PHYSICAL REVIEW B VOLUME 56, NUMBER 3 15 JULY 1997-I

Stability of classical electron orbits in triangular electron billiards

H. Linke, L. Christensson, and P. Omling
Solid State Physics, Nanometer Structure Consortium, Lund University, Box 118, S-221 00 Lund, Sweden

P. E. Lindelof
Niels-Bohr-Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

(Received 26 November 1996)

Using a triangular electron billiard, we investigate in detail which electron trajectories lead to the commensurability effects observed in the magnetoresistance. By comparing the magnetoresistance with simulations of classical electron trajectories, we are able to correlate the maxima of the resistance with specific electron trajectories. The findings are supported by a comparison with data obtained from other devices with different geometries. An explanation why certain trajectories are more important for the resistance than others is obtained from a numerical stability analysis using classical chaos theory. Short trajectories that have a small Liapunov exponent are most important for the resistance. Application of a bias voltage has a strong effect on the magnetoresistance. Here we suggest that voltage-induced small-angle electron-electron scattering acts like a small perturbation of the electron motion, to which chaotic trajectories are most sensitive. [S0163-1829(97)04928-X]

I. INTRODUCTION

The transport properties of electrons that are confined in devices smaller than the electron mean free path, called electron billiards, can to a large degree be explained within a classical, single-particle picture.[1] Electron focusing[2,3] and commensurability oscillations in periodical arrays of antidots[4,5] are well known examples of effects that can be understood in a billiard ball model. In these examples, only one type of electron trajectory needs to be considered for the successful explanation of the main features of the magnetoresistance. In the present paper, we address the question of what makes specific trajectories, among a large number of possible trajectories, important for the magnetoresistance of an electron billiard. We show that an analysis of the stability of the possible trajectories, with the help of classical chaos theory, can explain the magnetoresistance in great detail.

As an example we study a triangular billiard. We relate dominant maxima of the magnetoresistance to specific, simple trajectories that lead to the reflection of electrons. This is done with the help of simulations of classical electron trajectories. The interpretation of the commensurability effects is further supported by a comparison with data obtained from other devices of different geometry that selectively support a certain reflecting trajectory. The question why specific trajectories are of particular importance is answered with the help of numerical calculations of the Liapunov exponent of possible trajectories. We find that trajectories that can be related to maxima of the resistance are consistently less unstable towards small changes in the initial conditions than other, topologically similar trajectories that do not cause a dominant structure in the resistance. The stability of electron trajectories is an important criterion for their importance for the magnetoresistance.

Further, we observe that the commensurability effects are qualitatively changed by a bias voltage. We tentatively explain these changes by the increased electron-electron (e-e) interaction rate of electrons that are injected into the billiard with an energy in excess of the Fermi energy. An increased rate of e-e interaction may be viewed as a small perturbation of the electron orbits because e-e scattering is dominated by scattering in the forward direction. Therefore, trajectories with a small Liapunov exponent, which are relatively insensitive to small changes in the parameters of motion, are less sensitive to the increased e-e interaction rate than unstable orbits. This effect is observed as a splitting of peaks of the magnetoresistance induced by a bias voltage. These peaks consist of an unstable center region and more stable flanks. Apparently, the e-e interaction accentuates classical chaotic properties of the billiard. This result is particularly interesting because Ulloa and Pfannkuche recently predicted that the e-e interaction may induce a transition from regular to quantum chaotic behavior in quantum dots.[6]

The present paper is organized in the following way. In Sec. II we give experimental details. In Sec. III we present data of the magnetoresistance obtained from a triangular billiard and from a number of different test structures that help to interpret the observed commensurability effects. In Sec. IV we relate the experimental data to simulations that support the commensurability model. The relative importance of specific commensurate orbits is explained by their stability as compared to other trajectories in Sec. V. In Sec. VI we show that differences in the stability of different trajectories can be experimentally enhanced by inducing the e-e interaction with the help of a bias voltage. Section VII contains the conclusions.

II. EXPERIMENTAL DETAILS

Most of the electron billiards studied in this work had the shape of an isosceles triangle with contacts at the center of the base and at the tip (inset of Fig. 1). The geometry is described by b, the half length of the base, and the base angle α. More than ten different devices, with a lithographic

