

Spin-density wave in Ising-coupled antiferromagnetic chains

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The effect of anisotropy in the nearest-neighbor spin interactions that couple $N \geq 2$ consecutive spin- $\frac{1}{2}$ antiferromagnetic chains is studied theoretically by considering the limit where the coupling is purely of the Ising type. An analysis based on the equivalent Luttinger model reveals that the ground state is an Ising antiferromagnet in general. [S0163-1829(97)04646-8]

The recent discovery of the ‘‘ladder’’ materials spawned from research in high-temperature superconductivity has renewed interest in the physics of coupled antiferromagnetic chains.¹ The former systems are composed of magnetically isolated ladders of spin- $\frac{1}{2}$ moments that experience an effective exchange interaction between all nearest neighbors. Both experiment and theory find that the $S = \frac{1}{2}$ antiferromagnetic ladder shows a spin gap of order the exchange coupling constant between chains.^{1,2} This result, however, should be compared with that corresponding to a single chain, which exhibits no spin gap.³ In general, an analysis of the weak- and of the strong-coupling limits reveals that no spin gap appears for an odd number of chains.

Deviations from isotropy in the Heisenberg spin coupling that results from the exchange interaction can occur naturally, however, and have interesting theoretical consequences. It is known, for example, that Ising anisotropy produces a spin gap in the case of a single chain.^{4–6} In the present context of coupled antiferromagnetic chains, the following question then arises: What effect does anisotropy in the Heisenberg spin couplings that run *perpendicular* to the spin- $\frac{1}{2}$ chains have on the ground state? In response to this query, we shall study here the low-temperature physics of a finite number N of consecutive spin- $\frac{1}{2}$ XXZ chains³ coupled via a nearest-neighbor Ising interaction. The calculational strategy will be to first transcribe the problem to that of interacting spinless fermions utilizing the Jordan-Wigner transformation, and to then apply the well-known Abelian bosonization technique.^{6,7} On this basis, we arrive at the following conclusions valid for $N \geq 2$ Ising-coupled antiferromagnetic chains: (i) The ground state is in a pinned spin-density wave (SDW) state that is commensurate with the lattice,⁸ and that exhibits a spin gap; (ii) The SDW slides along the direction parallel to the chains in the case where it is incommensurate,⁹ which can result from the application of an external magnetic field.¹⁰ It is noteworthy that the bosonization analysis employed here is closely related to a

fermion analogy for the Lawrence-Doniach model description of layered superconductors.^{11,12} We now pass to the calculations.

The Hamiltonian for N Ising-coupled antiferromagnetic chains that are sequentially ordered can be divided into parallel and perpendicular parts, $H = H_{\parallel} + H_{\perp}$, where

$$H_{\parallel} = \sum_{l=1}^N \sum_i [J_{xy}^{\parallel} (S_{i,l}^x S_{i+1,l}^x + S_{i,l}^y S_{i+1,l}^y) + J_z^{\parallel} S_{i,l}^z S_{i+1,l}^z + h_z S_{i,l}^z] \quad (1)$$

and

$$H_{\perp} = \sum_{l=1}^{N-1} \sum_i J_z^{\perp} S_{i,l}^z S_{i,l+1}^z \quad (2)$$

describe, respectively, the spin couplings within and in between the corresponding XXZ chains. Here, the spin operator $S_{i,l}$ acts on spin- $\frac{1}{2}$ states lying at the i th site of chain l , and h_z denotes an external magnetic field directed along the z axis. We presume an antiferromagnetic sign, $J_{xy}^{\parallel} > 0$, for the intrachain XY coupling, and we shall set $\hbar = 1$ throughout. The Jordan-Wigner transformation^{3,6} then yields a system of ideal spinless fermions, with a degenerate energy spectrum $\varepsilon_k = -J_{xy}^{\parallel} \cos ka$ as a function of momentum k along each chain l , that interact through both the intrachain and the interchain Ising couplings J_z^{\parallel} and J_z^{\perp} . Here, a denotes the parallel lattice constant. In this language, an antiferromagnetic chain then corresponds to a half-filled band, $\varepsilon_k < 0$. Note that interchain XY coupling introduces unwieldy ‘‘string’’ contributions⁶ into the Hamiltonian, which is one reason why it has been omitted. The continuum limit can be taken *à la* Kogut and Susskind under these conditions,⁷ and we thereby obtain the following Luttinger model for the parallel and perpendicular pieces of the Ising-coupled antiferromagnetic chains:

