Pancake-antipancake gas in layered superconductors

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We consider the statistical mechanics of a layered superconductor in a magnetic field. The superconductor is described as a periodic system of superconducting layers coupled via a magnetic field. Each layer contains a gas of two-dimensional pancakes and antipancakes. To obtain the free energy of the pancake gas we used the method of collective variables which gives not only the main results of the Debye-Hückel theory, but corrections to it as well. We have calculated the magnetization which shows all the main features of the reversible magnetization of high-temperature superconductors. [S0163-1829(97)01146-6]

I. INTRODUCTION

Two-dimensional magnetic vortices—also known as pancakes—exist in high-temperature superconductors due to their layered structure. The pancakes were described in detail in a set of theoretical papers.[1–3] The interaction between two pancakes in the same layer grows as a logarithm of distance between them. This interaction is of principal importance in topological excitations in two-dimensional (2D) systems. The well-known manifestation of the 2D nature of pancakes is the Kosterlitz-Thouless (KT) phase transition[4,5] at which pancake-antipancake pairs dissociate and a gas of free pancakes at a temperature that exceeds the critical one $T>T_{\mathrm{KT}}$ is formed. The KT transition leads to an abrupt change of current-voltage characteristics of superconductors which is known as the Nelson-Kosterlitz jump of the temperature dependence of a in $V\sim I^{a(T)}$.[6] Such a behavior of IV's has been observed not only in superconductors with pronounced layered structure such as $Bi_2Sr_2CaCu_2O_x$,[7] $Bi_{1.75}Pb_{0.25}Sr_2Ca_2Cu_3O_{10}$,[8] and $Tl_2Ba_2CaCu_3O_8$,[9] but also in $YBa_2Cu_3O_{7-x}$,[10] $HgBa_2Cu_3O_{8+\delta}$,[11] and $HgBa_2CuO_{4+\delta}$ (Ref. 12) with their strong Josephson coupling between layers. Intervals between the mean field critical temperature T_{c0} and T_{KT} amount to several degrees.

Are there other properties of layered superconductors, which can be explained in the framework of the pancake gas approximation? We argue below that the reversible magnetization of high-T_c superconductors is one such property.

Numerous experiments were devoted to the magnetization of high-T_c superconductors. At high magnetic field layered superconductors demonstrate unusual dependencies $M(T)$, namely, all curves cross at the same temperature T^* and up to this temperature they are linear.[13] These dependences were described in Ref. 14 in which the contribution of vortex fluctuations to the free energy of a layered superconductor was taken into consideration. This model describes well magnetization curves only for $T<T^*$. A detailed analysis of magnetization curves of various superconductors is given in Ref. 15. In the critical region near T_c, the scaling model[16] describes magnetization by taking into account two-dimensional fluctuations. Both models complement each other but the theory still cannot fit the full set of $M(H)$ curves obtained in the region from 0.1 to 5 T.[13] The above peculiarities of magnetization curves appear in the region of high fields and temperatures where the magnetization is reversible.

In this paper we describe magnetization of layered superconductors using a model of the two-dimensional gas of pancakes and antipancakes. The main idea of the model is that the magnetization at high fields and temperatures is determined mainly by the presence of two-dimensional gas of pancakes. What distinguishes a pancake gas from liquid or lattice of vortices is the large entropy term in the free energy.

We assume that the equilibrium pancake density is formed by two mechanisms. In one of them, pancakes form in the superconductor under pressure of the external field. The other mechanism is dissociation of thermally excited pancake-antipancake pairs. The first mechanism provides the value of magnetic induction B that is equal approximately to external magnetic field H. The gas of pancakes has been considered in Ref. 17 to only describe reversible magnetization of irradiated high-T_c superconductors. A screening effect has been discussed for one pancake in the vortex liquid.[18]

We will show that the second mechanism of pancake generation takes place at any temperature and in any magnetic field. But its influence is noticeable near T_{KT} where the density of dissociated pancakes is comparable with B/ϕ_0. This mechanism allows us to describe the magnetization above T_{KT}.

Taking into consideration both mechanisms we can describe experimental magnetization curves up and down the temperature T^*. Possible consequences from this model will be discussed below.

II. GENERAL CONSIDERATION

At zero magnetic field, a pancake cannot be excited thermally because of its large self-energy. However, pancake-antipancake pairs, or ''pancake dipoles,'' with zero topological charge and with energy proportional to the logarithm of the dipole length, are present in the system at any nonzero temperature. Collective effects in the pancake system reveal themselves as the screening of pancake interactions above the temperature T_{KT} when dipole dissociates and a gas of free pancake forms.

