Electron-phonon scattering rates in semiconductor quantum wells with thin AlAs layers

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(Received 30 May 1997)

We have studied the effects on the phonon spectrum and on the electron–longitudinal-optical-phonon scattering rates in GaAs/AlAs quantum wells of additional thin AlAs layers in the wells. The confined and interface phonon modes and the electron-phonon scattering rates in these structures have been calculated using a dielectric continuum approach for the phonons. Model electron wave functions are used for the quantum well that are taken to be unaffected by the additional AlAs layer. The system with an additional AlAs layer is found to have intrasubband electron scattering rates that are increased modestly compared to those for the corresponding quantum wells due to the influence of the interface phonon modes associated with the additional layer. These results confirm that scattering rates in general depend only weakly on the effects of the structure of the quantum-well system on the phonon spectra. [S0163-1829(97)03140-8]

I. INTRODUCTION

Electron-phonon scattering in quantum-well systems controls carrier relaxation on the picosecond time scale and determines their optical and transport properties at room temperature. In recent years there has been interest in how the quantum-well structure affects these scattering rates and also on the possibility of modifying these scattering rates advantageously by structuring of such systems. For example, Zhu *et al.*[1] have proposed that additional thin AlAs layers in GaAs/Al_xGa_{1-x}As quantum wells increases carrier mobilities significantly.

It is now well known that the optical-phonon spectra of quantum wells such as the GaAs/AlAs system are modified significantly from plane-wave states and are characterized by interface and confined optical-phonon modes.[2,3] It has been shown that these modifications in quantum-well systems in general do not have a large effect on electron-phonon scattering rates.[4–6] Here we consider the effects of additional structure in quantum wells on the phonon spectra and thus on the electron-phonon scattering rates, which ultimately control mobilities and relaxation rates.

In recent work Tsuchiya and Ando[7] have investigated the effects of the modification of the electron wave functions on carrier mobilities in quantum-well systems with such additional AlAs layers. Here we address the effects of changes in the phonon spectra due to such additional AlAs layers. To do so, we use a model of the electron wave functions in which we take them to be unchanged by the additional AlAs layers. It is known that thin layers have large effects on the phonon spectra of these systems but have less effect on the electron states. Here we consider thin AlAs layers. We have studied theoretically the role of interface and confined phonons on the scattering of electrons both in simple GaAs/AlAs quantum wells (SQW's) and also in structurally modified GaAs/AlAs quantum wells (SMQW's). The structurally modified quantum wells contain a thin AlAs layer in the center of a GaAs well and are shown schematically in Fig. 1. The quantum wells range from 50 to 150 Å in width.

II. PHONON MODES AND THEIR POTENTIALS

We use a macroscopic dielectric continuum model to describe the optical phonons in these systems.[2] It has been shown, based on lattice dynamical calculations, that the dielectric continuum model gives a good representation of electron-phonon scattering rates in quantum-well systems. Here we use electromagnetic boundary conditions for the phonons[2] that have been shown to give a good representation of electron-phonon scattering rates.[5,8] Within each semiconductor material the displacement field satisfies $\nabla\cdot\mathbf{D}=0$ and the electric field is given by $\mathbf{D}=\varepsilon(\omega)E$. The dielectric function is taken to be $\varepsilon(\omega)=\varepsilon_{\infty,n}(\omega^2-\omega_{n,\mathrm{LO}}^2)/(\omega^2-\omega_{n,\mathrm{TO}}^2)$, where $n=1$ for GaAs $n=2$ for AlAs, and $\omega_{n,\mathrm{LO}},\omega_{n,\mathrm{TO}}$ are the longitudinal- and transverse-optical mode frequencies. Within this approach the frequencies of the confined LO modes are $\omega_{n,\mathrm{LO}}$. The interface modes satisfy the condition that $\nabla\cdot\mathbf{E}=0$ and the conditions that $E_\parallel$ and $D_\perp$ are continuous at the interfaces. The parameters used here for GaAs are $m_e^*=0.0665m_0$, $\hbar\omega_{\mathrm{LO}}=36.2$ meV, $\hbar\omega_{\mathrm{TO}}=33.2$ meV, $\varepsilon_{10}=12.35$, and $\varepsilon_{1\infty}=10.89$; for AlAs $\hbar\omega_{\mathrm{TO}}=44.9$ meV, $\hbar\omega_{\mathrm{LO}}=50.1$ meV, $\varepsilon_{2\infty}=8.16$, and $\varepsilon_{20}=10.06$. Here m_e^* is the electron effective mass.

