# Magnetic-field inversion in vortices in multilayers

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We present a description of very dense vortex lattices in highly anisotropic multilayers, for high fields parallel to the layers. We show that a magnetic-field inversion can occur away from the center of a vortex, provided the layers are sufficiently far apart. [S0163-1829(97)05745-7]

The structure of vortices in layered superconductors presents a fascinating array of peculiar characteristics, depending on the direction of the external magnetic field. Some of these have been studied quite some time ago, while others more recently.[1,2] The case where the external field is parallel to the layers is an especially interesting one. It was shown numerically,[3] for example, that in layered superconductors with inequivalent layers immersed in a magnetic field parallel to the layers, the field around a vortex can be inverted. The inversion can be achieved only when the in-plane penetration depth of the weakly superconducting layers is much greater than that of the strongly superconducting layers. This work was based on the assumption of an essentially uniform order-parameter amplitude along the layers.

The present paper aims to show that the above mentioned field inversion for vortices parallel to the layers can be a really generic effect in highly anisotropic multilayers, where the superconducting structure undulates along the $z$ axis, as long as the period of undulation is large enough, and as long as the applied magnetic field is high enough. We show in particular that if the period of undulation $d$ of the superconducting structure is much larger than a penetration depth $\lambda$ along the $z$ axis, then the field of a vortex is inverted away from the center of the vortex, for sufficiently high external fields.

What happens essentially is that the field $h$ of each vortex can go quite fast along the $z$ axis from a nonzero value $h(0)$ at the center of the vortex to the zero value of the Meissner state, provided the penetration depth $\lambda$ is small enough. In fact, for sufficiently large values of $d/\lambda$ the field $h$ could go to zero within a distance much shorter than $d$, where $d$ is the distance between layers. Indeed, it will do so for dense vortex lattices with $d \gg \lambda$, where there are vortex centers halfway between the layers, because the superconducting layers expel the magnetic field, limiting it to the interlayer regions. It turns out now that the value $h(0)$ of the field at the center of the vortex, which lies halfway between layers, is approximately equal to the value $H$ of the external field, for large $H$. Ideally, it would be zero on the superconducting layer due to screening, if the field can vary rapidly enough. This rapid variation is ensured if $d$ is much greater than $\lambda$, unlike the case of the usual superconductors or of high-temperature superconductors, where the field changes very slowly on the scale of $d$. Thus, if $d/\lambda$ is large enough, the field will start out at a high value halfway between layers and drop abruptly within just a few $\lambda$'s. It will have therefore a large negative initial slope, which makes it shoot down very abruptly. It will thus overshoot past zero towards negative values, before the existence of superconductivity on the layer forces it to come back up towards zero. The field therefore will decrease very rapidly from a large $h(0)$ to a negative value, then go to zero within the superconducting layer, and then rise again in the next interlayer region, where the external field can penetrate easily. A field inversion can occur therefore, if $d/\lambda$ and $H$ are sufficiently large, because the large negative slope of $h$ makes it drop down to negative values just before becoming zero around the superconducting layer.

In order to demonstrate the above-mentioned field inversion, we shall use a very general model that takes into account the existence of a nonzero order parameter between the layers.[4] We shall assume in particular that the Gibbs free energy of the multilayer in an external field $H$ parallel to the $x$ axis is

$$\int\int\int dx\,dy\,dz\Bigg[a(z)|\Psi|^2+\beta|\Psi|^4/2+\frac{\hbar^2}{2m}\left|-i\nabla_{\parallel}\Psi-\frac{2e}{\hbar c}A_{\parallel}\Psi\right|^2+\frac{\hbar^2}{2M}\left|-i\frac{\partial\Psi}{\partial z}-\frac{2e}{\hbar c}A_z\Psi\right|^2+\frac{1}{8\pi}(\nabla\times A-H)^2\Bigg]. \tag{1}$$

