

## Coulomb drag at the onset of Anderson insulators

Efrat Shimshoni

*Department of Mathematics-Physics, Oranim Haifa University, Tivon 36006, Israel*

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I study the Coulomb drag between two identical layers in the Anderson insulating state. The dependence of the transresistance  $\rho_t$  on the localization length  $\xi$  is considered, for both a Mott insulator and an Efros-Shklovskii (ES) insulator. In the former,  $\rho_t$  is monotonically increasing with  $\xi$ ; in the latter, the presence of a Coulomb gap leads to an opposite result:  $\rho_t$  is *enhanced* with a *decreasing*  $\xi$ , with the same exponential factor as the single layer resistivity. This distinction reflects the relatively pronounced role of excited density fluctuations in the ES state, implied by the enhancement in the rate of hopping processes at low frequencies. The magnitude of drag is estimated for typical experimental parameters in the different cases. It is concluded that a measurement of drag can be used to distinguish between the interacting and noninteracting insulating state. [S0163-1829(97)04243-4]

### I. INTRODUCTION AND PRINCIPAL RESULTS

A rich variety of phenomena in disordered electronic systems is associated with the Anderson localization.<sup>1</sup> The fundamental effect is a manifestation of quantum coherence: a subtle destructive interference of multiple partial waves in a random medium leads to exponentially localized electron eigenstates. In three-dimensional (3D) conductors, a metal-insulator transition (MIT) can be driven by tuning either the disorder potential or the carrier density across a threshold value. The transition is identified as second order: it is associated with a divergence of a correlation length  $\xi$ , which in the insulating side has the physical significance of a localization length. A true phase transition of this kind is generically absent in two dimensions (2D), at vanishing magnetic fields:<sup>2</sup> instead, a 2D system exhibits a smooth crossover between a “weak insulator” (in which at finite temperatures  $T$  the conductivity  $\sigma$  acquires a negative logarithmic correction), and a “strong insulator” [characterized by an exponentially small  $\sigma(T)$ ]. The presence of a magnetic field  $B$ , as well as spin-orbit scattering (Refs. 2 and 3), dramatically alter this behavior—they suppress localization, and, in principle, can recover the metallic state in 2D. Most prominently, in strong magnetic fields, singular extended states play an important role in the mechanism for the quantum Hall (QH) effect.<sup>4</sup> Recent experimental studies have posed new challenges in the understanding of Anderson insulators. For example, in QH systems, the observation of a weak-to-strong localization crossover near filling fraction  $1/2$ , is possibly an interesting indication for  $B=0$  localization of “composite fermions.”<sup>6</sup> In contrast, a true MIT has been observed at  $B=0$ ,<sup>7</sup> whose origin is yet obscure. In all the above, electron-electron interactions and their interplay with the disorder<sup>5</sup> play an important role, and complicates the theoretical analysis.

In view of the ongoing research activities described above, it is highly desired to have an extended array of different probing techniques. In the present paper, I suggest that Coulomb drag in a double layer system is potentially an

interesting probe that can diagnose subtle differences between distinct insulating states.

Coulomb drag<sup>8</sup> is a manifestation of the coupling between two spatially separated systems of charge carriers, due to Coulomb interactions across a barrier separating them. In the presence of a transport current in one layer, density fluctuations in that layer exert a frictional force on the other, and consequently induce a voltage in the latter (in an open circuit configuration)—even when tunneling between the layers is suppressed. The strength of this effect is characterized by the transresistivity,  $\rho_t \equiv E_2/j_1$ , where  $E_2$  is the parallel electric field induced in layer 2 in response to a current density  $j_1$  established in layer 1. Experimentally, drag has been observed in various semiconductor heterostructures,<sup>9</sup> theoretically, it has been a subject of much recent activity.<sup>10–16</sup> In particular, it was suggested as a useful test of certain electronic states in the layers, such as compressible states in QH systems,<sup>12,15</sup> and superfluid electron-hole condensates.<sup>16</sup>