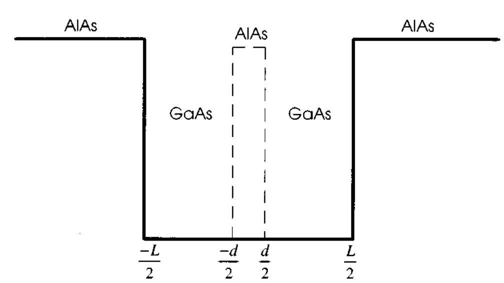

FIG. 1. Sketch of a simple quantum well (SQW) and a structurally modified quantum well (SMQW). The quantum-well material is GaAs. The barriers and the thin layer in the quantum well are AlAs. All calculations have been done with $d=8$ Å.

For a simple GaAs/AlAs quantum well of width L, the frequencies of the symmetric interface modes satisfy the condition

$$\varepsilon_{\mathrm{GaAs}}(\omega)\tanh\left(\frac{qL}{2}\right)=-\varepsilon_{\mathrm{AlAs}}(\omega) \tag{1}$$

and the antisymmetric modes satisfy

$$\varepsilon_{\mathrm{GaAs}}(\omega)\coth\left(\frac{qL}{2}\right)=-\varepsilon_{\mathrm{AlAs}}(\omega). \tag{2}$$

Applying the same conditions to the quantum well of width L with an additional AlAs layer of width d in the center yields, for the frequencies for the symmetric interface modes of a SMQW,

$$\begin{aligned}\varepsilon_{\mathrm{GaAs}}(\omega)&\left\{\varepsilon_{\mathrm{GaAs}}(\omega)(1+e^{-qd})\tanh\left(\frac{q(L-d)}{2}\right)+\varepsilon_{\mathrm{AlAs}}(\omega)\right\}\\&+\varepsilon_{\mathrm{AlAs}}(\omega)\left\{\varepsilon_{\mathrm{AlAs}}(\omega)(1-e^{-qd})\tanh\left(\frac{q(L-d)}{2}\right)\right.\\&\left.+\varepsilon_{\mathrm{GaAs}}(\omega)\right\}=0.\end{aligned} \tag{3}$$

The antisymmetric interface modes in the SMQW satisfy

$$\begin{aligned}\varepsilon_{\mathrm{GaAs}}(\omega)&\left\{\varepsilon_{\mathrm{GaAs}}(\omega)(1-e^{-qd})+\varepsilon_{\mathrm{AlAs}}(\omega)\coth\left(\frac{q(L-d)}{2}\right)\right\}\\&+\varepsilon_{\mathrm{AlAs}}(\omega)\left\{\varepsilon_{\mathrm{AlAs}}(\omega)(1+e^{-qd})\right.\\&\left.+\varepsilon_{\mathrm{GaAs}}(\omega)\coth\left(\frac{q(L-d)}{2}\right)\right\}=0.\end{aligned} \tag{4}$$

In Figs. 2 and 3, respectively, we show the dispersions of the symmetric and antisymmetric interface modes of the SQW and the SMQW. Their frequencies lie within the reststrahl regions of the two bulk materials. The solid lines correspond to a 50-Å SQW. The SMQW has modes that are shown by the dashed lines in addition to those shown by solid lines. The additional modes are those from the interfaces associated with the thin AlAs layer. For small wave vectors the frequencies of the interface modes approach the bulk mode frequencies, and for large wave vectors their frequencies approach those of a single interface, which lie in the middle parts of the reststrahl regions. The confined mode frequencies are at $\hbar\omega_{\mathrm{LO},n}$ in each material.

The LO phonons of quantum-well systems separate into confined modes, which are restricted to one or the other material, and interface modes. We have calculated the potentials of both the confined and interface phonons of the SQW and SMQW. The potentials are normalized by equating the energy density of the oscillating ions to the energy of a quantum-mechanical harmonic oscillator.[9] The confined and interface modes are either symmetric or antisymmetric with respect to a plane at the center of the well for both well configurations.