 © 1997 The American Physical Society

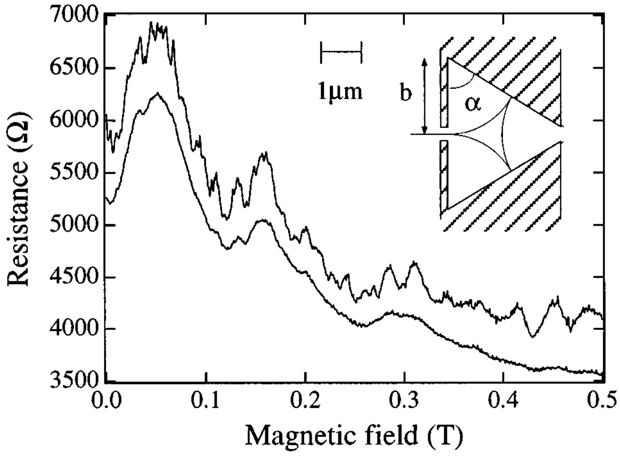

FIG. 1. Magnetoresistance of a triangular billiard (device I) at 0.3 K (upper curve) and at 4.5 K (lower curve). The inset shows the geometry of the billiard, which is described by the half base length b and the base angle α. Also indicated is the electron trajectory that is thought to be related to the absolute maximum of the resistance. The lower curve has been set off by $-500\ \Omega$ for clarity.

b ranging from 1.2 to 2.0 μm and base angles of the order of 60°, have been studied with consistent results. The samples were fabricated by electron-beam lithography and shallow wet etching from modulation-doped GaAs/Al_xGa_{1-x}As two-dimensional electron gas (2DEG) wafer material with a mobility of $\mu=114\ \mathrm{m^2\,V^{-1}\,s^{-1}}$ at 0.3 K. The distance between the devices and the voltage probes exceeded all characteristic lengths of transport in the 2DEG such that true two-point resistances were measured. All resistance measurements were carried out in a current-controlled, four-terminal geometry using the lock-in technique. For some experiments, a dc bias voltage was added to the ac component.

The triangular billiard from which most data presented in detail in this paper were obtained (device I) had a lithographic half base length of 1.5 μm and a lithographic base angle of $\alpha=60°$. Our experimental data are in agreement with an electric half base length $b=1.8\ \mu$m. The carrier concentration, as determined from measurements of the Hall effect in the unstructured 2DEG areas, was $n=3.0\times10^{15}\ \mathrm{m^{-2}}$, giving a Fermi energy $E_F=10$ meV and a mean free path of 10 μm, such that transport inside the device was ballistic. The point contacts had an estimated electric width of 100 nm and the estimated number of subbands in the point contacts was 3.

III. MAGNETORESISTANCE

Figure 1 shows the magnetoresistance of device I at the temperatures $T=0.3$ and at 4.5 K. For the measurement the current $i_{ac}=5$ nA was used and no dc bias was applied. The magnetoresistance was highly symmetric upon reversal of the magnetic field, which indicates good sample quality. The low-temperature data show reproducible fluctuations with an amplitude that corresponds to a conductance change of about 0.1 e^2/h. These conductance fluctuations are due to the interference of electrons inside the billiard.[7] At 4.5 K most of the quantum interference effects are smeared out due to a phase-destructive e-e interaction[8] and only the classical, ballistic

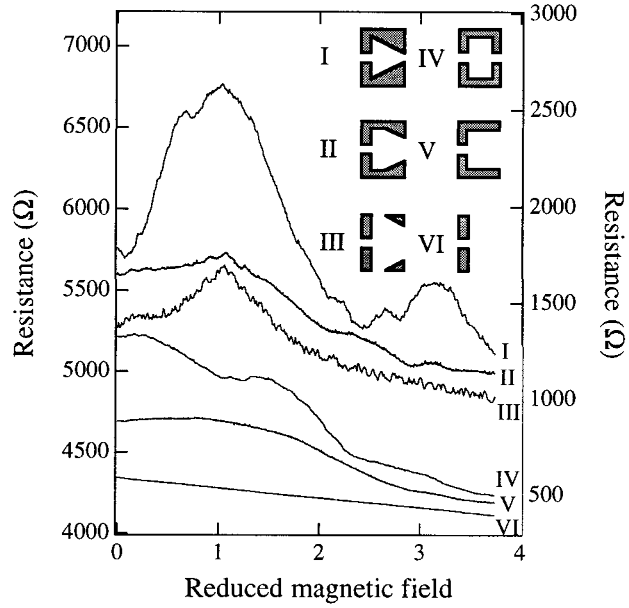

FIG. 2. Magnetoresistance at $T=4.4$ K of six devices with the geometries indicated in the figure. The magnetoresistance of the three devices I–III (left axis) shows an absolute maximum at $B/B_0=1$. Here B_0 is defined as the field at which an electron trajectory like the one in Fig. 1 is geometrically possible in each device. This type of electron trajectory is not possible in devices IV–VI and their magnetoresistance does not show the related peak (right axis). For clarity, curves I–VI have been set off by 0, 600, 1400, 300, 0, and $-600\ \Omega$, respectively.

effects are left. These classical effects are the main subject of this paper.

