electron spatial density distribution, thus the effect of modulation is partly compensated for.[3,4]

Next, let us turn to the dependence of energy levels on the modulation strength u_0. In Fig. 4, the lowest several energy levels of a 2DES are shown for two values of u_0. The upper four panels are for a PMLS, and the lower four panels are for a MMLS, respectively. Other system parameters are $v_e=0.8$, $\beta=3$, and $\nu=1.6$. In the left (right) four panels, $p=3$ ($p=13$), respectively. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. The dashed line indicates the position of the chemical potential. The overall

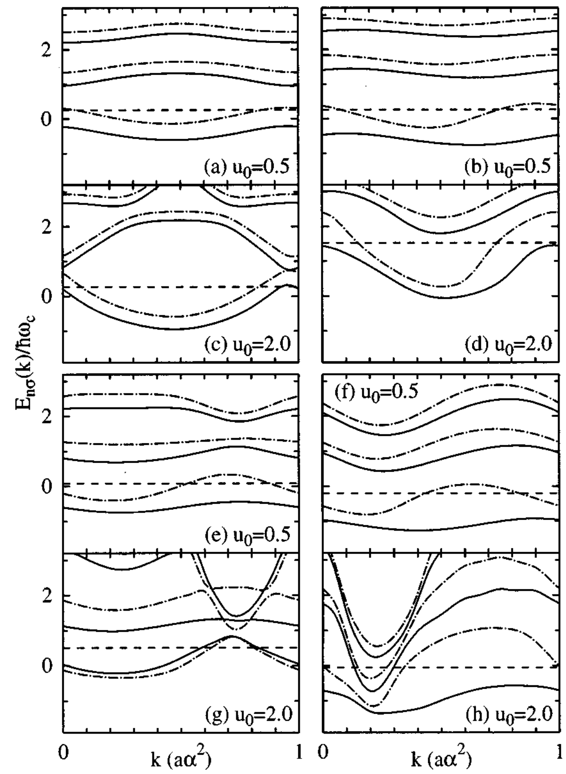

FIG. 4. The u_0 dependence of the energy levels. The upper (lower) four panels are for a PMLS (MMLS), respectively. $v_e=0.8$, $\beta=3$, and $\nu=1.6$. In the left (right) four panels, $p=3$ ($p=13$), respectively.

picture is the following: When u_0 is small, the variation of energy levels (the bandwidth) is also small. As u_0 increases, the variation increases either for a PMLS or a MMLS, with either a short or long modulation period. The spin splitting displays some interesting features.

In the case of a PMLS with a long modulation period, one observes that the energy levels shift upwards as a whole, as u_0 increases [compare panels (b) and (d)]. However, this behavior is absent in the short modulation period PMLS [see panels (a) and (c)]. We attribute this shift of energy levels as a whole to the enhancement of the self-consistent Hartree potential in the long modulation period PMLS. In the short modulation period PMLS, the self-consistent Hartree potential is suppressed due to the strong mixing of wave functions, i.e., the matrix elements of the self-consistent Hartree potential become smaller than that with a long modulation period as u_0 increases. The above argument is also corroborated by investigating the u_0 dependence of energy levels for other averaged electron densities (the results are, however, not shown here). In the upper four panels of Fig. 4, one also observes that the spin splitting does not change much as u_0 varies. This indicates that the exchange effect is less affected by the increase of the modulation strength in the regime studied here.

The shift of energy levels as a whole found above for the PMLS case, however, is not obviously observed for the MMLS case. This again can be attributed to the strong mixing of wave functions, so that the self-consistent Hartree potential is weakened. In the case of a MMLS with a short modulation period and a large modulation strength [compare panels (e) and (g)], for some values of k the energy of the lower spin state becomes higher than that of corresponding upper spin state. We attribute this flip of spin states to the sign change of the spatially varying magnetic field along the x axis when u_0 is large enough. As the local magnetic field is characterized by the ratio $B_1/B_0=(2\pi/p)u_0$, one sees that, for a fixed u_0, the sign change of the local magnetic field can only happen for a MMLS with a small value of p. Note that, as the matrix element of bare modulation does not depend on u_0 in a monotonic way, we are unable to give a ''simple'' expression for the critical value of u_0, at which the flip of spin states occurs. Note also that the flip of spin states is not observed for the PMLS case.[3,4]