It has been shown[5] that in order to describe the electron-phonon scattering rates properly the confined dispersionless LO phonons must satisfy Maxwell equations, and that additional boundary conditions on the phonon displacements are not necessary. Here we use confined phonon modes satisfying Maxwell equations given by the dielectric continuum model, which requires that the potentials go to zero at the interfaces. The electrostatic potentials of the confined phonons for the SQW are[10]

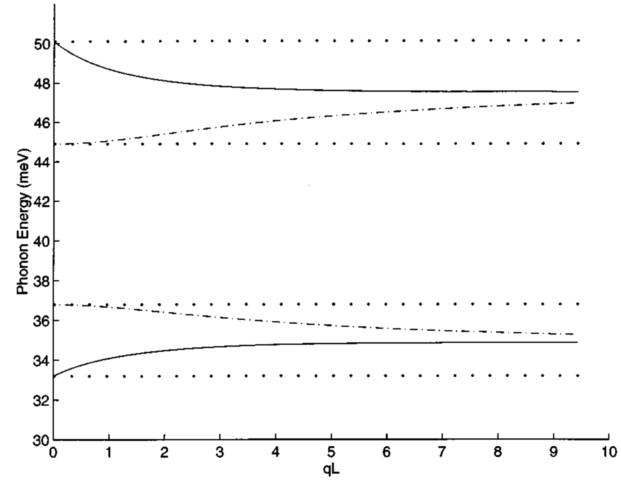

FIG. 2. Symmetric interface phonon energies as a function of wave vector for the SQW and a quantum well with a thin AlAs layer (SMQW). The two symmetric phonon modes for the simple well are given by solid curves. The four symmetric phonon modes for the structurally modified quantum well are given by the two dashed curves plus the two solid curves. The upper two dotted lines give $\hbar\omega_{\mathrm{LO}}$ and $\hbar\omega_{\mathrm{TO}}$ for bulk AlAs and the lower two dotted lines give $\hbar\omega_{\mathrm{LO}}$ and $\hbar\omega_{\mathrm{TO}}$ for bulk GaAs. All calculations have been done with $d=8$ Å.

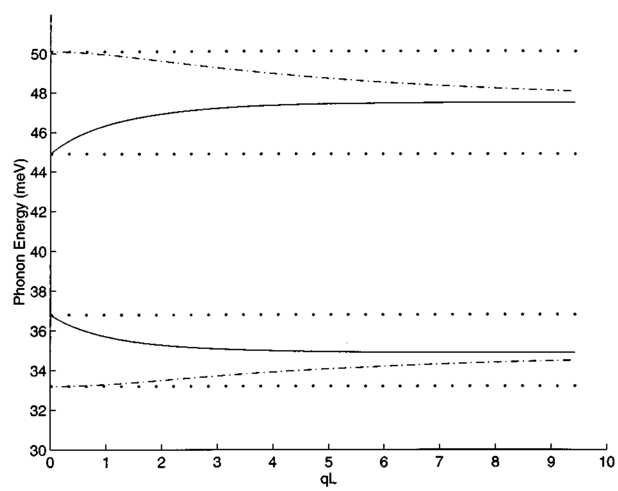

FIG. 3. Antisymmetric interface phonon energies as a function of wave vector for the SQW and for a quantum well with a thin AlAs layer (SMQW). The two antisymmetric phonon modes for the simple well are given by solid curves. The four antisymmetric phonon modes for the structurally modified quantum well are given by the dashed curves plus the two solid curves.

$$\phi^{\mathrm{SQW}}_{\mathrm{confined}}(q)=\sqrt{4\pi\hbar\omega_{\mathrm{LO}}\left(\frac{1}{\varepsilon_{\infty,\mathrm{GaAs}}}-\frac{1}{\varepsilon_{0,\mathrm{GaAs}}}\right)}\times\sum_{q,n}\sum_{\alpha=\pm}e^{i\mathbf{q}\cdot\mathbf{r}}t_n^{\mathrm{SQW}}(q)u_{n\alpha}(z)\times[a_{n\alpha}(q)+a^{\dagger}_{n\alpha}(-q)], \tag{5}$$

where

$$t_n^{\mathrm{SQW}}(q)=\frac{1}{\left[q^2+\left(\frac{n\pi}{L}\right)^2\right]^{1/2}},$$

$$u_{n,+}=\cos\left(\frac{n\pi z}{L}\right),\quad n=1,3,5,\ldots$$

$$u_{n,-}=\sin\left(\frac{n\pi z}{L}\right),\quad n=2,4,6,\ldots\ .$$

The plus and minus signs indicate symmetric and antisymmetric modes, respectively. For the SQW, the first confined phonon mode ($n=1$) has a nodeless potential, the second mode has one node at the center of the well, etc. All confined phonon potentials have nodes at the interfaces.