The dominant ballistic feature is the absolute maximum at $B\approx50$ mT, which we interpret as follows. Electrons passing through a point contact are known to be strongly focused in the forward direction.[9,10] Electrons that have entered the billiard through one contact will therefore, to a high probability, also pass the second point contact and leave the billiard without having collided with the boundaries. At zero magnetic field the exact shape of the billiard is therefore not of vital importance and the resistances of the two point contacts do not add.[11,12] As the magnetic field increases, the electron beam is deflected sideways. The electrons are scattered by the boundary and a certain fraction of the electrons, dependent on the magnetic field, will be reflected by the billiard. We suggest that the maximum of the resistance is reached when the cyclotron radius equals half the base length. Then electrons that enter the triangle centrally through the base contact are reflected very efficiently backward, with only two collisions with the boundaries (inset of Fig. 1). The field at the center of the peak corresponds to a cyclotron radius $l_c=mv_F/eB$ of 1.8 μm, which is close to the lithographic, half base length of 1.5 μm. Here m is the effective electron mass, v_F is the Fermi velocity, and e is the elementary charge. Apart from this peak and some structure at higher fields, the magnetoresistance decreases with increasing magnetic field as the cyclotron radius decreases, as is the case for a single point contact (Fig. 2). The smaller peaks at higher fields will be discussed in more detail later.

This explanation for the absolute maximum of the magnetoresistance emphasizes the specific trajectory shown in the inset of Fig. 1. If this simple picture is correct, any billiard of a different shape should show the same peak of the magnetoresistance provided that the new geometry also supports this trajectory. We have fabricated a number of different test structures and were in this way able to test the model for the peak. Figure 2 shows the geometry of six different devices and their magnetoresistance as a function of the reduced magnetic field B/B_0. Here B_0 is the field where the cyclotron radius equals the half base length of the structure. Spectra I–III correspond to structures that were designed in such a way that they support the trajectory that is suggested to give rise to the peak at $B/B_0=1$. Devices IV–VI do not support this trajectory and consistently do not have the peak. This is strong evidence that, in fact, the $B/B_0=1$ peak is related to the electron trajectory of Figs. 1 and 2. Obviously, this trajectory is of great importance for the electric properties of the billiard. The reason, as will be shown later, is that this trajectory is relatively stable towards small changes in the parameters of motion.

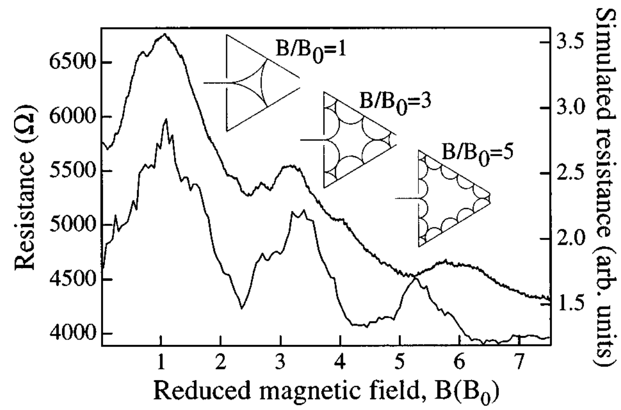

FIG. 3. Upper curve, the measured magnetoresistance of device I at 4.4 K, lower curve, data obtained from a classical simulation using a hardwall potential. Also indicated in the figure are the classical electron trajectories that can be related to the three maxima of the simulated data.

IV. SIMULATION

In order to further establish the origin of the magnetoresistance in specific trajectories that are commensurate with the shape of the billiard, we have carried out simulations of classical electron orbits. Electrons that start at different initial angles from the base contact at Fermi velocity were traced inside the billiard until they left it through one of the contacts. The starting point was varied over the entire base contact. Electrons that had not left the billiard after 100 boundary collisions were neglected. The ratio of electrons that left the billiard through the contact at the tip gave the transmission probability $t(B)$ for each magnetic field. A dimensionless value for the resistance was then obtained by taking $R(B)\propto t(B)^{-1}$. The electrons were treated entirely as classical, charged particles. A hard wall potential was assumed and no electron-electron interaction was taken into account. The focusing effect of the point contact[1,9] was simulated by taking the distribution of the initial angles as $P(\phi)=\frac{1}{2}f\cos(\phi)$ if $|\phi|<\arcsin(1/f)$ and $P(\phi)=0$ otherwise. Here f is a dimensionless normalization constant. The initial angle ϕ is defined as the deviation from the symmetry axis of the triangle. For the simulations we used $f=1.41$, which gives $|\phi|<45°$. This is a typical value found in experiments.[9] Our simulations were not sensitive to small changes in f. Impurity scattering was taken into account by changing the direction of motion randomly after an exponentially distributed random distance of travel with the experimentally determined mean free path as the characteristic distance.