$$H_{\parallel} = \sum_{l=1}^N \int dx [J_{xy}^{\parallel} a(L_l^{\dagger} i \partial_x L_l - R_l^{\dagger} i \partial_x R_l) + 4J_z^{\parallel} a L_l^{\dagger} R_l^{\dagger} R_l L_l + h_z (L_l^{\dagger} L_l + R_l^{\dagger} R_l)] \quad (3)$$

and $H_{\perp} = H_{\perp,1} + H_{\perp,3} + H_{\perp,4}$, with a backscattering term

$$H_{\perp,1} = \sum_{l=1}^{N-1} \int dx J_z^{\perp} a (L_l^{\dagger} R_{l+1}^{\dagger} L_{l+1} R_l + \text{H.c.}), \quad (4)$$

an interchain umklapp term

$$H_{\perp,3} = \sum_{l=1}^{N-1} \int dx J_z^{\perp} a (R_l^{\dagger} R_{l+1}^{\dagger} L_{l+1} L_l e^{i4k_F x} + \text{H.c.}), \quad (5)$$

and an interchain forward-scattering term

$$H_{\perp,4} = \sum_{l=1}^{N-1} \int dx J_z^{\perp} a : (L_l^{\dagger} L_l + R_l^{\dagger} R_l) :: (L_{l+1}^{\dagger} L_{l+1} + R_{l+1}^{\dagger} R_{l+1}) :. \quad (6)$$

Here $e^{-ik_F x} R_l(x)$ and $e^{ik_F x} L_l(x)$ denote field operators for right and left moving spinless fermions that move along the l th chain, with a Fermi surface at $\pm k_F$, while the symbols “: :” represent normal ordering.^{6,7} The intrachain umklapp term^{4,5} that has been omitted from Eq. (3) will be discussed at the end of the paper [see Eq. (15)].

We first consider two chains,¹³ in which case the above Luttinger model can be treated exactly. This is achieved by observing that the chain index can be interpreted as a pseudospin label. Equations (3)–(6) then describe the Luther-Emery model for pseudospin- $\frac{1}{2}$ fermions in this instance.^{14–16} Since such fermions experience pseudospin-charge separation, we have that the coupled chains factorize following $H_{\parallel} + H_{\perp} = H_{\rho} + H_{\sigma}$, where

$$H_{\rho} = 2\pi v_{\rho} \sum_{q>0} \sum_{j=R,L} \rho_j(q) \rho_j(-q) + g_{\rho} \sum_q \rho_R(q) \rho_L(-q) + H_{\perp,3}, \quad (7)$$

$$H_{\sigma} = 2\pi v_{\sigma} \sum_{q>0} \sum_{j=R,L} \sigma_j(q) \sigma_j(-q) + g_{\sigma} \sum_q \sigma_R(q) \sigma_L(-q) + H_{\perp,1} \quad (8)$$

are the respective commuting portions of the Hamiltonian. Here, $\rho_j(q) = 2^{-1/2} [\rho_j(q,1) + \rho_j(q,2)]$ and $\sigma_j(q) = 2^{-1/2} [\rho_j(q,2) - \rho_j(q,1)]$ are the particle-hole operators for pseudocharge and pseudospin excitations, with $\rho_R(q,l) = \sum_k a_l^{\dagger}(q+k) a_l(k)$ and $\rho_L(q,l) = \sum_k b_l^{\dagger}(q+k) b_l(k)$. The operators $a_l(k)$ and $b_l(k)$, respectively, annihilate right and left moving electrons of momentum k on the l th chain. Also, the Fermi velocities and interaction strengths for each component are renormalized by the interchain forward-scattering process (6) to