As pointed out in Ref. 12, interlayer drag can probe the dynamics and response properties of electronic systems in a domain that is inaccessible to transport measurements in a single layer. As it stems from a frictional force, Coulomb drag is dominated by the interaction between relatively long-lived density fluctuations in the two layers. Therefore, its low temperature behavior is sensitive to the ability of the electronic systems to create and maintain such fluctuations, reflected by the density-density response functions  $\chi_{1,2}(q, \omega)$  at *low frequencies* ( $\omega \rightarrow 0$ ) and *finite wave vectors*<sup>17</sup>  $q$ . To see this, note that to leading order in the screened interlayer interaction  $U$ , the transresistivity can be expressed as<sup>11,18</sup>

$$\rho_t = \frac{\beta \hbar^2}{\pi n^{(1)} n^{(2)} e^2} \int \frac{d^2 q}{(2\pi)^2} q^2 \times \int_0^\infty d\omega \frac{|U(q, \omega)|^2 \text{Im} \chi_1(q, \omega) \text{Im} \chi_2(q, \omega)}{4 \sinh^2(\beta \hbar \omega/2)}; \quad (1)$$

here  $\beta = 1/k_B T$ , where  $T$  is the temperature,  $n^{(1)}, n^{(2)}$  are the carrier densities in layers 1, 2, and

$$U(q, \omega) = \frac{V(q)e^{-qd}}{[1 + V(q)\chi_1(q, \omega)][1 + V(q)\chi_2(q, \omega)] - V^2(q)e^{-2qd}\chi_1(q, \omega)\chi_2(q, \omega)}, \quad (2)$$

where  $V(q) = 2\pi e^2/\epsilon q$  ( $\epsilon$  being the background dielectric constant), and  $d$  the interlayer distance. In Eq. (1), the integration over  $\omega$  is effectively cutoff by  $T$ , while the cutoff on  $q$  is set by  $1/d$ ; the former is *stricter* at low  $T$ .

In the vicinity of a MIT, the behavior of drag is expected to be particularly interesting, due to the significant role played by density fluctuations. Variations in the conductivity (of one layer or both) have competing effects on the *availability* of density fluctuations and on their *stability*. In the conducting state (e.g., in the diffusive regime<sup>11</sup>), the drag is *enhanced* as the diffusion coefficient is reduced, since then the decay rate of density fluctuations is slowed down. On the other hand, in the insulating state the creation of density fluctuations is suppressed at low  $q$  and  $\omega$ , and this can lead to a *reduction* of  $\rho_t$  with a reduced localization length—provided that the simultaneous variations in the screening properties are not dominant.<sup>14,19</sup> Thus, in contrast with the single-layer resistivity,  $\rho_t$  is potentially a nonmonotonous function of the parameter which drives the transition; a similar behavior is expected also near a smooth (noncritical) crossover between a weak and strong insulator. At the transition, density fluctuations and their dynamics become critical, and one expects a pronounced peak in the drag (similarly to the behavior predicted in Ref. 12 for QH transitions).

In this paper I study the Coulomb drag in Anderson insulators, in order to verify under what circumstances the above described qualitative picture holds. I calculate  $\rho_t$  in a double-layer system, where the electronic states in the layers are assumed for simplicity to be identical and uncorrelated. Two possible insulating states are considered: a Mott insulator,<sup>20</sup> where the in-layer long-range Coulomb interactions are suppressed, and an Efros-Shklovskii (ES) state<sup>21</sup> where the interactions are significant. In both cases, the dominant transport mechanism is assumed to be variable range hopping<sup>22</sup> among localized sites, and  $\rho_t$  is evaluated as a function of the localization length  $\xi$ .

