

Exact solution of the open Heisenberg chain with two impurities

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We propose an integrable model of the spin-1/2 Heisenberg chain coupled to two impurity moments. With the open boundary conditions at the impurity sites, the model can be exactly solved for arbitrary impurity spin and arbitrary exchange constants between the bulk and the impurities. The absence of redundant terms in the Hamiltonian makes the model very reasonable. The Hamiltonian is diagonalized via algebraic Bethe ansatz. It is found that the impurity spins can only be screened (partially for $S > 1/2$) for antiferromagnetic coupling between the impurity and the bulk. Otherwise the impurity spins cannot be screened. The residual entropy of the ground state and the Kondo temperature are also derived explicitly based on the Bethe ansatz and the local Fermi-liquid theory. [S0163-1829(97)00345-7]

I. INTRODUCTION

Recently, considerable attention has been focused on the problem of impurities embedded in a quantum chain. Using simpler bosonization and renormalization-group techniques, Kane and Fisher have shown that a potential scattering center in a Luttinger liquid is driven to a strong-coupling fixed point by the repulsive interaction in the bulk.¹ This work showed that a single impurity in a one-dimensional quantum system behaves rather different from that in a Fermi liquid, and directly stimulated the study on the problem of local perturbations to a Luttinger liquid and especially on the Kondo problem in a Luttinger liquid.²⁻⁵ It is well known now that the spin dynamics of the Kondo problem is equivalent to that of a spin chain with an impurity.⁶

The integrable impurity problem of the Heisenberg chain with periodic boundary condition was first considered by Andrei and Johannesson.⁷ They studied the integrable case of a spin $S > 1/2$ embedded in a spin-1/2 Heisenberg chain. Subsequently, the problem was generalized to arbitrary spins by Lee, Schlottmann, and Schlottmann.⁸ Now the quantum inverse scattering method becomes a standard method to construct integrable impurity models.⁹ A recent example is the integrable impurity problem in the supersymmetric t - J model.¹⁰ In the QISM, the Hamiltonian of the model is usually written as the logarithmic derivative of a homogeneous transfer matrix at a special value of the spectral parameter. Inputting some inhomogeneous vertex matrices (provided these matrices satisfy the same Yang-Baxter relation of the homogeneous ones), then we obtain an inhomogeneous transfer matrix. Its logarithmic derivative at some special value of the spectral parameter thus gives a Hamiltonian with local interactions. However, to construct an integrable impurity model with periodic boundary condition, there is a prize to pay, namely some unphysical terms must be present in the Hamiltonian, though they may be irrelevant. Their presence is required by the integrability.

On the other hand, the open boundary problem for the quantum chain has been renewed due to Kane and Fisher's observation.¹ It is found that the open boundary theory is very useful to formulate both the thermodynamics and the transport properties of the quantum chains with

impurity.¹¹⁻¹³ The open boundary problem of the integrable models was first considered by Gaudin,¹⁴ who studied the nonlinear Schrödinger model and the spin-1/2 Heisenberg chain with simple open boundaries. Subsequently, his method was generalized to the Hubbard model by Schulz¹⁵ and the spin chain with boundary fields by Alcaraz *et al.*¹⁶ In addition, Sklyannin formulated the same result based on the QISM. In his theory, a new relation which now is called as the reflecting Yang-Baxter relation¹⁷ was used.

In this paper, we study the problem of an open spin-1/2 Heisenberg chain coupled with two impurity spins sited at the ends of the system. The Hamiltonian we shall consider reads

$$H = \frac{1}{2} J \sum_{j=1}^{N-1} \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + J_R \vec{\sigma}_N \cdot \vec{S}_R + J_L \vec{\sigma}_1 \cdot \vec{S}_L, \quad (1)$$

where $\vec{\sigma}_j$ are the Pauli matrices; $\vec{S}_{R,L}$ are the impurity moments with an arbitrary spin S ; N is the site number of the bulk; J is a positive constant and $J_{R,L}$ are two arbitrary real constants which describe the coupling between the bulk and the impurities. We remark that the present model is very reasonable for the absence of redundant terms which cannot be justified on physical grounds.

The structure of the present paper is the following: In the subsequent section, we construct the transfer matrix corresponding to the Hamiltonian (1), thus the integrability of the present model can be directly justified. Based on the QISM, the Bethe ansatz equation and the eigenvalue of the Hamiltonian will be derived. In Sec. III, the ground state properties for different regions of parameter $J_{R,L}$ will be discussed. Section IV is attributed to the residual entropy of the ground state and the low temperature specific heat. Concluding remarks will be given in Sec. V.