Figure 3 shows the result of the simulation for device I and the experimental data as a function of the reduced magnetic field. The global maximum at $B/B_0=1$ and the overall decrease of the magnetoresistance at higher fields are clearly reproduced. In addition, the simulation has local maxima also at higher odd values of B/B_0 that are also visible in the experimental data. This series of peaks is correlated with a certain type of trajectories with an incident angle $\phi=0°$, as is also indicated in the figure. These trajectories lead to the reflection of electrons entering the billiard from the base contact. The positions of the peaks at $B/B_0=1,3$ in the simulation and in the experimental data agree well. However, the peak in the experimental data at $B/B_0\approx 5$ is shifted to higher fields compared to the simulation. We suggest that this could be due to deviations of the shape of the electron billiard from a perfect triangle. In particular, one can expect that the corners of the electric device were rounded, which would be most important at $B/B_0\approx 5$ because the relevant trajectory reaches deep into the corners.

It is interesting that no peaks are observed at even values of B/B_0, although similar trajectories exist as at odd values of B/B_0. It will be shown in Sec. V that the trajectories at even B/B_0 are more unstable than at odd values, and we believe that this is the reason for the missing structure.

It should be noted that all trajectories in Fig. 3 start and end at the base contact. No equally simple trajectories exist for electrons entering the triangle from the tip contact. One may therefore suspect that the magnetoresistance is different for different directions of the current in the asymmetric potential under consideration. However, in the linear-response regime the resistance does not change if the directions of the electric and the magnetic field are reversed simultaneously.[13] The apparent contradiction is resolved if transmitted trajectories are considered instead of reflected because transmitted trajectories are symmetric upon time reversal, i.e., they are the same if both the current direction and the magnetic field are reversed. This concept has been applied in the simulations. In fact, the simulations give qualitatively the same result independently of the source contact used. It is interesting that the discussion of reflected trajectories coming from only one side can, nevertheless, explain so much about the behavior of the resistance. The reason is that the probability of reflection r is directly related to the probability of transmission as $t=(1-r)$. Therefore it is equivalent to consider reflected or transmitted electron orbits; both approaches lead to the same result.

V. STABILITY ANALYSIS

In the previous sections, details of the magnetoresistance of triangular billiards were explained in terms of a small number of electron trajectories. In this section we investigate the reason for the importance of these specific trajectories with the help of classical chaos theory. We show that these trajectories are only weakly unstable with respect to changes in the initial conditions, while other, similar trajectories that do not give rise to structure in the magnetoresistance are relatively unstable.

In order to quantify the stability of the different electron trajectories against small perturbations we have calculated numerically the Liapunov exponent λ of the different trajectories. The Liapunov exponent describes the sensitivity of the parameters of motion to small changes in the initial conditions.[14,15] If $\lambda>0$, two trajectories that start close to each other in phase space diverge exponentially. Such trajectories are called chaotic, while for regular behavior $\lambda\rightarrow 0$. In the following, λ is the larger of the two Liapunov exponents of angle and position of the electron. The value for λ has been determined in such a way that the difference in phase space is, on average, increased by the factor $\exp(\lambda)$ after each boundary collision. The final difference in phase space is larger for longer trajectories with more boundary collisions than for short trajectories with few boundary collisions. Therefore, we define a stability parameter for each specific trajectory as $S=\exp(-\lambda N)$, where N is the number of boundary collisions of the trajectory until the electron leaves the billiard.

Figure 4(a) shows a logarithmic plot of S for trajectories that start at the center of the base contact as a function of the reduced magnetic field and ϕ, the initial angle of motion. The white regions in the plot represent initial conditions that lead to the transmission of electrons. Gray areas represent reflected trajectories. The darker the color, the more stable the reflected trajectory. Therefore, dark areas are important for the resistance. S is not symmetric in the initial angle because the magnetic field breaks the symmetry of the triangle.

Major areas of stable, reflecting trajectories are found centered around $\phi=0$ and $B/B_0=1,3,5,\ldots$. This is in very good agreement with the experimental data that also show resistance maxima at odd values of B/B_0. At even values of B/B_0, where the magnetoresistance has minima, we find only transmitting or unstable, reflecting trajectories. In general, the number of transmitting trajectories increases with increasing magnetic field. This is in agreement with the decreasing resistance and reflects the formation of edge channels. At sufficiently large fields, when the cyclotron radius is smaller than the width of the point contact, all electrons will be transmitted by the billiard.

