Ground state of a double-layer charged Bose system

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The ground state of a double-layer charged Bose system is investigated beyond the random-phase approximation by including the many-body correlation effects. The correlations within the layer are accounted for within the self-consistent theory of Singwi, Tosi, Land, and Sjölander, while we have neglected the effect of interlayer correlations. The static susceptibility, the elementary excitation spectrum, and the static screened potentials are calculated and their dependence on the boson number density and the layer spacing is examined. Results are compared with the recent work of Tanatar and Das where correlations are not included. It is found that the inclusion of correlation effects combined with the interlayer interactions can significantly alter the nature of the ground state and, in particular, can favor an inhomogeneous charge-density-wave ground state. A comparison is also made in terms of screened potentials with the double-layer electron system that enables one to have a deeper insight into the role played by the exchange and Coulomb correlations separately. [S0163-1829(97)00846-1]

I. INTRODUCTION

The study of a layered charged Bose system has drawn considerable interest in recent years due to its recognition as a possible model for high-T_c superconductors that are layered materials. The system consists of a fluid of identical charged particles obeying Bose-Einstein statistics and confined dynamically to a plane in the presence of a uniform neutralizing background. The model, proposed a long time ago by Schafroth[1] in three dimensions, has received renewed importance due to the failure of the BCS theory[2] to account for the unusual behavior exhibited by the high-T_c superconductors. To the present level of understanding, the superconducting phase transition in the high-T_c materials is expected to be related with the Bose-Einstein condensation of charged bosons (bound pairs of fermions). There are some experimentally observed facts that support the Bose condensation as a possible mechanism for the high-T_c superconductivity. For example, the magnetic penetration depth below T_c measured in the muon-spin-relaxation experiment[3] and the variation of T_c with mass and density are well explained in terms of Bose condensation. Further, the specific-heat behavior near T_c is similar as near the λ-transition point in liquid helium-4.[4] The different experimental properties of high-T_c superconductors in connection with different theoretical ideas have been recently reviewed by Micnas and co-workers[5] and Harshman and Mills.[6] In view of the above interests, the study of a two-dimensional (2D) charged Bose fluid has become an important subject.

On the theory side, Hines and Frankel[7] first studied the 2D charged Bose problem to calculate its dielectric response and the elementary excitation spectrum within the random-phase approximation (RPA). Other important contributions include the variational calculation by Sim and co-workers[8] and the diffusion Monte Carlo study by Magro and Ceperley.[9] In connection with the high-T_c superconductivity, the 2D charged Bose system has recently been considered by Alexandrov and Mott[10] and Gold.[11] Alexandrov and Mott have given an explanation for the unusual enhancement of the thermal conductivity in the superconducting phase in high-T_c metal oxides by considering the bosons as the charge carriers with charge $2e$, where e is the charge on the electron. Gold[11(b)] has stressed the question of mechanism for the formation of charged bosons and has calculated T_c in terms of Bose condensation in agreement with the recent experimental measurements.[12] It may be mentioned here that it is a multilayer electron system that has been found to exhibit high-T_c superconductivity. This has led to a surge of investigations of multilayer electron systems both at the theoretical[13,14] and experimental[15] levels. These systems are found to exhibit a variety of features due entirely to the interlayer Coulomb interactions. The possibility of charge-density-wave (CDW) ground states, the enhancement of Wigner crystallization density[13] and the observation of fractional quantum Hall states[15] in the presence of a perpendicular magnetic field are among the typical examples.

Tanatar and Das,[16] motivated by the interesting behavior exhibited by the double-layer electron system, considered a double-layer charged Bose system interacting via long-range Coulomb and short-range contact potentials. The bosons were assumed to be in the condensate phase at absolute zero temperature. The collective density excitation spectrum and the static screened potentials were calculated for different layer spacings. The calculations were based on the RPA that takes account dynamic screening but does not include the Coulomb correlation effects. However, the corrections to the

RPA due to correlations are much more important in 2D.[17] The inclusion of the Coulomb correlations in the RPA treatment of the double-layer charged Bose system is one of the motivations for the present work. In particular, our interest lies in investigating the effect of many-body correlations and of interlayer interaction on the static and dynamic structure of the system. We describe the intralayer correlation effects within the self-consistent theory of Singwi, Tosi, Land, and Sjölander (STLS) (Ref. 18) that we have recently[19] used for studying the ground state of a single 2D charged Bose layer, but, treat the interlayer correlations within the RPA. This is justified up to some extent as the interlayer potential is not singular at the origin and, consequently, the interlayer correlations are expected to be weaker than the intralayer correlations. The static density susceptibility, the elementary excitation spectrum, and the static screened potentials are calculated for different layer spacings and boson number densities. Results are compared with the calculation of Tanatar and Das. We have also compared our results for the screened potential with the double-layer electron system. The comparison enables one to see the role played by exchange and Coulomb correlations separately and may have interesting consequences in connection with the theory of high-T_c superconductivity.

The paper is planned as follows: In Sec. II, we derive an expression for the wave vector and frequency-dependent density response function appropriate to the double-layer system within the linear response framework. The static susceptibility, elementary excitation spectrum and screened potentials are calculable directly from the response function. Results and discussion are presented in Sec. III. In Sec. IV, we present the concluding remarks.