Here $z$ is the direction normal to the layers.
We shall measure $x,y,z$ in terms of $d$, the period of the superconducting structure along the $z$ axis. We render $a(z)$ dimensionless by taking out a dimensionful constant $\alpha$, so that $a(z)/\alpha=\alpha(z)$. We measure $\Psi^2$ in units of $\alpha/\beta$, the vector potential $A$ in units of $\hbar c/2ed$, the magnetic fields in units of $\hbar c/2ed^2$, and the energies in units of $d^3\alpha^2/\beta$. Thus the Gibbs free-energy density takes the dimensionless form

$$g=\Bigg[\alpha(z)|\Psi|^2+|\Psi|^4/2+\nu\Gamma^2\left|-i\nabla_{\parallel}\Psi-A_{\parallel}\Psi\right|^2+\nu\left|-i\frac{\partial\Psi}{\partial z}-A_z\Psi\right|^2+\nu\Gamma^2\frac{\lambda^2}{d^2}(\nabla\times A-H)^2\Bigg], \tag{2}$$

where $\nu=\hbar^2/2M\alpha d^2$, $\Gamma^2=M/m$, and $\lambda^2=mc^2\beta/16\pi e^2\alpha$. The penetration depth $\lambda$ should not be confused with the experimentally measured penetration depth. Note furthermore that $\lambda$ depends on the choice of the dimensionful constant $\alpha$, which is unspecified so far, while $\lambda^2/\nu d^2$ does not.

 

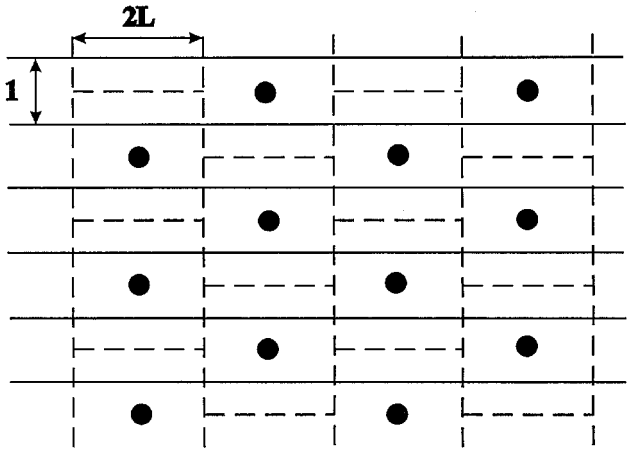


FIG. 1. The vortex lattice at very high fields. Each cell has sides $2L$ and 2.

In this paper we shall examine very dense vortex lattices. In such lattices the vortex cells are practically rectangular. Vortices exist in every interlayer spacing, but in a squeezed triangular formation, due to vortex repulsion (see Fig. 1). The vortices in the vortex lattice are parallel to the $x$ axis, because the external field is assumed to be along that axis. The order parameter $\Psi$ becomes maximum on each layer, the distance between successive layers being $d$, but vortices will appear only every two layers along the $z$ axis, due to their mutual repulsion. Thus each rectangular vortex cell will have a length of $2L$ along the $y$ axis, and $2d$ along the $z$ axis. As the field $H$ increases, the vortex cells get more narrow; in other words $L$ decreases, but the structure along the $z$ axis remains unchanged. Indeed, due to their mutual repulsion the distance between two vortices along the $z$ axis cannot be reduced below $2d$. We have thus a triangular lattice, with vortices situated in every interlayer spacing. The spatial periodicity of the vortex lattice results in the quantization of the magnetic flux contained in each unit cell. We anticipate therefore that $L \propto 1/H$.

We expect each vortex to be axisymmetric very close to its center. The vector potential and the magnetic field vary typically over distances of the order of $\lambda$ along the $z$ axis, but of order $\Gamma\lambda$ along the $y$ axis. We shall work consistently in the limit $d \gg \lambda$, and $\Gamma \to \infty$. In other words, our results will hold for highly anisotropic multilayers, with the layers sufficiently far apart.

Very close to the vortex center, the field is axisymmetric. Thus, in the vicinity of the center, we expect the lines of constant field to be ellipses centered at the vortex center, i.e., $h = h((z^2 + y^2/\Gamma^2)^{1/2})$. Since the magnetic field decays to zero pretty fast, within just a few $\lambda$'s along the $z$ axis, the various vortices do not overlap in the $z$ direction. Note also that $\alpha(z) \approx \alpha((z^2 + y^2/\Gamma^2)^{1/2})$ for $|z| \gg L/\Gamma > |y|/\Gamma$. Thus, if $L \ll \Gamma$, $\alpha(z)$ is axisymmetric practically everywhere. It is thus reasonable to assume that the axisymmetry of the magnetic field will be approximately true over most of the cell, as long as $\Gamma$ is very large. We can easily verify that the *ansatz*

$$A_y = -\frac{\sin\phi}{\Gamma} A(\rho), \tag{3}$$

$$A_z = \cos\phi A(\rho), \tag{4}$$

with $A_x = 0$, can lead to such an axisymmetric single vortex field. The anisotropic polar coordinates $\rho$, $\phi$ on the $y$-$z$ plane are defined through the equations

$$\rho = \sqrt{z^2 + y^2/\Gamma^2}, \tag{5}$$

$$\sin\phi = \frac{z}{\rho}, \tag{6}$$

$$\cos\phi = \frac{y}{\Gamma\rho}, \tag{7}$$

assuming that the origin is the center of the vortex.