Figure 4(b) shows the integral of the stability parameter of reflected trajectories in the range $-45°<\phi<45°$. This plot gives a measure of the averaged stability of the trajectories that lead to reflection as a function of magnetic field, weighted by their number. As one would expect, maxima of the average stability are centered around odd values of B/B_0. However, the stability maxima have a local minimum at the center because the trajectories at exactly odd values of B/B_0 are on average less stable than those corresponding to the outer regions of the peaks [Fig. 4(a)]. The local minimum of the average stability at $B/B_0=1$ is also reproduced in simulations of the magnetoresistance when an infinite mean free path is used. However, it is not visible when a finite mean free path is used (Fig. 3). The reason is that trajectories

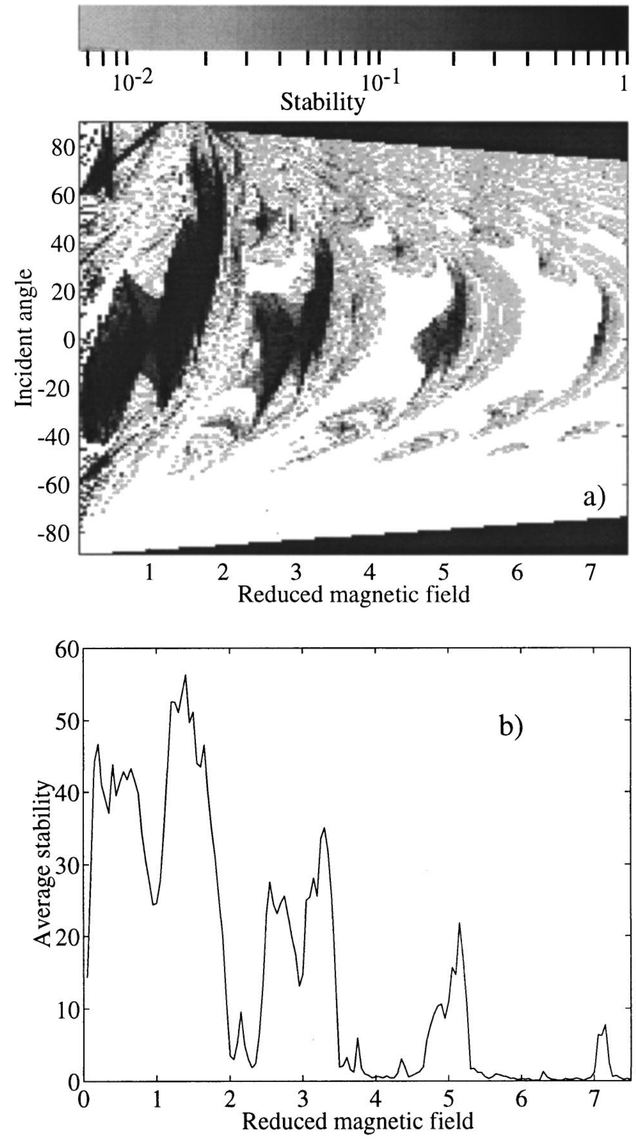

FIG. 4. (a) Logarithmic plot of the stability parameter S of reflected trajectories that start at the center of the base contact as a function of the initial angle and the reduced magnetic field. The darker the color, the more stable the trajectory. White areas represent initial conditions that lead to transmission of electrons. The three stable areas centered around odd values of B/B_0 correspond to the three maxima observed in the magnetoresistance. (b) The stability parameter of reflected trajectories averaged over the incidence angles $-45°<\phi<45°$ as a function of the magnetic field, weighted by the number of reflected trajectories at each field. The centers of the stable areas around $B/B_0=1$ and 3 are less stable than the outer regions.

that are longer than the mean free path are unimportant in reality also if they are very stable.

To conclude this section, an analysis of the relative stability of different trajectories in an electron billiard can explain commensurability effects in great detail. Only relatively stable, short, commensurate trajectories are important for the magnetoresistance. In the next section we will show that the commensurability effects are qualitatively changed when a bias voltage is applied. We suggest that the differences in the stabilities of the different trajectories are en-

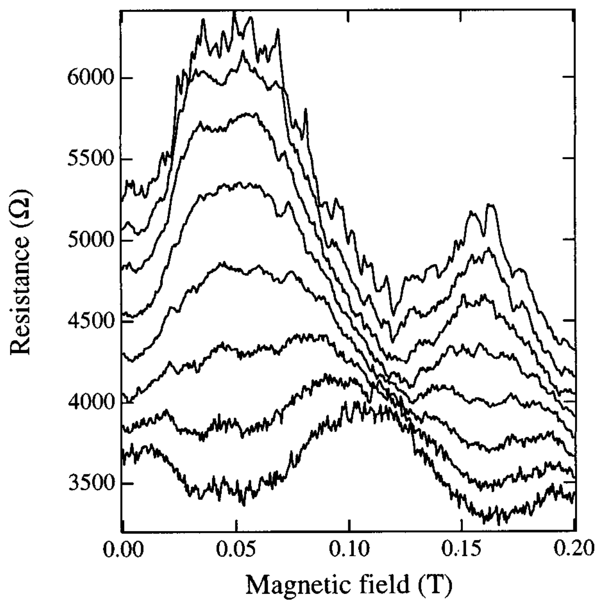

FIG. 5. Magnetoresistance of device I at different bias voltages. From the top the bias voltage is given by 0, 0.5, 1.0, . . . ,3.5 mV. The electron-electron interaction perturbs the ballistic trajectories. The less stable centers of the resistance peaks are quenched faster than the more stable flanks. From the top, the curves have been set off by 0, -200, -400, . . . ,-1200 Ω for clarity.

hanced when electron motion is perturbed by the e-e interaction.