II. THEORETICAL FORMALISM

A. Expression for the density response function

Consider two identical layers of charged bosons separated by a perpendicular distance d. The bosons are free to move in the x-y plane parallel to the layer but are confined in the z direction. At absolute zero temperature, the system is assumed to be in the condensate phase. We wish to study the density response of the system to an external potential $V^{\mathrm{ext}}(q,\omega)$ varying in space and time with wave vector $\mathbf{q}$ and frequency ω. Within the linear response theory, the total potential experienced by particles in one layer consists of the external potential plus the potential induced due to the density fluctuations in the second layer. Thus, the induced density in the layer labeled '1' can be written as

$$\delta n_1(q,\omega)=\chi_{11}(q,\omega)\{V_1^{\mathrm{ext}}(q,\omega)+V_{12}(q)[1-G_{12}(q)] \times \delta n_2(q,\omega)\}. \quad (1)$$

$\chi_{11}(q,\omega)$ $[=\chi_{22}(q,\omega)]$ is the density response function for the single isolated charged Bose layer. Hereafter, we shall use the notation $\chi_{11}(q,\omega)=\chi_{22}(q,\omega)=\chi(q,\omega)$. $V_{12}(q)$ is the interaction potential between charged bosons in the layers '1' and '2' and is given by

$$V_{12}(q)=V_{21}(q)=V(q)e^{-qd}, \quad (2)$$

where $V(q)=2\pi e^2/q$, is the intralayer Coulomb interaction potential. $G_{12}(q)$ represents the local-field correction arising due to the short-range interlayer Coulomb correlations. However, in the present study, we assume that there is negligible tunneling between the layers, so that we can treat the interlayer interaction to be the bare Coulomb interaction, i.e., we take $G_{12}(q)=0$ in Eq. (1). From Eq. (1), the density response function (matrix) is obtained as

$$[\chi^T(q,\omega)]^{-1}=\begin{bmatrix}\chi^{-1}(q,\omega) & -V_{12}(q)\\ -V_{12}(q) & \chi^{-1}(q,\omega)\end{bmatrix}. \quad (3)$$

For $\chi(q,\omega)$, we shall use the STLS expression given as

$$\chi(q,\omega)=\frac{\chi_0(q,\omega)}{1-V(q)[1-G(q)]\chi_0(q,\omega)}. \quad (4)$$

In Eq. (4), $\chi_0(q,\omega)$ is the response function for the noninteracting charged Bose gas and at absolute zero it is given by

$$\chi_0(q,\omega)=\frac{2n\epsilon_q}{[(\omega+\iota\eta)^2-\epsilon_q^2]}, \quad (5)$$

where $\epsilon_q=\hbar^2q^2/2m$, is the free particle energy, n is the boson number density, η is a positive infinitesimal quantity, and $G(q)$ is the local-field correction accounting for the intralayer short-range correlations between charged bosons.

The static density response matrix corresponds to $\omega=0$ in Eq. (3). On diagonalizing the resulting matrix, the diagonal component of static density susceptibility defined by $\chi^T(q)=-\chi^T(q,\omega=0)$ is obtained as

$$\chi_\pm^T(q)=\frac{\chi(q)}{1\pm V_{12}(q)\chi(q)}. \quad (6)$$

The plus sign corresponds to the case when the density fluctuations in the two layers have the same phase $[\delta n_1(q)=\delta n_2(q)]$ and the minus sign to the case when they are out of phase by π $[\delta n_1(q)=-\delta n_2(q)]$.

B. Elementary excitation spectrum and static screened potentials

The energy spectrum $E(q)$ $[=\hbar\omega(q)]$ of elementary excitations is determined from the solution of the equation

$$\det[\chi^T(q,\omega)]^{-1}=0, \quad (7)$$

i.e.,

$$\left(\frac{1}{\chi(q,\omega)}\right)^2-V_{12}^2(q)=0. \quad (8)$$

Substituting for $\chi(q,\omega)$ and $V_{12}(q)$ in the above equation and then solving the resulting equation, $E(q)$ is obtained as

$$E(q)=\{\epsilon_q^2+\omega_p^2(q)[1-G(q)\pm e^{-qd}]\}^{1/2}, \quad (9)$$

where $\omega_p(q)=(2\pi ne^2q/m)^{1/2}$, is the 2D plasmon frequency. Thus, for a double-layer system there are two plasmon modes, an in-phase mode (+ sign) and an out-of-phase mode (− sign). It is evident from Eq. (9) that the in-phase mode has higher energy as compared to the out-of-phase mode and the inclusion of intralayer correlations will result in the lowering of the excitation energy over the RPA where $G(q)=0$.

