Electronic spin resonance of a weakly interacting Bose gas

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Study of the physical properties of Bose condensed gases has recently become a very active topic due to the experimental realizations of Bose-Einstein condensation in magnetically trapped gases of alkali atoms. In this paper we theoretically study the microwave electronic spin resonance of a weakly interacting Bose gas. For a Bose gas above the critical temperature for Bose-Einstein condensation, the microwave radiation induced by the driving microwave field in the electronic spin resonance exhibits the ordinary Rabi oscillation damped by the inhomogeneous Doppler dephasing. For a Bose gas with a condensate, the induced radiation is composed of two components. One is the strong coherent Rabi oscillation from the condensate. The other is a modulated Rabi oscillation due to the noncondensed part of the gas. We show that the modulation of Rabi oscillation of the noncondensed gas is directly related to the elementary collective excitation of atoms in the gas. [S0163-1829(97)02246-7]

I. INTRODUCTION

In recent years, the study of different physical properties exhibited by an ultracold atomic gas in the presence of an electromagnetic field has been one of the most active research topics in quantum optics and atom optics. The experimental success with generating Bose-Einstein condensates of atoms in magnetic traps[1–6] is offering a new challenge in this area. Current research focuses on several aspects for different purposes. Studies of weak light scattering from Bose condensed gases are stimulated by seeking for optical signatures of Bose-Einstein condensates.[7–15] Theoretical studies towards generation and applications of ultracold atomic samples and coherent atomic sources such as atom lasers in atom optics started before the realizations of Bose-Einstein condensation (BEC) in magnetic traps[16–22] and are attracting strong interest both theoretically and experimentally.[23–30] For example, very recently the MIT group has demonstrated an output coupler for a trapped Bose-Einstein condensate which could be regarded as a rudimentary version of a pulsed atom laser.[23] The other aspects involve understanding the BEC in atomic traps and collective quantum dynamics of the trapped Bose gas.[5,6,31–37] In this paper we propose to study the Rabi oscillation[38] of a Bose gas in the regime of electronic spin resonance (ESR). To observe the Rabi oscillation in the microwave ESR, a strong stationary microwave field is used to excite the electronic spin-flip transitions of atoms in the Bose gas. In the ESR regime, the heating and loss of atoms due to incoherent spontaneous emission can be avoided since the electron-spin magnetic dipole transitions have extremely long spontaneous-emission lifetimes. But for a magnetically trapped atomic gas, the spin-flip transitions can lead to a loss of atoms due to the spin-dependent trapping potential.[39] Such a loss limits the observation of Rabi oscillation in the ESR regime for atomic gases confined by spin-dependent traps. However, the difficulty can technically be overcome by confining atoms in other spin-independent traps before applying the microwave field. For example, we can employ the techniques developed in atom optics to confine atoms in a U-tube hollow fiber cavity[40,41] or a gravity cavity.[42]

In this paper, to simplify our analysis, we are not concerned with the techniques for trapping atoms. We consider a homogeneous, weakly interacting Bose gas in the thermodynamic limit as our example system. Furthermore the Bose gas discussed here is not limited to an ensemble composed of alkali atoms. It may include other ensembles such as an exciton gas in semiconductors[43,44] or even superfluid helium.[45] The latter cases are closer to the homogeneous Bose gas.

This paper is organized as follows: in Sec. II we briefly introduce the vector quantum field theory for a two-state spin system which we have developed for the electronic spin echo.[46] In Sec. III, we study the magnetic resonance coupling of a weakly interacting Bose gas to a stationary microwave field through the transition between the ground-state Zeeman sublevels. The induced microwave signal by the driving microwave field is calculated in terms of the quantum field equation given in Sec. II. The Rabi oscillation of the signal is displayed and the dependence of the signal on the temperature of the gas is analyzed. The difference between the Rabi oscillation of the signal from a Bose gas with a condensate and that without a condensate is discussed. The conclusions are drawn in Sec. IV.