II. ALGEBRAIC BETHE ANSATZ

In the framework of QISM, the integrable Hamiltonian with the open boundary condition is usually obtained from a monodromy matrix¹⁷

$$U(\lambda) = K_-(\lambda) T(\lambda) K_+(\lambda) T^{-1}(-\lambda), \quad (2)$$

where $K_{\pm}(\lambda)$ are the reflecting matrices which satisfy the reflecting Yang-Baxter relation

$$S_{12}(\lambda - \mu)K_{\pm}^1(\lambda)S_{12}(\lambda + \mu)K_{\pm}^2(\mu) = K_{\pm}^2(\mu)S_{12}(\lambda + \mu)K_{\pm}^1(\lambda)S_{12}(\lambda - \mu), \quad (3)$$

with $K_{\pm}^{1,2}$ and S_{12} acting on the space $V_{1,2}$ and $V_1 \otimes V_2$, respectively. S_{12} is the scattering matrix which satisfy the traditional Yang-Baxter relation.¹⁸ $T(\lambda)$ is the monodromy matrix for the periodic system which satisfies the Yang-Baxter relation

$$S_{12}(\lambda - \mu)T_1(\lambda)T_2(\mu) = T_2(\mu)T_1(\lambda)S_{12}(\lambda - \mu). \quad (4)$$

As demonstrated by Sklyannin,¹⁷ $U(\lambda)$ satisfies the same reflecting Yang-Baxter relation (3) as K_{\pm} does. Thus the trace of $U(\lambda)$ gives an infinite number of conserved quantities. We remark that Sklyannin and many other authors used c number $K_{\pm}(\lambda)$ to construct their models, where $K_{\pm}(\lambda)$ only induce the boundary fields. The operator ones were first used by the present author and co-workers⁵ to study the Kondo problem in one-dimensional strongly correlated electron systems.

To construct the algebraic Bethe ansatz of the present model, we define

$$T_{\tau}(\lambda) = L_{R\tau}^+(\lambda)L_{N\tau}(\lambda)L_{N-1\tau}(\lambda) \cdots L_{1\tau}(\lambda), \\ \tilde{T}_{\tau}(\lambda) = L_{1\tau}(\lambda)L_{2\tau}(\lambda) \cdots L_{N\tau}(\lambda)L_{R\tau}^-(\lambda), \quad (5)$$

where $L_{j\tau}(\lambda) = i\lambda + 1/2(1 + \vec{\sigma}_j \cdot \vec{\tau})$, and $L_{R\tau}^{\pm}(\lambda) = i\lambda \pm c_R + \frac{1}{2} + \vec{\tau} \cdot \vec{S}_R$ with $\vec{\tau}$ an auxiliary Pauli matrix. Furthermore, we put

$$K_-(\lambda) = 1, \\ K_+(\lambda) = \left(i\lambda + c_L + \frac{1}{2} + \vec{\tau} \cdot \vec{S}_L \right) \left(i\lambda - c_L + \frac{1}{2} + \vec{\tau} \cdot \vec{S}_L \right), \quad (6)$$

where $c_{R,L}$ are two constants. Obviously, $K_{\pm}(\lambda)$ defined in Eq. (6) satisfy the reflecting relation (3). The monodromy matrix $U(\lambda)$ is defined for our model as

$$U_{\tau}(\lambda) = K_-(\lambda)T_{\tau}(\lambda)K_+(\lambda)\tilde{T}_{\tau}(\lambda), \quad (7)$$

which satisfy the reflecting equation

$$S_{12}(\lambda - \mu)U_{\tau_1}(\lambda)S_{12}(\lambda + \mu)U_{\tau_2}(\mu) = U_{\tau_2}(\mu)S_{12}(\lambda + \mu)U_{\tau_1}(\lambda)S_{12}(\lambda - \mu), \quad (8)$$

with $S_{12}(\lambda \pm \mu) = L_{\tau_1\tau_2}(\lambda \pm \mu)$. The Hamiltonian (1) is obtained by the following relation

$$H = \frac{-iJ}{4 \prod_{r=R,L} [(S+1/2)^2 - c_r^2]} \frac{dX(\lambda)}{d\lambda} \Bigg|_{\lambda=0} - \frac{N}{2}J \\ - \frac{1}{2} \sum_{r=R,L} \frac{J}{(S+1/2)^2 - c_r^2}, \quad (9)$$

where $X(\lambda) = \text{tr}_{\tau} U_{\tau}(\lambda)$ and $J_{R,L}$ are parametrized by $c_{R,L}$ as $J_{R,L} = J/[(S+1/2)^2 - c_{R,L}^2]$.