$$v_{\rho,\sigma} = J_{xy}^{\parallel} \pm J_z^{\perp} / 2\pi, \quad (9)$$

$$g_{\rho,\sigma} = 4J_z^{\parallel} \pm J_z^{\perp}, \quad (10)$$

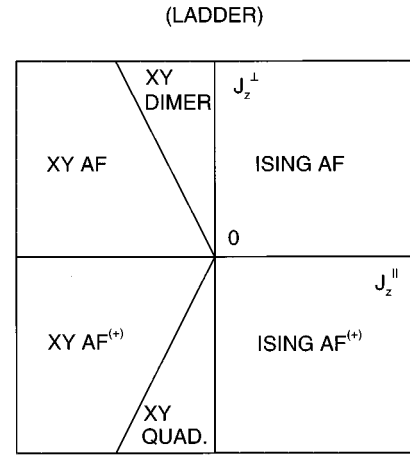


FIG. 1. Shown is the phase diagram for two spin- $\frac{1}{2}$ XXZ chains coupled via a weak Ising interaction (J_z^{\perp}). The Ising antiferromagnetic (AF) regions ($\Delta_{\rho,\sigma} \neq 0$) are characterized by strict long-range spin order along each chain, while the XY AF regions ($\Delta_{\rho,\sigma} = 0$) show dominant XY spin correlations $\langle S_{x,l}^+ S_{0,l}^- \rangle \propto e^{i2k_F x} (\alpha/x)^{(K_{\rho}^{-1} + K_{\sigma}^{-1})/4}$ along each chain l [see Eq. (13)]. The sign of the intrachain Ising coupling (J_z^{\parallel}) switches between these two phases by passing through an intermediate XY dimer phase ($\Delta_{\rho} \neq 0, \Delta_{\sigma} = 0$) characterized by dominant interchain dimer correlations $\langle S_{x,1}^+ S_{x,2}^- S_{0,1}^+ S_{0,2}^- \rangle \propto (\alpha/x)^{K_{\sigma}^{-1}}$ for antiferromagnetic coupling $J_z^{\perp} > 0$. Quadrupolar XY spin correlations $\langle S_{x,1}^+ S_{x,2}^+ S_{0,2}^- S_{0,1}^- \rangle \propto (\alpha/x)^{K_{\rho}^{-1}}$ dominate in the opposing region ($\Delta_{\rho} = 0, \Delta_{\sigma} \neq 0$) at ferromagnetic couplings. Note that the superscript (+) indicates local ferromagnetic order in between chains.

where the $+(-)$ signs above correspond to the $\rho(\sigma)$ label. Application of the bosonic representation⁷

$$R_l(x) \cong (2\pi\alpha)^{-1/2} \exp[i(4\pi)^{1/2} \phi_R(x,l)], \quad (11)$$

$$L_l(x) \cong (2\pi\alpha)^{-1/2} \exp[-i(4\pi)^{1/2} \phi_L(x,l)] \quad (12)$$

for the spinless fermions, where $\phi_j(x,l) = \lim_{\alpha \rightarrow 0} 2\pi L_x^{-1} \sum_q q^{-1} \exp(-\frac{1}{2}\alpha|q| - iqx) \rho_j(q,l)$ are the bosonic fields corresponding to right and left moving fermions ($j=R,L$), reveals that the spectrum of the pseudospin sector (8) has a gap, $\Delta_{\sigma} \neq 0$, for $-g_{\sigma} < |J_z^{\perp}|$.¹⁶ (In general, $\alpha^{-1} \sim a^{-1}$ is the momentum cutoff of the Luttinger model.) At half filling ($h_z = 0$), where $4k_F = 2\pi/a$ and the umklapp process (5) is at work, similar considerations indicate that a gap, $\Delta_{\rho} \neq 0$, opens in the pseudocharge spectrum (7) for $-g_{\rho} < |J_z^{\perp}|$. Comparison of Eq. (10) with these conditions yields the phase diagram shown in Fig. 1 for two Ising-coupled spin- $\frac{1}{2}$ XXZ chains. We remind the reader that the spinless fermions corresponding to the pseudocharge (7) and the pseudospin (8) systems are noninteracting along the respective Luther-Emery lines $g_{\rho} = 6\pi v_{\rho}/5$ and $g_{\sigma} = 6\pi v_{\sigma}/5$, at which point the gaps have value $\Delta_{\rho,\sigma} = (a/\alpha)(|J_z^{\perp}|/2\pi)$. On the other hand, the Coulomb gas analogy¹⁶ as well as mean-field theory¹² indicate that the latter vanish exponentially as the Luttinger-liquid state is approached; e.g., $\Delta_{\rho,\sigma} \sim |J_z^{\perp}| \exp[-2\pi v_{\rho,\sigma} / (|J_z^{\perp}| + g_{\rho,\sigma})]$.

We now address the issue of the physical character of the two-chain system in the case where all spin couplings are antiferromagnetic, which means that $\Delta_{\rho,\sigma} \neq 0$ (see Fig. 1).

