Current-voltage characteristics of diluted Josephson-junction arrays: Scaling behavior at current and percolation threshold

Enzo Granato
Laboratório Associado de Sensores e Materiais, Instituto Nacional de Pesquisas Espaciais, 12201 São José dos Campos, São Paulo, Brazil

Daniel Domínguez
Centro Atómico Bariloche, 8400 San Carlos de Bariloche, Rio Negro, Argentina

(Received 26 November 1996; revised manuscript received 30 June 1997)

Dynamical simulations and scaling arguments are used to study the current-voltage (I-V) characteristics of a two-dimensional model of resistively shunted Josephson-junction arrays in presence of percolative disorder, at zero external field. Two different limits of the Josephson-coupling concentration p are considered, where p_c is the percolation threshold. For $p>p_c$ and zero temperature, the I-V curves show power-law behavior above a disorder-dependent critical current. The power-law behavior and critical exponents are consistent with a simple scaling analysis. At p_c and finite temperature T, the results show the scaling behavior of a $T=0$ superconducting transition. The resistance is linear but vanishes for decreasing T with an apparent exponential behavior. Crossover to nonlinearity appears at currents proportional to $T^{1+\nu_T}$, with a thermal-correlation length exponent ν_T consistent with the corresponding value for the diluted XY model at p_c. [S0163-1829(97)08445-2]

I. INTRODUCTION

There has recently been an interest, both experimental[1,2] and theoretical,[3–11] in the resistive behavior of diluted Josephson-junction arrays (JJA's). These systems provide a useful model for several universal transport properties of granular high-T_c materials[12] and can also be physically realized as superconducting arrays[1,2] or wire networks[13] with accurately controlled parameters using microfabrication techniques. Although most investigations have considered the combined effects of disorder and magnetic fields which leads to the vortex glass state,[14] there are also interesting questions even in the absence of an external field as a result of disorder. The effects of percolative disorder on the resistive transition and nonlinear current-voltage (I-V) relation, at finite temperatures, have been studied in artificial two-dimensional arrays[1] and also in the context of the high-T_c oxides.[15] For an ordered two-dimensional JJA, which is isomorphic to the XY model, it is well known that the resistive transition is in the Kosterlitz-Thouless (KT) universality class where the low-temperature phase has a nonlinear I-V relation, $V\propto I^{\tilde{a}(T)}$, due to current-induced vortex-pair unbinding. The temperature-dependent exponent $\tilde{a}(T)$ decreases with increasing temperature and reaches $\tilde{a}(T_c)=3$ at the transition.[16] In the presence of percolative disorder but well above the percolation threshold, this behavior still persists with a broadened resistive transition and disorder renormalized critical temperature $T_c(p)$ and exponent $\tilde{a}(T,p)$, where p is the fraction of superconducting grains.[1] Right at the percolation threshold p_c, however, as for the XY model,[17] the transition temperature is expected to vanish but a divergent correlation length $\xi\sim T^{-\nu_T}$ and thermal critical exponent ν_T can still be defined for $T>0$. A natural question arises as to what should be the behavior of the I-V curves for $T>0$ in this limit since the correlation length is finite. In fact, one expects the increasing correlation length as $T\rightarrow 0$ to have important consequences at finite T for the nonlinear behavior of the I-V curves.[14,18] The linear resistance is expected to be finite for any $T>0$ but crossover to nonlinearity can appear for increasing currents due to the finite correlation length with a universal behavior determined by the $T=0$ transition. It is important to understand this behavior in detail because, in two dimensions and in the presence of an external magnetic field, an additional $T=0$ (vortex-glass-like) transition is expected[3] near p_c which should also have a similar effect but it could be difficult to separate these two effects experimentally. Also, in principle, the universal behavior at p_c may be useful to identify the proximity to the percolation threshold in systems where p_c is uncertain as in granular materials modeled as diluted JJA.

There are also other interesting effects of disorder on the current-voltage relation which appear in a different limit and are not well understood. Above the percolation threshold p_c and zero temperature, the I-V characteristics shows a power-law behavior, $V\sim(J-J_c)^a$, above a critical current density J_c. For example, an exponent $a\sim 3$ was obtained from numerical simulations by Xia and Leath[6] in $d=2$. Clearly, this behavior at $T=0$ and finite current density J_c is unrelated to the condition $\tilde{a}(T_c)=3$ at the KT transition mentioned above and requires different considerations. In a linearized model of the critical current as a phase-coherence breakdown phenomenon,[6] it has been argued that the critical current of a diluted array vanishes in the thermodynamic limit. In this case, the apparent finite J_c and the exponent a, presumably universal, are in fact an artifact of finite system sizes. However, it is possible that nonlinearities in the array problem could lead to a finite value even in the large system limit.[7,8] In the latter scenario J_c is finite but is expected to vanish at