For the SMQW, the electrostatic potentials due to the confined modes are

$$\phi^{\mathrm{SMQW}}_{\mathrm{conf},+}(q)=\sum_{q,n}Ce^{i\mathbf{q}\cdot\mathbf{r}}t_n^{\mathrm{SMQW}}(q)[a(q)+a^{\dagger}(-q)]\times\cos\left[\frac{2\pi n}{L}\left(|z|-\frac{L}{4}\right)\right],\quad n=1,3,5,\ldots \tag{6}$$

$$\phi^{\mathrm{SMQW}}_{\mathrm{conf},-}(q)=\sum_{q,n}Ce^{i\mathbf{q}\cdot\mathbf{r}}t_n^{\mathrm{SMQW}}(q)[a(q)+a^{\dagger}(-q)]\times\sin\left[\frac{2\pi n}{L}\left(|z|-\frac{L}{4}\right)\right],\quad n=2,4,6,\ldots, \tag{7}$$

where

$$t_n^{\mathrm{SMQW}}(q)=\frac{1}{\left[q^2+\left(\frac{2\pi n}{L}\right)^2\right]^{1/2}},$$

$$C=\left(\frac{4\pi\hbar}{A\omega_{\mathrm{GaAs,LO}}}\right)^{1/2}\frac{1}{\left[\frac{L-d}{\omega^2_{\mathrm{GaAs,LO}}\left(\frac{1}{\varepsilon_{\infty,\mathrm{GaAs}}}-\frac{1}{\varepsilon_{0,\mathrm{GaAs}}}\right)}+\frac{d}{\omega^2_{\mathrm{AlAs,LO}}\left(\frac{1}{\varepsilon_{\infty,\mathrm{AlAs}}}-\frac{1}{\varepsilon_{0,\mathrm{AlAs}}}\right)}\right]^{1/2}},$$

where A is the area of the quantum well. The first confined mode has a node in the potential at the interface between the GaAs well material and the additional AlAs layer in addition to nodes at the interfaces of the AlAs barriers. Higher-order confined modes have additional nodes in the GaAs well. It is expected that these nodes will result in substantially reduced scattering from the confined modes due to reduced electron-phonon matrix elements given by $t_n^{\mathrm{SMQW}}(q)$.

The potentials of the interface phonons decay exponentially away from the interfaces and extend into both materials. For the SQW there are symmetric and antisymmetric interface modes, both of which correspond to longitudinal-optical modes.[8] These symmetric (+) and antisymmetric (−) electrostatic potentials are

$$\phi^{\mathrm{SQW}}_{\mathrm{IF},\pm}(q)=C(q)\left(\frac{e^{qz}\pm e^{-qz}}{e^{qL/2}\pm e^{-qL/2}}\right)e^{-qL/2}e^{i\mathbf{q}\cdot\mathbf{r}}\times[a(q)+a^{\dagger}(-q)], \tag{8}$$

where

$$C(q)=\left(\frac{2\pi\hbar}{\omega(q)}\right)^{1/2}\frac{1}{\left[2Aqe^{-qL}\left\{\beta^2_{\mathrm{AlAs}}+\beta^2_{\mathrm{GaAs}}\tanh\left(\frac{qL}{2}\right)\right\}^{1/2}\right]}$$

and

$$\beta^2_{\mathrm{AlAs;GaAs}}=\frac{\varepsilon_{\infty,\mathrm{AlAs;GaAs}}(\omega^2_{\mathrm{LO,AlAs;GaAs}}-\omega^2_{\mathrm{TO,AlAs;GaAs}})}{(\omega^2-\omega^2_{\mathrm{TO,AlAs;GaAs}})^2}.$$