We shall therefore assume that this *ansatz* for the vector potential holds over the whole cell. Since $\alpha(z)$ is really $\alpha(\rho)$ for very anisotropic multilayers ($\Gamma \to \infty$), provided $|y| \ll \Gamma$, the order parameter amplitude and the magnetic field are essentially functions of $\rho$ only, justifying thus the *ansatz* of Eqs. (3) and (4).

The magnetic field of the vortex will be along the $x$ axis:

$$h(\rho) = \frac{1}{\Gamma\rho}\frac{\partial Q}{\partial \rho}, \tag{8}$$

with

$$Q(\rho) = -1 + \rho A(\rho). \tag{9}$$

The approximate axisymmetry of the magnetic field and of $\alpha(z)$ imply that the magnitude of the order parameter is approximately axisymmetric as well:

$$\Psi(\rho, \phi) = \psi(\rho) e^{i\phi}. \tag{10}$$

The Gibbs free-energy density takes the following form, in terms of $\psi$ and $Q$:

$$g = \alpha(z)\psi^2 + \frac{1}{2}\psi^4 + \nu\left(\frac{\partial\psi}{\partial\rho}\right)^2 + \frac{\nu}{\rho^2} Q^2\psi^2 + \frac{\nu\lambda^2}{d^2}\left(\frac{1}{\rho}\frac{\partial Q}{\partial\rho} - \Gamma H\right)^2. \tag{11}$$

The superconducting structure is represented by $\alpha(z)$. If this structure has a period $d$, then $\alpha(z)$ is periodic with a period $d$. We expect that the vortices are situated halfway between the layers. Therefore $\alpha(z)$ will have an extremum at the center of the vortex. Combined with the periodicity, this implies that $\alpha(z)$ is an even function of $z$.

We repeat that since $\Gamma \to \infty$, we can assume that $\alpha(z)$ is really $\alpha(\rho)$, as long as $|y| < L \ll \Gamma$. Thus $\psi$, $Q$, $A$, and $h$ are essentially functions of $\rho$.

For a very dense vortex lattice, i.e., a high external field, the unit cell will be a truncated ellipse, with $y \ll \Gamma$ within it (see Fig. 2), and it will resemble a rectangle with sides $2L$ and 2, where $L \ll \Gamma$. In that case $|z| \leqslant \rho \leqslant (z^2 + L^2/\Gamma^2)^{1/2} \approx |z|$, hence $\rho \approx |z|$. The flux in the unit cell will then be $2L\,2\int_0^1 h\,d\rho = 2\pi$. The field varies very little along the $y$ axis when the $y$ coordinate is much less than $\Gamma$, so when $L$ has become quite small the field $h$ will be varying along the $z$ direction only, remaining almost constant along the $y$ direc-

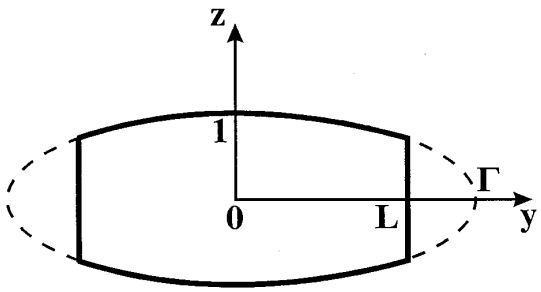


FIG. 2. A truncated elliptical cell. The distances are always in units of $d$.

tion. The order parameter will also be a function of $z$ only, remaining practically constant in the $y$ direction. We can say equivalently that $\psi$ is a function of $\rho$ only.