 © 1997 The American Physical Society

the percolation threshold as $J_c \sim (p-p_c)^v$ with an exponent $v=\nu_p(d-1)$, where ν_p is the percolation exponent and d is the dimensionality of the system. Unlike the behavior at finite temperatures, little is known however about the precise value of a and the universality class of the current-induced transition leading to the power-law behavior for $p>p_c$, under the assumption that there is in fact a well defined J_c. Nevertheless, it is important to understand these features in detail in order to be able to separate these effects, which arise only from disorder, from others induced by a combination of disorder and strong magnetic fields as in the case of the vortex glass which also leads to values of a of the same order[19] at $T=0$. Indeed, in experimental systems, these effects can appear simultaneously and it is possible that some of the power-law behavior observed in the I-V characteristics of granular high-T_c materials that tend to be regarded as a manifestation of vortex-glass behavior is actually a result of intrinsic geometrical disorder and would persist even in the absence of the external field.

In this paper, we present dynamical simulations of the I-V characteristics of diluted two-dimensional JJA, at zero external field, in the two different limits discussed above which allow for a scaling analysis in terms of a single length scale. (i) At $T=0$ and $p>p_c$, we study the power-law behavior in the I-V curves above J_c as resulting from a current-driven dynamical transition. We discuss scaling arguments for this behavior which leads to $a=\nu_I(z+1)$, where ν_I is a correlation length exponent and z is a dynamical exponent, and compare with numerical results. (ii) At the percolation threshold $p=p_c$ and finite T we study the scaling of the I-V curves as resulting from a $T=0$ superconducting transition. The results are consistent with a linear resistance at finite temperatures and nonlinear behavior appearing at current densities proportional to $T^{1+\nu_T}$, where ν_T is the thermal-correlation length exponent for the diluted XY model at p_c.

II. MODEL AND SIMULATION

We consider a model of resistively shunted JJA's consisting of coupled superconducting islands located at the nodes of a square network with Josephson and normal currents flowing between them. The nodes are located at $\mathbf{r}=m\hat{x}+n\hat{y}$ with unit lattice constant. The current $I_\mu(\mathbf{r})$ flowing between $\mathbf{r}$ and $\mathbf{r}+\hat{\mu}$, is modeled as[6,19,20]

$$I_\mu(\mathbf{r})=I^0_{\mathbf{r}\mu}\sin(\Delta_\mu\theta(\mathbf{r},t))+\frac{\hbar}{2eR_{\mathbf{r}\mu}}\frac{d\Delta_\mu\theta(\mathbf{r},t)}{dt}+\eta_\mu(\mathbf{r},t). \tag{1}$$

Here $\Delta_\mu\theta(\mathbf{r},t)$ is a discrete gradient of the superconducting phases $\theta(\mathbf{r},t)$, $I^0_{\mathbf{r}\mu}$ is the critical current of the junctions, and $R_{\mathbf{r}\mu}$ is a shunt resistance between the islands. The white-noise random variable $\eta_\mu(\mathbf{r},t)$ represents thermal Johnson fluctuations in the current with covariance,

$$\langle \eta_\mu(\mathbf{r},t)\,\eta_{\mu'}(\mathbf{r}',t')\rangle=\frac{2kT}{R_\mu}\,\delta_{\mathbf{r},\mathbf{r}'}\delta_{\mu,\mu'}\delta(t-t'). \tag{2}$$

We assume that disorder affects only the Josephson coupling. In the absence of the coupling, $I^0_{\mathbf{r}\mu}=0$, and so only normal current flows between the neighboring grains. Disorder effects in $R_{\mathbf{r}\mu}$ are assumed to be less important. This model has been applied to the study of transport properties of high-T_c oxides[15] and, in its site-dilution version, to composite superconductors.[8] For proximity coupled artificial arrays,[1,2] this model is also a reasonable approximation since, as pointed out in Ref. 2, the underlying normal-conducting layer over which the superconducting grains are deposited provides a roughly constant R_μ. Dilution of the junctions is introduced by taking critical currents $I^0_{\mathbf{r}\mu}=I_0$ with probability p or $I^0_{\mathbf{r}\mu}=0$ with probability $1-p$ and constant shunt resistance, $R_\mu=R_0$. After combining Eq. (1) with current conservation at each node, $\Delta_\mu\cdot I_\mu(\mathbf{r})=I^{\text{ext}}(\mathbf{r})$, the dynamical equation of motion for $\theta(r,t)$ becomes

$$\dot{\theta}(\mathbf{r},t)=-\sum_{\mathbf{r}'}G(\mathbf{r},\mathbf{r}')\{I^{\text{ext}}(\mathbf{r}')-\Delta_\mu\cdot[I^0_{\mathbf{r}'\mu}\sin(\Delta_\mu\theta(\mathbf{r}',t))+\eta_\mu(\mathbf{r}',t)]\}, \tag{3}$$