Although the model can be solved for arbitrary $J_{R,L}$, we consider only the $c_R = c_L = c$ ($J_R = J_L = J_i$) case in this paper. The general case can be formulated following the same procedure without any difficulty. We introduce the notation

$$U_{\tau}(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}.$$

Some useful commutation relation can be formulated from Eq. (8) as

$$[A(\lambda), A(\mu)] = [A(\lambda), D(\mu)] = [B(\lambda), B(\mu)] = 0,$$

$$A(\lambda)B(\mu) = \frac{(\lambda + \mu)(\lambda - \mu + i)}{(\lambda - \mu)(\lambda + \mu - i)} B(\mu)A(\lambda) \\ - \frac{2i\mu}{(\lambda - \mu)(2\mu - i)} B(\lambda)A(\mu) \\ + \frac{i}{(\lambda + \mu - i)(2\mu - i)} B(\lambda)\bar{D}(\mu), \quad (10) \\ \bar{D}(\lambda)B(\mu) = \frac{(\lambda - \mu - i)(\lambda + \mu - 2i)}{(\lambda - \mu)(\lambda + \mu - i)} B(\mu)\bar{D}(\lambda) \\ + \frac{2i(\lambda - i)}{(\lambda - \mu)(2\mu - i)} B(\lambda)\bar{D}(\mu) \\ - \frac{4i(\lambda - i)\mu}{(2\mu - i)(\lambda + \mu - i)} B(\lambda)A(\mu),$$

where $\bar{D}(\lambda)$ is defined as $\bar{D}(\lambda) = (2\lambda - i)D(\lambda) + iA(\lambda)$. Therefore, the trace of $U_{\tau}(\lambda)$ can be expressed as

$$X(\lambda) \equiv \text{Tr}_{\tau} U_{\tau}(\lambda) = \frac{1}{2\lambda - i} \bar{D}(\lambda) + \frac{2\lambda - 2i}{2\lambda - i} A(\lambda). \quad (11)$$

Define the pseudovacuum state $|0\rangle$ as

$$\sigma_j^+ |0\rangle = S^+ |0\rangle = 0. \quad (12)$$

The elements of $U_{\tau}(\lambda)$ acting on the pseudovacuum state behave as

$$C(\lambda)|0\rangle = 0,$$

$$A(\lambda)|0\rangle = a(\lambda)|0\rangle = \left[\left(i\lambda + S + \frac{1}{2} \right)^2 - c^2 \right]^2 (i\lambda + 1)^{2N} |0\rangle, \quad (13)$$

$$\bar{D}(\lambda)|0\rangle = \bar{d}(\lambda)|0\rangle = (i\lambda)^{2N} \left[\left(i\lambda - S + \frac{1}{2} \right)^2 - c^2 \right]^2 |0\rangle.$$

Therefore, $X(\lambda)$ can be treated as the generating functional of an infinite number of conserved quantities (including the Hamiltonian) and $B(\lambda)$ is the creation operator of their eigenstates. An eigenstate of $X(\lambda)$ with M spins down can be constructed as

$$|\Omega\rangle = \prod_{j=1}^M B(\lambda_j)|0\rangle. \quad (14)$$

With the relations (10) and (13), we obtain the eigenvalue of $X(\lambda)$ acting on the state $|\Omega\rangle$ as

$$\begin{aligned} X(\lambda)|\Omega\rangle &= \Lambda(\lambda; \lambda_1, \dots, \lambda_M)|\Omega\rangle, \\ \Lambda(\lambda; \lambda_1, \dots, \lambda_M) &= \frac{\bar{d}(\lambda)}{2\lambda - i} \prod_{j=1}^M \frac{(\lambda - \lambda_j - i)(\lambda + \lambda_j - 2i)}{(\lambda - \lambda_j)(\lambda + \lambda_j - i)} \\ &\quad + \frac{2(\lambda - i)a(\lambda)}{2\lambda - i} \prod_{j=1}^M \\ &\quad \times \frac{(\lambda + \lambda_j)(\lambda - \lambda_j + i)}{(\lambda + \lambda_j - i)(\lambda - \lambda_j)}. \end{aligned} \quad (15)$$