In an external magnetic field, the superconductor is a to-

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pologically charged system. Magnetic flux penetrates superconductors in the form of vortices. In this paper we consider a region of high temperatures and inductions where the magnetic flux is carried mainly by the gas of pancakes and antipancakes and we can neglect the contribution of vortex lines. In this case the number of pancakes whose flux is directed along the external field is larger than the number of antipancakes. The number of ''particles'' in the system is not fixed and has to be found from conditions of thermodynamic equilibrium.

We assume that a layered superconductor is a periodic system of Josephson decoupled superconducting planes with $\lambda_{ab} \gg \xi_{ab}$ and with distance s between planes. We choose the axis z along the crystal axis **c** perpendicular to the layers. The free energy of superconductor is

$$F=\frac{\phi_0^2}{16\pi^3\Lambda}\sum_{\mu}\int d^3r\left(\nabla\theta_\mu-\frac{2\pi}{\phi_0}\mathbf{A}\right)^2\delta(z-\mu s)+\int d^3r\frac{[\nabla\mathbf{A}]^2}{8\pi}, \tag{1}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential of magnetic field, $\theta_\mu(\mathbf{x})$ is the phase of order parameter in μth layer, $\mathbf{r}=(\mathbf{x},z)$, and $\Lambda=2\lambda_{ab}^2/s$.

Varying the potential (1) relative to **A** and θ leads to the equilibrium equations

$$\frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\mathbf{A}}{\partial\mathbf{x}}\right)-\Delta\mathbf{A}(\mathbf{r})=\sum_{\mu}\frac{4\pi}{c}I_\mu(\mathbf{x})\,\delta(z-\mu s), \tag{2}$$

$$\frac{4\pi}{c}I_\mu(\mathbf{x})=\frac{2}{\Lambda}\left[\frac{\phi_o}{2\pi}\nabla\theta_\mu(\mathbf{x})-\mathbf{A}_\mu(\mathbf{x})\right]. \tag{3}$$

The phase changes by 2π along any closed curve containing the pancake center $\mathbf{x}_0$, and by -2π for an antipancake:

$$\left[\frac{\partial}{\partial\mathbf{x}},\nabla\theta(\mathbf{x})\right]=\pm 2\pi\hat{z}(\mathbf{x}-\mathbf{x}_0).$$

Excluding the vector potential $\mathbf{A}_\mu$ from Eq. (3) we find an equation for currents $\mathbf{I}_\mu$. If the right-hand side of the equation is a gradient of the phase of a single vortex then it determines a current generated by this vortex only. Multiplying it by the phase gradient of any other vortex we find the equation for the interaction potential

$$\Delta U^{\mu\mu'}(\mathbf{x})+\sum_{\mu''}\int d\mathbf{y}K^{\mu\mu''}(\mathbf{x}-\mathbf{y})U^{\mu''\mu'}(\mathbf{y})=4\pi Tp\,\delta(\mathbf{x})\,\delta_{\mu\mu'}, \tag{4}$$

with the kernel

$$K^{\mu\mu'}(\mathbf{x})=\int\frac{d\mathbf{q}}{(2\pi)^2}\frac{q}{\Lambda}e^{-qs|\mu-\mu'|+i(\mathbf{qx})}.$$

Here

$$p=\frac{\phi_0^2 s}{32\pi^2\lambda^2 T}.$$

The partition function of the gas of N_+^μ pancakes and N_-^μ antipancakes with interaction (4) is

$$Z[N_+,N_-]=e^{-\beta F}=\prod_{\alpha}\frac{1}{N_+^\alpha!N_-^\alpha!}\left(\frac{S}{\pi\xi^2}e^{-\beta E_0}\int\frac{d\mathbf{x}}{S}\right)^{N_+^\alpha+N_-^\alpha}\times\exp\left[-\tfrac{1}{2}\beta\sum_{i,j}\sum_{\alpha,\alpha'}U^{\alpha,\alpha'}(\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'})\right]. \tag{5}$$

Here E_0 is the vortex core energy, F is the free energy of the system, $\beta=1/T$, S is the area of the superconducting layer, and ξ is the coherence length.