with $G(\mathbf{r},\mathbf{r}')$ the $d=2$ lattice Green function. Dimensionless quantities are used with time in units of $\tau_J=\hbar/2eR_0I_0$, currents in units of I_0, voltages in units of R_0I_0, and temperatures in units of $\hbar I_0/2ek_B$. We choose periodic boundary conditions along the x direction and open boundary conditions along the y direction. The array has $L\times L$ bonds, corresponding to $L\times(L+1)$ nodes. The total current I is injected uniformly along the y direction with $i^{\text{ext}}(m,n)=J(\delta_{n,1}-\delta_{n,L+1})$ where J is the current density $J=I/L$. Equation (3) is solved with the second-order Runge-Kutta-Helfand-Greenside algorithm for stochastic differential equations[20] with a time step of $\Delta t=0.1\tau_J$. Temporal averages are taken over a time of $2000\tau_J$ after a transient time of $500\tau_J$. The matrix multiplication by $G(\mathbf{r},\mathbf{r}')$ is performed by means of a fast-Fourier transform and cyclic reduction algorithm as used in Refs. 19 and 20. The voltage drop along the y direction is given by

$$V=\frac{1}{L}\sum_{m=1}^{L}\langle\dot{\theta}(m,n=L+1)-\dot{\theta}(m,n=1)\rangle \tag{4}$$

in dimensionless units, where $\langle\ldots\rangle$ is a time average, and the average electric field is given by $E=V/L$.

III. RESULTS AND DISCUSSION

Figure 1 shows some of the I-V characteristics obtained numerically at $T=0$. Our averaging time is a factor of 2 larger than the one used in Ref. 8 to study a site-dilution version of the same model in $d=3$. As a test of the numerical method we included in Fig. 1 the calculation for $p=1$ where the array behaves as a single junction with a critical current density $J_c=1$. Above the percolation threshold $p_c=\frac{1}{2}$, there is an infinite cluster of superconducting junctions through the system and an apparent finite critical current density J_c below which the voltage is very small. For $p<p_c$, only isolated finite clusters occur and the resistance is Ohmic for small currents.

Figure 2(a) shows the behavior of the critical current density J_c and linear resistance $R_L=\lim_{J\to 0}E/J$ as a function of p at zero temperature, for the largest system size used in the simulations, $L=64$. J_c decreases with p and presumably vanishes at the percolation threshold p_c, while R_L is nonzero

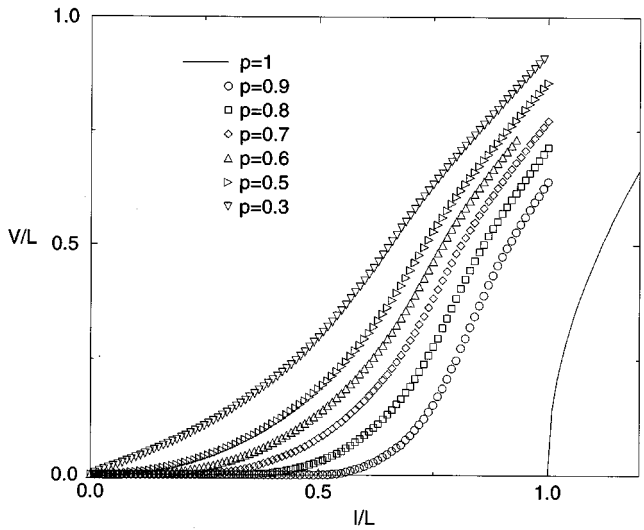

FIG. 1. Current-voltage characteristics for a $L=64$ array, at zero temperature, as a function of the Josephson-coupling concentration p.

only for $p<p_c$ and also appears to vanish at p_c. Unfortunately, closer to p_c our data is not accurate enough to test the expected power-law behavior[7,8] for the critical current $J_c(p)=c(p-p_c)^{\nu}$, with $\nu=\nu_p(d-1)=\frac{4}{3}$, and similar behavior for the linear resistance $R_L\sim(p_c-p)^s$, with $s\sim1.3$.

The exponent a obtained from a power-law fit to the current-voltage characteristics just above J_c, $E\sim(J-J_c)^a$, as a function of p is indicated in Fig. 2(b). At $p=1$ the system behaves as a single junction and $a=\frac{1}{2}$ exactly[21] which agrees with the numerical simulation. In presence of disorder for decreasing p, this exponent jumps to a roughly constant value, $a\sim2.5(2)$, for the largest system size $L=64$. For the smaller system size its p dependence is more significant but we consider this as an effect of small system sizes where the true asymptotic limit has not been reached. The value of a for $L=32$ is in fair agreement with other simulations[6] at a fixed value of $p=0.9$ for comparable system sizes even though the model used differs from ours in that dilution affects both the Josephson coupling and the associated shunt resistance simultaneously. We believe that the difference between the models should not affect the behavior of a far above p_c where there is an infinite cluster of superconducting junctions and only finite clusters of nonsuperconducting junctions. A more accurate estimate of a would require a precise determination of J_c, many averages over disorder and long simulations due to the divergent relaxation time near J_c as discussed below. Despite the uncertainties in the estimate of a, the behavior in Fig. 2(b) for the largest system size suggests that a could be a universal critical exponent independent of the degree of disorder parametrized by p as long as $p_c<p<1$.