However, the spectral parameters λ_j are not independent of each other but satisfy the following Bethe ansatz equation:

$$\begin{aligned} \left(\frac{\lambda_j + i/2}{\lambda_j - i/2}\right)^{2N} \left(\frac{\lambda_j - ic + iS}{\lambda_j - ic - iS}\right)^2 \left(\frac{\lambda_j + ic + iS}{\lambda_j + ic - iS}\right)^2 \\ = \prod_{r=\pm 1} \prod_{l \neq j}^M \frac{\lambda_j - r\lambda_l + i}{\lambda_j - r\lambda_l - i}. \end{aligned} \quad (16)$$

The eigenvalue of the Hamiltonian (1) acting on the state $|\Omega\rangle$ can be obtained from Eqs. (9) and (15) as

$$E(\lambda_1, \dots, \lambda_M) = - \sum_{j=1}^M \frac{J}{\lambda_j^2 + 1/4} + \frac{1}{2}(N-1)J + 2SJ_i. \quad (17)$$

III. GROUND STATE

Since the parameter J_i depends only on c^2 , c may take either real or imaginary values. In this section, we discuss the ground state properties of different c values.

(i) $c > S + 1/2$ case. In this case, the coupling between the bulk and the impurity is ferromagnetic. All the λ modes take real values in the ground state to minimize the energy. Taking the logarithm of Eq. (16) we obtain

$$\left(\frac{\lambda_j + i/2}{\lambda_j - i/2}\right)^{2N} \left(\frac{\lambda_j - ic + iS}{\lambda_j - ic - iS}\right)^2 \left(\frac{\lambda_j + ic + iS}{\lambda_j + ic - iS}\right)^2 = \left(\frac{\lambda_j - i|c| + iS + i}{\lambda_j - i|c| + iS - i}\right)^2 \left(\frac{\lambda_j + i|c| - iS + i}{\lambda_j + i|c| - iS - i}\right)^2 \prod_{r=\pm 1} \prod_{l \neq j}^{M-2} \frac{\lambda_j - r\lambda_l + i}{\lambda_j - r\lambda_l - i}. \quad (22)$$

With the same procedure discussed in case (i), we obtain $M = (N+2)/2$ or the residual magnetization of the ground state $M_g = 2S - 1$ as expected. This means the impurity moment is partially screened, a similar result to that of the Kondo problem.⁶

(iii) $0 < |c| < S$. In this case, the system also falls into the regime of antiferromagnetic coupling ($J_i > 0$). No bound state can exist in the ground state. The function $Z_N(\lambda)$ can be defined in a similar way to case (i) but with a different $\phi(\lambda) = 2\theta_{2(S+c)}(\lambda) + 2\theta_{2(S-c)}(\lambda) + \theta_2(\lambda) + \theta_1(\lambda)$. With the condition $Z_N(\pm\infty) = \pm(M+1/2)/N$ we have again $M = (N+2)/2$ and the residual magnetization $M_g = 2S - 1$.

(iv) When c takes an imaginary value, J_i is always positive. Suppose $c = ib$. The Bethe ansatz equation (15) then becomes

$$\left(\frac{\lambda_j + i/2}{\lambda_j - i/2}\right)^{2N} \left(\frac{\lambda_j - b + iS}{\lambda_j - b - iS}\right)^2 \left(\frac{\lambda_j + b + iS}{\lambda_j + b - iS}\right)^2 = \prod_{r=\pm 1} \prod_{l \neq j}^M \frac{\lambda_j - r\lambda_l + i}{\lambda_j - r\lambda_l - i}. \quad (23)$$

$$\frac{I_j}{N} = \frac{1}{\pi} \left[\theta_1(\lambda_j) + \frac{1}{2N} \left(\phi(\lambda_j) - \sum_{l=-M}^M \theta_2(\lambda_j - \lambda_l) \right) \right], \quad (18)$$

where $\theta_n(\lambda) = 2 \tan^{-1}(2\lambda/n)$, $\phi(\lambda) = 2\theta_{2(S+|c|)}(\lambda) - 2\theta_{2(|c|-S)}(\lambda) + \theta_2(\lambda) + \theta_1(\lambda)$, I_j are some integers. Note above we have included the negative modes by putting $\lambda_j = -\lambda_{-j}$ ($\lambda_0 = 0$). Define

$$Z_N(\lambda) = \frac{1}{\pi} \left[\theta_1(\lambda) + \frac{1}{2N} \left(\phi(\lambda) - \sum_{l=-M}^M \theta_2(\lambda - \lambda_l) \right) \right]. \quad (19)$$