To determine equilibrium numbers N_+ and N_- of the vortices in the system we have to calculate the grand partition function:

$$\Xi=e^{-\beta\Omega}=\sum_{N_+^\alpha,N_-^\alpha}e^{\beta(\mu_+^\alpha N_+^\alpha+\mu_-^\alpha N_-^\alpha)}Z[N_+,N_-], \tag{6}$$

where $\mu_\pm$ are the chemical potentials of pancakes and antipancakes, and Ω is the thermodynamic potential of the system with a variable number of particles. The equilibrium number of particles is determined as $N_i=\partial\Omega/\partial\mu_\pm$ at equilibrium values of $\mu_\pm$.[19]

Equations (5),(6) differ from usual formulas of statistical mechanics of classical particles because pancakes are massless particles. Thus pancakes do not have kinetic energy and their partition function coincides with the path integral.

III. FREE ENERGY OF PANCAKE GAS

In the first approximation the free energy of a layered superconductor in an external magnetic field contains two contributions. The first one is the magnetic field energy. This energy does not depend on details of the field distribution in high fields. It can be obtained by averaging the exponent (5) over space variables

$$U_B=\sum_{\alpha\alpha'}\int\frac{d\mathbf{x}_i^\alpha}{S}\int\frac{d\mathbf{x}_j^{\alpha'}}{S}U^{\alpha\alpha'}(\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'})=\sum_{\alpha}\frac{(N_+^\alpha-N_-^\alpha)^2\phi_0^2}{8\pi S^2}sS=\frac{B^2}{8\pi}V. \tag{7}$$

Here V is the volume of superconductor. The differences of the numbers of pancakes and antipancakes are the same in all layers.

The other contribution to energy depends on mutual positions of pancakes and is a configurational correction. To evaluate this contribution we use the method presented in the Appendix and consisting in a representation of the partition function (5) in terms of a path integral over a scalar field variable. As shown in the Appendix, Eq. (A8), the main part of the interaction energy of the pancakes system per layer is

$$u=4p\kappa^2(n_+-n_-)^2+p(n_++n_-)[1-\ln 8p(n_++n_-)] \tag{8}$$

which is the main result of the Debye-Hückel theory. Here we use dimensionless energy $u=\beta U\pi\xi^2/S$, and density of pancakes $n_\pm=N_\pm\pi\xi^2/S$. $\kappa=\lambda_{ab}/\xi_{ab}$. The first term in this expression is the energy of magnetic flux (7) and the other one is the main configurational correction (A8). It agrees with the result using the Debye-Hückel method. In the appendix we calculate corrections to the energy and show that they are small.

The self-energy of the superconductor also contains the sum of core contributions $\beta E_0 \equiv pe_0$, where e_0 is the temperature-independent value ~ 3.[20] The actual value of e_0 will be found by analyzing experimental data.

The free energy has to be complemented by the entropy term. Taking into account expressions (7), (8) the dimensionless free energy $f=\beta F \pi \xi^2/S$ can be written in the form

$$f(n_+,n_-)=u+(n_++n_-)pe_0+n_+(\ln n_+-1)+n_-(\ln n_--1). \quad (9)$$

Substituting this energy in (5) and (6) we obtain the grand partition function. The sum (6) contains exponents with powers

$$\beta[\mu_+N_++\mu_-N_--F(N_+,N_-)]=\frac{S}{\pi\xi^2}[8p\kappa^2h(n_+-n_-)-f(n_+,n_-)]. \quad (10)$$

Since the equilibrium densities of pancakes and antipancakes are results of a dynamic balance between recombination and dissociation of vortex dipoles, the condition of the balance is the vanishing chemical potential of dipoles $\mu=\mu_+-\mu_-$. We have taken into consideration this relation and used dimensionless parameter $h=\beta\mu_+/(8p\kappa^2)$ instead of μ_+. Note that the equilibrium value of the chemical potential is proportional to the Maxwell field H which coincides with the applied field for a long cylindrical sample:

$$\mu_+=\frac{\phi_0 Hs}{4\pi},$$

$H=h\phi_0/\pi\xi^2$.

We assume that the main contribution to the sum (6) is ensured by densities n_+,n_- which are minima of expression (10) which follow from the solution of the system of equations

$$\partial f/\partial n_+=\ln n_+-p\ln(n_++n_-)-p(\ln 8p-e_0)+8p\kappa^2(n_+-n_-)=8p\kappa^2h, \quad (11)$$

$$\partial f/\partial n_-=\ln n_--p\ln(n_++n_-)-p(\ln 8p-e_0)-8p\kappa^2(n_+-n_-)=-8p\kappa^2h. \quad (12)$$

By solving these equations we find the equilibrium densities n_+, n_- in the mean field approximation.