II. VECTOR QUANTUM FIELD THEORY

A sample containing magnetic dipoles might be expected to interact with the magnetic component of a microwave radiation in the presence of a static magnetic field. The static magnetic field leads to different Zeeman levels in terms of the magnetic quantum numbers. The microwave radiation can induce transitions between these Zeeman levels. In terms of the ESR selection rules, for a single-electron system with spin $S=1/2$, only the spin-flip transitions between two Zeeman levels is involved. As an example, we consider the ground state $^2S_{1/2}$ of alkali atoms with total nuclear spin $I=3/2$. The static magnetic field prepares the atoms in the doubly spin-polarized state $|\uparrow_e\uparrow_n\rangle=|M_s=1/2,M_I=3/2\rangle$. Hence only the transition between the states $|\uparrow_e\uparrow_n\rangle$

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$=|M_s=1/2,M_I=3/2\rangle$ and $|\downarrow_e\uparrow_n\rangle=|M_s=-1/2,M_I=3/2\rangle$ can occur in the ESR regime. Similarly for an exciton gas, we are only concerned with two electronic spin states $|\uparrow_e\rangle=|M_s=1/2\rangle$ and $|\downarrow_e\rangle=|M_s=-1/2\rangle$. In this paper, we will limit our discussion to a Bose gases with only two internal Zeeman levels. In this case, the Bose gas can be treated as a two-component vector quantum field,[46]

$$\psi(\vec{r},t)=\psi_1(\vec{r},t)|\downarrow_e\uparrow_n\rangle+\psi_2(\vec{r},t)|\uparrow_e\uparrow_n\rangle. \tag{2.1}$$

Including the two-body interaction, the quantum field equations for the two-state field components have the forms[46]

$$\begin{aligned} i\hbar\frac{\partial\psi_1}{\partial t}&=\left\{-\frac{\hbar^2\nabla^2}{2m}-\mu\right\}\psi_1-\frac{\hbar\Omega}{2}e^{-i\vec{k}_H\cdot\vec{r}}\psi_2\\ &\quad+\hbar\chi_T(2\psi_1^\dagger\psi_1+\psi_2^\dagger\psi_2)\psi_1,\\ i\hbar\frac{\partial\psi_2}{\partial t}&=\left\{-\frac{\hbar^2\nabla^2}{2m}-\mu-\hbar\Delta\right\}\psi_2\\ &\quad-\frac{\hbar\Omega}{2}e^{i\vec{k}_H\cdot\vec{r}}\psi_1+\hbar\chi_T(2\psi_2^\dagger\psi_2+\psi_1^\dagger\psi_1)\psi_2, \end{aligned} \tag{2.2}$$

where $\vec{k}_H$ is the wave vector of the microwave field and μ the chemical potential of the gas. Equation (2.2) has been transformed into the rotating frame with the frequency ω_0 of the microwave field $\vec{H}_1(t)=2\vec{B}\cos(\omega_0 t)$. The parameter $\Delta=\omega_0-\omega_a$ denotes the detuning of the microwave field frequency from the atomic Lamor frequency and the resonance condition $\Delta=0$ is considered in this paper. The Rabi frequency $\Omega=2\vec{\mu}_M\cdot\vec{B}(t)/\hbar$ describes the electronic spin-flip transition between Zeeman levels. The nonlinear terms in equation (2.2) arise from interparticle interactions in the gas. For a Bose gas composed of alkali atoms, these terms correspond to the triplet state collisions between atoms. For an exciton gas, they phenomenologically describe the interaction between excitons. The nonlinear coefficient is defined as $\chi_T=4\pi\hbar a_T/m$ with a_T denoting the s-wave scattering length. The factor of 2 in the nonlinear terms originates from the Bose enhancement of collisions between atoms in the same states.

III. RABI OSCILLATION OF A BOSE GAS

To study the magnetic resonance dynamics of the Bose gas driven by a stationary microwave field, we solve the nonlinear vector quantum field equations (2.2) at a different temperature of the gas. Above the critical temperature T_c for BEC, the induced microwave radiation from the gas is affected by two types of broadening mechanisms. One is the random thermal motion of the individual atom which results in an inhomogeneous Doppler broadening through the sample. The other is the collisional relaxation due to the nonlinear terms in Eq. (2.2). At high temperatures, the collisional relaxation is negligible compared to the Doppler broadening. At low temperatures, both the Doppler broadening and the collisional relaxation can be neglected. So the collisional relaxation can be excluded in our discussions for temperatures above the critical temperature. In this case,