The Gibbs free energy will be the integral of $g$ over the area of the truncated cell of Fig. 2. Thus the Gibbs free energy per unit $x$ length will equal

$$\int_0^{L/\Gamma} d\rho\ 2\pi\Gamma\rho g + \int_{L/\Gamma}^1 d\rho\ \Gamma\rho g\left[2\pi - 4\cos^{-1}\left(\frac{L}{\Gamma\rho}\right)\right]. \tag{12}$$

The equations that minimize this functional for $0<\rho<L/\Gamma$ are

$$\alpha(z)\psi+\psi^3+\frac{\nu Q^2\psi}{\rho^2}=\nu\frac{\partial^2\psi}{\partial\rho^2}+\frac{\nu}{\rho}\frac{\partial\psi}{\partial\rho}, \tag{13}$$

$$\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial Q}{\partial\rho}\right)=\frac{d^2}{\lambda^2}\frac{Q\psi^2}{\rho}. \tag{14}$$

Indeed, these equations minimize the first integral in Eq. (12). Clearly, since $L\ll\Gamma$, these equations really hold only at the vicinity of the vortex center. We can easily show then that $Q(0)=-1$ and $\psi(0)=0$. Furthermore, $\partial h/\partial\rho$ is zero at $\rho=0$ and $\psi$ is linear in $\rho$ near the origin.

Let us now look at the case $L/\Gamma<\rho<1$. If $L\ll\Gamma$, then the second integral of Eq. (23) can be approximated by the integral $\int_{L/\Gamma}^1 4Lg\ d\rho$, because $\cos^{-1}(L/\Gamma\rho)\approx(\pi/2)-(L/\Gamma\rho)$ on most of the interval $[L/\Gamma,1]$. Thus we have to find the field equations that minimize $\int_{L/\Gamma}^1 g\ d\rho$. These equations are

$$\alpha(\rho)\psi+\psi^3+\frac{\nu Q^2\psi}{\rho^2}=\nu\frac{\partial^2\psi}{\partial\rho^2}, \tag{15}$$

where $\rho\approx|z|$ within the cell, and

$$\frac{\partial}{\partial\rho}\left(\frac{\Gamma h-\Gamma H}{\rho}\right)=\frac{d^2}{\lambda^2}\frac{Q\psi^2}{\rho^2}. \tag{16}$$

This last equation can be rewritten in the form

$$\rho\Gamma\frac{\partial h}{\partial\rho}-\Gamma h+\Gamma H=\frac{d^2}{\lambda^2}Q\psi^2. \tag{17}$$

We stress again that Eqs. (15) and (17) hold *only* in the interval $L/\Gamma<\rho<1$.

Let us now discuss the boundary conditions. We already said that $Q(0)=-1$ at the center of the vortex. Also $\psi(0)=0$. Since the vortex is situated halfway between the layers, and since $\psi$ varies little along the $y$ axis, due to the small $L$ and the large $\Gamma$, the order parameter must be zero on all the planes that are located halfway between neighboring superconducting layers. But the edges of the cells lie also on such planes. Hence $\psi(1)=0$. The field $h$ has to fit with that of the next cell, so we must also have that $\partial h/\partial\rho$ be zero at the edge of the cell, i.e., $\dot{h}(1)=0$. Combined with the boundary condition $\psi(1)=0$ and with Eq. (17) this implies that $h(1)=H$. The boundary conditions for Eqs. (15) and (17) are thus $\psi(1)=0$, $h(1)=H$ at the edge of the cell.

Since the interval $[0,L/\Gamma]$ is quite small, we can consider the boundary conditions $\psi(0)=0$, $Q(0)=-1$ as appropriate for Eqs. (15) and (17) as well, even though those equations apply strictly to the interval $[L/\Gamma,1]$ *only*.

We conclude then that Eqs. (15) and (17) describe fully the dense vortex lattice if the $y$ length of the cell is much smaller than $\Gamma$, subject to the boundary conditions $\psi(0)=\psi(1)=0$, $Q(0)=-1$, $h(1)=H$.

We shall explore the qualitative consequences of this description through a simple variational model before conducting a more careful numerical study. Thus, we shall adopt the trial order parameter

$$\psi^2(\rho)=\psi_0^2\delta\left(\rho-\frac{1}{2}\right), \tag{18}$$

where $\psi_0$ is determined by the details of $\alpha(z)$, and we shall solve for $Q$.