The free energy (9) and this system of equations are the main result of our paper. In zero magnetic field we obtain $n_+=n_-=n$ and

$$(1-p)\ln n=p(\ln 16p-e_0),$$

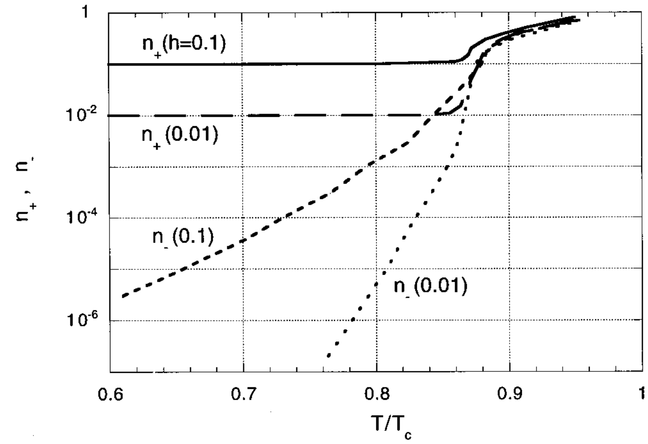

FIG. 1. Equilibrium densities of pancakes (n_+) and antipancakes (n_-) vs temperature. For calculation we used $T_{KT}/T_c=0.9$, $\kappa=100$, $e_0=3$, and two values of external field $H=h\phi_0/\pi\xi^2$.

which determines the nonzero density $n(T)$ at $T>T_{KT}$. In the case of a nonzero field h the densities of pancakes and antipancakes are different and the dependences $n_\pm(T)$ can be used to calculate the magnetization.

Numerical solutions of the equilibrium equations (11),(12) are shown in Fig. 1 for two values of the external field. Several features of the densities $n_+(T)$ and $n_-(T)$ are worth noting. At any temperatures and field, the density n_+ of the pancakes exceeds the density of the antipancakes. The value of induction n_+-n_- is slightly less than the applied field h. At small temperatures the density of the antipancakes n_- is exponentially small. This means that we can neglect the dipole dissociation at low temperatures and all the main results of Ref. 14 can be confirmed. On the contrary, at temperatures $T>T_{KT}$, the majority of the pancakes results in dipole dissociation, while the field influence on the gas density is negligible.

IV. MAGNETIZATION OF LAYERED SUPERCONDUCTOR

Equilibrium value of magnetization $M=(B-H)/4\pi$ of a layered superconductor is related in a simple way to equilibrium densities (11) and (12):

$$m=8p\kappa^2(n_+-n_--h)=-\frac{1}{2}\ln\frac{n_+}{n_-}, \quad (13)$$

or in common units

$$M=-\frac{T}{2s\phi_0}\ln\frac{n_+}{n_-}. \quad (14)$$

Note that $M<0$ since $n_+>n_-$.

Magnetization curves (14) obtained by solving the equilibrium equations (11),(12) are presented in Fig. 2. They reflect all the main peculiarities of the unusual behavior of the magnetization of layered high-T_c superconductors at low temperatures $T<T_{KT}$ as well as at high ones $T>T_{KT}$.

Experimental curves $M(T)$ of real superconductors cross at point M^*.[13] Numerically calculated curves cross at the temperature $T^*=T_{KT}$ and at magnetization value

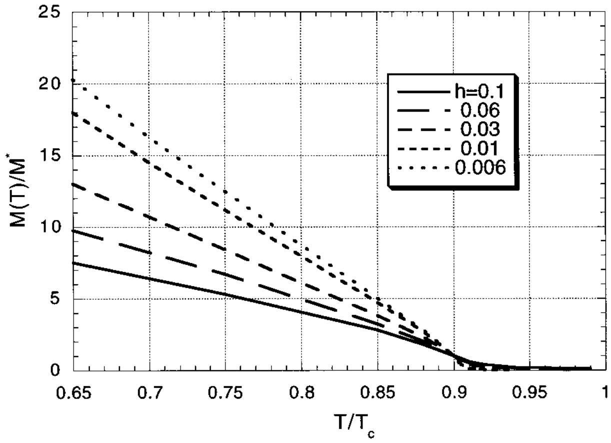

FIG. 2. The temperature dependence of magnetization. We used $T_{KT}/T_c=0.9$, $\kappa=100$, $e_0=3$. The values of $H=h\phi_0/\pi\xi^2$ are shown. $M^*=M(T_{KT})$.