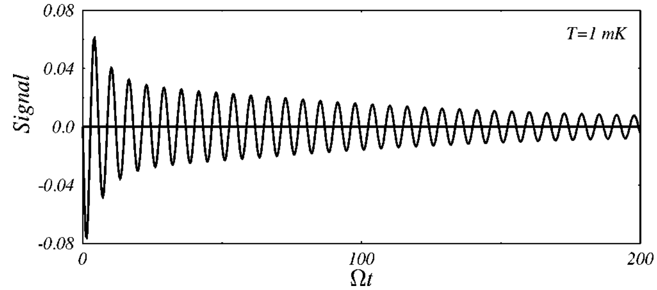

FIG. 1. The microwave induced electron-spin-resonance Rabi oscillation of a Bose gas above the critical temperature for BEC. The temperature for the gas is chosen to be $T=1$ mK.

Eq. (2.2) becomes a linear vector field equation and its solution can easily be obtained in the momentum space by the following Fourier transformation:

$$\begin{aligned} \psi_1(\vec{r},t)&=\frac{1}{\sqrt{V}}\sum_{\vec{k}} a_{\vec{k}}(t)\exp[i(\vec{k}-\vec{k}_H)\cdot\vec{r}],\\ \psi_2(\vec{r},t)&=\frac{1}{\sqrt{V}}\sum_{\vec{k}} b_{\vec{k}}(t)\exp(i\vec{k}\cdot\vec{r}), \end{aligned} \tag{3.1}$$

where V is the volume occupied by the gas. Substituting equation (3.1) into equation (2.2), we obtain the expression

$$\frac{\partial}{\partial t}\begin{pmatrix} a_{\vec{k}}(t)\\ b_{\vec{k}}(t)\end{pmatrix}=\begin{pmatrix} -i(\omega_{\vec{k}}-\delta_{\vec{k}}) & i\Omega/2\\ i\Omega/2 & -i\omega_{\vec{k}}\end{pmatrix}\begin{pmatrix} a_{\vec{k}}(t)\\ b_{\vec{k}}(t)\end{pmatrix}, \tag{3.2}$$

where $\hbar\omega_{\vec{k}}=\hbar^2\vec{k}^2/2m$ is the kinetic energy of the gas, and $\delta_{\vec{k}}=\hbar\vec{k}\cdot\vec{k}_H/m$ the Doppler shift due to the center of mass motion of the atoms in the gas. In terms of Eqs. (3.1) and (3.2), the signal induced by the driving microwave field is given by

$$\begin{aligned} \vec{H}_e(t)&=\vec{\mu}_M\langle\psi_1^\dagger(\vec{r},t)\psi_2(\vec{r},t)\rangle\exp(-i\omega_0 t)+\text{c.c.}\\ &=\frac{\vec{\mu}_M}{2}\exp(i\vec{k}_H\cdot\vec{r}-i\omega_0 t-i\pi/2)\\ &\quad\times\frac{1}{V}\sum_{\vec{k}} N_{\vec{k}}\frac{\Omega}{\sqrt{\Omega^2+\delta_{\vec{k}}^2}}\sin(\sqrt{\Omega^2+\delta_{\vec{k}}^2}t). \end{aligned} \tag{3.3}$$

We numerically calculate the summation in the momentum space. The atomic number $N_{\vec{k}}=1/\{\exp[(E_k-\mu)/kT]-1\}$ is chosen to be a Bose-Einstein distribution. The signals are shown in Figs. 1 and 2. In Fig. 1, we see a damped oscillation with the Rabi frequency. The damping is caused by the inhomogeneous Doppler broadening which depends on the temperature of the gas. The damping rate is determined by the Doppler width $\delta_D=2\sqrt{\ln 2}k_H\sqrt{2k_BT/m}$. With the decrease of the temperature, the damping rate decreases. As a result, we see a Rabi oscillation with almost equal amplitude at the low temperature as shown in Fig. 2.