For a double-layer system, there will be two different kinds of screened potentials, namely, the intralayer and interlayer potentials. The screened potential matrix in the static limit ($\omega=0$) is defined as

$$[V_{ij}^{\rm sc}(q,0)]=\frac{[V_{ij}(q)]}{[\epsilon_{ij}(q,0)]}, \tag{10}$$

where $[\epsilon_{ij}(q,0)]$ stands for the static dielectric response matrix that in the STLS theory is related to the density response matrix by the relation

$$\frac{1}{[\epsilon_{ij}(q,0)]}=[\delta_{ij}]+[V_{ij}(q)][\chi_{ij}^{T}(q,0)]. \tag{11}$$

Using Eqs. (3), (10), and (11), the screened intralayer and interlayer potentials are obtained, respectively, as

$$V_{11}^{\rm sc}(q,0)=V_{11}(q)+\frac{\chi(q,0)}{1-\chi^2(q,0)V_{12}^2(q)}\,[V_{11}^2(q)+V_{12}^2(q)+2\chi(q,0)V_{11}(q)V_{12}^2(q)], \tag{12}$$

$$V_{12}^{\rm sc}(q,0)=V_{12}(q)+\frac{\chi(q,0)}{1-\chi^2(q,0)V_{12}^2(q)}\,\{2V_{11}(q)V_{12}(q)+\chi(q,0)[V_{12}^3(q)+V_{11}^2(q)V_{12}(q)]\}. \tag{13}$$

In the RPA, where $G(q)=0$, $\chi(q,0)$ is given by

$$\chi(q,0)=\frac{\chi_0(q,0)}{1-V(q)\chi_0(q,0)} \tag{14}$$

and the screened potentials are simplified to

$$V_{11}^{\rm sc}(q,0)=\frac{V_{11}(q)-[V_{11}^2(q)-V_{12}^2(q)]\chi_0(q,0)}{[1-V_{11}(q)\chi_0(q,0)]^2-[V_{12}(q)\chi_0(q,0)]^2} \tag{15}$$

and

$$V_{12}^{\rm sc}(q,0)=\frac{V_{12}(q)}{[1-V_{11}(q)\chi_0(q,0)]^2-[V_{12}(q)\chi_0(q,0)]^2}. \tag{16}$$

The screened potentials in the real space can be obtained by the inverse Fourier transformation, i.e.,

$$V_{11}^{\rm sc}(r)=\int\frac{d\mathbf{q}}{(2\pi)^2}\,e^{\iota\mathbf{q}\cdot\mathbf{r}}V_{11}^{\rm sc}(q,0). \tag{17}$$

Substituting for $V_{11}^{\rm sc}(q,0)$ and performing the integration, we have

$$V_{11}^{\rm sc}(r)=e^2\left\{\frac{1}{r}+\int_0^\infty d\,qJ_0(qr)\left[\frac{\chi(q,0)V_{11}(q)}{1-\chi^2(q,0)V_{12}^2(q)}\times[1+e^{-2qd}+2\chi(q,0)V_{11}(q)e^{-2qd}]\right]\right\}, \tag{18}$$

where $J_0(qr)$ is the zeroth-order Bessel's function of first kind. In the same way, $V_{12}^{\rm sc}(r)$ is obtained as

$$V_{12}^{\rm sc}(r)=e^2\left\{\frac{1}{\sqrt{r^2+d^2}}+\int_0^\infty d\,qJ_0(qr)\times\left[\frac{\chi(q,0)V_{11}(q)}{1-\chi^2(q,0)V_{12}^2(q)}\,[2e^{-qd}+\chi(q,0)V_{11}(q)\times(e^{-3qd}+e^{-qd})]\right]\right\}. \tag{19}$$

In Eqs. (18) and (19), the first term represents, respectively, the bare intralayer and interlayer interaction potentials and it is the second term that accounts for the screening effects. In the next section, we present numerical results for the various quantities of interest.

III. RESULTS AND DISCUSSION

In the numerical calculations and the results presented, we use a system of units in which $\hbar=1$ and lengths are expressed in units of q_s^{-1} with $q_s=\sqrt{2}/[r_s a_0]$. r_s is the dimensionless density parameter related to the boson number density n as $r_s=r_0/a_0$, where $r_0=1/\sqrt{n\pi}$ and $a_0=\hbar^2/[me^2]$, is the Bohr atomic radius. For the intralayer quantities, the STLS results obtained recently by us are used.

A. Static density susceptibility

The static density susceptibility constitutes an important quantity as it provides information about the static structure in the system. It is evident from Eq. (6) that the out-of-phase component of the density susceptibility $\chi_-^T(q)$ can diverge for a critical spacing d_c even when the single-layer susceptibility $\chi(q)$ is finite. This can happen because $\chi(q)$ and $V_{12}(q)$ are both positive. This implies that even if the single isolated layer has a uniform density distribution, the interlayer interactions can induce in the system a particular density modulation. The corresponding critical wave vector q_c and the critical layer spacing d_c can be specified from the solution of the equation

$$1-V_{12}(q_c)\chi(q_c)=0. \tag{20}$$

Substituting for $\chi(q)$ from Eq. (4), we obtain q_c as

$$q_c^3=2^{3/2}r_s[G(q_c)+e^{-q_c d_c}-1]. \tag{21}$$

Equation (21) cannot be solved analytically for q_c and we calculate it numerically. However, it may be noted that in the RPA $[G(q)=0]$ Eq. (21) has no solution with $q_c>0$. Further, when $G(q)$ is finite, the condition

$$G(q_c)+e^{-q_c d_c}-1>0$$

must be satisfied for q_c to be nonzero. This, in turn defines the critical spacing d_c. Thus, it can be concluded that the intralayer correlations combined with the interlayer interactions are together responsible for the divergence in $\chi_-^T(q)$. The divergent behavior of $\chi_-^T(q)$ means that below a critical spacing the system will be unstable to a phase transition into an inhomogeneous CDW state. Results for $\chi_-^T(q)$ are shown in Fig. 1 at three densities corresponding to $r_s=1$, 5, and 10 for different values of d. Clearly, $\chi_-^T(q)$ exhibits a sharp