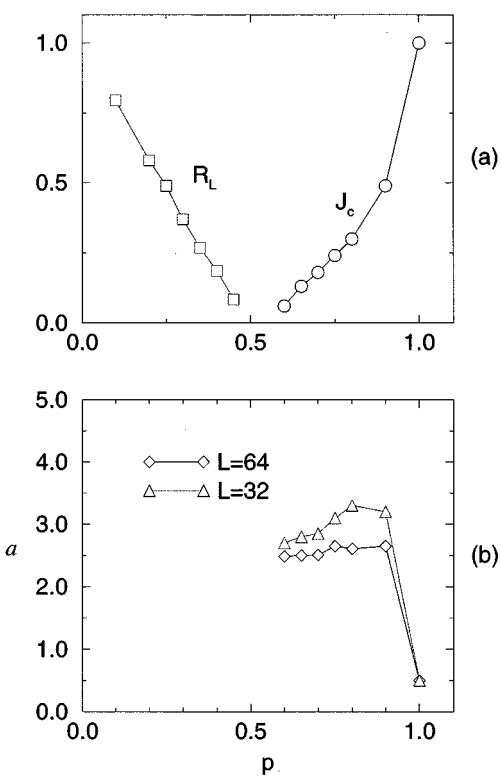

FIG. 2. (a) Critical current J_c and linear resistance R_L as a function of p for $L=64$. (b) Exponent a of the power-law behavior $E\sim(J-J_c)^a$.

At or sufficiently close to p_c, where the percolation correlation length is the dominant length scale, one expects a different behavior. In fact, by matching the scaling of the I-V curves below and above p_c, it has been shown that[8] at p_c the power-law exponent $a=1+s/[\nu(d-1)]$ with $J_c=0$ which gives $a\sim1.98$ in $d=2$. We find $a=2.1(2)$ for $L=64$ [not indicated in Fig. 2(b)] which is consistent with this value.

Some insight into the possible universal behavior for $p>p_c$ can be obtained by regarding the onset of resistive behavior for $J>J_c$ as a dynamical critical phenomenon driven by the external current, where power-law behavior appears naturally as a result of scaling. The required scaling assumptions are similar to those proposed by Fisher *et al.*[14] in a different context. Above the critical current, the superconducting coherence length ξ is finite, leading to resistive behavior. We assume that it diverges as a power law $\xi\sim(J-J_c)^{-\nu_I}$ near J^c, where ν_I is a critical exponent characterizing the current-driven transition. From the definition of the electric field we have $E=-\partial_t A$, where A is the vector potential which enters the Hamiltonian of the JJA in the dimensionless form $\int_r^{r'} A\cdot dr$. The typical time scale is given by the relaxation time which diverges near J_c as $\tau\sim\xi^z$, where z is a dynamic critical exponent. From dimensional analysis we then expect the scaling $E\sim1/(\xi\tau)\sim\xi^{-1-z}$ and a power-law behavior of the current-voltage curve

$$E\sim(J-J_c)^a,\quad a=(z+1)\nu_I, \tag{5}$$

above J_c. Sufficiently close to the percolation threshold, there is another characteristic length scale, $\xi_p\sim(p-p_c)^{-\nu_p}$, the percolation correlation length, and one expects a crossover to another critical behavior.[8] We have only considered the case of a single relevant length scale and focus in the regime $p>p_c$.

The exact values of the critical exponents ν_I and z are not known but the following qualitative estimate of these exponents appears consistent with the numerical results. We ex-

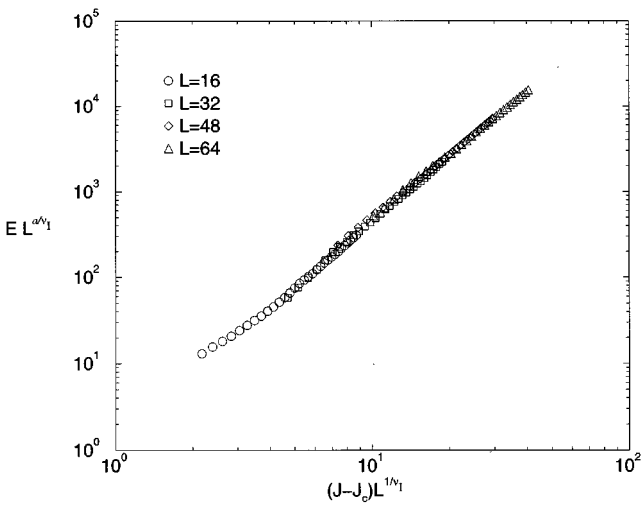

FIG. 3. Finite-size scaling plot of the power-law behavior $E \sim (J-J_c)^a$ for $p=0.7$ at zero temperature, using $J_c=0.19$, $\nu_I=0.9$, and $a=2.5$.