TABLE I. Listed is the correlation exponent η obtained via the bosonization technique (Ref. 3) for various order parameters, $O(x)$, in the spin ladder; i.e., $\langle O(x)O^\dagger(0) \rangle \propto (\alpha/x)^\eta$. Antiferromagnetic Ising coupling inbetween chains is assumed (see Fig. 1). Note that the value $\eta=0$ indicates strict long-range order, while $\eta=\infty$ indicates short-range order. Below, we have $K_{\rho,\sigma} = (2\pi v_{\rho,\sigma} - g_{\rho,\sigma})^{1/2} / (2\pi v_{\rho,\sigma} + g_{\rho,\sigma})^{1/2}$.

Order parameter	η (XY AF)	η (XY dimer)	η (Ising AF)
$S_{i,l}^y$	$\frac{1}{4}(K_\rho^{-1} + K_\sigma^{-1})$	∞	∞
$S_{i,l}^z$	$K_\rho + K_\sigma$	K_σ	0
$S_{i,1}^+ S_{i,2}^-$	K_σ^{-1}	K_σ^{-1}	∞
$S_{i,1}^+ S_{i,2}^+$	K_ρ^{-1}	∞	∞

Since the longitudinal paramagnetic susceptibility is simply given by the pseudocharge compressibility, we have that this quantity follows the activated behavior $\chi_z \propto \exp(-2\Delta_\rho/k_B T)$ at low temperatures. Also, pseudocharge-spin separation implies that the specific heat is given by the sum $C_v = C_\rho + C_\sigma$ of the respective charge and spin contributions, each of which follow the activated behavior $C_{\rho,\sigma} \propto \exp(-2\Delta_{\rho,\sigma}/k_B T)$ at low temperatures. The question of long-range order at zero temperature can be attacked with the bosonization method (11) and (12). Following Luther and Peschel,³ the static transverse spin correlator on the same chain may be calculated using the formula

$$\begin{aligned} \langle S_{x,1}^+ S_{0,1}^- \rangle &= e^{i2k_F x} \langle e^{i\pi^{1/2}[\phi_R(x,1) - \phi_L(x,1)]} e^{-i\pi^{1/2}[\phi_R(0,1) - \phi_L(0,1)]} \rangle / 4\pi\alpha \\ & \quad (13) \end{aligned}$$

that is valid to lowest order in $e^{i2k_F x}$. Pseudospin-charge separation then implies that this correlator has the asymptotic form $\langle S_{x,1}^+ S_{0,1}^- \rangle = e^{i2k_F x} G_{TS}^{(\rho)}(x) G_{TS}^{(\sigma)}(x) / 4\pi\alpha$, where $G_{TS}^{(\rho)}(x) = (\alpha/x)^{\theta_{\sigma,\rho}} e^{-x/\xi_\sigma}$ is the autocorrelation function for pseudotriplet superconductivity,^{15,16} with Cooper pairs of effective *unit* charge, and where $G_{TS}^{(\rho)}(x)$ is equal to the previous modulo the symbolic replacement $\rho \leftrightarrow \sigma$. Here, $\theta_{\sigma,\rho} = \frac{1}{4}$ and $a/\xi_{\sigma,\rho} = \Delta_{\sigma,\rho}/v_{\sigma,\rho}$. Hence, we arrive at the result $\langle S_{x,1}^+ S_{0,1}^- \rangle \sim e^{i2k_F x} (\alpha/x)^{1/2} e^{-x/\xi_{xy}}$, where $\xi_{xy}/a = (\Delta_\rho/v_\rho + \Delta_\sigma/v_\sigma)^{-1}$ is the (finite) XY correlation length that signals short-range transverse spin correlations. Lorentz invariance in the dynamical correlator $G_{TS}^{(\rho,\sigma)}(x,t)$ for pseudotriplet superconductivity then implies that the dynamical transverse spin correlator (13) exhibits a spin gap $\Delta_{xy} = \Delta_\rho + \Delta_\sigma$. Similar calculations reveal that the longitudinal spin correlator has the asymptotic form $\langle S_{x,1}^z S_{0,1}^z \rangle = \cos(2k_F x) G_{SDW}^{(\rho)}(x) G_{SDW}^{(\sigma)}(x) / (4\pi\alpha)^2$, where $G_{SDW}^{(\rho)}(x) \rightarrow 1$ is the autocorrelator for pseudo-SDW order,⁷ and where $G_{SDW}^{(\rho)}(x)$ is obtained again through a trivial symbolic replacement. We therefore find that such Ising-coupled antiferromagnetic chains display *strict* long-range order of the Ising type, like that displayed by an isolated Heisenberg chain in the presence of Ising anisotropy.⁴⁻⁶ (See Table I for a listing of the relevant static correlation exponents). In conclusion, the system is in a pinned SDW state that is commensurate with the lattice, and that necessarily exhibits a spin gap.