Then $Z_N(\lambda_j) = I_j/N$ gives the Bethe ansatz equation (18). For the ground state, I_j must take consecutive numbers around zero symmetrically. A density function for the ground state in the thermodynamic limit can be defined as

$$\rho_N(\lambda) = \frac{dZ_N(\lambda)}{d\lambda}, \quad (20)$$

with the condition $\int_{-\Lambda}^{\Lambda} \rho_N(\lambda) d\lambda = (2M+1)/N$, where Λ is the cutoff of λ modes. As discussed in many earlier papers,¹⁹ the eigenenergy is minimized at $\Lambda = \infty$ up to the order $O(N^{-2})$. With this condition, we obtain that $M = N/2$, which gives the self-magnetization of the ground state as

$$M_g = \frac{1}{2}N - M + 2S = 2S. \quad (21)$$

Such a result indicates that the impurity moments cannot be screened due to the ferromagnetic coupling between the bulk and the impurity.

(ii) $S < |c| < S + 1/2$. In this case, $J_i > 0$ and thus the exchange interaction between the impurity and the bulk falls into the antiferromagnetic regime. Two imaginary modes of λ at $\lambda = i(|c| - S)$ can exist in the ground state. Note this mode carries energy $\epsilon_i = -1/[1/4 - (|c| - S)^2]$, which is smaller than those carried by any real modes. The real modes thus satisfy the following Bethe ansatz equation:

For the ground state, all λ take real values and their cutoff Λ still tends to infinity in the thermodynamic limit. As discussed in case (i), a similar function $Z_N(\lambda)$ can be defined but with a different $\phi(\lambda) = 2\theta_{2S}(\lambda - b) + 2\theta_{2S}(\lambda + b) + \theta_2(\lambda) + \theta_1(\lambda)$. Therefore $M = (N + 2)/2$ for the ground state, which still leaves a residual magnetization of $2S - 1$.

From the above discussion we conclude that the impurities can be screened (partially for $S > 1/2$) only in the case where $J_i > 0$. The impurity moments cannot be screened when $J_i < 0$, unlike the situation for the Kondo problem in a Luttinger liquid predicted by Furusaki and Nagaosa.³

IV. RESIDUAL ENTROPY AND LOW-TEMPERATURE SPECIFIC HEAT

The thermodynamics of the present model can be constructed with the standard method proposed by Yang and Yang²⁰ based on the string hypothesis.²¹ Here we omit the details which can be found in some nice reviews.²² An interesting feature of our model is that the residual entropy S_g may take different values depending on the bond deformation between the bulk and the impurities:

$$S_g = \begin{cases} \ln \frac{[2|c|] + 2S}{[2|c|] - 2S} & \text{if } |c| > S + \frac{1}{2}, \\ \ln \frac{(2S + [2|c|])(2 + [2|c|] - 2S)}{([2|c|] - 2S)(2 - [2|c|] + 2S)} & \text{if } S + \frac{1}{2} > |c| > S, \\ 2 \ln \sqrt{4S^2 - [2|c|]^2} & \text{if } S > |c| > 0, \\ 2 \ln(2S) & \text{if } c \text{ is imaginary,} \end{cases}$$

where $[2|c|]$ is the maximum integer equal or less than $2|c|$. For $|c| < 1/2$ or imaginary and $S = 1/2$, $S_g = 0$, which means the impurity spin can be completely screened in the ground state and thus the system flows to a local Fermi-liquid fixed point at low energy scales. The low temperature thermodynamics can be formulated based on the local Fermi liquid theory for the Kondo problem.²³ Since the spectrum are described by the quantum numbers I_j , $p_N(\lambda_j) = \pi Z_N(\lambda_j)$ can be treated as the momenta of the quasiparticles in the Luttinger-Fermi liquid picture for the integrable models.²⁴ In the thermodynamic limit, the ground state energy can be expressed as

$$E_g = \frac{1}{2} N \int \epsilon_0(\lambda) \rho_N(\lambda) d\lambda + \text{const}, \quad (24)$$

up to the order of $O(N^{-2})$, where $\epsilon_0(\lambda) = -J/(\lambda^2 + 1/4)$. From the definition of $\rho_N(\lambda)$ in Eq. (20), we can rewrite Eq. (24) as