Now we consider Rabi oscillation of the Bose gas below the critical temperature where a condensate exists. When there is a condensate, Eq. (2.2) can be solved by separating the condensate component and noncondensed component in terms of the generalized Bogoliubov transformation for the vector quantum fields,

$$\psi_1(\vec{r},t)=\phi_1(t)\exp(-i\vec{k}_H\cdot\vec{r}) + \frac{1}{\sqrt{V}}\sum_{\vec{k}\neq 0} a_{\vec{k}}(t)\exp[i(\vec{k}-\vec{k}_H)\cdot\vec{r}],$$

$$\psi_2(\vec{r},t)=\phi_2(t)+\frac{1}{\sqrt{V}}\sum_{\vec{k}\neq 0} b_{\vec{k}}(t)\exp(i\vec{k}\cdot\vec{r}), \qquad (3.4)$$

where ϕ_1 and ϕ_2 describe the condensates corresponding to two different electronic spin states. The operators $a_{\vec{k}}$ and $b_{\vec{k}}$ describe the corresponding noncondensed component in momentum space. Substituting Eq. (3.4) into Eq. (2.2), we obtain the following equation for the condensates:

$$i\frac{\partial}{\partial t}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}=\begin{pmatrix}\chi_T(\rho_0+|\phi_1|^2) & -(\Omega/2)\exp(-i\vec{k}_H\cdot\vec{r})\\ -(\Omega/2)\exp(i\vec{k}_H\cdot\vec{r}) & \chi_T(\rho_0+|\phi_2|^2)\end{pmatrix} \times\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}. \qquad (3.5)$$

In deriving Eq. (3.5), we assume a homogeneously extended condensate and neglect the collision-induced correction from the noncondensed components if the condensate is big enough. The total density $\rho_0=|\phi_1|^2+|\phi_2|^2$ for the condensate is a conserved quantity in terms of Eq. (3.5). For a stationary driving microwave field with an amplitude $\Omega \gg \chi_T|\phi_i|^2$, Eq. (3.5) has the following approximate Rabi oscillation solutions if the initial condensate is prepared in the doubly spin-polarized state:

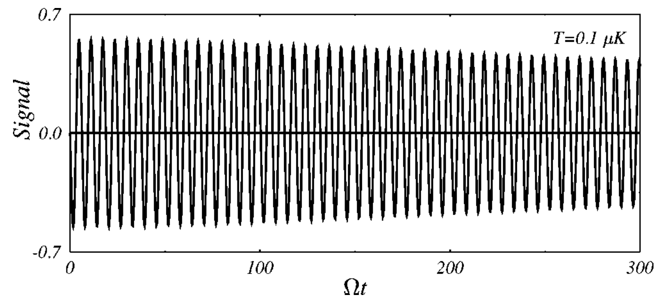

FIG. 2. The microwave induced electron-spin-resonance Rabi oscillation of a Bose gas above the critical temperature for BEC. The temperature for the gas is chosen to be $T=0.1$ μK.

$$\phi_1(t)\approx i\sqrt{\rho_0}\sin\left(\frac{\Omega t}{2}\right)\exp(-i\vec{k}_H\cdot\vec{r}-i\chi_T\rho_0 t+i\theta_0),$$

$$\phi_2(t)\approx\sqrt{\rho_0}\cos\left(\frac{\Omega t}{2}\right)\exp(-i\chi_T\rho_0 t+i\theta_0), \qquad (3.6)$$

where θ_0 is the phase of the initial condensate. In the approximations made above, the condensate exhibits a Rabi oscillation which is the same as in a single-atom case. This result agrees with the recent experiment for an output coupler[23] of a trapped condensate.

On the other hand, in the low-temperature Bose condensed gas, except the condensate, a noncondensed part always exists. The noncondensed part is due to the collective excitation and the incompletely condensed factor. In terms of Eqs. (3.4) and (2.2), the noncondensed part in the driving microwave field is described by the following equation in the rotating frame with frequency ω_c:

$$\frac{\partial}{\partial t}\begin{pmatrix}a_{\vec{k}}(t)\\ b_{\vec{k}}(t)\\ a^{\dagger}_{-\vec{k}}(t)\\ b^{\dagger}_{-\vec{k}}(t)\end{pmatrix}=\begin{pmatrix}-i\omega_- & i\Omega^*_{\rm eff}(t)/2 & 2i\omega_c\sin^2\left(\frac{\Omega t}{2}\right) & \frac{\omega_c}{2}\sin(\Omega t)\\ i\Omega_{\rm eff}(t)/2 & -i\omega_1 & \frac{\omega_c}{2}\sin(\Omega t) & -2i\omega_c\cos^2\left(\frac{\Omega t}{2}\right)\\ -2i\omega_c\sin^2\left(\frac{\Omega t}{2}\right) & \frac{\omega_c}{2}\sin(\Omega t) & i\omega_+ & -i\Omega^*_{\rm eff}(t)/2\\ \frac{\omega_c}{2}\sin(\Omega t) & 2i\omega_c\cos^2\left(\frac{\Omega t}{2}\right) & -i\Omega^*_{\rm eff}(t)/2 & i\omega_1\end{pmatrix}\begin{pmatrix}a_{\vec{k}}(t)\\ b_{\vec{k}}(t)\\ a^{\dagger}_{-\vec{k}}(t)\\ b^{\dagger}_{-\vec{k}}(t)\end{pmatrix}, \qquad (3.7)$$