Equation (16), along with the boundary condition $Q(0)=-1$, yields

$$Q(\rho)=-1+\Gamma H\rho^2/2+\Gamma\kappa\rho^3/3 \tag{19}$$

for $0<\rho<1/2$, as well as

$$h(\rho)=H+\mu\rho \tag{20}$$

for $\frac{1}{2}<\rho<1$, $\kappa$ and $\mu$ being integration constants.
The boundary condition $h(1)=H$, i.e., $\dot{h}(1)=0$, gives $\mu=0$. Thus we get

$$h(\rho)=H+\kappa\rho \quad \text{if } 0<\rho<\tfrac{1}{2}$$
$$=H \quad \text{if } \tfrac{1}{2}<\rho<1. \tag{21}$$

Let us now integrate Eq. (16) from $\frac{1}{2}-\epsilon$ to $\frac{1}{2}+\epsilon$, $\epsilon$ being a positive number tending to zero. Using the *ansatz* of Eq. (18) we get the boundary condition at $\rho=\frac{1}{2}$:

$$\Gamma h(\tfrac{1}{2}+)-\Gamma h(\tfrac{1}{2}-)=2\omega Q(\tfrac{1}{2}), \tag{22}$$

with $\omega=d^2\psi_0^2/\lambda^2$. We combine now Eqs. (19), (21), and (22) to get

$$\Gamma\kappa=\frac{3\omega(8-\Gamma H)}{6+\omega}. \tag{23}$$

Hence

$$\Gamma h(\tfrac{1}{2}-)=\Gamma H+\frac{\Gamma\kappa}{2}=\frac{\Gamma H(12-\omega)+24\omega}{12+2\omega}, \tag{24}$$

while $h(\frac{1}{2}+)=H$. Hence $h$ changes discontinuously across the superconducting layer. A similar change was presented in Ref. 2.

We now note that $h(\frac{1}{2}-)$ will be negative if and only if $\omega>12$ and $\Gamma H>24\omega/(\omega-12)$. Hence this naive model shows that the field $h$ of the vortex will be inverted if the external field $H$ is sufficiently high, and if $d/\lambda$ is sufficiently large.

We shall now obtain further results with the aid of numerical work done with the choice

$$\alpha(z)=1-\sum_n ae^{-b(z-n-1/2)^2}. \tag{25}$$

We shall assume that there is a vortex at $z=0$, and another at $z=2$. The superconducting layers are at $z=\pm\frac{1}{2}$, $z=\pm\frac{3}{2}$, $z=\pm\frac{5}{2}$, etc. We have solved Eqs. (15) and (17) numerically along the $z$ axis, in the interval [0,1], assuming solutions $\psi(\rho)$ and $Q(\rho)$ with $\rho\approx|z|$ and $\phi\approx\pi/2$. Since $\Gamma$ is large, our solutions are valid away from the $z$ axis as well, in regions with $y\ll\Gamma$. The boundary conditions are, at the center of the vortex, $\psi(0)=0$, $Q(0)=-1$, as mentioned earlier, while $\psi(1)=0$, $h(1)=H$ at the edge of the cell.

We repeat here that the boundary conditions at the origin are actually the boundary conditions arising from Eqs. (13) and (14), since Eqs. (15) and (17) are valid in $[L/\Gamma,1]$ *only*. If $L$ is much smaller than $\Gamma$ though, we may consider them as appropriate for Eqs. (15) and (17).

We find that for sufficiently high $d/\lambda$, and for sufficiently large $H$, the field is inverted along a substantial interval, but definitely before $\frac{1}{2}$. This is happening because the slope of $h$ is very large when $d/\lambda$ and $H$ are large, and hence $h$ drops so low that it can become negative. We see also that the field is practically zero along the superconducting layer, giving thus an almost ideal Meissner effect there. After crossing the layer, the field $h$ rises again, till it reaches the value $H$. In this final interval beyond $\rho=\frac{1}{2}$ we seem to have the normal state. The behavior described here can be seen in Figs. 3–7.

The curves in Figs. 3 and 4 were obtained for the input parameters $\Gamma=100$, $H=2$, $\nu=0.0001$, $a=2.5$, $b=10$, and $d/\lambda=40$, and they correspond to $L=2.2$, $L/\Gamma=0.022$. The solution holds in $[L/\Gamma,1]$ strictly, so $h$ has been extended towards the origin in Figs. 3(a), 5, and 7, using the fact that it has zero slope there. The curves in Figs. 5 and 6 were obtained for the input parameters $\Gamma=100$, $H=2$, $\nu=0.0001$, $a=2.5$, $b=10$, and $d/\lambda=200$, and they correspond to $L=2.8$, $L/\Gamma=0.028$. Finally, the curve in Fig. 7 has the input parameters $\Gamma=50$, $H=15$, $\nu=0.0001$, $a=2.5$, $b=10$, and $d/\lambda=40$, and it corresponds to $L/\Gamma=0.006$.