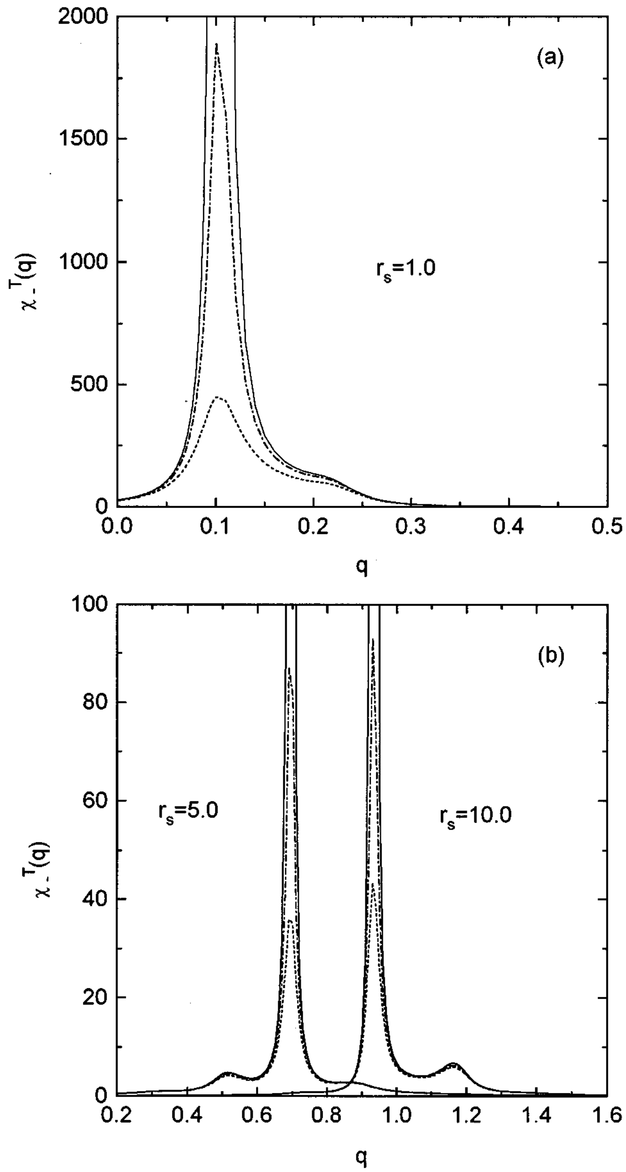

FIG. 1. The static density susceptibility $\chi_-^T(q)$ vs q. (a) For $r_s=1$ and $d=0.4945$ (dashed line), 0.4943 (dash-dot line), 0.4942 (solid line). (b) For $r_s=5$ and $d=0.7125$ (dashed line), 0.7119 (dash-dot line), 0.7115 (solid line) and for $r_s=10$ and $d=0.8275$ (dashed line), 0.8272 (dash-dot line), 0.8270 (solid line).

peak with its sharpness increasing with decreasing d. At $r_s=10$, $\chi_-^T(q)$ diverges at $q_c\approx0.93$ with $d_c\approx0.8269$, while at $r_s=5$, $q_c\approx0.69$; $d_c\approx0.7115$ and at $r_s=1$, $q_c\approx0.102$; $d_c\approx0.4942$. Thus, the value of d for the CDW instability decreases with decrease in r_s, i.e., with increase in density. This arises due to the weakening of the intralayer correlations with increase in density. Also, the critical wave vector q_c associated with the instability decreases with decrease in r_s. This suggests that q_c corresponds to a CDW state as if it had been the wave vector associated with the Wigner crystal it would not have changed with r_s. Further, we notice that $\chi_-^T(q)$ approaches the single-layer result for $d\gg d_c$. This is expected as the interlayer interaction effects become very weak at large layer spacings. It may be mentioned here that Gold[20] has also predicted the presence of the CDW state in the double-layer charged Bose system. On comparison, we find that our results are in agreement with the conclusions of Gold.