The resulting transresistance indicates a dramatic difference between the Mott and ES insulators. While the former exhibits a suppression of  $\rho_t$  with a decreasing  $\xi$ , as implied by the naive argument pointed out earlier, in the latter the dominant contribution to  $\rho_t$  is *enhanced* with the same exponential factor as the single layer resistivity. As will be shown below, this follows from the relative enhancement of hopping processes at  $\omega \rightarrow 0$ , due to the presence of a Coulomb gap in the ES state. Drag measurement is therefore a suggestive experimental means of distinguishing the two types of insulating states. In contrast, the single-layer resistivity exhibits a qualitatively similar dependence on  $\xi$  and  $T$ :  $\rho(T) \sim e^{(T_0/T)^\alpha}$ , where in the 2D Mott insulator  $\alpha = 1/3$  and  $T_0 \sim \xi^{-2}$ , and in the ES state  $\alpha = 1/2$  and  $T_0 \sim \xi^{-1}$ .

In the following sections I detail the calculation of  $\rho_t$ : in Sec. II, for the Mott insulator, and in Sec. III for the ES insulator. The conclusions and experimental implications are summarized in Sec. IV.

## II. MOTT INSULATORS

I consider two parallel 2D layers separated by a perfect insulating barrier of thickness  $d$ , in both of which the electronic state is a noninteracting Mott insulator. The transresistance is evaluated using Eqs. (1) and (2), where  $\chi_1(q, \omega) = \chi_2(q, \omega)$  is the density response function, which has been derived diagrammatically by Vollhardt and Wölfle.<sup>23</sup> To leading order in  $q$  and  $\omega$ , it is given by

$$\chi_{1(2)}(q, \omega) = \frac{dn}{d\mu} \frac{Dq^2}{Dq^2 - [i\omega + \tau(\omega^2 - \omega_0^2)]}. \quad (3)$$

Here  $dn/d\mu$  is the density of states,  $\tau$  is the elastic mean free time, and  $D = v_F^2\tau/2$  ( $v_F$  being the Fermi velocity) is the diffusion coefficient in the conducting state. The localization effect is represented by the frequency  $\omega_0$ : as pointed out in Ref. 23, Eq. (3) is consistent with a simple hydrodynamical model, in which localization is manifested as an effective restoring force acting on density fluctuations, with a characteristic oscillator frequency  $\omega_0$ . The localization length is directly related to  $\omega_0$  through  $\xi = v_F/\sqrt{2}\omega_0$ .

I next assume that  $T$ , the effective upper cutoff on  $\omega$  in Eq. (1), is sufficiently low that  $\omega \ll \omega_0, \omega_0^2\tau$ . The real and imaginary parts of  $\chi(q, \omega)$  (hereon, the layers indices are dropped) are then given, respectively, by

$$\text{Re } \chi(q, \omega) \approx \frac{dn}{d\mu} \frac{q^2}{q^2 + \xi^{-2}}, \quad (4)$$

$$\text{Im } \chi(q, \omega) \approx \frac{dn}{d\mu} \frac{Dq^2\omega}{D^2(q^2 + \xi^{-2})^2}. \quad (5)$$

Substitution in Eqs. (1) and (2) yields the final expression for  $\rho_t$ . The result depends crucially on whether  $\xi$  is smaller or greater than the layers separation  $d$ , and exhibits a crossover between the two limiting cases considered below.

Deep in the insulating state,  $\xi \ll d$ , one can neglect the  $q^2$  term in the denominators of Eqs. (4) and (5) (in the effective range of wave vectors,  $q < 1/d$  and hence  $q \ll 1/\xi$ ). The interlayer interaction can be then approximated by its unscreened form,  $U(q, \omega) \approx V(q)e^{-qd}$ . The integrations in Eq. (1) are straightforward and result in

$$\rho_t \approx \frac{5}{32\pi} \frac{h}{e^2} \left( \frac{k_B T}{\hbar D n} \right)^2 (q_{\text{TF}} d)^2 \left( \frac{\xi}{d} \right)^8, \quad (6)$$

where  $n \equiv n^{(1)} = n^{(2)}$ , and  $q_{\text{TF}} = 2\pi e^2(dn/d\mu)/\epsilon$  is the Thomas-Fermi screening wave vector.