Note that the field inversion and the situation described by these figures, where the vortex at $z=0$ and the normal state at $z=1$ are connected through a superconducting region, were also discussed recently[5] in the context of ordinary superconductors in high fields. It should be noted furthermore that the above figures were also obtained by imposing the boundary conditions $\psi(0)=\psi(1)=0$, $Q(0)=-1$, and $Q(1)=q$, finding the solution and the corresponding Gibbs free energy, and then finding which $Q(1)$ gave the minimum

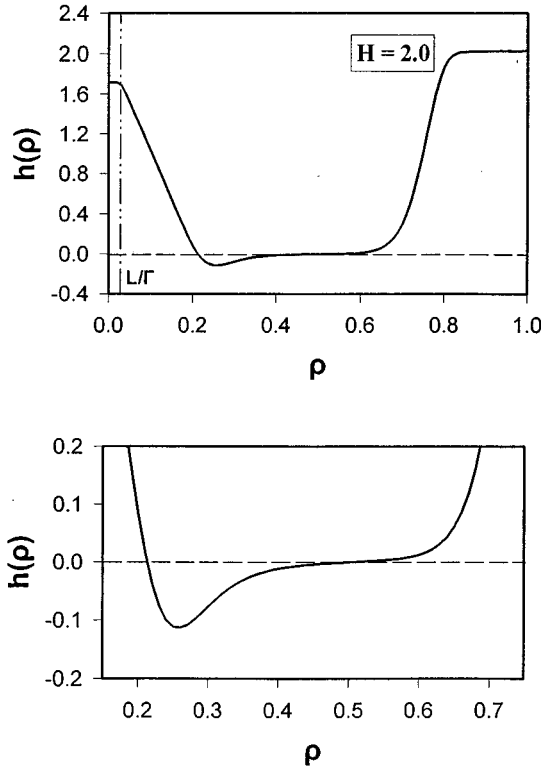


FIG. 3. (a) The field $h$ as a function of $\rho$, for $H=2$, $\Gamma=100$, $a=2.5$, $b=10$, $d/\lambda=40$, and $\nu=0.0001$. (b) Detail of this graph.

Gibbs free energy. We thus confirmed that the boundary condition $h(1)=H$ arises naturally from the minimization of the Gibbs free energy.

As $H$ goes up, $Q(1)$ and $h(1)$ will increase. Thus the normal state at the cell boundaries gradually proceeds inwards, until $Q(1)$ becomes sufficiently large to destroy the superconductivity on the layers.

It should be mentioned that the above conclusions do not depend on the details of $\alpha(z)$. They depend on the fact that

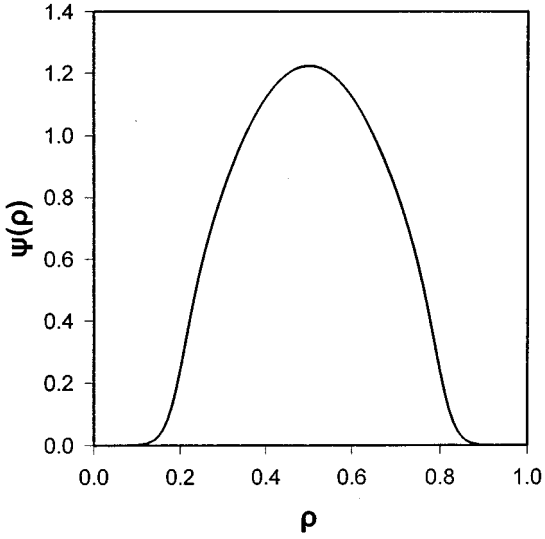


FIG. 4. The order parameter amplitude $\psi$ as a function of $\rho$, for the parameters of Fig. 3.