VI. INFLUENCE OF ELECTRON-ELECTRON SCATTERING

The electron-electron interaction is a function of the temperature of the electron gas and of the electron excess energy Δ with respect to the Fermi level.[16] In order to increase the e-e interaction rate experimentally, it is convenient to add a dc bias voltage to the ac voltage used for the lock-in technique.[17,18] In this way one can tune the excess energy Δ of the electrons that are detected by the lock-in technique. In addition, a significant fraction of the additional kinetic energy of electrons entering the triangle is transferred to the other electrons inside the billiard, heating the electron gas up. Both effects lead to an increase in the e-e interaction rate with increasing bias voltage.

Figure 5 shows the magnetoresistance of the triangular billiard at different bias voltages. The curves are offset for clarity.[19,20] At small voltages, the conductance fluctuations disappear because of the phase-destructive effect of e-e interactions.[8] A bias voltage of 1 mV has been found to be a typical value for phase destruction in small devices.[21] Here we are interested in the strong, qualitative change in the commensurability effects as a function of bias voltage. With increasing bias voltage, the maxima at odd B/B_0 are quenched and gradually local minima of the resistance are formed at the same positions.

We tentatively explain this striking effect as follows. Electron-electron scattering of excess electrons above the Fermi level is dominated by small-angle scattering.[22] We suggest, therefore, that e-e scattering causes small perturbations of the parameters of motion. The Liapunov exponent λ measures the sensitivity of trajectories towards such small perturbations. Therefore, we believe that trajectories with a large stability parameter $S=\exp(-\lambda N)$ are less sensitive to e-e scattering than such with a small S. The stability analysis in Sec. V has shown [Fig. 4(a)] that fewer of the trajectories that underlie the center of the magnetoresistance peaks at odd values of B/B_0 are stable compared to the flanks of the peaks. Therefore, the contribution to the resistance of the trajectories underlying the center of the peak is reduced faster by e-e scattering than the contributions of the trajectories that are related to the flanks of the peak. This is observed in our experiment as a splitting of the peaks at $B/B_0=1,3$.

As already mentioned, at zero bias the finite mean free path makes the contribution of long trajectories unimportant, such that the local minimum of the averaged stability at odd B/B_0 is not observed experimentally (Fig. 1). However, as the bias voltage is increased, the perturbation due to e-e scattering is large enough to make small instabilities important also for short trajectories and the local minimum can be observed. In this sense, e-e scattering appears to accentuate classical chaotic behavior of electron billiards.

An influence of e-e scattering on ballistic transport has been observed previously in electron focusing experiments.[18,23] In these investigations it was found that a good approximation for the momentum relaxation rate due to the e-e interaction is given by the energy relaxation rate for the ideal electron gas $\tau_{ee}(\Delta,T)$, which has been calculated by Chaplik[24] and Giuliani and Quinn.[16] This observation is in line with the theory by Gurzhi, Kalinenko, and Kopeliovich, who predict that the angular distribution of a collimated beam with excess energy Δ is widened by a factor of $(\Delta/E_F)^{1/2}$ within a time of the order of $\tau_{ee}(\Delta,T)$.[22] We can therefore, as a rough estimate, approximate the mean free path with respect to small-angle e-e scattering by $l_{ee}=v_F\tau_{ee}$. If we neglect heating effects inside the billiard,[25–27] τ_{ee} for $k_BT=0$ and $\Delta\ll\hbar^2k_Fq_{\mathrm{TF}}/m=390$ eV is given by

$$\tau_{ee}(x)^{-1}=\frac{E_F}{4\pi\hbar}\,x^2\left[\ln\left(\frac{2q_{\mathrm{TF}}}{k_F}\right)+\frac{1}{2}-\ln(x)\right].$$

Here $x=\Delta/E_F$, $q_{\mathrm{TF}}=2me^2/\varepsilon\hbar^2$ is the Thomas-Fermi screening wave vector, and $\varepsilon=12.7$ is the dielectric constant. Figure 6 shows l_{ee} as a function of the electron excess energy. Also indicated in the figure are the dimension of the device b and the elastic mean free path l_e. We assume that the electron excess energy is approximately given by the bias voltage. The electron-electron interaction is expected to be important for ballistic effects only if $l_{ee}<l_e$ or $|V|>0.7$ mV. Strong effects are expected when $l_{ee}<b$ or $|V|>1.5$ mV. This is in agreement with the experimental observations and we conclude that the e-e interaction rate is of the right order of magnitude to explain the observed ballistic effects.