When the localization length is increased (e.g., by controlling a parameter that drives the insulator into a transition to the metallic state), eventually the distance  $d$  is exceeded. In the limit  $\xi \gg d$ , the screening of  $U(q, \omega)$  becomes significant as long as  $q > 1/\xi^2 q_{\text{TF}}$  [see Eqs. (2) and (4)]. The inte-

gration over  $q$  in (1) is therefore divided into two regions: for  $0 < q < 1/\xi^2 q_{\text{TF}}$ ,  $U(q, \omega)$  is approximated by the unscreened form, while for  $q > 1/\xi^2 q_{\text{TF}}$ ,

$$U(q, \omega) \approx \frac{\pi e^2 q [1 + 1/(\xi q)^2]^2}{\epsilon q_{\text{TF}}^2 \sinh(qd)}. \quad (7)$$

The dominant contribution to  $\rho_t$  in this regime is

$$\rho_t \approx \frac{1}{48\pi} \frac{h}{e^2} \left( \frac{k_B T}{\hbar D n} \right)^2 \frac{\ln(q_{\text{TF}} \xi)}{(q_{\text{TF}} d)^2}. \quad (8)$$

Note that in both Eqs. (6) and (8)  $\rho_t$  is an increasing function of  $\xi$  (typically  $q_{\text{TF}} \sim 1/d$ , hence  $q_{\text{TF}} \xi \gg 1$ ). However, for  $\xi \gg d$  the dependence on  $\xi$  is weaker: there is a considerable contribution to the frictional drag from density fluctuations on length scales within  $\xi$ . In addition, in this regime the enhancement of screening competes with the enhancement of conductivity.

Finally, when  $\xi$  is further increased, eventually the approximations made in the beginning of this section break down. For  $\xi \rightarrow \infty$ , the form of  $\chi(q, \omega)$  coincides with that of the diffusive state considered in Ref. 11: in the low  $T$  limit ( $k_B T \ll \hbar/\tau$ ), Zheng and MacDonald find  $\rho_t \sim T^2 \ln(k_B T \tau / \hbar) / D^2$ . When a control parameter is varied so that  $\xi$  gradually increases from  $\xi \ll d$  to  $\xi \rightarrow \infty$ ,  $\rho_t$  interpolates between the expressions (6), (8) and the diffusive behavior.

### III. EFROS-SHKLOVSKII INSULATORS

The effect of Coulomb interactions within the layers on the dissipative processes associated with the hopping mechanism is quite subtle.<sup>21</sup> The dominant processes, at small but finite  $\omega$ ,  $q$ , and  $T$ , are transitions of electrons between two localized sites that are close in energy,<sup>24,25</sup> typically, different pairs of such sites are sparsely distributed. In the presence of Coulomb interactions, on one hand the transitions are enhanced due to the greater probability to find a *singly* occupied pair; on the other hand, a Coulomb gap is introduced in the single-electron density of states near the Fermi level. The a.c. conductivity  $\sigma(q, \omega)$ , assisted by resonant transitions, is more strongly affected by the former, and is therefore enhanced compared to the noninteracting case. This has crucial consequences on the drag between layers at the ES insulating state.

Similarly to the previous section, the transresistance is evaluated employing Eqs. (1) and (2) with the appropriate form of  $\chi(q, \omega)$ , assumed to be identical in the two layers. In this case the effect of  $\chi(q, \omega)$  on the interlayer interaction can be neglected, and  $U$  is approximated by the unscreened form,  $U(q, \omega) \approx V(q) e^{-qd}$ . As will become evident below, the  $\omega$  integration in Eq. (1) has an infrared divergence, and hence the prominent contribution to  $\rho_t$  arises from the lower cutoff. This cutoff is introduced at finite  $T$  by dephasing, associated with phonon-mediated relaxational processes, which suppresses resonant transitions between sites separated by a distance  $r_\omega$  larger than the dephasing length  $L_\phi$ . The pair arm  $r_\omega$  diverges<sup>24</sup> with the frequency as

$$r_\omega = \xi \ln(\omega_0 / \omega), \quad \omega_0 \equiv \frac{e^2}{\epsilon \xi \hbar}, \quad (9)$$

and  $L_\phi$  is set by the hopping distance

$$L_\phi = \xi \left( \frac{T_0}{T} \right)^{1/2}, \quad \left( T_0 = \frac{\hbar \omega_0}{k_B} \right). \quad (10)$$