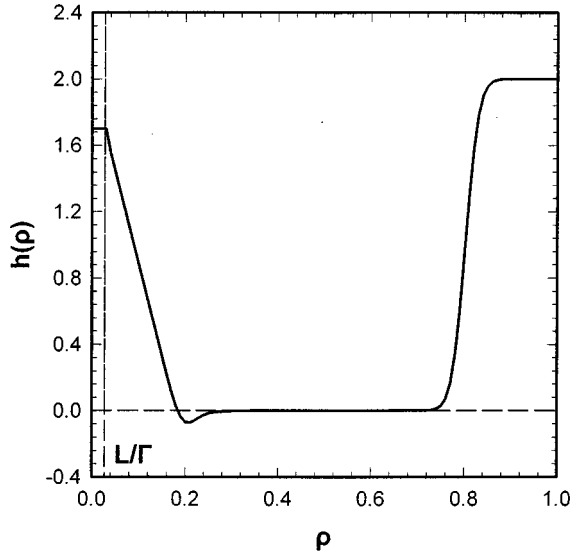


FIG. 5. The field $h$ as a function of $\rho$, for $H=2$, $\Gamma=100$, $a=2.5$, $b=10$, $d/\lambda=200$, and $\nu=0.0001$.

$\alpha(z)$ is even and periodic, with extrema at 0 and 1, and a layer at $\frac{1}{2}$. Nor do they involve the $y$ distance between the vortices, since $\Gamma$ is large.

We also note that the inversion results from the combination of two factors: the large negative slope of $h$ at the origin, due to the large values of $d/\lambda$ and $H$, and the expulsion of the field from the layer. It should be therefore quite independent of the boundary conditions at $z=1$. Indeed the equation for $h$ has an attractive fixed point at $h=0$. This is shown clearly in Figs. 3, 5, and 7. We see that the field becomes zero the moment it starts crossing into the layer, maintaining this value up to the point when it starts exiting the layer. We have thus a perfect Meissner effect inside the layer. The inversion occurs around the point where the field starts entering the layer. It decreases linearly near the origin, with a large negative slope. This large slope makes it overshoot past zero, before turning upwards again, towards the zero value of the Meissner state.

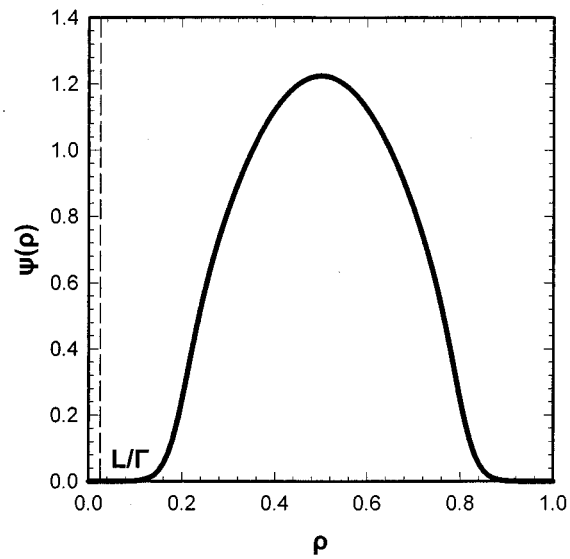


FIG. 6. The order parameter amplitude $\psi$ as a function of $\rho$, for the parameters of Fig. 5.

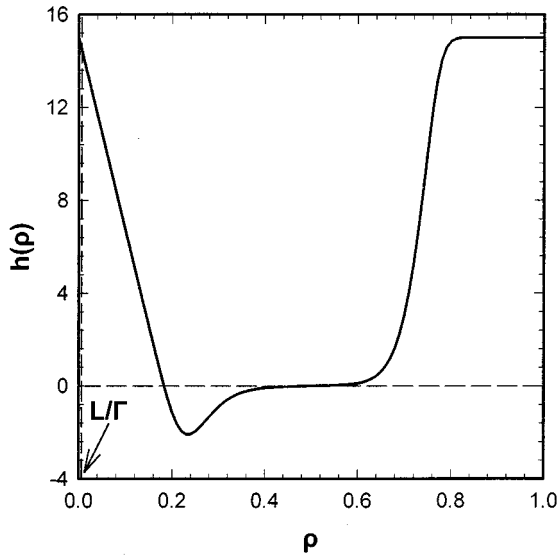


FIG. 7. The field $h$ as a function of $\rho$, for $H=15$, $\Gamma=50$, $a=2.5$, $b=10$, $d/\lambda=40$, and $\nu=0.0001$.