In the case of the SMQW, additional symmetric and antisymmetric interface modes are introduced by the AlAs layer as shown in Figs. 2 and 3. For small wave vectors, these additional modes are AlAs TO-like and GaAs LO-like modes for the symmetric case and AlAs LO-like and GaAs TO-like modes for the antisymmetric case. The electrostatic potentials of these interface modes for the SMQW are

$$\phi^{\mathrm{SMQW}}_{\mathrm{IF,GaAs}}(q)=\widetilde{C}(q)e^{-qL/2}e^{i\mathbf{q}\cdot\mathbf{r}}[a(q)+a^{\dagger}(-q)]\times\left[\cosh q\left(z+\frac{L}{2}\right)+x\sinh q\left(z+\frac{L}{2}\right)\right],$$

$$\frac{-L}{2}\leqslant z<\frac{-d}{2} \tag{9}$$

$$\phi_{\mathrm{IF,AlAs}}^{\mathrm{SMQW}}(q)=\widetilde{C}(q)\,\frac{x^2-1}{x}\,e^{i\mathbf{q}\cdot\mathbf{r}}(e^{-qL}-e^{-qd})\times[a(q)+a^{\dagger}(-q)]\begin{Bmatrix}\cosh(qz)\\ \sinh(qz)\end{Bmatrix},\qquad \frac{-d}{2}\leqslant z\leqslant\frac{d}{2}\quad(10)$$

$$\phi_{\mathrm{IF,GaAs}}^{\mathrm{SMQW}}(q)=\widetilde{C}(q)\left(\frac{x^2-1}{4x}\right)\times(e^{-qL}-e^{-qd})e^{i\mathbf{q}\cdot\mathbf{r}}[a(q)+a(q)^{\dagger}]\times\begin{Bmatrix}(x-1)\cosh q(d-z)-(x+1)\cosh qz\\ (x+1)\sinh qz-(x-1)\sinh q(d-z)\end{Bmatrix},\qquad \frac{d}{2}<z\leqslant\frac{L}{2}\quad(11)$$

where $x=\varepsilon_{\mathrm{GaAs}}(\omega)/\varepsilon_{\mathrm{AlAs}}(\omega)$ and

$$\widetilde{C}(q)=[Aq\{R+S+T\}]^{-1/2},$$

with

$$R=\beta_{\mathrm{AlAs}}^2e^{-qL}\left[1+\frac{e^{qd/2}\cosh\frac{qd}{2}}{2x(x+1)}\left(x-\tanh\frac{q(d-L)}{4}\right)\times\left(x-\coth\frac{q(d-L)}{4}\right)\left(x\tanh\frac{qd}{2}-1\right)\right],$$

$$S=\frac{1}{2}\,\beta_{\mathrm{GaAs}}^2\Big(e^{-qL}[(x^2+1)\sinh q(L-d)+2x\cosh q(L-d)-2x]+\frac{1}{16x^2}(e^{-qL}-e^{-qd})^2\times(x^2-1)^2\{(x-1)^2[\sinh qd-\sinh q(2d-L)]+2(x^2-1)\sinh q(d-L)+(x+1)^2[\sinh qL-\sinh qd]\}\Big),$$

$$T=\beta_{\mathrm{AlAs}}^2\,\frac{1}{2x^2}\,(1-x^2)^2(e^{-qL}-e^{-qd})^2\,\sinh qd.$$

III. ELECTRON-PHONON SCATTERING RATES

Here we are interested in the effects of the changes in the phonon states on the scattering rates. In order to see these effects we consider thin AlAs layers that should affect the phonon properties more than the electronic states. We take the electron states in both the SQW and the SMQW to be simple effective mass sine and cosine functions corresponding to infinite potentials at the barrier–quantum-well interfaces and to be unaffected by the thin AlAs layer.[2]

The electron-phonon scattering rates for single-phonon emission are given by

$$W(k)=\frac{2\pi}{\hbar}\int dN_f\,\delta(E-E')|\langle k',1|H_{ep}|k,0\rangle|^2.\quad(12)$$

Here the initial electron energy is E, the final energy is E', k and k' are the initial and final wave vectors for the electron, H_{ep} is the Hamiltonian for the electron-phonon interactions, and there are zero phonons and one phonon in the initial and final states, respectively.