We have also considered the possibility that the voltage-induced changes of the magnetoresistance might be due to a difference between the Fermi velocity of electrons entering the billiard through the base and those entering through the tip. The model considered is similar to the explanation for half plateaus that are observed in the quantized conductance of narrow constrictions when a bias voltage is

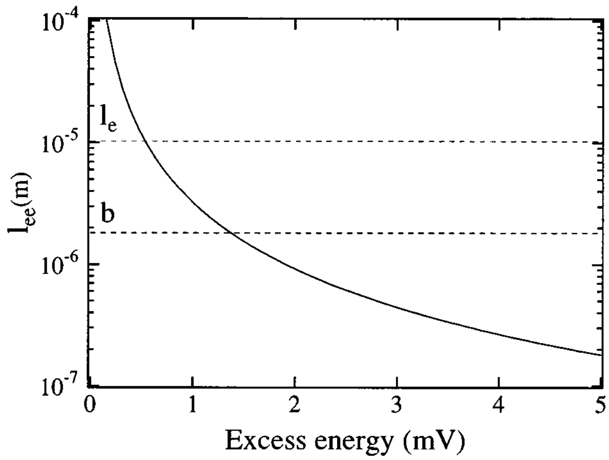

FIG. 6. Calculated mean free path with respect to energy relaxation due to the e-e interaction as a function of the excess energy of the electrons. The energy relaxation rate is used as an approximation for the small-angle e-e scattering rate (see the text).

applied.[28–30] We assume that a portion αU of the bias voltage U drops at the base contact of the billiard and a portion $(1-\alpha)U$ at the contact at the tip, where α is a parameter between 0 and 1. Then the Fermi velocity of an electron entering the billiard through the base contact is given by $[2(E_F+\alpha eU)/m]^{1/2}$ and that of electrons entering the billiards via the tip contact by $\{2[E_F+(1-\alpha)eU]/m\}^{1/2}$.The difference in Fermi velocity increases with increasing bias voltage. The maxima of the magnetoresistance that are related to classical orbits will then split into two peaks and the changes in the magnetoresistance might be qualitatively similar to those shown in Fig. 5. However, the observed changes are too strong to be explained by this effect alone. Even if all voltage drops at one contact (α=1), the peak at 50 mT would shift towards higher fields only by a factor $(1+eU/E_F)^{1/2}$ or by 15% at the highest voltage used in the experiment. In the experiment, however, the field value at the maximum is changed by more than a factor of 2. Clearly, although they may be important, voltage-induced changes of the Fermi velocity alone cannot explain the dramatic changes shown in Fig. 5.

VII. CONCLUSION

We have observed commensurability effects in triangular electron billiards. By comparing the magnetoresistance of different geometries and simulations of classical electron trajectories, we have been able to correlate specific electron trajectories with maxima in the resistance. An explanation why certain trajectories are more important for the resistance than others was established on the basis of a numerical stability analysis. Short trajectories with a small Liapunov exponent are most important for the resistance.

The experiments indicate that the commensurability effects of a billiard are influenced by an electron-electron interaction that is induced by a bias voltage. It is speculated that unstable electron orbits are most sensitive to the perturbations caused by the e-e interaction than stable orbits, which accentuates the classical chaotic behavior. A similar effect has recently been predicted for the case of quantum chaos in a theoretical investigation of the influence of the e-e interaction on the level statistics in a quantum dot.[6] It would be an exciting experiment to correlate the influence of e-e scattering on classical trajectories with the changes in the level statistics in the same device.

Another interesting problem is the question concerning the degree to which the conductance of a highly asymmetric potential such as a triangle is symmetric. At higher voltages, nonlinear effects may be important and the conductance may experience nonsymmetric contributions.[31] In fact, such effects are observed at low temperatures and zero magnetic field. This is the focus of ongoing investigations.[32]

ACKNOWLEDGMENTS

We thank Hongqi Xu for fruitful discussions. This work was performed within the Nanometer Structure Consortium in Lund, Sweden. Financial support by the Swedish Natural Science Research Council, by the Swedish Research Council for Engineering Sciences, by NUTEK, and by the Göran Gustafsson Foundation for Research in Natural Sciences and Medicine is gratefully acknowledged. We thank Claus B. Sørensen, the III-V NANOLAB, and CNAST for the MBE-grown structures.