It is also found that, as u_0 increases, the flip of spin states emerges gradually. As a consequence, the spin splitting of the corresponding energy level can become very small [see panel (g), two lowest energy levels]. This indicates the breakdown, in this regime, of the perturbation picture, where the understanding of the exchange effect is based upon the picture of a uniform 2DES with the lateral modulation viewed as a perturbation. This breakdown apparently should be attributed to the competition between the local magnetic field and the exchange effect, as this competition becomes more important in this regime. The calculations carried out for other averaged electron densities (but not shown here) shows that the collapse of spin splitting found above is not related to the well-known shrinkage of spin splitting of a fully occupied Landau level.[6] Note that the reduction of spin splitting found for the short modulation period MMLS cannot be clearly identified for the short modulation period PMLS, because the magnetic field is spatially uniform in the PMLS.

In Fig. 4, one observes that the energy level can display small oscillations versus k when u_0 becomes relatively large [see panel (h)]. This is only observed for the long modulation period MMLS, and it is found that these oscillations arise from the spatial oscillation of the electron density distribution, which is most likely to occur in a LS with a longer modulation period. In the regime studied in this paper, this energy oscillatory behavior is, however, not observed for the PMLS case in contrast with some previous studies.[4] This is because the modulation strength studied in the paper is relatively small, as we aim at obtaining a clear and traceable picture. Comparing panels (f) and (h) for the case of the MMLS, one finds that, as u_0 increases, the spin splitting of the two lowest (partially occupied) energy levels, where the chemical potential lies between, also increases. It is found that this is due to the enhancement of the exchange effect, which is well known for a uniform 2DES.[6] The calculations carried out for other averaged electron densities confirm this argument. In comparison with the PMLS case, one finds that the exchange effect in the MMLS depends more sensitively on u_0, because of the spatially varying magnetic field.

Finally, let us study the dependence of energy levels on the averaged electron density or averaged filling factor ν.[3,6] In Fig. 5, the lowest several energy levels are shown for a PMLS with different values of ν. Other system parameters are $v_e=0.8$, $\beta=3$, and $u_0=0.6$. In the left (right) seven panels, $p=3$ ($p=13$), respectively. The solid (dash-dotted) curves are for the lower (upper) spin states, respectively. The dashed line indicates the position of the chemical potential. Fig. 6 shows the ν dependence of energy levels with the same set of parameters as in Fig. 5, but for a MMLS instead.