The expressions for intrasubband scattering (subband $1\rightarrow1$) by emission of confined phonons in the SQW and in the SMQW are

$$W_{11}^{\mathrm{SQW}}(k)=\frac{2\pi}{\hbar}\,(N_{\mathrm{LO}}+1)\,\frac{\lambda^2}{(2\pi)^2L}\times\sum_n\int d^2q|G_{n+}^{\mathrm{SQW},11}|^2[t_n^{\mathrm{SQW}}(q)]^2\times\delta(E(k)-E'(k')),\quad(13)$$

where

$$E(k)-E'(k')=\frac{[\hbar(k'+q)]^2}{2m}-\frac{(\hbar k')^2}{2m}-\hbar\omega_{\mathrm{LO}},$$

$$\lambda^2=4\pi e^2\hbar\omega_{\mathrm{LO,GaAs}}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right),$$

$$[t_n^{\mathrm{SQW}}(q)]^2=\left[q^2+\left(\frac{\pi n}{L}\right)^2\right]^{-1},$$

$$G_{n+}^{\mathrm{SQW},11}=-2(-1)^{(n+1)/2}\left[\frac{1}{n\pi}+\frac{n\pi}{(2\pi)^2-(n\pi)^2}\right],\qquad n=1,3,5,\ldots$$

and

$$W_{11}^{\mathrm{SMQW}}(k)=\frac{2\pi e^2}{\hbar}\,(N_{\mathrm{LO}}+1)\,\frac{A}{(2\pi)^2}\,C^2\times\sum_n\int d^2q|G_{n+}^{\mathrm{SMQW},11}|^2[t_n^{\mathrm{SMQW}}(q)]^2\times\delta(E(k)-E'(k')),\quad(14)$$

where

$$C^2=4\pi\,\frac{\hbar\omega_{\mathrm{LO,GaAs}}}{A}\left(\frac{1}{\varepsilon_{\infty,\mathrm{GaAs}}}-\frac{1}{\varepsilon_{0,\mathrm{GaAs}}}\right)\times\left[(L-d)+d\,\frac{\omega_{\mathrm{LO,GaAs}}^2\left(\frac{1}{\varepsilon_{\infty,\mathrm{GaAs}}}-\frac{1}{\varepsilon_{0,\mathrm{GaAs}}}\right)}{\omega_{\mathrm{LO,AlAs}}^2\left(\frac{1}{\varepsilon_{\infty,\mathrm{AlAs}}}-\frac{1}{\varepsilon_{0,\mathrm{AlAs}}}\right)}\right]^{-1},$$

$$[t_n^{\mathrm{SMQW}}(q)]^2=\left[q^2+\left(\frac{2\pi n}{L}\right)^2\right]^{-1},$$

and

$$G_{n+}^{\mathrm{SMQW},11}=\frac{-2}{n\pi}(-1)^{(n+1)/2},\quad n=1,3,5,\ldots\ .$$

Calculation of the intrasubband scattering rates of electrons due to emission of interface phonons in SQW yields

$$W_{\mathrm{IF}}^{\mathrm{SQW},11}(k)=\frac{2\pi}{\hbar}\left(\frac{A}{4\pi^2}\right)(N_{\mathrm{LO}}+1)\int d^2q \times\delta(E(k)-E'(k'))|G_{+}^{\mathrm{SQW},11}(q)|^2[t^{\mathrm{SQW}}(q)]^2, \tag{15}$$

where

$$E(k)-E'(k')=\frac{[\hbar(k'+q)]^2}{2m}-\frac{(\hbar k')^2}{2m}-\hbar\omega_{\mathrm{LO}}(q),$$

$$G_{+}^{\mathrm{SQW},11}(q)=\frac{2}{qL}\sinh\left(\frac{qL}{2}\right)\left[\frac{\left(\frac{2\pi}{qL}\right)^2}{1+\left(\frac{2\pi}{qL}\right)^2}\right],$$

and

$$[t^{\mathrm{SQW}}(q)]^2=\left(\frac{\pi e^2\hbar}{q\omega(q)A\cosh^2\left(\frac{qL}{2}\right)}\right)\times\left[\frac{\varepsilon_{\infty,\mathrm{AlAs}}(\omega_{\mathrm{LO,AlAs}}^2-\omega_{\mathrm{TO,AlAs}}^2)}{\omega(q)^2-\omega_{\mathrm{TO,AlAs}}^2}-\frac{\varepsilon_{\infty,\mathrm{GaAs}}(\omega_{\mathrm{LO,GaAs}}^2-\omega_{\mathrm{TO,GaAs}}^2)}{\omega(q)^2-\omega_{\mathrm{TO,GaAs}}^2}\tanh\left(\frac{qL}{2}\right)\right]^{-1}.$$