pect that at a characteristic current density $J>J_c$, phase coherence can change significantly in a correlation volume of order ξ^d. In this volume, the typical variation of the phase is $\sim 2\pi$. By comparing the coupling term of the external current to the phase gradient $\int (J-J_c)\nabla\theta$ which appears in the continuous version of the JJA Hamiltonian and the quadratic approximation to the Josephson energy term $K^*\int(\nabla\theta)^2$, where K^* is an effective stiffness, one finds that $(J-J_c)$ should scale as $(J-J_c)\sim 1/\xi$ and so $\nu_I=1$. Thus, the only remaining parameter is the dynamical exponent z. If we neglect nonlinearities and assume relaxation dynamics as in time-dependent Ginzburg-Landau models, we expect $z=2$, and therefore $a=(z+1)\nu_I=3$. On the other hand, recent work[22] suggests that for the dynamics described by Eq. (3), where there is current conservation at each lattice site, the dynamical exponent is $z\sim 0.9$ and so $a=1.9$. The data in Fig. 2(b) for the largest system size $L=64$ is intermediate between these two values. Since in Eq. (5) a depends both on ν_I and z, we need additional results to test this analysis.

We have performed afinite-size scaling analysis at a fixed value of $p=0.7$ to verify the scaling behavior of Eq. (5) and extract an independent numerical estimate of ν_I. In a finite system, the correlation length is limited by the system size L and finite-size scaling leads to

$$EL^{a/\nu_I}=f((J-J_c)L^{1/\nu_I}), \tag{6}$$

where f is a scaling function. From Eq. (6), all data in the scaling plot $EL^{a/\nu_I}\times(J-J_c)L^{1/\nu_I}$ should collapse on to the same curve if ν and a are chosen correctly as shown in Fig. 3 for system sizes ranging from $L=16$ to 64. We find that a reasonable scaling behavior is obtained for $J_c\sim 0.19(2)$, $\nu_I\sim 1.1(3)$, and $a\sim 2.4(2)$ where the error estimates are obtained by averaging various estimates of J_c, ν_I, and a which give equally acceptable scaling plots. Using the relation in Eq. (5), we find $z=1.2(6)$. The value of ν_I is in agreement within the errors with the one predicted above but z is not accurate enough to allow any comparison. Improved estimates and further analytical work are necessary for a detailed study of the critical behavior. The close analogy to other critical phenomena near threshold[23] may provide an interesting approach and eventually identify the relevant universality class.

It is interesting to note that if we assume $\nu_I=1$ in Eq. (5), the result $a=z+1$ is similar to the one inferred by Prester[12] based on an analogy between the onset of dissipation at the critical current and the resistance of a random resistor network which leads to $a=t+1$, where t is the conductivity exponent of a mixture of resistors and insulators. This would suggest $z=t$. In two dimensions, where[17] $t\sim 1.3$, this gives $z=1.3$ and $a=2.3$ which is in fact consistent with our numerical estimates.

We now turn to finite-temperature effects. We have only studied the behavior at percolation threshold where the superconducting transition is known to occur[17] at $T=0$. Again, this is the simplest case where there is a single dominant length scale in the system which at finite temperature is the thermal correlation length ξ_T. This correlation length diverges for decreasing temperature as $\xi_T\propto 1/T^{\nu_T}$ where ν_I is the thermal correlation length exponent of the diluted XY model which is isomorphic to the JJA at zero current. One can anticipate that the increasing correlation length will have important effects in the nonlinear resistance for decreasing temperatures.[14,18] At any $T>0$, the linear resistance, $R_L=\lim_{J\to 0}E/J$, is nonzero since the superconducting correlation length is finite and is expected to be thermally activated. However, in the presence of a finite current density J, an additional length scale $L_I\propto kT/J$ is set by the external current[14] due to temperature fluctuations. This can be obtained by comparing the extra energy arising from the coupling to the external current, $JL_I\Delta\theta$ within a length scale L_I and for a typical phase variation $\Delta\theta\sim 2\pi$, to the thermal energy kT. For $\xi_T<L_I$, which holds for sufficiently small J at any finite T, the linear resistance is basically unchanged since the smaller length scale ξ_T dominates the activation energy. However, for current densities larger than $J_{\rm nl}\propto T^{1+\nu_T}$, nonlinear behavior sets in as L_I/ξ_T in this case. So, the range in J where E/J is roughly a constant should decrease with temperature and the characteristic current density $J_{\rm nl}$ where it crosses over to nonlinear behavior decreases as a power law $T^{1+\nu_T}$ with a universal exponent. Associated with the divergent correlation length ξ_T one also defines a relaxation time τ that, owing to the zero-temperature transition, does not follow the usual form $\tau\propto\xi^z$ and can have an exponential temperature dependence. Since the electric field scales as $E\sim 1/(\xi\tau)$, the current density as $J\sim kT/\xi$, and using $\xi=1/T^{\nu_T}$ the nonlinear resistance behaves as[14]