[1] C. W. J. Beenakker and H. v. Houten, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, Boston, 1991), Vol. 44.
[2] Y. V. Sharvin, Zh. Eksp. Teor. Fiz. **48**, 984 (1965) [Sov. Phys. JETP **21**, 655 (1965)].
[3] H. v. Houten, C. W. J. Beenakker, J. G. Williamson, M. E. I. Broekaart, P. H. M. v. Loosdrecht, B. J. v. Wees, J. E. Mooij, C. T. Foxon, and J. J. Harris, Phys. Rev. B **39**, 8556 (1989).
[4] M. L. Roukes and A. Scherer, Bull. Am. Phys. Soc. **34**, 633 (1989).
[5] K. Ensslin and P. M. Petroff, Phys. Rev. B **41**, 12 307 (1990).
[6] S. E. Ulloa and D. Pfannkuche, Superlatt. Microstruct. **21**, 21 (1997).
[7] C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506 (1992).
[8] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[9] L. W. Molenkamp, A. A. M. Staring, C. W. J. Beenakker, R. Eppenga, C. E. Timmering, J. G. Williamson, C. J. P. M. Harmans, and C. T. Foxon, Phys. Rev. B **41**, 1274 (1990).
[10] H. v. Houten and C. W. J. Beenakker, in *Nanostructures Physics and Fabrication*, edited by M. A. Reed and W. P. Kirk (Academic, New York, 1989).
[11] D. A. Wharam, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L887 (1988).
[12] C. W. J. Beenakker and H. v. Houten, Phys. Rev. B **39**, 10 445 (1989).
[13] M. Büttiker, IBM J. Res. Dev. **32**, 317 (1988).
[14] P. Bergé, Y. Pomeau, and C. Vidal, *Order within Chaos*—

Towards a Deterministic Approach to Turbulence (Cambridge University Press, Cambridge, 1990).

[15] S. H. Strogatz, *Nonlinear Dynamics and Chaos—With Application to Physics, Biology, Chemistry, and Engineering* (Addison-Wesley, Reading, MA, 1994).

[16] G. F. Giuliani and J. J. Quinn, Phys. Rev. B **26**, 4421 (1982).

[17] A. Yacoby, U. Sivan, C. P. Umbach, and J. M. Hong, Phys. Rev. Lett. **66**, 1938 (1991).

[18] T. Schäpers, M. Krüger, J. Appenzeller, A. Förster, B. Lengeler, and H. Lüth, Appl. Phys. Lett. **66**, 3606 (1995).

[19] Offsetting the curves at different voltages camouflages that the magnetoresistance at $B=0$ decreases weakly with increasing bias voltage, which has not been investigated in detail in this work. Important effects may be the different occupation of subbands in the point contacts (Ref. 20) or an increase of the focusing effect of the point contacts due to the electric field.

[20] L. P. Kouwenhoven, B. J. v. Wees, C. J. P. M. Harmans, J. G. Williamson, H. v. Houten, C. W. J. Beenakker, C. T. Foxon, and J. J. Harris, Phys. Rev. B **39**, 8040 (1989).

[21] H. Linke, P. Omling, H. Xu, and P. E. Lindelof, Phys. Rev. B **55**, 4061 (1997).

[22] R. N. Gurzhi, A. N. Kalinenko, and A. I. Kopeliovich, Phys. Rev. B **52**, 4744 (1995).

[23] L. W. Molenkamp, M. J. P. Brugmans, H. v. Houten, and C. T. Foxon, Semicond. Sci. Technol. **7**, B228 (1992).

[24] A. V. Chaplik, Zh. Eksp. Teor. Fiz. **60**, 1845 (1971) [Sov. Phys. JETP **33**, 997 (1971)].

[25] Heating effects due to the bias voltage are most likely important in a closed structure such as a billiard (Ref. 26). However, the error made by neglecting heating is reduced by the fact that only temperature changes of $T > eU/k_B$ are important (Ref. 27).

[26] R. Taboryski, A. Kristensen, C. B. Sørensen, and P. E. Lindelof, Phys. Rev. B **51**, 2282 (1995).

[27] G. Fasol, Appl. Phys. Lett. **59**, 2430 (1991).

[28] L. I. Glazman and A. V. Khaetskii, Europhys. Lett. **9**, 263 (1989).

[29] N. K. Patel, J. T. Nicholls, L. Martin-Moreno, M. Pepper, J. E. F. Frost, D. A. Ritchie, and G. A. C. Jones, Phys. Rev. B **44**, 13 549 (1991).

[30] H. Xu, Phys. Rev. B **47**, 15 630 (1993).

[31] R. Landauer, in *Nonlinearity in Condensed Matter*, edited by A. R. Bishop, D. K. Campbell, P. Kumar, and S. E. Trullinger (Springer-Verlag, Berlin, 1987).

[32] H. Linke, P. Omling, H. Xu, and P. E. Lindelof, in *Proceedings of the 23rd International Conference on the Physics of Semiconductors*, edited by M. Scheffler and R. Zimmermann (World Scientific, Singapore, 1996), p. 1593.