where $\omega_{\pm}=\omega_{\vec{k}}\pm\delta_{\vec{k}}+3\omega_c\sin^2(\Omega t/2)$, $\omega_1=\omega_{\vec{k}}+3\omega_c\cos^2(\Omega t/2)$, $\omega_c=\chi_T\rho_0$ is the collective excitation frequency, and $\Omega_{\rm eff}(t)=\Omega+i\omega_c\sin(\Omega t)$ is the effective Rabi frequency. From Eq. (3.7) we see several interesting effects. For the noncondensed component, a time-dependent correction to the Rabi frequency is induced via the interatomic interaction. The induced term is proportional to the strength of the oscillation field emitted by the driven condensate in terms of Eq. (3.6). This means that the noncondensed component can ''see'' the Rabi oscillation of the driven condensate through the interatomic interaction. In addition, the nonlinear interatomic interaction induces collective excitations which result in the coupled terms between the operator pairs $(a_{\vec{k}},b_{\vec{k}})$ and $(a^{\dagger}_{-\vec{k}},b^{\dagger}_{-\vec{k}})$. Being different from those discussed in spin echo,[46] the collective excitations shown in Eq. (3.7) are time dependent. The time dependence is due to the Rabi oscillation of the condensate driven by a stationary microwave field. Neglecting Doppler effects at the low temperatures, Eq. (3.7) can be solved analytically. This gives a signal emitted by the noncondensed component,

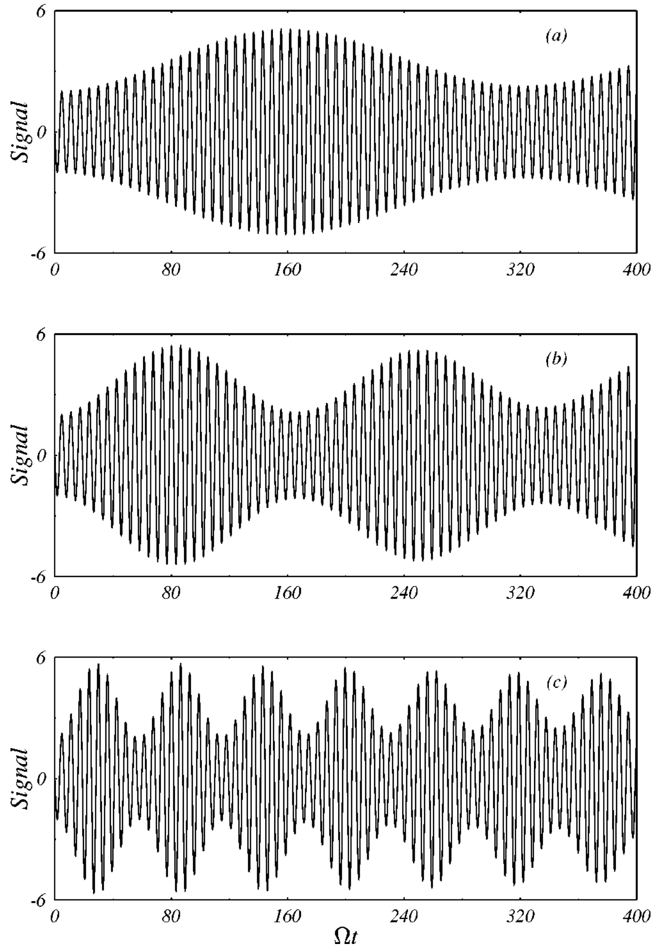

FIG. 3. The microwave induced electron-spin-resonance Rabi oscillation of a Bose gas below the critical temperature for BEC. The parameters chosen for the simulation are the temperature $T=50$ nK, and the densities for the condensate: (a) $\rho_0=2\times10^{14}$, (b) $\rho_0=4\times10^{14}$, (c) $\rho_0=1.2\times10^{15}$.