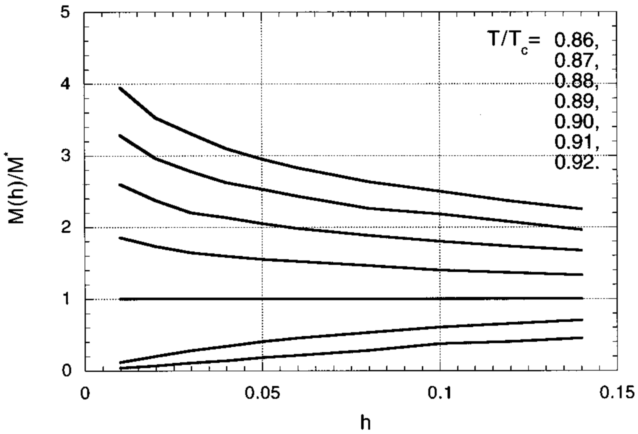

FIG. 3. The field dependence of M near the temperature of KT transition, $T_{KT}=0.9T_c$. $M^*=M(T_{KT})$.

$$M^*=M(T_{KT})=-\frac{T_{KT}}{s\phi_0}\cosh^{-1}\left(\frac{e^{e_0}}{16}\right). \qquad (15)$$

Experimental curves $M(T)$ are almost linear functions in T in a wide temperature region up to T_{KT}.[13] This feature is clearly seen in the theoretical dependences of Fig. 2. At $T<T_{KT}$, M is given by

$$M=-\frac{\phi_0}{32\pi^2\lambda^2(T)}\ln\frac{H_0}{H}. \qquad (16)$$

For conventional superconductors, the field H_0 differs from the upper critical field by a factor of order unity. In our model

$$H_0=e^{e_0}\frac{4\pi\kappa^2T}{s\phi_0} \qquad (17)$$

differs from H_{c2}; in particular, unlike H_{c2}, it increases with temperature.

Now we will discuss the consequences of our model in detail. Expression (15) contains dimensionless value of energy e_0, the half energy of a 2D dipole with intervortex distance 2ξ,

$$e_0\approx\ln4+1.56\approx3.$$

Here the term 1.56 is the energy of the nonuniform order parameter in the core.[21] Note that the core energy of an Abrikosov vortex is 0.16 of our dimensionless units and the value $e_0\approx1.6$ is noticeably less than the value for layered superconductors. For BSCCO (Refs. 13,22,23) $M^*=0.3$ Oe, $T^*=88$ K, $s=15$ Å; using expression (15) we get $e_0=3$.

Magnetization curves[24] experimentally obtained at different temperatures $T<T_{KT}$ have almost the same shapes and can be put on each other in the scale $M(H)/M(H_s)$ (see Fig. 3). Here H_s is the same field for all sets of curves. Upon careful examination it can be seen that these curves have only one common point $H=H_s$. At other fields we see a small but systematical deviation of curves which depends on temperature. From Eqs. (16) and (17) we can get the relation

$$\frac{M(H)}{M(H_s)}=\frac{\ln[H_0(T_{KT})]-\ln(H)+\ln(T/T_{KT})}{\ln[H_0(T_{KT})]-\ln(H_s)+\ln(T/T_{KT})}.$$

As a rule, field region in experiments is several T and $H_0(T_{KT})\approx100$ T. Thus the variation of T/T_{KT} from 0.4 to 1 leads to small change of this function which correctly describes the drift of experimental curves at increasing temperature. We should note that these experimental data have been explained in Ref. 15 by using a nonlocal correction to the London approach.

The value $H_0(T)$ obtained from experimental magnetization data can be used to determine the upper critical field of a superconductor; namely,

$$H_{c2}(T)=H_0(T)4pe^{-e_0}.$$

Using experimental data $M(T)$ from Refs. 13,15 we extract a value of the upper critical field for BSCCO $H_{c2}(0)\approx70$ T and for TSCCO $H_{c2}(0)\approx$ 120 T.

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APPENDIX: METHOD OF COLLECTIVE VARIABLES

The main difficulty in evaluating the partition function (5) is the long-range nature of the pancake interaction. As a rule, to calculate the free energy of such a system, the summation of the ring graph series[25,26] is used. However, there are two unresolved questions in this method. The first one is the method accuracy. The interlayer interaction evokes the second. To overcome these difficulties we propose a method which allows us to construct nonsingular perturbation theory for a system of long-range interacting particles.