$$\frac{E}{J}=\frac{1}{\tau T}\,g\left(\frac{J}{T^{1+\nu_T}}\right) \tag{7}$$

in two dimensions, where g is a scaling function. If the linear resistance R_L is finite for any $T>0$ then $g(0)$ is a constant which can be set to unity, $g(0)=1$. When the nonlinear resistance E/J is normalized by its linear value R_L at the same temperature one can then write

$$\frac{E}{JR_L}=g\left(\frac{J}{T^{\nu_T+1}}\right). \tag{8}$$

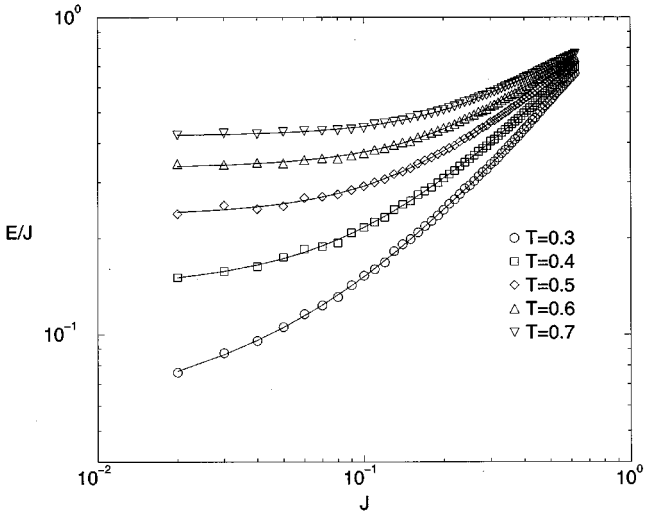

FIG. 4. Nonlinear resistance E/J as a function of temperature for $L=64$ at $p=0.5$. Continuous lines are a guide to the eyes.

It is clear from Eq. (8) that the characteristic current at which nonlinear behavior is expected to set in varies as $T^{1+\nu_T}$ as mentioned before.

The nonlinear resistance E/J obtained numerically at $p=p_c$ for the largest system size $L=64$ is shown in Fig. 4. The curves shows the expected behavior. For small current densities J, there is a linear contribution where E/J tends to a finite value which depends on the temperature. This is more clear for the highest temperature $T=0.7$ where the range of J in which E/J is roughly a constant is more pronounced. For increasing current densities it crosses over to a nonlinear behavior. As temperature decreases nonlinearity appears at smaller currents and the linear behavior is less clear. For the lowest temperature $T=0.3$ the linear behavior presumably occurs at current densities smaller than $J=0.02$ but numerical calculations in this range require very long equilibration times which prevent us from confirming this behavior. In fact, as discussed below, the relaxation time τ increases very rapidly (possibly exponentially) with decreasing temperatures.

The temperature dependence of the linear resistance estimated at the lowest current is indicated in Fig. 5(a). It seems to be consistent with an activated behavior with an energy barrier $E_b \sim 0.91(3)$. Our data are not accurate enough and the temperature range is too limited to exclude more complicated behavior as a temperature-dependent $E_b(T)$. The links-nodes picture of the percolation cluster[17] might suggest a single-junction behavior which in fact gives an Arrhenius behavior at sufficiently higher temperatures[21] but this would give a much larger barrier of $E_b=2$ and requires that the Josephson coupling of the effective single junction is not renormalized. In addition, the scaling form in Eq. (8) which is found in our case as discussed below, does not hold for the single junction as can be verified from the closed-form solution of the current-voltage relation.[21] It is unclear at the moment what is the appropriate model to describe the temperature dependence of the R_L. In any case, if the apparent exponential behavior of R_L holds down to very low temperatures it implies, from Eq. (7), that the relaxation time diverges exponentially, $\tau \sim \exp(E_b/T)/T$ and so the current-voltage characteristics at very low temperatures and currents may be inaccessible by direct simulation.