Hence, coherent frequency-driven hopping occurs at  $\omega > \omega_c$ , where

$$\omega_c = \omega_0 e^{-L_\phi / \xi} = \omega_0 e^{-(T_0 / T)^{1/2}}. \quad (11)$$

To proceed with the calculation of  $\rho_t$  using Eq. (1), I relate  $\text{Im} \chi(q, \omega)$  to the a.c. conductivity through

$$\text{Im} \chi(q, \omega) = \frac{q^2}{\omega e^2} \sigma(q, \omega). \quad (12)$$

$\sigma(q, \omega)$  at a finite  $q$  was calculated for a 2D ES insulator by Aleiner and Shklovskii,<sup>26</sup> who obtained

$$\sigma(q, \omega) \sim C_1 \epsilon \xi \omega \quad \text{for } q \ll r_\omega^{-1},$$

$$\sigma(q, \omega) \sim \frac{C_2 \epsilon \xi \omega}{(q r_\omega)^2} \quad \text{for } r_\omega^{-1} \ll q \ll \xi^{-1},$$

$$\sigma(q, \omega) \sim \frac{C_3 \epsilon \xi \omega}{(q r_\omega)^2 (q \xi)^{2-\eta}} \quad \text{for } \xi^{-1} \ll q; \quad (13)$$

here  $C_j$  ( $j=1,2,3$ ) are numerical constants of order unity, and  $\eta$  is nonvanishing in case the localized single-electron states have multifractal structure. Note that Ref. 26 considers the limit  $\hbar \omega \gg k_B T$ , while in the present case the relevant range of frequencies [which dominates Eq. (1)] is  $\hbar \omega < k_B T$ ; however, following Refs. 25 and 27, it can be shown that the result differs only by the values of the  $C_j$ 's. Inserting Eq. (13) into (12) yields approximate expressions for  $\text{Im} \chi(q, \omega)$  in three different regimes of  $q$ . I then evaluate  $\rho_t$ , similarly to the previous section, distinguishing the limit cases  $\xi \ll d$  and  $\xi \gg d$ .

In the case  $\xi \ll d$ , the high  $q$  regime ( $q > \xi^{-1}$ ) is exponentially suppressed, and the  $q$  integration in Eq. (1) yields (for carrier density  $n$  in the two layers)

$$\rho_t \sim \frac{C_2^2 \beta \hbar^2}{8 n^2 e^2} \left( \frac{\xi}{d} \right)^2 \int_{\omega_c}^{\omega_d} \frac{d\omega}{\sinh^2(\beta \hbar \omega / 2) r_\omega^4}. \quad (14)$$

The upper limit  $\omega_d \equiv \omega_0 e^{-2d/\xi}$  corresponds to  $r_\omega = 2d$ ; the integration over the frequency range  $\omega > \omega_d$  gives a subdominant contribution that is neglected here. The final expression for  $\rho_t$  is dominated by the lower cutoff (for sufficiently low  $T$ ,  $L_\phi \gg d$ ):

$$\rho_t \sim \frac{C_2^2}{4\pi} \frac{h}{e^2} \frac{1}{(nd\xi)^2} \left( \frac{T}{T_0} \right)^3 \exp \left\{ \left( \frac{T_0}{T} \right)^{1/2} \right\}. \quad (15)$$

In the opposite regime, where the localization length  $\xi$  greatly exceeds  $d$ , one should account for the contribution of short length-scale density fluctuations with  $1/\xi \ll q \ll 1/d$ . The integration over  $q$  is facilitated by the approximation  $r_\omega \gg d$  (note that in the relevant range of  $\omega$ ,  $r_\omega > \xi$ ), and gives

$$\rho_t \sim \frac{\beta \hbar^2 \xi^2}{2n^2 e^2} \int_{\omega_c}^{\omega_d} \frac{d\omega}{\sinh^2(\beta \hbar \omega/2)} \times \left\{ \frac{(C_1^2/6 - C_2^2)}{r_\omega^6} + \frac{[C_2^2 + C_3^2/2(1-\eta)]}{r_\omega^4 \xi^2} \right\}. \quad (16)$$