B. Elementary excitation spectrum

It will be interesting to investigate the spectrum of elementary excitations near the CDW transition. For this, we choose the value of d well above and near the CDW instability and calculate the collective excitation modes by using Eq. (9). In Fig. 2, we show the calculated dispersion for the two excitation modes at density $r_s=10$. To show the importance of correlation effects, results are compared with the RPA calculation. We notice that the in-phase mode has higher excitation energy, while the out-of-phase mode represents the low-lying excited state of the system and has acousticlike linear dispersion at small q. Further, the many-body correlations depress the dispersion curves over the RPA results and near the transition point [$d=0.8275$, Fig. 2(b)] the energy of the out-of-phase mode becomes very close to zero for $q\approx0.93$. The appearance of this mode indicates that it costs very little energy to excite the system into a state with a periodic modulation of density of wave vector $q\approx0.93$. For $d=0.8269$, the minima in the excitation energy of the acoustic mode becomes exactly equal to zero and there would be a strong tendency for the system to be unstable against transition into a periodic CDW ground state. It may be noted that the values of critical q and d are exactly the same as encountered in the calculations of static susceptibility. A similar behavior of $E(q)$ is also found at $r_s=1$ and 5. Thus, we demonstrate clearly that the excitation spectrum is dramatically affected near the CDW instability. We further notice that the many-body correlations have significant effect on the dispersion curves even at small values of q. To illustrate this, we calculate the dispersion in the long-wavelength limit, i.e., $q\rightarrow0$. Taking $q\rightarrow0$ limit in Eq. (9), we obtain

$$E_+(q)\approx\omega_p(q)\sqrt{2}\left[1-\frac{qd}{4}\left(1+\frac{\gamma}{d}\right)\right] \tag{22}$$

and

$$E_-(q)\approx\omega_p(q)\sqrt{qd}\left[1-\frac{1}{2}\left(\frac{\gamma}{d}\right)\right], \tag{23}$$

where

$$\gamma=-\frac{1}{2}\int_0^\infty dq[S(q)-1],$$

accounts for the correlation effects and is a positive definite quantity and $S(q)$ is the intralayer static structure factor. The corresponding limiting behavior in RPA is given by