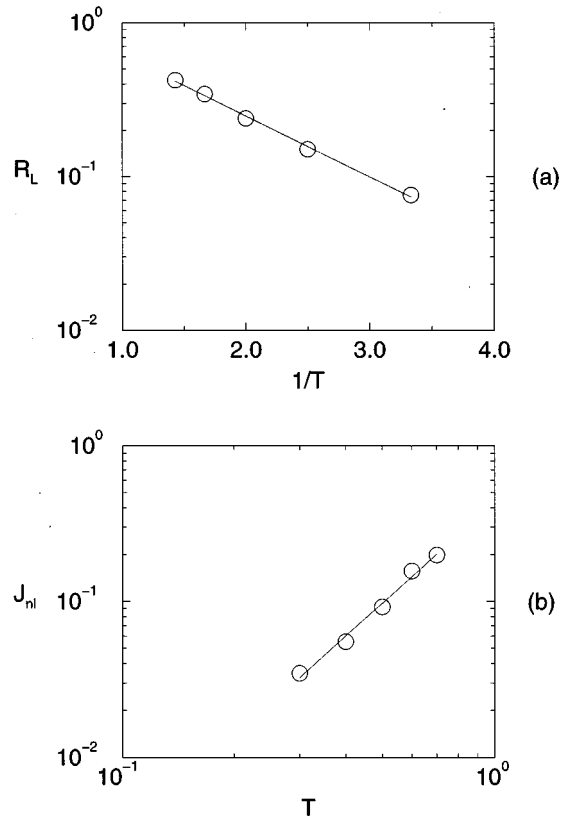

FIG. 5. (a) Arrhenius plot for the temperature dependence of the linear resistance R_L, estimated at $J=0.02$, for $L=64$ at $p=0.5$. (b) Crossover current density J_{nl} where nonlinear behavior appears as a function of temperature. The slope gives an estimate of $1+\nu_T=2.2(2)$.

In Fig. 5(b), we show the temperature dependence of J_{nl} for the data in Fig. 4 in a log-log plot. It is consistent with the power-law behavior $J_{nl} \propto T^{1+\nu_T}$ and provides a direct estimate of the thermal exponent $\nu_T=1.2(2)$. To estimate J_{nl} we defined the crossover to nonlinear behavior as the value of J where E/JR_L starts to deviate from a fixed value equal to 1.2. The slope in the plot of Fig. 5(b) should not depend on this value as long as it is not too large compared to unity and we checked that other choices give the same results within the error estimates. In Fig. 6, we show the scaling plot of the data in Fig. 4 according to the scaling behavior of Eq. (8). The scaling plot is obtained by adjusting a single parameter ν_T that gives the best data collapse. This is consistent with the scaling behavior discussed above and gives an independent estimate of $\nu_T \sim 1.4(2)$. From these results we obtain a final estimate of the thermal critical exponent $\nu_T=1.3(3)$. This value of ν_T is in fact consistent with the thermal correlation exponent of the diluted XY model at percolation threshold,[17] $\nu=0.98-1.03$.

Our analysis for the temperature effects on the nonlinear resistance is strictly valid at p_c, where the percolation correlation length ξ_p is infinite but the thermal correlation length ξ_T is finite. In order to compare to the available experimental data of Harris *et al.*[1] on artificial arrays for p close to percolation threshold, which is in fact consistent

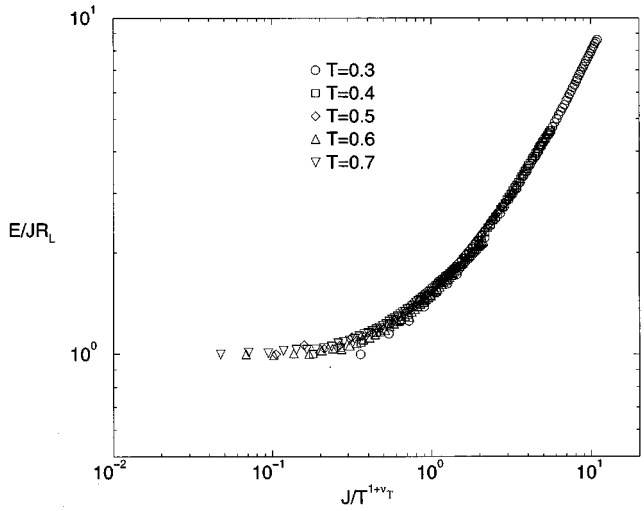

FIG. 6. Scaling plot of the data in Fig. 4 for $\nu_T = 1.4$.

with a vanishing transition temperature, we must take into account the competing effects of ξ_T and ξ_p which is a more complicated problem. However, sufficiently close to p_c the analysis should still be valid at high temperatures when $\xi_T \ll \xi_p$. Unfortunately, the scattering of the data at small currents and the limited range of temperatures where linear resistive behavior is apparent prevent us from performing the same scaling analysis as described above.