The energy of the 2D vortex interaction depends on the layer labels and the sign is defined by the mutual directions of the pancake fluxes

$$\beta U^{\alpha\alpha'}(\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'})=Q_i^\alpha Q_j^{\alpha'}u^{\alpha\alpha'}(\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'}).$$

The dimensionless charge $Q_i^\alpha=\pm\sqrt{2p}$ controls the energy sign and the dimensionless potential obeys the equation

$$-\Delta u^{\alpha\alpha'}(\mathbf{x})+\sum_{\alpha''}\int d\mathbf{y}K^{\alpha\alpha''}(\mathbf{x}-\mathbf{y})u^{\alpha''\alpha'}(\mathbf{y})=4\pi\delta(\mathbf{x})\delta_{\alpha\alpha'}. \tag{A1}$$

To transform the partition function, we substitute the unit representation by a functional integral over a scalar variable ϕ:

$$1=A^{-1}\int D\phi\exp\Bigg\{-\frac{1}{8\pi}\sum_\alpha\int d\mathbf{y}\Bigg[[\nabla\phi^\alpha(\mathbf{y})]^2+\sum_{\alpha'}\int d\mathbf{y}'\phi^\alpha(\mathbf{y})K^{\alpha\alpha'}(\mathbf{y}-\mathbf{y}')\phi^\alpha(\mathbf{y}')\Bigg]\Bigg\} \tag{A2}$$

under integral sign in Eq. (5). To regularize the integral we divide it by the constant A which is equal to the integral and may formally be expressed as a functional determinant of the operator $\hat{M}_0=1/4\pi(\nabla^2+\hat{K})$: $A^{-1}=(\det\hat{M}_0)^{1/2}$.

The next step is to carry out the change of the variable ϕ:

$$\phi^\alpha(\mathbf{y})\Rightarrow\phi^\alpha(\mathbf{y})+i\sum_{\alpha'}\sum_i Q_i^{\alpha'}u^{\alpha\alpha'}(\mathbf{y}-\mathbf{x}_i^{\alpha'}). \tag{A3}$$

The change is chosen in such a way that after integrating by parts over $\mathbf{y}$ and using Eq. (A1) it leads to a compensation of the sum of interaction potentials in the power of the exponential function (5) and transforms the partition function to the form

$$Z=\prod_\alpha\left[\frac{1}{N_+^\alpha!N_-^\alpha!}\left(\frac{Se^{-\beta E_0}}{\pi\xi^2}\right)^{N_+^\alpha+N_-^\alpha}\int\frac{d\mathbf{x}}{S}\right]\int D\phi e^{-S[\phi]},$$

with the effective functional

$$S[\phi]=\frac{1}{2}\sum_{\alpha\alpha'}\phi^\alpha\hat{M}_0^{\alpha\alpha'}\phi^{\alpha'}+i\sum^{\alpha}\sum_i Q_i^\alpha\phi^\alpha(\mathbf{x}_i^\alpha)=S_1+S_2.$$

For further transformations of the functional integral it is convenient to go over a Fourier representation of $S[\phi]$ in which the operator $\hat{M}_0$ is diagonal for functional variables. Expanding the exponential function e^{-S_2} in a power series we can keep only even terms because S_1 is quadratic in ϕ^α. Furthermore, the diagonality of $\hat{M}_0(\mathbf{q})$ produces selection rules corresponding to the Wick theorem. In accordance with these rules only the combinations $\phi^\alpha(\mathbf{q})\phi^{\alpha'}(-\mathbf{q})$ contribute to the integral. The contribution of all pairings $\phi^\alpha(\mathbf{q})\phi^{\alpha'}(-\mathbf{q})$ are the same. So we can keep a single pair in each term and multiply it by a number of pairings $(2b-1)!!$ for the bth term. As a result, the series takes the form

$$\int D\phi e^{-S[\phi]}=\int D\phi\exp\left\{-\frac{1}{2}\sum_{\mathbf{q}}\sum_{\alpha\alpha'}\phi^\alpha(\mathbf{q})M_0^{\alpha\alpha'}(\mathbf{q})\phi^{\alpha'}(-\mathbf{q})\right\}\times\sum_{b=0}^\infty\frac{(-1)^b}{(b)!2^b}\left(\sum_{\alpha\alpha'}\sum_{ij}Q_i^\alpha Q_j^{\alpha'}\times\sum_{\mathbf{q}}\phi^\alpha(\mathbf{q})\phi^{\alpha'}(-\mathbf{q})e^{i(\mathbf{q},\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'})}\right)^b. \tag{A4}$$

Now we have to carry out the integration over particle coordinates. It produces additional selection rules which are just the momentum conservation laws.

It is convenient first to separate the contribution into the free energy of terms $\mathbf{q}=0$. This can be done directly in the exponents of the partition function (5) as an average over space variables. In zero magnetic field, it results in no contribution. In field it gives the magnetic energy. As a result of the averaging, any term in series (A4) containing variables $\phi^\alpha(\mathbf{q}=0)$ which can arise due to integration over pancake coordinates does not contribute to the integral.