$$E_+(q)\approx\omega_p(q)\sqrt{2}\left(1-\frac{qd}{4}\right) \tag{24}$$

and

$$E_-(q)\approx\omega_p(q)\sqrt{qd}\left(1-\frac{1}{4}qd\right). \tag{25}$$

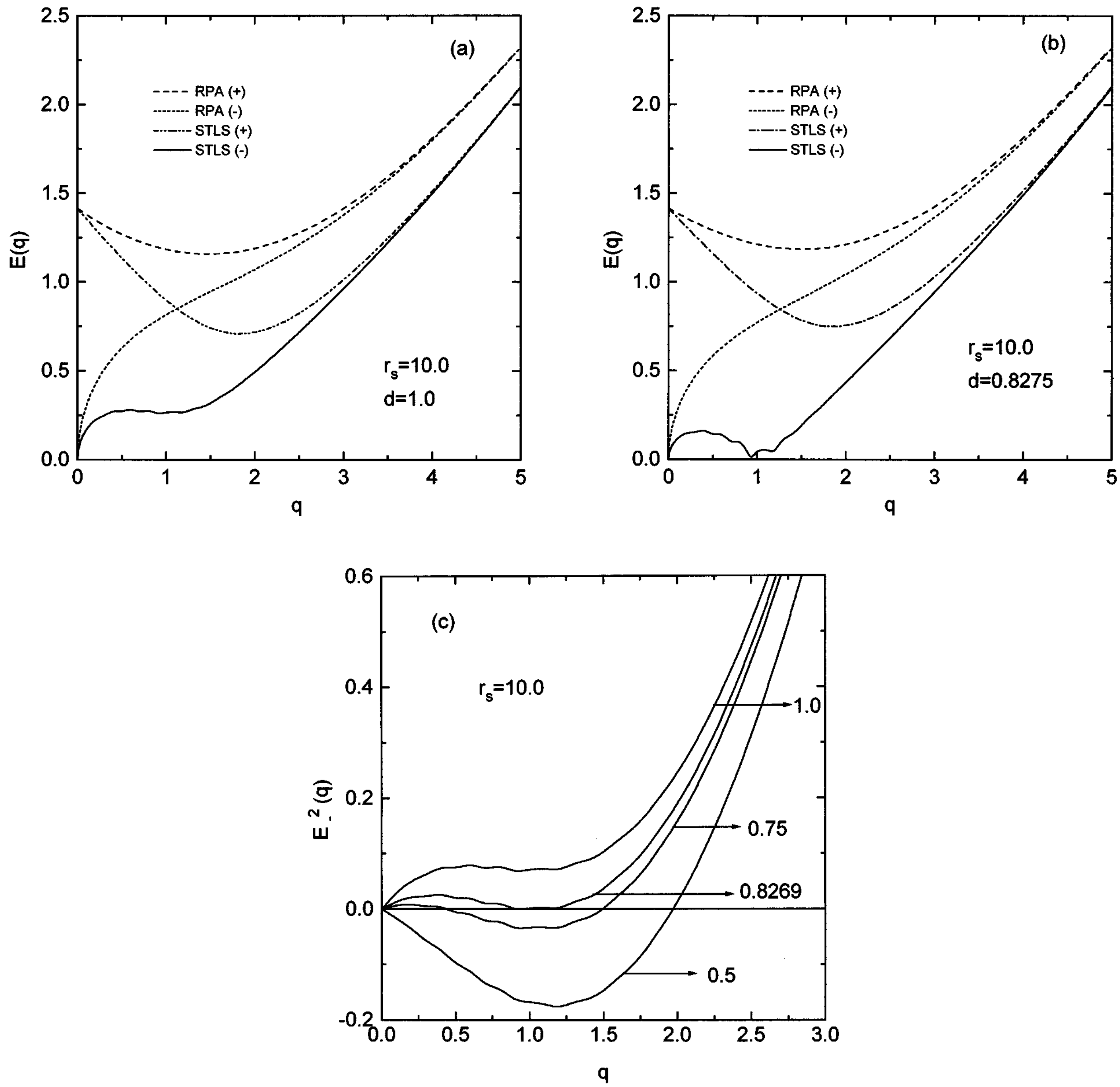

FIG. 2. The elementary excitation spectrum $E(q)$ [in units of $\omega_p(q)$] vs q at $r_s=10$. (a) For $d=1.0$. (b) For $d=0.8275$. The curves labeled STLS (+) and STLS (−) represent our results for the in-phase and out-of-phase modes, respectively. The curves labeled RPA (+) and RPA (−) represent the corresponding results in the random-phase approximation. (c) $E_-^2(q)$ vs q for $d=1.0$, 0.8269, 0.75, and 0.5.

The significance of correlations at small q is self-evident from Eqs. (22)–(25). We further notice that for $d<d_c$, $E_-(q)$ becomes imaginary over a range of q values that broadens with decreasing d. To illustrate this behavior, $E_-^2(q)$ is plotted in Fig. 2(c) at $d=1.0$, 0.8269, 0.75, and 0.5 for $r_s=10$. It is apparent that at $d=0.75$, $E_-^2(q)$ cuts the q axis twice, namely, at $q\approx0.439$ and 1.495. This implies that there are two values of q satisfying the CDW criterion fixed by Eq. (20). This is equivalent to saying that Eq. (21) has all three roots ($q=0$, 0.439, 1.495) distinct at $d=0.75$. However, with further decrease in d (for example, at $d=0.5$), the two nonzero roots become equal. Thus, our calculations show that the system is unstable to two different CDW ground states for a range of layer spacing below the critical spacing. A similar type of behavior has recently been found in an electron double layer by Lu *et al.*[14]

C. Static screened potentials

We have so far seen that the ground state and the elementary excitation spectrum are markedly affected by the interlayer interactions and the many-body correlations. It is natural to expect the similar signatures in the screening properties also. For this, we calculate the static screened intralayer and interlayer potentials by using Eqs. (18) and (19) and examine their dependence on density r_s and the layer spacing d. In Fig. 3(a), the screened intralayer potential $V_{11}^{sc}(r)$ is shown at $r_s=1$, 5, and 10 for $d=1$. Also shown for comparison is the RPA result at $r_s=10$. From the curves shown, it is clear that $V_{11}^{sc}(r)$ exhibits an attractive part and its depth is strongly enhanced over the RPA value (approximately by a factor of 5 at $r_s=10$) by the many-body correlations. Also, the many-body enhancement of attraction increases with the decrease in density. The screened interlayer potential $V_{12}^{sc}(r)$ is plotted