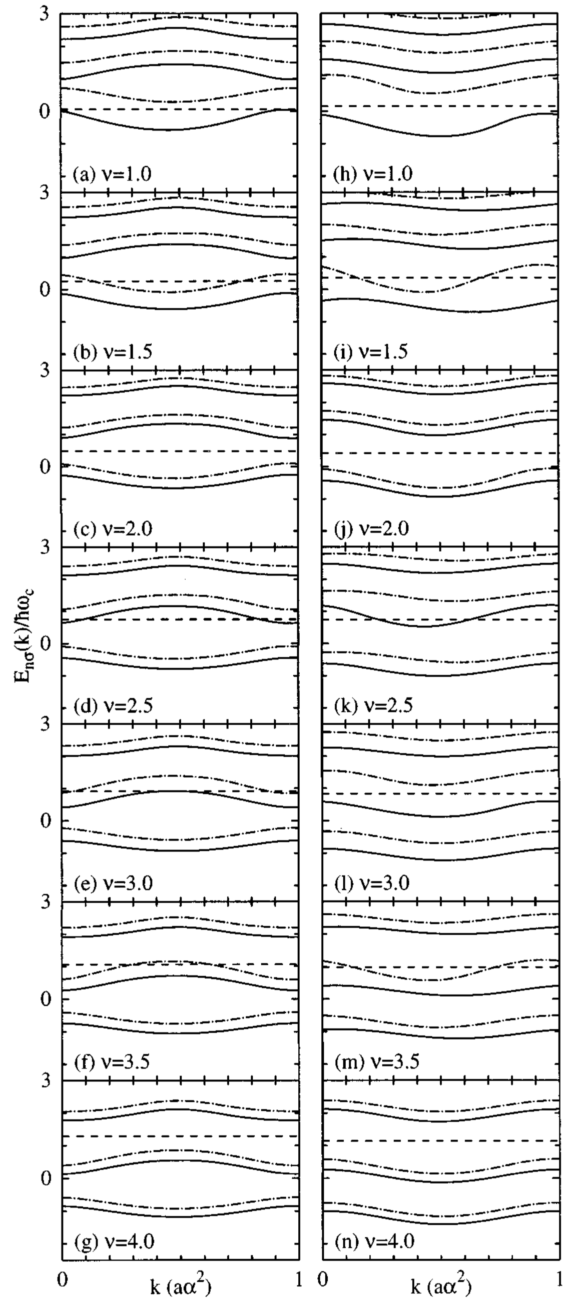

FIG. 5. The ν dependence of the energy levels for a PMLS. $v_e=0.8$, $\beta=3$, and $u_0=0.6$. In the left (right) seven panels, $p=3$ ($p=13$), respectively.

It is clear that, even with a moderate modulation strength $u_0=0.6$, the behavior of spin splitting shown in these two figures can be qualitatively understood by recalling the results of a uniform 2DES (Ref. 6) in the presence of a perpendicular magnetic field with the modulation viewed as a perturbation. As the average electron density (ν) increases, the chemical potential increases and sweeps through each energy level. When the chemical potential lies between a pair of lower and upper spin states, i.e., the corresponding

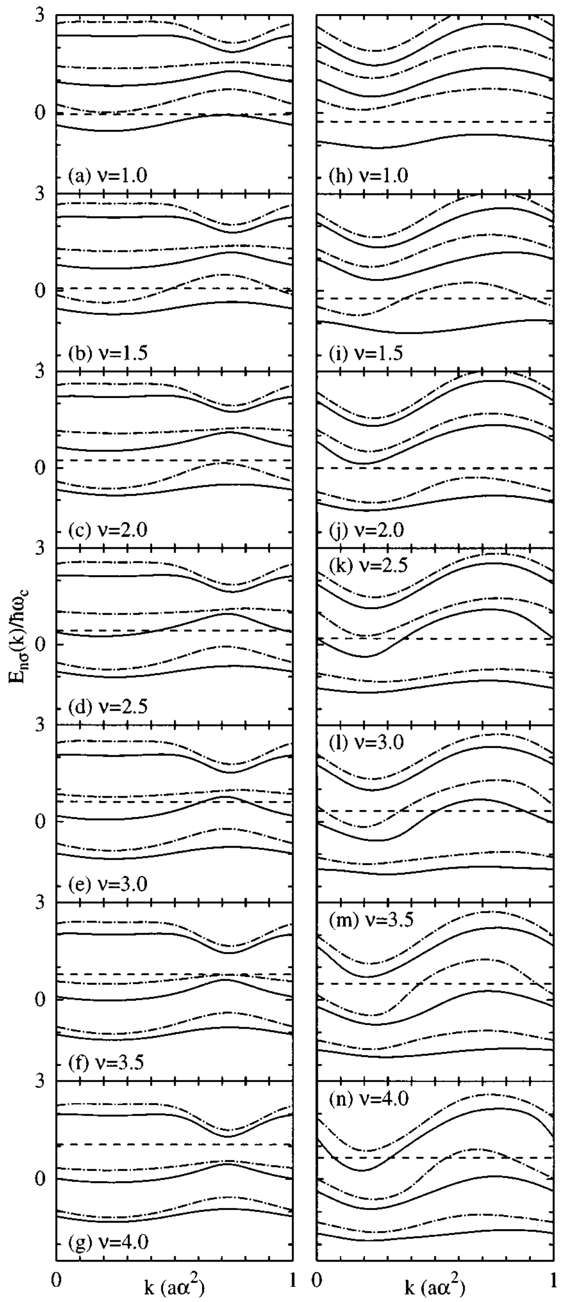

FIG. 6. The same as Fig. 5, but for a MMLS.