First we can select and sum up the sequence of the space independent terms in Eq. (A4). They consist of pairs with equal coordinates $\mathbf{x}_i^\alpha=\mathbf{x}_j^{\alpha'}$ (and of course $\alpha=\alpha'$). The summation of this sequence leads to a quadratic form of ϕ in exponentials:

$$\sum_{b=0}^\infty\frac{(-1)^b}{(b)!2^b}\left(\sum_\alpha(N_+^\alpha+N_-^\alpha)Q^2\sum_{\mathbf{q}}\phi^\alpha(\mathbf{q})\phi^\alpha(-\mathbf{q})\right)^b=\exp\left\{-\frac{1}{2}\sum_\alpha(N_+^\alpha+N_-^\alpha)Q^2\sum_{\mathbf{q}}\phi^\alpha(\mathbf{q})\phi^\alpha(-\mathbf{q})\right\}.$$

This term is most significant in our consideration. Adding it to the unperturbed functional in the exponents (A4) we obtain a new perturbation theory which is determined by the unperturbed action renormalized by collective effects

$$\int D\phi e^{-S[\phi]}=\int D\phi\exp\left\{-\frac{1}{2}\sum_{\mathbf{q}}\sum_{\alpha\alpha'}\phi^\alpha(\mathbf{q})[M_0^{\alpha\alpha'}+\delta_{\alpha\alpha'}(N_+^\alpha+N_-^\alpha)]\phi^{\alpha'}(-\mathbf{q})\right\}\times\sum_{b=0}^\infty\frac{(-1)^b}{b!2^b}\left(\sum_{\alpha\alpha'}\sum_{i\neq j}Q_i^\alpha Q_j^{\alpha'}\times\sum_{\mathbf{q}}\phi^\alpha(\mathbf{q})\phi^{\alpha'}(-\mathbf{q})e^{i(\mathbf{q},\mathbf{x}_i^\alpha-\mathbf{x}_j^{\alpha'})}\right)^b. \tag{A5}$$

Unlike Eq. (A4), integrals over $\mathbf{q}$ in the each term (A5) converges as $q\to 0$.

The leading contribution to the configurational correction to the system energy is obtained by the integration of the exponential function (A5). This portion of the energy is

$$\int D\phi e^{-S[\phi]} \approx \exp\left\{ -\frac{S}{2}\int \frac{d\mathbf{q}}{(2\pi)^2} \ln \det[\delta_{\alpha\alpha'} + Q^2(N_+^\alpha + N_-^\alpha)(M_0^{-1})^{\alpha\alpha'}] \right\}. \quad (A6)$$

It corresponds to the well-known sum of the ring graph series[25,26] and accounts for the collective effects in the system of long-range interacting particles. The rest of the terms are due to the interaction of screened particles.

As follows from Eq. (A1) and the definition of the charge Q,

$$4\pi Q^2(M_0^{-1})^{\alpha\alpha'}(q) = U^{\alpha\alpha'}(q).$$

So, inverting the matrix $\hat{M}_0$ we find the configurational energy of the pancake-antipancake gas

$$\beta U_{\rm corr} = \frac{S}{2}\int \frac{d\mathbf{q}}{(2\pi)^2} \ln \det\left[\left(1 + \frac{4\pi p(N_+^\alpha + N_-^\alpha)}{Sq^2} \right)\delta_{\alpha\alpha'} - \frac{4\pi p(N_+^\alpha + N_-^\alpha)}{Sq^2} W_{\alpha\alpha'}(q) \right], \quad (A7)$$

where

$$W_{\alpha\alpha'}(\mathbf{q}) = \frac{\sinh qs}{\Lambda q} \frac{[G(q) - \sqrt{G(q)^2 - 1}]^{|\alpha - \alpha'|}}{\sqrt{G(q)^2 - 1}},$$

$$G(q) = \cosh qs + \frac{1}{\Lambda q}\sinh qs.$$

The diagonal terms in the square brackets are inlayer pancake interactions, while nondiagonal terms are interactions of pancakes in different layers. Since $W_{\alpha\alpha'}(\mathbf{q}) \sim \sqrt{s/\Lambda} \ll 1$, we can evaluate the infinite determinant in Eq. (A7). The leading contribution to the determinant is the product of the diagonal terms. The correction to the inlayer interaction due to $W_{\alpha\alpha}(\mathbf{q})$ is small and may be neglected. The main correction to the energy due to interlayer interactions arises in the term of second order $W_{\alpha\alpha'}(\mathbf{q})$, which can be neglected also.