$$\Delta\vec{H}_e(t)=\vec{\mu}_M\left(\langle\psi_1^\dagger(\vec{r},t)\psi_2(\vec{r},t)\rangle-\frac{\rho_0}{2}e^{i\vec{k}_H\cdot\vec{r}-i\pi/2}\sin(\Omega t)\right)\times\exp(-i\omega_0 t)+\text{c.c.}$$
$$\approx\frac{\vec{\mu}_M}{2}e^{i\vec{k}_H\cdot\vec{r}-i\omega_0 t-i\pi/2}\frac{1}{V}\times\sum_{\vec{k}\neq0}\left[N_{\vec{k}}+\frac{6N_{\vec{k}}\omega_c^2}{\omega_M^2}\sin^2(\omega_M t)\right]\sin(\Omega t)+\text{c.c.}\tag{3.8}$$

The signal shown in Eq. (3.8) is a Rabi oscillation modulated by an oscillating sine function with a modulation frequency $\omega_M=\sqrt{(\omega_{\vec{k}}+4\frac{1}{4}\omega_c)(\omega_{\vec{k}}+\frac{3}{4}\omega_c)}$. The modulation frequency depends on the kinetic energy of the noncondensed component and the collective excitation frequency ω_c. It reflects the elementary excitation spectrum of the Bose condensed gas. We numerically solve Eq. (3.7). The exactly calculated signal is plotted in Figs. 3(a)–3(c) which clearly shows the modulation of the Rabi oscillation. The collective excitation frequency depends on the density of the condensate ρ_0. The higher the density of the condensate is, the higher the collective excitation frequency. As a numerical example, we use the data from MIT experiment for the $^{23}N_a$ Bose-Einstein condensate:[3] $m=3.8\times10^{-23}g$, $a_T=4.9\times10^{-7}$ cm, $\rho_0=4\times10^{14}$ cm^{-3}, which give the collective excitation frequency $\omega_c=6.8\times10^4$ Hz. The modulated Rabi oscillation corresponding to the $^{23}N_a$ BEC is plotted in Fig. 3(b) with the Rabi frequency chosen to be $\Omega=100\omega_c$. By increasing or decreasing the density of the condensate, one can change the modulation in the Rabi oscillation as shown in Figs. 3(a) and 3(c).

In real observation, the modulation component is usually buried in the total radiation field. However it can be observed and analyzed by microwave spectroscopy techniques. Hence this suggests that the microwave spectroscopy techniques could be employed to detect the elementary collective excitation and other information involving the complicated interatomic interaction in a Bose condensed gas.

IV. CONCLUSIONS

In this paper, we studied the physical properties of a weakly interacting Bose gas driven by a stationary microwave field. The Bose gas was treated as a two-component vector quantum field. We derived the coupled quantum field equations to describe the magnetic resonance interaction of the gas with the microwave field. By solving the coupled quantum field equations, we studied the induced microwave radiation. For a Bose gas above the BEC critical temperature, the induced microwave radiation exhibits the ordinary Rabi oscillation[38] due to the existence of the driving microwave field. For a Bose gas composed of a condensate, the Rabi oscillation exhibits modulated structures. We show that the modulation of Rabi oscillation depends on elementary collective excitation of the Bose condensed gas.

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[1] M. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).

[2] C. C. Bradley, C. A. Sackett, J. J. Tollet, and R. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).

[3] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).

[4] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **77**, 416 (1996).

[5] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 420 (1996).

[6] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **77**, 988 (1996).

[7] B. Svistunov and G. Shlyapnikov, Zh. Éksp. Teor. Fiz. **97**, 821 (1990) [Sov. Phys. JETP **70**, 460 (1990)].