$$E_g = \frac{N}{4\pi} \int \epsilon(\lambda) \rho_N^{(0)}(\lambda) d\lambda + \text{const}, \quad (25)$$

where $\epsilon(\lambda) = -\pi J / \cosh(\pi\lambda)$ is the dressed energy function, which can be treated as the energy of the ‘‘quasiparticles.’’ Note that $\rho_N(\lambda)$ for c imaginary can be solved up to $O(N^{-1})$ as

$$\rho_N(\lambda) = \rho_0(\lambda) + \frac{1}{N} \rho_i(\lambda) + \frac{1}{N} \rho_b(\lambda),$$

$$\rho_0(\lambda) = \frac{1}{\cosh(\pi\lambda)}, \quad (26)$$

$$\rho_b(\lambda) = \frac{1}{2 \cosh(\pi\lambda)} + \int \frac{e^{-(1/2)|\omega|} e^{-i\lambda\omega}}{4\pi \cosh[(1/2)\omega]} d\omega,$$

$$\rho_i(\lambda) = \frac{1}{\cosh(\lambda - b)} + \frac{1}{\cosh(\lambda + b)},$$

where $\rho_0(\lambda)$, $\rho_i(\lambda)/N$, and $\rho_b(\lambda)/N$ are the contributions of the bulk, the impurity, and the open boundary to the density, respectively. The density of states at the Fermi surface can be determined in the Fermi-liquid picture as

$$N(0) = \frac{1}{\pi} \left. \frac{dp_N(\lambda)}{d\epsilon(\lambda)} \right|_{\lambda=\infty} = \frac{\rho_N(\lambda)}{\epsilon'(\lambda)} \Big|_{\lambda=\infty}. \quad (27)$$

Therefore, the impurity contribution to the low temperature specific heat reads

$$C_i = \frac{2\pi}{3NJ} \cosh(\pi b) T, \quad (28)$$

while for $|c| < 1/2$, $\rho_i(\lambda)$ can be solved by a similar way as

$$\rho_i(\lambda) = \frac{1}{\cosh(\lambda - ic)} + \frac{1}{\cosh(\lambda + ic)}. \quad (29)$$

Thus the impurity specific heat reads

$$C_i = \frac{2\pi}{3NJ} \cos(\pi c) T. \quad (30)$$

Notice that the Kondo temperature is nothing but the effective Fermi temperature in the local Fermi-liquid theory. Therefore, we conclude that the Kondo temperature for the cases discussed above is given by

$$T_k = \pi J \cos^{-1}(\pi c). \quad (31)$$

Such a result directly shows the crossover from an exponential law to an algebraic one when c goes from imaginary to real as pointed out earlier by Lee and Toner.² Notice that $c = \sqrt{1 - J/J_i}$ and for the Kondo problem in a Hubbard chain, the bandwidth of the spinons (proportional to J) is about $4t^2/U$. Where $2t$ is the bandwidth of the fermions, U is the on-site Coulomb repulsion. For the strong correlation limit $U \gg t$, $J \leq J_i$, the Kondo temperature shows a power law of $t^2/(UJ_i)$. While for the weak correlation limit $U \leq t$, $J \geq J_i$, the Kondo temperature shows an exponential law of t^2/UJ_i .

V. CONCLUDING REMARKS

In conclusion, we establish an exactly solvable model of Heisenberg chain coupled with two impurity moments. This model is relevant to the Kondo problem in a Luttinger liquid. With the algebraic Bethe ansatz method, the Hamiltonian is exactly diagonalized. It is found that the local moments can be screened (partially for $S > 1/2$) only in the case where the coupling between the bulk and the impurities is antiferromagnetic ($J_i > 0$). Such a result strongly contradicts that of the classical system (Ising model), where the ground state is a pure Néel state with total magnetization of zero (provided

N even) for both $J_i > 0$ and $J_i < 0$ and arbitrary S . The present result shows that the quantum fluctuation plays a crucial role in the one-dimensional quantum system. The residual entropy of the ground state is derived from the thermal Bethe ansatz. It strongly depends on the parameter c , a similar result to that of the Kondo problem.⁵ Based on the local Fermi-liquid theory²³ and the Landau-Luttinger description for the integrable models,²⁴ we derive the low temperature specific heat for some special cases. The Kondo temperatures are exactly carried out for these cases. The exact result does

show the crossover of the Kondo temperature from an exponential law to a power law when the parameter c goes from imaginary to real, a phenomenon obtained by Lee and Toner.²

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