Similarly to the short  $\xi$  limit, the lower cutoff dominates. The final expression for  $\rho_t$  for  $\xi \gg d$  is

$$\rho_t \sim \frac{[C_2^2 + C_3^2/2(1-\eta)]}{\pi} \frac{\hbar}{e^2} \frac{1}{n^2 \xi^4} \left( \frac{T}{T_0} \right)^3 \exp \left\{ \left( \frac{T_0}{T} \right)^{1/2} \right\}. \quad (17)$$

This result essentially differs from Eq. (15) only by the algebraic dependence on  $\xi$ —short length-scale fluctuations are effectively cut off by  $\xi$  rather than  $d$  (the latter sets a higher upper cutoff on  $q$ ). Hence, there is no dependence on the interlayer separation in this case.

#### IV. DISCUSSION AND SUMMARY

As shown in the calculations detailed above, Coulomb drag between layers in the Anderson insulator state can serve as a clear signature of the presence or absence of a Coulomb gap in the layers. Most importantly, the transresistance in a double layer of Mott insulators is *suppressed* with a decreasing temperature and localization length; in contrast, the presence of a Coulomb gap in ES insulating layers leads to a *divergence* of  $\rho_t$  at low  $T$  and  $\xi$ .

The physical origin of this distinction is the relative enhancement of resonant hopping processes in the presence of a Coulomb gap, which ensures a greater probability that a ‘‘destination site’’ is empty. In the single-layer a.c. conductivity  $\sigma(q, \omega)$ , this difference is indicated by the power-law dependence on  $\omega$  ( $\omega$  versus  $\omega^2$ ), though in both cases  $\sigma$  vanishes at  $\omega \rightarrow 0$ . However, the low frequency limit of the dynamical structure factor,

$$S(q, \omega) = \frac{\hbar}{1 - e^{-\hbar\omega\beta}} \text{Im} \chi(q, \omega), \quad (18)$$

is crucially distinct: since  $S \sim \sigma/\omega^2$ , in the Mott insulator  $S(q, \omega \rightarrow 0)$  is finite, while in the ES state it diverges as  $1/\omega$ . As expressed by Eq. (1), the frictional drag directly probes the dynamics of density fluctuations through the convolution of  $S(q, \omega)$  in the two layers. Therefore, in the ES insulator  $\rho_t$  depends on the *lower* frequency cutoff, associated with the dephasing length, hence diverges at low  $T$  and  $\xi$ . In the Mott insulator this anomaly does not exist, and  $\rho_t$  decreases with  $\xi$  due to the suppression of excited density fluctuations—in agreement with the naive intuitive argument.

In both types of insulators,  $\rho_t(\xi)$  depends on whether  $\xi$  is smaller or larger than the layers separation  $d$ . The predictions of this paper can be summarized as follows.

(a) In the Mott insulator,  $\rho_t \sim T^2$  similarly to the free electron gas case. The localization is manifested as a strong dependence on  $\xi$  for  $\xi \ll d$ , while for  $\xi \gg d$  the subtle role played by screening effects yields a weaker dependence:

$$\rho_t \propto \left( \frac{k_B T}{\hbar D n} \right)^2 (q_{\text{TF}} d)^2 \left( \frac{\xi}{d} \right)^8 \quad (\xi \ll d), \quad (19)$$

$$\rho_t \propto \left( \frac{k_B T}{\hbar D n} \right)^2 \frac{1}{(q_{\text{TF}} d)^2} \ln(q_{\text{TF}} \xi) \quad (\xi \gg d). \quad (20)$$