energy level (or the Landau level, if no confusion) is partially occupied, the exchange effect is enhanced and the spin splitting increases. When the chemical potential lies in a gap, i.e., two spin states of a energy level are fully occupied, the enhancement to the exchange effect is suppressed. Here this qualitative picture is demonstrated to be correct either for a PMLS or a MMLS, with either a short or long modulation period.

The feature of nonuniform spin splitting observed and discussed in preceding figures can also be seen here in those density dependence plots. In the case of the PMLS, $E_{n\sigma}(k)$ becomes almost symmetrical about $k=k_p/2$, and the spin splitting is almost uniform, when the chemical potential lies in the gap. When the chemical potential intersects one energy level, that level displays a k asymmetry, and the spin splitting becomes nonuniform. As we discussed above, this uniform to nonuniform (symmetry to asymmetry) behavior change is more pronounced for the PMLS with a long modulation period. In the case of the MMLS, because of the presence of a spatially varying magnetic field, the spin splitting is generally nonuniform. One can observe that the spin splitting is typically larger in the MMLS with a long modulation period in comparison with that with a short modulation period. The spin splitting becomes smaller when the chemical potential lies in the energy gap. The underlying physics for these behaviors has been discussed in previous paragraphs, thus it will not be repeated here. The results shown in Figs. 5 and 6 are complementary to the results displayed in preceding figures.

In Figs. 5 and 6, we have not observed the pinning of Fermi level, either for a PMLS or a MMLS, that was found in some earlier studies.[3] This is because, in the present paper, we adopted the self-consistent Hartree-Fock approximation, and also limited ourselves to the regime of $\nu>1$ where the electron correlation effect should be less important. Those earlier studies considered the regime of strong electron correlation. It should be pointed out that our results are consistent with some other theoretical investigations.[4]

IV. SUMMARY

In summary, we have investigated comparatively the electronic state of a PMLS against that of a MMLS in a perpendicular magnetic field. In the presence of a moderate modulation, the electronic state of a PMLS is qualitatively different from that of a MMLS, if the exchange effect is not taken into account. The inclusion of the exchange effect brings qualitative changes to the electronic state of a PMLS where the spin splitting generally becomes nonuniform, and the difference between a PMLS and a MMLS becomes less obvious. The exchange effect is found to be suppressed when the system temperature becomes high.

It is found that, when the coupling between Landau levels is taken into account, the energy levels $E_{n\sigma}(k)$ of a PMLS is qualitatively different from that without Landau-level coupling. For a PMLS with a long modulation period, the spin splitting generally becomes nonuniform, because of the self-consistent Hartree potential. One can observe that, even with a moderate modulation strength, the narrowing of spin splitting for some values of k is accompanied by the widening of spin splitting for some other values of k. In a short modulation period PMLS, the self-consistent Hartree potential is found to be less effective due to the strong mixing of wave functions.

For a MMLS with a short modulation period, as the modulation strength increases, the spin splitting could even become smaller than the bare spin splitting, and a flip of spin states could emerge. The flip of spin states is not observed in a MMLS with a long modulation period, nor in a PMLS. In a long modulation period MMLS, the energy levels can display small oscillations in k, as the modulation strength becomes large enough, due to the oscillation of electron spatial distribution.

It is also found that, even in the regime of intermediate modulation strength, the density dependence of the spin splitting of energy levels, either for a PMLS or a MMLS, can be qualitatively understood based on the picture of a 2DES in a perpendicular magnetic field with the modulation viewed as a perturbation.

ACKNOWLEDGMENT

This work was partly supported by the Chinese National Natural Science Foundation.

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