Thermoelectric and hot-electron properties of a silicon inversion layer

R. Fletcher
Physics Department, Queen's University, Kingston, Ontario, Canada K7L 3N6

V. M. Pudalov
Institute for High Pressure Physics, Troitsk, Moscow District, 142092, Russia

Y. Feng
Institute for Microstructural Sciences, National Research Council, Ottawa, Ontario, Canada K1A 0R6

M. Tsaousidou and P. N. Butcher
Physics Department, University of Warwick, Coventry CV4 7AL, United Kingdom

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Electron-phonon coupling of a two-dimensional electron gas in a Si metal-oxide-semiconductor field-effect transistor in the temperature range $0.3\ \mathrm{K}<T<4\ \mathrm{K}$ has been investigated using phonon-drag thermopower S^g and electron energy loss rate $F(T)$. At low temperatures (the Bloch limit) we find $S^g \propto T^6$, as expected for electron-phonon scattering mediated by a screened deformation potential, and the magnitude is in excellent agreement with a calculation using no adjustable parameters; the calculation continues to give good agreement at higher temperatures. $F(T)$ has been calculated using the same input parameters as for S^g. Reasonably good agreement is found with the observed values for $T>1.5$ K, but at lower temperatures the measured $F(T)$ is much larger than predicted and also exhibits a much weaker temperature dependence. Possible reasons are suggested. [S0163-1829(97)05143-6]

I. INTRODUCTION

The present work investigates the low-temperature coupling of electrons in a Si inversion layer to bulk phonons in the substrate using phonon-drag thermopower S^g and electron energy loss rate $F(T)$. Both of these quantities are sensitive measures of electron-phonon (e-p) coupling but in different ways. S^g is determined by the momentum relaxation time for e-p scattering, whereas $F(T)$ measures the e-p relaxation time for energy loss.

Thermopower has two contributions: phonon drag S^g, which has a complex temperature dependence, and diffusion S^d, which is linear in temperature for degenerate electrons. We are not primarily interested in S^d except insofar as we need to separate it from S^g in the experimental results. S^d has been calculated for different electron scattering mechanisms,[1] the main ones for metal-oxide-semiconductor field-effect transistors (MOSFET's) being interface roughness, background impurities, and remote impurities. For the present purposes, the most interesting point is that the magnitude and sign of S^d can be varied by adjusting the carrier density and can even be made zero; we have done this in the present experiments.

Theoretical work[2,3] on S^g has led to a good understanding of the thermoelectric properties of a number of two-dimensional electron gases (2DEG's). In particular, experimental data on MOSFET's by Gallagher *et al.*[4] were well reproduced by the theory. At low temperatures S^g becomes very small, but we have taken advantage of the fact that S^d can be adjusted to zero to accurately measure S^g over a wide temperature range. This has enabled us to probe S^g in the Bloch limit of e-p scattering, i.e., when the magnitude of the average phonon wave vector Q becomes much smaller than the diameter of the Fermi circle $2k_F$. The Bloch limit has previously been observed in GaAs heterostructures,[5] but there the interaction is dominated by piezoelectric coupling, whereas in Si inversion layers it occurs via a deformation potential. The temperature dependence of S^g should be different in the two cases.

There has been extensive previous work on energy loss rates in Si inversion layers (e.g., see Refs. 6–9), but none below 1 K. Serious disagreements between calculation and experiment were found. In retrospect it is clear that a significant problem in the comparisons was that none of these early calculations included screening of the scattering potential so that the resulting calculated values were too high and had the wrong temperature dependence. This was especially so in the Bloch limit where screening gives an extra factor T^2.

The present experiments provide an example of simultaneous measurements on a pair of quantities, each of which should independently determine e-p coupling. We shall see that S^g behaves as predicted, as does $F(T)$ at higher temperatures. However, the latter is found to be unexpectedly large at low temperatures.

II. THEORY

We first examine S^g. Electrons in the 2DEG with wave vector $\mathbf{k}=(k_x,k_y)$ are quasielastically scattered by three-dimensional (3D) phonons in the substrate of wave vector $\mathbf{Q}$ and polarization i; the phonon wave vector is expressed as $\mathbf{Q}=(\mathbf{q},q_z)$, where $\mathbf{q}$ is the component in the plane of the 2DEG and q_z the perpendicular component. Following Smith,[10] the result for S^g, which includes a correction for

nondegeneracy of the 2DEG, can be written

$$S^g = -\frac{(2m^*)^{3/2} g_v \Lambda}{16(2\pi)^3 k_B T^2 n e \varrho} \times \sum_i v_i \int_0^\infty \int_{-\infty}^\infty \frac{\Xi^2(\mathbf{Q}) q^2 Q^2 \Delta(q_z) G(\mathbf{Q})}{\epsilon^2(q) \sinh^2(\hbar\omega_\mathbf{Q}/2k_B T)} dq\, dq_z, \tag{1}$$