The above results imply that this SDW state is incommensurate⁹ ($k_F \neq \pi/2a$) for large enough Zeeman energy splittings¹⁰ $|h_z| \gg \Delta_\rho$, in which case umklapp processes become irrelevant. Then by Eq. (7), the pseudocharge sector is in a pure Luttinger liquid state, $\Delta_\rho \rightarrow 0$. Also, since the bosonic field $\phi_\rho = 2^{-1/2}(\phi_1 + \phi_2)$ now represents a Goldstone mode, where $\phi_l(x) = \phi_L(x,l) + \phi_R(x,l)$, we have that the SDW can freely *slide* along the chain direction. The list of physical properties that we outlined above must then be revised as follows. Since the pseudocharge system is now compressible, the longitudinal paramagnetic susceptibility no longer shows a spin gap. In particular, we have that $\chi_z = [(\pi/2)J_{xy} + J_z^\parallel + \frac{1}{2}J_z^\perp]^{-1}$ at zero temperature. Likewise, the low-temperature specific heat is now dominated by the pseudocharge component $C_\rho \propto T$. Last, the sliding Goldstone mode results in only *algebraic* longitudinal spin correlations $\langle S_{x,1}^z S_{0,1}^z \rangle \sim \cos(2k_F x) (\alpha/x)^{K_\rho}$, with exponent $K_\rho = (2\pi v_\rho - g_\rho)^{1/2} / (2\pi v_\rho + g_\rho)^{1/2}$. Note that here S_z refers to the deviation of the magnetization with respect to its average value (Refs. 9 and 10), $\chi_z h_z$. And although the transverse spin correlations remain finite in the the present incommensurate case, the XY spin-correlation length, $\xi_{xy}/a = v_\sigma/\Delta_\sigma$, is now larger.

We shall now treat the general case of $N \geq 2$ chains. The previous results, Eqs. (9) and (10), obtained for the case of two Ising-coupled chains indicate that the interchain forward-scattering process (6) can be neglected in the limit of weak coupling, $|J_z^\perp| \ll |J_{xy}^\parallel|, |J_z^\parallel|$. This shall be assumed throughout in the present discussion. To begin with, we shall also neglect all umklapp processes. The Luttinger model for the Ising-coupled antiferromagnetic chains in such case reduces to the sum of Eqs. (3) and (4), which describes a generalized backscattering model.¹⁴ A mean-field analysis¹² of this model for large N finds that long-range order of the charge-density wave type, $\langle L_i^\dagger(x) R_i(x) \rangle \propto \exp[i(4\pi)^{1/2} \phi_i(x)]$, is stable for $|J_z^\perp| > -2J_z^\parallel$. Notice that this agrees with the exact results for two chains (see Fig. 1). In addition, the application of the Abelian bosonization technique, Eqs. (11) and (12), yields the action

$$\begin{aligned} S_{LD} = i \int dx_0 dx_1 & \left\{ \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi'_i)^2 \right. \\ & \left. - \xi_0^{-2} \sum_{i=1}^{N-1} \cos[(4\pi K)^{1/2} (\phi'_{i+1} - \phi'_i)] \right\} \quad (14) \end{aligned}$$

related to the Lawrence-Doniach model of layered superconductivity,¹¹ where $\phi'_i(x,t)$ represents the time evolution of the bosonic field operator $\phi_i(x)$ after renormalization.⁶ Here, $K = e^{2\psi}$ is one-half the exponent for Ising order, with the angle ψ set by the condition $\tanh 2\psi = -2J_z^\parallel/\pi J_{xy}^\parallel$, while $x_\mu = (iv_F t, x)$ is a two vector, with a renormalized Fermi velocity $v_F = J_{xy}^\parallel a \operatorname{sech} 2\psi$. Also, $\xi_0 \propto (J_z^\perp)^{-1/2}$ denotes the XY spin-correlation length. An analysis based on the equivalence of Eq. (14) to a layered Coulomb gas¹¹ indicates that this term is relevant in the renormalization group sense for $|J_z^\perp| > -4J_z^\parallel$. Notice that this condition is roughly consistent with the phase diagram corresponding to two chains (see Fig. 1), as well as with the mean-field result just cited. Also, it is clear from Eq. (14)

that collective sliding along the chain direction, $\phi_l(x) \rightarrow \phi_l(x) + \delta\phi_\rho$, represents a Goldstone mode. All of these pieces of information when put together then strongly suggest that the ground state of weakly Ising-coupled chains corresponds to an unpinned SDW state if umklapp processes are absent.