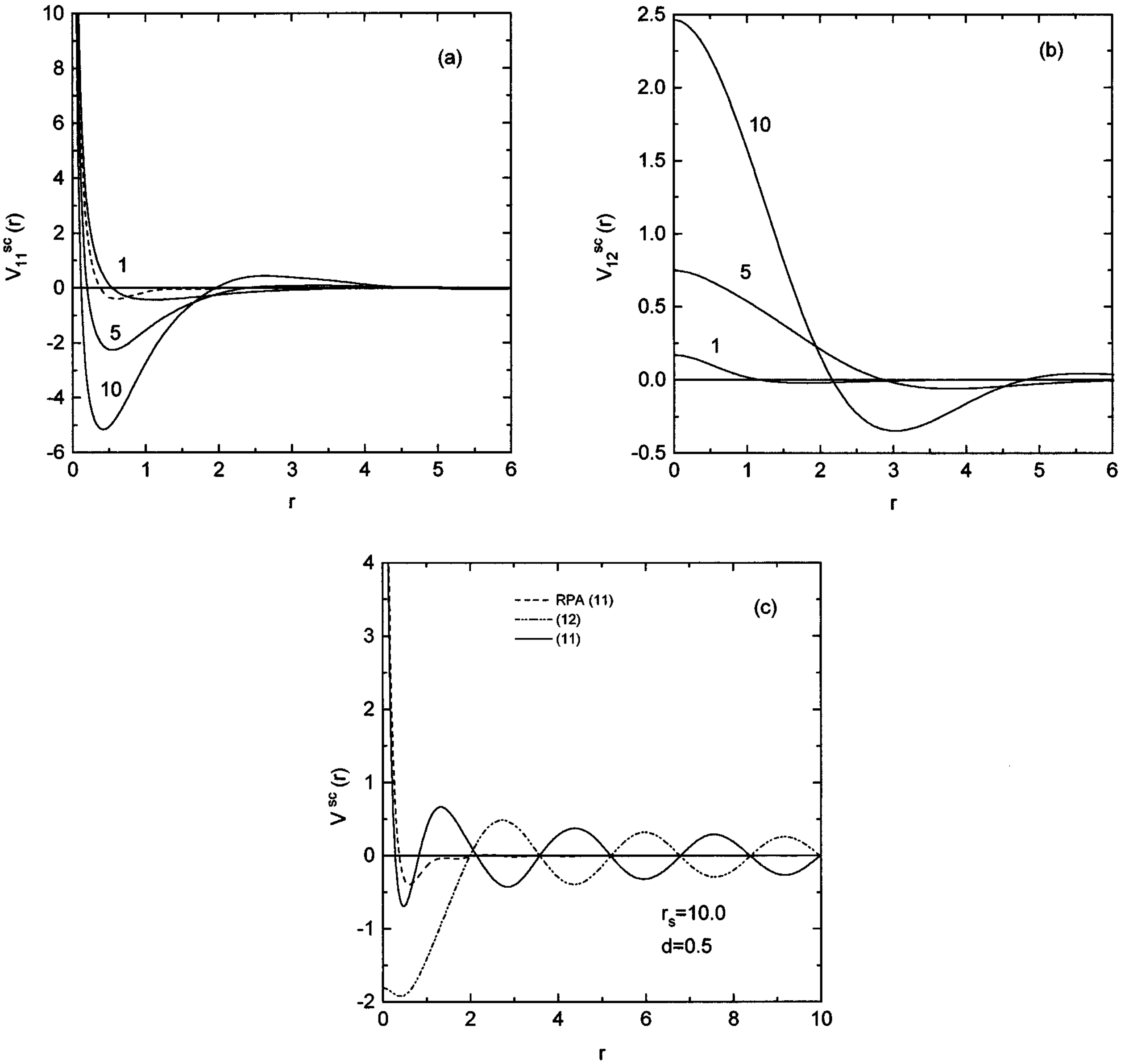

FIG. 3. The static screened potentials $V^{sc}(r)$ (in units of $e^2 q_s$) vs r for the Bose system. (a) The screened intralayer potential for $d=1.0$ and $r_s=1$, 5, and 10; the dashed curve is the RPA result at $r_s=10$. (b) The screened interlayer potential for $d=1.0$ and $r_s=1$, 5, and 10. (c) The screened inter- and intralayer potentials for $d=0.5$ and $r_s=10$. The value of d lies well in the charge-density-wave instability region. The curves labeled 11 and 12 are the present results for the screened intralayer and interlayer potentials, respectively. The curve labeled RPA (11) is the RPA result for the intralayer potential.

in Fig. 3(b) at $r_s=1$, 5, and 10 for $d=1$. In $V_{12}^{sc}(r)$, r defines the parallel separation between particles in the two layers, i.e., $V_{12}^{sc}(r)$ represents the screened interaction between a particle at origin in the layer '1' and a particle in the layer '2' at a distance r parallel to the layer '1'. Unlike $V_{11}^{sc}(r)$, $V_{12}^{sc}(r)$ is not singular at $r=0$. We have also calculated the screened potentials for an isolated charged Bose layer and, on comparison, it was found that the depth of the attractive part increases due to the presence of interlayer interactions.

Now, we select a value of d lying well in the CDW instability region. For this case $(d=0.5)$, the screened potentials are plotted in Fig. 3(c) at $r_s=10$. We observe that both $V_{11}^{sc}(r)$ and $V_{12}^{sc}(r)$ exhibit a periodic oscillatory behavior about zero with very small damping at large r. The phases of two potentials differ exactly by π. This clearly indicates the existence of a periodic density modulated ground state in each layer. This is what we expect as d is taken to be less than the critical spacing for the CDW instability. The period of the oscillatory potential yields the length scale for the density modulated state. The phase difference π between $V_{11}^{sc}(r)$ and $V_{12}^{sc}(r)$ is in agreement with the fact that it is the out-of-phase component of static susceptibility that shows divergence. For comparison, the RPA result (dashed line) for $V_{11}^{sc}(r)$ is also shown. It can be seen from Figs. 3(a) and 3(c) that the RPA results for $V_{11}^{sc}(r)$ are almost independent of d. This once again confirms the relative importance of correlations effects.

It is interesting to compare our results for the screened potentials with the double-layer electron system. Before we

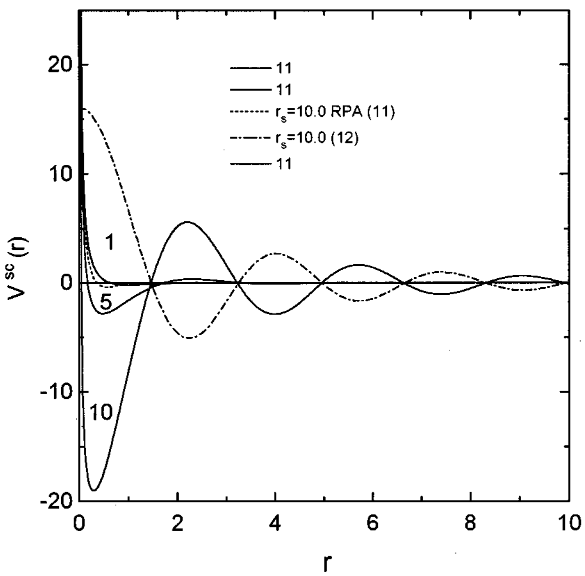

FIG. 4. The static screened potential $V^{sc}(r)$ (in units of e^2q_F) vs r for the electron system for $d=1.0$ and $r_s=1$, 5, and 10. The curves are labeled the same as in Fig. 3(c).