For the SMQW, the scattering rate due to interface phonons is

$$W_{\mathrm{IF}}^{\mathrm{SMQW},11}(k)=\frac{2\pi}{\hbar}\left(\frac{A}{4\pi^2}\right)(N_{\mathrm{LO}}+1)\int d^2q \times\delta(E(k)-E'(k'))|G_{+}^{\mathrm{SMQW},11}(q)|^2 \times[t^{\mathrm{SMQW}}(q)]^2, \tag{16}$$

where

$$G_{+}^{\mathrm{SQW},11}(q)=\int_{-L/2}^{-d/2}dz\ \psi_1^*(z)\phi_{\mathrm{IF,GaAs}}^{\mathrm{SMQW}}(z)\psi_1(z)+\int_{-d/2}^{d/2}dz\ \psi_1^*(z)\phi_{\mathrm{IF,AlAs}}^{\mathrm{SMQW}}(z)\psi_1(z)+\int_{d/2}^{L/2}dz\ \psi_1^*(z)\phi_{\mathrm{IF,GaAs}}^{\mathrm{SmQW}}(z)\psi_1(z),$$

$$[t^{\mathrm{SMQW}}(q)]^2=\left(\frac{\pi e^2\hbar(1-x^2)^2(e^{-qL}-e^{-qd})^2}{8x^2q\omega(q)A[f(q)+g(q)+h(q)]}\right).$$

Here

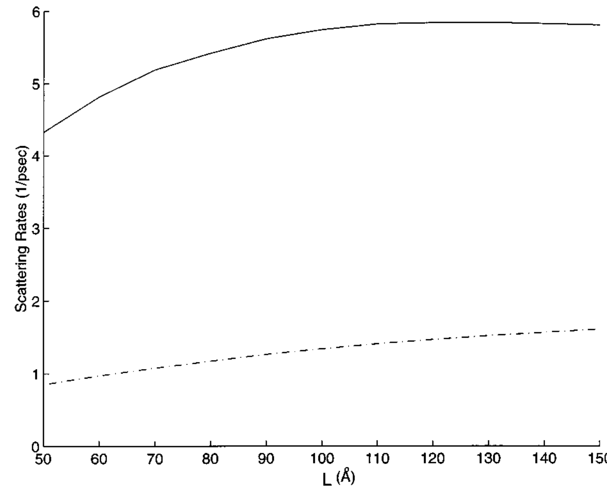

FIG. 4. Scattering rates from confined phonons as a function of well width. The solid curve is for the SQW and the dash-dotted curve is for the SMQW.

$$f(q)=\frac{\varepsilon_{\infty,\mathrm{AlAs}}(\omega_{\mathrm{LO,AlAs}}^2-\omega_{\mathrm{TO,AlAs}}^2)}{4\pi[\omega(q)^2-\omega_{\mathrm{TO,AlAs}}^2]}e^{-qL}\times\left\{1+\left[\frac{e^{qd/2}\cosh\left(\frac{qd}{2}\right)}{2x(x+1)}\left[x-\tanh\left(\frac{q(d-L)}{4}\right)\right]\times\left[x-\coth\left(\frac{q(d-L)}{4}\right)\right]\left[x\tanh\left(\frac{qd}{2}\right)-1\right]\right]^2\right\},$$

$$g(q)=\frac{\varepsilon_{\infty,\mathrm{GaAs}}(\omega_{\mathrm{LO,GaAs}}^2-\omega_{\mathrm{TO,GaAs}}^2)}{2\times4\pi[\omega(q)^2-\omega_{\mathrm{TO,GaAs}}^2]}\times\Big[e^{-qL}[(x^2+1)\sinh q(L-d)+2x\cosh q(L-d)-2x]+\left(\frac{1}{16x^2}\right)(e^{-qL}-e^{-qd})^2(x^2-1)^2\times\{(x-1)^2[\sinh qd-\sinh q(2d-L)]+2(x^2-1)\times\sinh q(d-L)+(x+1)^2[\sinh qL-\sinh qd]\}\Big],$$