Suppose now that we include the umklapp processes present (at half filling) in zero field. The addition of the interchain process (5) as well as the intrachain one⁴⁻⁶ then leads to the bosonic action

$$S = i \int d^2x \left(\sum_{l=1}^N \left\{ \frac{1}{2} (\partial_\mu \phi'_l)^2 - \xi_2^{-2} \cos[2(4\pi K)^{1/2} \phi'_l] \right\} - \sum_{l=1}^{N-1} 2\xi_0^{-2} \cos[(4\pi K)^{1/2} \phi'_{l+1}] \cos[(4\pi K)^{1/2} \phi'_l] \right), \quad (15)$$

where ξ_2 represents the XY spin-correlation length of an isolated XXZ chain. If we then fix a particular chain l and integrate out the neighboring fields,¹⁷ we obtain the effective action

$$\bar{S}_l = i \int d^2x \left\{ \frac{1}{2} (\partial_\mu \phi'_l)^2 - \xi_1^{-2} \cos[(4\pi K)^{1/2} \phi'_l] - \xi_2^{-2} \cos[2(4\pi K)^{1/2} \phi'_l] \right\}, \quad (16)$$

which is a sine-Gordon model¹⁸ with a first harmonic due to the intrachain umklapp process. Note that the coefficient ξ_1^{-2} of the base term is not free, it being proportional to the appropriate average $\langle \cos[(4\pi K)^{1/2} \phi'_{l-1}] \rangle + \langle \cos[(4\pi K)^{1/2} \phi'_{l+1}] \rangle$ that results from the integration. Within the equivalent Coulomb gas description, this interchain term corresponds to unit charges, while the former intrachain term proportional to ξ_2^{-2} corresponds to *double* charges. Since double charges in general dissociate into unit charges if the system is in the (spin-gap) plasma phase,¹⁹ we

conclude that the first harmonic term is irrelevant in the presence of the base term. In the limit of weak coupling, $J_z^\perp \rightarrow 0$, this means that a spin gap ($\xi_1 < \infty$) opens for $J_z^\parallel > -(3\pi/10)J_{xy}^\parallel$, and that bound states appear for $J_z^\parallel > 0$.¹⁸ In contrast, a spin gap will open only in the presence of Ising anisotropy, $J_z^\parallel > J_{xy}^\parallel$, for the case of an isolated XXZ chain ($J_z^\perp = 0$).⁶ On the basis of what is known for a single XXZ chain,^{4,5} we conclude that Ising-coupled spin- $\frac{1}{2}$ XXZ chains with all-antiferromagnetic couplings are generally in an Ising antiferromagnetic state. Notice that this claim relies on the validity of the reduction (16) to a single spin chain, which clearly fails in the case of an *odd* number of Heisenberg-coupled chains.¹⁷ Nevertheless, the fact that we obtain a sliding SDW state in the absence of umklapp processes, coupled with the fact that umklapp processes only reinforce (commensurate) SDW order, strongly indicates that such a reduction is indeed correct.

To conclude, we find that anisotropy in the spin coupling that may exist in between antiferromagnetic chains can dramatically change the nature of the ground state of each chain. In particular, the present Luttinger-model-based analysis demonstrates that consecutive spin- $\frac{1}{2}$ antiferromagnetic chains flow to the Ising antiferromagnetic fixed point in the presence of *any* amount of Ising coupling inbetween nearest-neighbor chains (see Fig. 1). This contrasts with the case of a single chain, where the Ising antiferromagnetic state exist only in the presence of Ising *anisotropy*, $J_z^\parallel > J_{xy}^\parallel$. It also contrasts with the situation where all couplings are Heisenberg, in which case no spin gap exists for an odd number of chains. The latter suggests that a transition between the XY and the Ising antiferromagnetic fixed points occurs at some intermediate perpendicular anisotropy, $J_{xy}^\perp \leq J_z^\perp$, in such case.

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