(b) In the ES insulator, the resulting  $\rho_t(\xi, T)$  is

$$\rho_t \propto \frac{1}{n^2 (\xi d)^2} \left( \frac{T}{T_0} \right)^3 \exp \left\{ \left( \frac{T_0}{T} \right)^{1/2} \right\} \quad (\xi \ll d), \quad (21)$$

$$\rho_t \propto \frac{1}{n^2 \xi^4} \left( \frac{T}{T_0} \right)^3 \exp \left\{ \left( \frac{T_0}{T} \right)^{1/2} \right\} \quad (\xi \gg d), \quad (22)$$

where  $k_B T_0 = e^2/\epsilon \xi$ . Note that  $\rho_t$  diverges at low  $T$  and  $\xi$  with the same exponential factor as the in-layer resistance  $\rho$ . The algebraic prefactor typically suppresses  $\rho_t$  with respect to  $\rho$  by 3 to 4 orders of magnitude. This ensures that although the drag is a huge effect in this case, the two-layer resistivity tensor is still almost diagonal, and the weak-coupling assumption underlying Eq. (1) is justified.

Experimentally, a crossover from an ES to a Mott insulator can be in principle controlled by a metallic back gate, which effectively attenuates the range of interactions in the layer. The above predictions imply that at the ES state, the onset of an insulating behavior should be accompanied by a sharp increase of  $\rho_t$  towards the insulating regime; when the insulating state is well approximated by the Mott behavior,  $\rho_t$  should be peaked near the transition to the insulator, and strongly attenuated when  $\xi$  is reduced below  $d$ . A similar qualitative behavior is also expected in the case where some of the simplifying assumptions of this paper are violated—for example, when the layers are not identical, and in particular if only one of them undergoes a transition to the insulator. However, note that in the case where one of the layers is a good conductor, it may serve as a back gate which suppresses Coulomb interactions in the other.

In order to observe an appreciable drag at low  $T$  in the Mott state, the desired experimental setup should involve low mobility, low density samples. For  $\ell \sim 1 \mu\text{m}$ ,  $n \sim 10^{10} \text{ cm}^{-2}$ ,  $d = 200 \text{ \AA}$ , band mass of  $m \approx 0.07 m_e$ ,  $\epsilon = 13$  (typical to GaAs) and  $T = 1 \text{ K}$ , I get  $\rho_t$  in the order of a few tens of  $m\Omega$  in the regime  $\xi > d$ . When the localization length is reduced to  $20 \text{ \AA}$ ,  $\rho_t$  is attenuated by a factor of  $10^{-5}$ . Assuming the same parameters in the ES state, I estimate  $\rho_t \sim 575 \Omega$  for  $\xi \sim 1000 \text{ \AA}$ ; at  $\xi \sim 100 \text{ \AA}$ , the transresistance rises to  $\rho_t \sim 100 \text{ k}\Omega$ .

Finally, the experimental testing of the effects predicted in this paper involves a number of difficulties. Primarily, once the in-layer resistance is comparable to that of the barrier separating the two layers, tunneling across the barrier is no longer negligible; its contribution should be carefully eliminated. In addition, to obtain a sizable signal, the voltage imposed on the drive layer in an insulating state may be large enough to produce nonlinear response effects. The role of interlayer coupling mediated by phonons (see, e.g., Gramila *et al.* in Ref. 9 and Ref. 14), and possible thermoelectric effects (Solomon *et al.* in Ref. 9, Laikhtman *et al.* in Ref.

10), are not discussed in the present paper. These are generally expected to be subdominant at low  $T$ ; however, at the onset of an insulating state (and particularly near a MIT) such effects may be enhanced as well. Nevertheless, the estimates made above indicate that Coulomb drag near the onset of an insulator is an appreciable effect, and a sensitive probe of the significance of Coulomb interactions in insulating states.

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- <sup>19</sup>As noted in Ref. 14, the strength of Coulomb drag is dictated by an interplay between  $\text{Im } \chi(q, \omega)$  (directly related to the a.c. conductivity in the layers), and their dielectric properties which affect the interlayer interaction [see Eq. (2)]; this interplay becomes quite subtle in case both factors vary considerably. In an Anderson insulator, both the conduction and screening are rapidly suppressed, so that *a priori* it is not obvious that the former is the overwhelming factor.
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