We have checked that the inversion is independent of the boundary conditions at $z=1$ by solving Eqs. (15) and (17) from $z=0$ to $z=\frac{1}{2}$, subject to the boundary conditions $Q(0)=-1$, $\psi(0)=0$, and $\partial h/\partial\rho=0$, $\partial\psi/\partial\rho=0$ at $\rho=\frac{1}{2}$. We choose $\Gamma=50$, $H=50$, $\nu=0.0001$, $a=2.5$, $b=10$, and $d/\lambda=40$. The result, plotted in Fig. 8, shows again the inversion and the attractive fixed point at $h=0$.

We have shown then that in highly anisotropic multilayers, and for fields parallel to the layers, the field is inverted away from the center of a vortex, as long as $d\gg\lambda$, i.e., as long as the vector potential is able to perform the necessary variations within the interlayer space, and as long as $H$ is

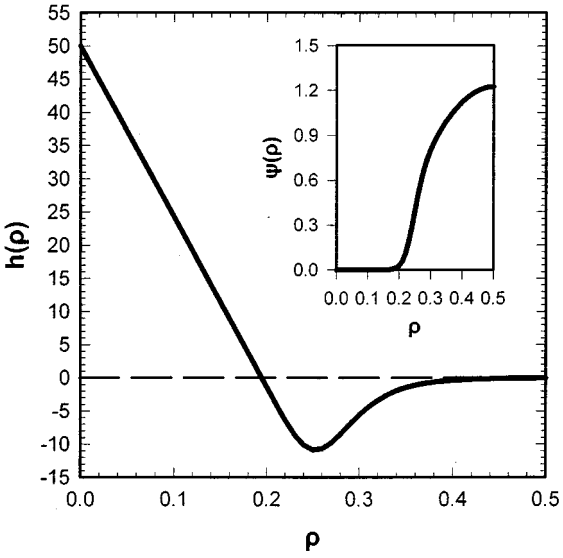


FIG. 8. The field $h$ as a function of $\rho$, for $H=50$, $\Gamma=50$, $a=2.5$, $b=10$, $d/\lambda=40$, and $\nu=0.0001$, obtained with the boundary conditions $\partial\psi/\partial\rho=0$ and $\partial h/\partial\rho=0$ at $\rho=\frac{1}{2}$.

large. This result is insensitive to the boundary conditions at the edge.

The field inversion discussed here could be seen experimentally in highly anisotropic multilayers, provided the layers are very far apart ($d \gg \lambda$), at high fields. Construction of such multilayers would be especially interesting because it would be one of the few instances where the magnetic field would have ample space for varying. In all superconductors examined so far, even the high-$T_c$ ones, the penetration depth is much longer than the interlayer distance.

A field inversion has already been presented in Ref. 3. However it was derived using a discretized model. The variation of $\Psi$ along the $z$ axis, which is essential for the inversion to occur, is introduced in that work by having inequivalent layers, with various constant order-parameter amplitudes. In our work the variation of $\Psi$ arises naturally from $\alpha(z)$, which creates the undulating structure. Furthermore, the result of Ref. 3 was derived near $H_{c1}$, and it does not assume that $d > \lambda$. Thus the inversion in Ref. 3 occurs mostly along the $y$ axis, unlike our inversion.

A field inversion has been also presented in Ref. 6. It applies however to nonlayered anisotropic materials, and to tilted fields. Furthermore, the $\lambda$ there is large, unlike our $\lambda$.

Our field inversion occurs between the vortex and the normal state. It might occur though, for sufficiently large $d/\lambda$, even earlier, when the elliptical cells just start touching. Thus, as the ellipse begins its truncation along the $y$ axis, it may be preferable for the field at the edges of the elliptic cell to become negative. Then the cell could be truncated along the $y$ axis without changing its flux. This field inversion would make possible a drastic reduction of the cell, without affecting the flux of each cell. Indeed, the flux of the cell would increase if we were to cut out some of the cell on the right and on the left, since the field would be negative in those pieces. We would then need to cut more on the left and on the right, pieces with positive $h$ now, in order to bring the flux down to its quantized value. Thus the cell would be truncated considerably without affecting its flux. If such a field inversion at the edges of the cell were to occur, then we would have neighboring vortices with an inverted field in between. In such a case though, the lattice would be an array of parallel chains of vortices, and each chain would have a vortex in every interlayer spacing. The existence of such chains would require an attraction between the vortices along the $z$ direction, an attraction that would be made possible precisely because of the field inversion.

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