draw the comparison, it may be mentioned here that the ground state of an isolated 2D electron system has been studied by the quantum Monte Carlo method due to Tanatar and Ceperley.[21] Therefore, for the electron system we deduce the necessary intralayer inputs from the accurate Monte Carlo study. Results for $V_{11}^{sc}(r)$ and $V_{12}^{sc}(r)$ for the electron system are shown in Fig. 4 at $r_s=1$, 5, and 10 for the same spacing as in Fig. 3(a), i.e., $d=1.0$. Here, lengths are expressed in units of q_F^{-1}, q_F being the Fermi wave vector. By comparing the curves in Figs. 3 and 4, it can be noticed that the many-body enhancement of the attractive part in screened potentials is relatively larger in the electron system. Also, the potentials in Fig. 4 exhibit a damped periodic oscillatory behavior that is less pronounced in the Bose case for the corresponding spacing, i.e., $d=1.0$. From the comparative study, we may conclude that the CDW instability should occur at a larger critical spacing in the electron system as compared to the Bose system. This we also anticipate on the physical grounds as the intralayer correlations in the electron system are determined by both exchange and Coulomb correlations while the former are absent in the Bose case. The above study of screened interactions may have its relevance to superconductivity in layered compounds.

IV. SUMMARY AND CONCLUSIONS

We have studied the ground state of a double-layer charged Bose system beyond the random-phase approximation by including the corrections due to the many-body correlations. The intralayer correlation effects are incorporated through the static local-field factor calculated within the self-consistent theory of Singwi, Tosi, Land, and Sjölander. We have neglected the effect of interlayer correlations. The static density susceptibility, the elementary excitation spectrum, and the static screened potentials are calculated and their dependence on the layer spacing and the boson number density is critically examined. We find that the system will be unstable against a phase transition into an inhomogeneous charge-density-wave ground state below a critical spacing between the layers. It is noted that the critical spacing for the CDW instability decreases with increase in the density. We have compared our results with the RPA where we find no transition into the CDW ground state. Also, no such transition has been found to exist in an isolated layer in the density range $1<r_s<10$ examined. Thus, we may conclude that the many-body correlations and the interlayer interactions are together responsible for transition into the CDW ground state.

We have also compared our results for the screened potentials with the double-layer electron system. For the same spacing, the screened potentials are relatively more negative in the electron system and this may lead to the formation of bound pairs of electrons and hence, bosons. The comparison we have discussed could have its relevance in connection with the theory of high-T_c superconductivity.

At the end we would like to add that in our analysis of the boson ground state the effect of short-range interlayer correlations has not been taken into consideration. This approximation seems quite reasonable provided the layer spacing d is larger as compared to the particle spacing (r_0) within the layer. However, in our calculations $d/r_0<1$ and, consequently, the interlayer correlations are not sufficiently weak to be negligible as compared to intralayer correlations. One may notice that the inclusion of interlayer correlations beyond RPA would act to reduce the strength of bare interlayer potential $V_{12}(q)$ and this will in turn lead to an upward shift [Eq. (9)] of the dispersion curve corresponding to the out-of-phase acoustic mode. Thus, we expect on physical grounds that the inclusion of interlayer correlations beyond the RPA will contribute in a direction to oppose the tendency towards the inhomogeneous CDW ground state. For the electronic double layer (and also for the electron superlattice), Neilson and co-workers[22] and Lu *et al.*[14] have analyzed the effect of interlayer correlations on the existence of the CDW ground state within different theoretical schemes and have arrived at different conclusions. Neilson and co-workers have treated the interlayer correlations within the self-consistent STLS approximation, while Lu *et al.* have used the quasilocalized charge approximation[23] where the interlayer and intralayer contribution to correlations are determined by satisfying the third frequency-moment-sum rule of density response function. The STLS calculations show that the correlations between layers act to oppose the development of the inhomogeneous CDW ground state, but, the effect of these is not to destroy the existence of the CDW state. However, in contrast, Lu *et al.* find that the transition into the CDW ground state may be quenched with the consistent accounting of interlayer correlations and the acoustic excitation spectrum is dramatically modified by the appearance of an energy gap that has a stabilizing effect on the CDW instability. In our opinion, the use of STLS may be a poor approximation for short-range interlayer correlations when d/r_0 is substantially less than unity. In view of the above discussion, it seems that the transition into an inhomogeneous CDW ground state could be an artifact of the neglect of interlayer correlations or of its inadequate treatment.

Further, in the present study we have assumed the condensate phase as the ground state for the bosons. However, in view of recent diffusion Monte Carlo (DMC) study of 3D charged Bose systems by Moroni and co-workers,[24] this assumption appears restrictive in the high-density limit ($r_s \leqslant 1$). The DMC study reveals that the condensate fraction decreases with decreasing density, i.e., increasing r_s. The inclusion of the effects of depletion of condensate phase due to Coulomb correlations and of interlayer correlations need further study.

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