Thus, the main portion of the configurational energy of a layered superconductor is

$$\beta U_{\rm corr} = \frac{S}{2}\sum_\alpha \int \frac{d\mathbf{q}}{(2\pi)^2} \ln\left(1 + \frac{1}{\delta^2 q^2} \right) = \sum_\alpha \pi p(N_+^\alpha + N_-^\alpha)[1 - \ln 8p(n_+^\alpha + n_-^\alpha)], \quad (A8)$$

where $\delta^{-2} = 4\pi p(n_+ + n_-)$ is the screening length and $n_\pm = N_\pm \pi\xi^2/S$ are the pancake densities.

The next correction to the configurational energy may be obtained by integration over pancake coordinates and summation of the series (A5):

$$-\Delta(\beta U_{\rm corr}) \sim Sp\sum_\alpha (n_+^\alpha + n_-^\alpha).$$

This correction does not affect such properties of the pancake-antipancake system as the KT transition temperature. It may renormalize the core energy which is anyway a phenomenological parameter in our model. Next corrections are proportional to the second or higher power of the concentrations.

[1] K. B. Efetov, Zh. Éksp. Teor. Fiz. **76**, 1781 (1979) [Sov. Phys. JETP **34**, 905 (1979)].
[2] J. R. Clem, Phys. Rev. B **43**, 7837 (1991).
[3] K. H. Fisher, Physica C **178**, 161 (1991).
[4] V. L. Beresinskii, Sov. Phys. JETP **34**, 610 (1972).
[5] J. M. Kosterlitz and D. G. Thouless, J. Phys. C **6**, 1181 (1973).
[6] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
[7] S. N. Artemenko, I. G. Gorlova, and Yu. I. Latyshev, JETP Lett. **49**, 654 (1989).
[8] J. H. Cho, D. C. Johnston, M. Ledvij, and V. G. Kogan, Physica C **212**, 419 (1993).
[9] D. H. Kim, A. M. Goldman, J. H. Kang, and R. T. Kampwirth, Phys. Rev. B **40**, 8834 (1989).
[10] Q. Y. Ying and H. S. Kwok, Phys. Rev. B **42**, 2242 (1990).
[11] L. Fruchter, V. Vulcanescu, A. Bertinotti, D. Colson, G. Le Bras, and J.-F. Marucco, Physica C **259**, 131 (1996).
[12] G. Le Bras, V. Viallet, A. Bertinotti, A. Forget, J. Hamman, J.-F. Marucco, D. Colson, V. Vulcanescu, and L. Fruchter, Physica C **271**, 205 (1996).
[13] P. H. Kes, C. J. van der Beek, M. P. Maley, M. E. McHenry, D. A. Huse, M. J. V. Menken, and A. A. Menovsky, Phys. Rev. Lett. **67**, 2383 (1991).
[14] L. N. Bulaevskii, M. Ledvij, and V. Kogan, Phys. Rev. Lett. **68**, 3773 (1992).
[15] V. G. Kogan, A. Gurevich, J. H. Cho, D. C. Johnston, Ming Xu, J. R. Thompson, and A. Martynovich, Phys. Rev. B **54**, 12 386 (1996).
[16] Z. Tešanović, L. Xing, L. Bulaevskii, Q. Li, and M. Suenaga, Phys. Rev. Lett. **69**, 3563 (1992); Z. Tešanović and A. V. Andreev, Phys. Rev. B **49**, 4064 (1994).
[17] L. N. Bulaevskii, V. M. Vinokur, and M. P. Maley, Phys. Rev. Lett. **77**, 936 (1996).
[18] M. Slutzky, R. G. Mints, and E. H. Brandt, Phys. Rev. B **56**, 453 (1997).
[19] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1980), Pt. 1.
[20] A. Yu. Martynovich, Physica C **227**, 22 (1994).
[21] S. Scheidl and G. Hackenbroich, Europhys. Lett. **20**, 511 (1992).
[22] C. J. van der Beek, M. V. Indenbom, V. Berseth, M. Konczykowski, T. W. Li, P. H. Kes, and W. Benoit, Czech. J. Phys. **46**, 1655 (1996).
[23] Sung-Ik Lee, Mun-Seog Kim, Jin-Nam Park, Seong-Cho Yu, and Nam H. Hur, Czech. J. Phys. **46**, 1781 (1996).
[24] J. H. Cho, Z. Hao, and D. C. Johnston, Phys. Rev. B **46**, 8679 (1992).
[25] J. Mayer and M. Goeppert Mayer, *Statistical Mechanics* (Wiley, New York, 1977).
[26] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley, New York, 1975).