IV. CONCLUSION

In summary, we have studied the I-V characteristics in a model of resistively shunted two-dimensional diluted JJA, at zero external field, by numerical simulations and scaling arguments. At $T=0$, the I-V curves show power-law behavior above a critical current density which decreases with dilution. The power-law behavior follows from a simple scaling analysis which leads to $a=(z+1)\nu_I$, where z is the dynamical exponent and ν_I is the superconducting correlation length exponent. Numerically we find $\nu_I = 1.1(3)$, consistent with a scaling argument which gives $\nu_I = 1$, and $a = 2.4(2)$. The value of a is in agreement with the relation $a = t+1$, in two dimensions, which has been proposed[12] in relation to granular high-T_c materials in zero field. At the percolation threshold and finite T, the results are consistent with the scaling behavior of a $T=0$ superconducting transition. Crossover to nonlinear behavior appears at currents proportional to $T^{1+\nu_T}$, where ν_T is a correlation length exponent for the diluted XY model at percolation threshold. The behavior at percolation threshold is analogous to the zero-temperature vortex glass of disordered superconductors in a magnetic field, except for the value of ν_T. For experiments in arrays and granular high-T_c materials, this behavior clearly demonstrates the importance of carefully comparing the expected power-law behavior of I-V characteristics resulting from field-induced effects to the zero-field case.

ACKNOWLEDGMENTS

We thank S. Shenoy and M. Prester for helpful discussions. The authors acknowledge the support from the International Center for Theoretical Physics, where part of the work was done.

[1] D. C. Harris, S. T. Herbert, D. Stroud, and J. C. Garland, Phys. Rev. Lett. **25**, 3606 (1991).
[2] A.-L. Eichenberger, J. Affolter, M. Willemin, M. Mombelli, H. Beck, P. Martinoli, and S. E. Korshunov, Phys. Rev. Lett. **77**, 3905 (1996).
[3] S. John and T. C. Lubensky, Phys. Rev. Lett. **55**, 1014 (1985); M. Benakli, E. Granato, S. R. Shenoy, and M. Gabay (unpublished).
[4] R. M. Bradley, D. Kung, S. Doniach, and P. N. Strenski, J. Phys. A **20**, L911 (1987).
[5] P. L. Leath and W. Tang, Phys. Rev. B **39**, 6485 (1989).
[6] W. Xia and P. L. Leath, Phys. Rev. Lett. **25**, 1428 (1989); P. L. Leath and W. Xia, Phys. Rev. B **44**, 9619 (1991).
[7] C. J. Lobb, P. M. Hui, and D. Stroud, Phys. Rev. B **36**, 1956 (1987).
[8] Ki H. Lee and D. Stroud, Phys. Rev. B **45**, 2417 (1992).
[9] Y. H. Li and S. Teitel, Phys. Rev. Lett. **67**, 2894 (1991).
[10] X. C. Zeng, D. Stroud, and J. S. Chung, Phys. Rev. B **43**, 3042 (1991).
[11] L. B. Kiss and P. Svedlindh, Phys. Rev. Lett. **71**, 2817 (1993).
[12] M. Prester, Phys. Rev. B **54**, 606 (1996).
[13] J. Simonin and A. López, Phys. Rev. Lett. **56**, 2649 (1986); J. M. Gordon, A. M. Goldman, and B. Whitehead, *ibid.* **59**, 2311 (1987); R. G. Steinmann and B. Pannetier, Europhys. Lett. **5**, 559 (1988).
[14] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991); R. A. Hyman, M. Wallin, M. P. A. Fisher, S. M. Girvin, and A. P. Young, *ibid.* **51**, 15 304 (1995).
[15] T. Onogi, R. Sugano, and Y. Murayama, Solid State Commun. **78**, 103 (1991).
[16] C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1993).
[17] R. B. Stinchcombe, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, London, 1983), Vol. 7; D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).
[18] Similar behavior is found for otherwise unrelated problems such as, an *ordered* JJA at irrational frustration [E. Granato, Phys. Rev. B **54**, R9655 (1996)] and vortex-glass models in two dimensions (Ref. 14). The common feature leading to the similar behavior is a divergent correlation $\xi \sim T^{-\nu}$.
[19] D. Domínguez, Phys. Rev. Lett. **72**, 3096 (1994).
[20] D. Domínguez, J. V. José, A. Karma, and C. Wiecko, Phys. Rev. Lett. **67**, 2367 (1991).
[21] A. R. Bishop and S. E. Trullinger, Phys. Rev. B **17**, 2175 (1978).
[22] P. H. E. Tiesinga, T. J. Hagenaars, J. E. van Himbergen, and J. V. José, Phys. Rev. Lett. **78**, 519 (1997).
[23] O. Narayan and D. S. Fisher, Phys. Rev. B **49**, 9469 (1994); D. Cule and T. Hwa, Phys. Rev. Lett. **77**, 278 (1996).