[8] H. D. Politzer, Phys. Rev. A **43**, 6444 (1991).
[9] M. Lewenstein and L. You, Phys. Rev. Lett. **71**, 1339 (1993).
[10] L. You, M. Lewenstein, and J. Cooper, Phys. Rev. A **50**, R3565 (1994).
[11] Olivier Morice, Yvan Castin, and Jean Dalibard, Phys. Rev. A **51**, 3896 (1995).
[12] J. Javanainen, Phys. Rev. Lett. **75**, 1927 (1995).
[13] J. Javanainen and J. Ruostekoski, Phys. Rev. A **52**, 3033 (1995).
[14] R. Graham and D. F. Walls, Phys. Rev. Lett. **76**, 1774 (1996).
[15] A. Imamoglu and T. A. B. Kennedy, Phys. Rev. A **55**, R849 (1997).
[16] Weiping Zhang, Phys. Lett. A **176**, 225 (1993).
[17] G. Lenz, P. Meystre, and E. M. Wright, Phys. Rev. Lett. **71**, 3271 (1993).
[18] Weiping Zhang, D. F. Walls, and Barry Sanders, Phys. Rev. Lett. **72**, 60 (1994).
[19] Weiping Zhang and D. F. Walls, Phys. Rev. A **49**, 3799 (1994).
[20] G. Lenz, P. Meystre, and E. M. Wright, Phys. Rev. A **50**, 1681 (1994).
[21] Weiping Zhang, Pierre Meystre, and E. M. Wright, Phys. Rev. A **52**, 498 (1995).
[22] Weiping Zhang, Aust. J. Phys. **49**, 819 (1996).
[23] M.-O. Mewes, M. R. Andrews, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **78**, 582 (1997).
[24] H. M. Wiseman and M. J. Collett, Phys. Lett. A **202**, 246 (1995).
[25] R. J. C. Spreeuw, T. Pfau, U. Janicke, and M. Wilkens, Europhys. Lett. **32**, 469 (1995).
[26] M. Olshanii, Y. Castin, and J. Dalibard, in *Proceedings of the XIIth Conference on Laser Spectroscopy*, edited by M. Inguscio, M. Allegrini, and A. Sasso (World Scientific, Singapore, 1995).
[27] A. M. Guzman, M. Moore, and P. Meystre, Phys. Rev. A **53**, 977 (1996).
[28] H. M. Wiseman, A.-M. Martins, and D. F. Walls, Quantum Semiclassic. Opt. **8**, 737 (1996).
[29] M. Holland, K. Burnett, C. Gardiner, J. I. Cirac, and P. Zoller, Phys. Rev. A **54**, R1757 (1996).
[30] G. M. Moy, J. J. Hope, and C. M. Savage (unpublished).
[31] G. Baym and C. J. Pethick, Phys. Rev. Lett. **76**, 6 (1996).
[32] Yu Kagan, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **76**, 2670 (1996).
[33] K. G. Singh and D. S. Rokhsar, Phys. Rev. Lett. **77**, 1667 (1996).
[34] Mark Edwards, P. A. Ruprecht, K. Burnett, R. J. Dodd, and C. W. Clark, Phys. Rev. Lett. **77**, 1671 (1996).
[35] E. M. Wright, D. F. Walls, and J. C. Garrison, Phys. Rev. Lett. **77**, 2158 (1996).
[36] G. J. Milburn, E. M. Wright, and D. F. Walls (unpublished).
[37] G. J. Milburn, J. Corney, D. Harris, E. M. Wright, and D. F. Walls, in *Atom Optics*, edited by M. G. Prentiss and W. D. Phillips, SPIE Proc. Vol. 2995 (SPIE, Bellingham, WA, 1997), p. 232.
[38] I. I. Rabi, Phys. Rev. **51**, 652 (1937).
[39] C. Foot, Nature (London) **375**, 447 (1995).
[40] D. J. Harris and C. M. Savage, Phys. Rev. A **51**, 3967 (1995).
[41] K. G. H. Baldwin, Aust. J. Phys. **49**, 855 (1996).
[42] H. Wallis, J. Dalibard, and C. Cohen-Tannoudji, Appl. Phys. B: Photophys. Laser Chem. **54**, 407 (1992).
[43] A. Griffin, D. W. Snoke, and S. Stringari, *Bose-Einstein Condensation* (Cambridge University Press, Cambridge, 1995).
[44] J. L. Lin and J. P. Wolfe, Phys. Rev. Lett. **71**, 1222 (1993).
[45] F. London, *Superfluids* (Wiley, New York, 1954), Vol. 2.
[46] Weiping Zhang and Guo-Qiang Liu, in *Atom Optics* (Ref. 37), p. 240.