$$h(q)=\frac{\varepsilon_{\infty,\mathrm{AlAs}}(\omega_{\mathrm{LO,AlAs}}^2-\omega_{\mathrm{TO,AlAs}}^2)}{4\pi[\omega(q)^2-\omega_{\mathrm{TO,AlAs}}^2]}\times\left\{\left(\frac{1}{2x^2}\right)(1-x^2)^2(e^{-qL}-e^{-qd})^2\sinh qd\right\},$$

and $x=\varepsilon_{\mathrm{GaAs}}(\omega)/\varepsilon_{\mathrm{AlAs}}(\omega)$. $\psi_1(z)$ is the electron envelope function for the first electron subband.

The scattering rates for phonon emission due to confined phonons in both types of quantum well are shown in Fig. 4, and the total rates from confined and interface modes are shown in Fig. 5. We find that scattering of electrons from confined phonons increases with increasing well width for both the SQW and the SMQW, as shown in Fig. 4. The

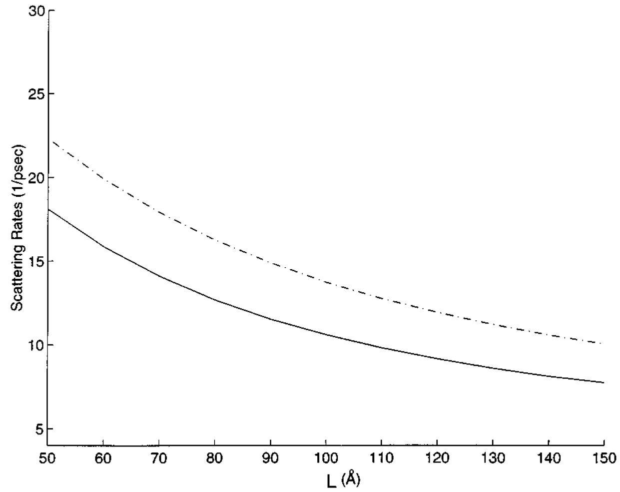

FIG. 5. Total scattering rates for emission of confined and interface phonons as a function of well width. The solid curve corresponds to a SQW and the dashed curve to a SMQW.

scattering rates due to confined phonons for both systems approach those for bulk GaAs for large well widths.

The scattering rates for confined phonons is reduced for the SMQW as compared to SQW however. This can be understood from the effects of the AlAs layer on the confined phonon potentials. The first confined phonon mode for the SQW has a nodeless potential, and this mode gives the largest scattering for confined phonons. Then higher-order modes, which have nodes in their potentials, give smaller scattering. For the SMQW, the first mode has a potential with a node at the edge of the AlAs thin layer. The scattering strength of this mode is comparable to that of the second mode ($n=2$) of the SQW, which accounts for the decreased scattering from confined modes in the SMQW.

We see from Fig. 5 that the total scattering rate from both confined and interface modes is somewhat higher for the SMQW than for the SQW. This results from the increased electron-interface phonon scattering in the SMQW as compared to that of the SQW due to the additional interface modes associated with the thin AlAs layer. The results in Fig. 5 are for AlAs layer thickness $d=8$ Å. The scattering rates in the SMQW are found to increase modestly with increasing d. For example, in going from $d=4$ to 10 Å, the scattering rate increases by about 4%. Here we consider relatively thin AlAs layers because we address only the modifications in the phonon spectrum. For sufficiently thin layers, we take the electron wave functions to be unaffected by the layer. The AlAs LO-like interface phonons remain the dominant scattering modes for both the SQW and the SMQW.

IV. CONCLUSION

We find that the introduction of a thin AlAs layer in the quantum well significantly reduces scattering of electrons from confined phonons, but that this decrease is more than compensated for by increased scattering from interface phonons associated with the AlAs layer. The net overall effect is a modest increase in the total scattering rate due to the AlAs layer. Our results suggest that the enhancement of mobility reported by Zhu *et al.*[1] is not expected to arise from decreased electron-phonon scattering due to the AlAs layer.

ACKNOWLEDGMENTS

This research was supported in part by the Office of Naval Research. C.R.M. would like to thank Thomas Forbang for assistance in this work.

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