where ϱ is the mass density of Si, n is the carrier density, e is the magnitude of the electronic charge, Λ is the phonon mean free path, g_v is the valley degeneracy, and $\Xi(\mathbf{Q})$ denotes the effective acoustic scattering potential. For Si, e-p coupling is complicated by the anisotropy of the electronic energy spectrum and we follow Smith and Butcher[3] and Ridley[11] in using $\Xi_{/\!/}(\mathbf{Q}) = \Xi_u(D + q_z^2/Q^2)$ and $\Xi_t(\mathbf{Q}) = \Xi_u q_z q/Q^2$ for the deformation potentials appropriate to longitudinal and transverse phonons, respectively, where Ξ_d and Ξ_u are the deformation potential constants for pure dilation and pure shear strain and $D = \Xi_d/\Xi_u$. We note that $\Xi_t(\mathbf{Q})$ includes the coupling to both transverse branches. The form factor $\Delta(q_z)$ allows for the finite thickness of the 2DEG and is given by $|\int \phi^*(z) e^{iq_z z} \phi(z) dz|^2$. For $\phi(z)$ we use the Fang-Howard variational function[12] for which $\Delta(q_z) = b^6/(b^2 + q_z^2)^3$, where b is a variational parameter. The static dielectric screening function $\epsilon(q)$ is given by[3,13] $1 + (Q_s/q)\xi(q) F_s(q)$. Here $Q_s = g_v m^* e^2/2\pi\epsilon_0 \kappa \hbar^2$ is the screening wave vector, κ is the average permittivity of Si and SiO_2, and ϵ_0 is the permittivity of free space; $\xi(q)$ is unity for $q \leqslant 2k_F$ and $1 - [1 - (2k_F/q)^2]^{1/2}$ for $q > 2k_F$; $F_s(q)$ is the screening form factor, which for the Fang-Howard function is given by[13]

$$F_s(q) = \frac{1}{16}\left(1 + \frac{\kappa_{ox}}{\kappa_{sc}}\right)\left(1 + \frac{q}{b}\right)^{-3}\left(8 + 9\frac{q}{b} + 3\frac{q^2}{b^2}\right) + \frac{1}{2}\left(1 - \frac{\kappa_{ox}}{\kappa_{sc}}\right)\left(1 + \frac{q}{b}\right)^{-6}, \tag{2}$$

where κ_{ox} and κ_{sc} are the permittivities of SiO_2 and Si, respectively. Finally, $G(\mathbf{Q})$ is the energy integral

$$G(\mathbf{Q}) = \frac{1 - \exp(-\hbar\omega_\mathbf{Q}/k_B T)}{\hbar\omega_\mathbf{Q}} \times \int_\gamma^\infty d\varepsilon\, \frac{f_0(\varepsilon)[1 - f_0(\varepsilon + \hbar\omega_\mathbf{Q})]}{\sqrt{\varepsilon - \gamma}}, \tag{3}$$

where $\varepsilon = \hbar^2 k^2/2m^*$ is the electronic energy, $f_0(\varepsilon)$ is the Fermi-Dirac function, and γ is given by $(\hbar\omega_\mathbf{Q} - E_q)^2/4E_q$, with $E_q = \hbar^2 q^2/2m^*$.

At a low enough temperature when $Q \ll 2k_F$ the system enters the Bloch limit and there are many simplifications that can be made. Taking $\hbar\omega_\mathbf{Q}$ as a small quantity, the product of the electron occupation factors in Eq. (3) can be approximated by $\hbar\omega_\mathbf{Q}[1 - \exp(-\hbar\omega_\mathbf{Q}/k_B T)]^{-1} \delta(\varepsilon - \varepsilon_F + \hbar\omega_\mathbf{Q}/2)$, where ϵ_F is the Fermi energy. Then, in the limit of $T \to 0$ (for which $Q \to 0$), $G(\mathbf{Q})$ reduces to $(2m^*)^{1/2}/\hbar k_F$. In the same limit, assuming that the 2DEG is thin enough for q_z/b and q/b to be much less than 1, we can write $\Delta(q_z) \to 1$ and $\epsilon^2(q) \to (Q_s/q)^2$. Finally, using $u = \hbar q v_i/k_B T$ and $w = \hbar q_z v_i/k_B T$, Eq. (1) can be approximated by

$$S^g = -\frac{m^{*2} \Lambda k_B^7 T^6 \Xi_u^2}{16\pi^2 k_F^3 \hbar^9 e \varrho Q_s^2} \times \sum_i \frac{1}{v_i^7} \int_{-\infty}^\infty dw \int_0^\infty du\, \frac{a_i^2 u^4 (u^2 + w^2)}{\sinh^2(\sqrt{u^2 + w^2}/2)}, \tag{4}$$

where $a_{/\!/} = D + w^2/(u^2 + w^2)$ and $a_t = uw/(u^2 + w^2)$ for longitudinal and transverse modes, respectively. There are no unknown quantities in this expression and, provided Λ is independent of T, $S^g \propto T^6$. In this limit $S_g \propto n^{-3/2}$ via the factor k_F^{-3}, but the result is independent of m^* (because $Q_s \propto m^*$) and the details of the electronic wave function. With piezoelectric coupling, which is appropriate in GaAs structures, the limiting low-temperature dependence is T^4 and the magnitude remains much larger to lower temperatures.[5]

For completeness, we briefly examine the diffusion contribution S^d to the thermopower of a degenerate 2DEG. This is given by

$$S^d = -\frac{\pi^2 k_B^2 T}{3e\varepsilon_F}(1 + p), \tag{5}$$

where p is a constant whose value depends on the type of scattering. In a calculation for a particular MOSFET, Karavolas and Butcher[1] found that p passes through -1 in the region of $n = 9 \times 10^{15}$ m^{-2}, and this was experimentally confirmed by Karavolas *et al.*[14] In these circumstances the scattering term (i.e., that part dependent on p) and the entropy term (the remainder) cancel, leaving $S^d \sim 0$; thus S_g will dominate in this region.

The theoretical result for the energy loss rate is taken from Ma *et al.*[15] [see their Eq. (A2)], but we have modified this to allow for nondegeneracy as with S^g. When the substrate is at zero temperature and the electron temperature is T the energy loss rate $F(T)$ can be written as

$$F(T) = \frac{(2m^*)^{3/2} g_v}{16\pi^3 \hbar n \varrho} \times \sum_i v_i \int_0^\infty \int_{-\infty}^\infty \frac{\Xi^2(\mathbf{Q}) Q^3 \Delta(q_z) G(\mathbf{Q})}{\epsilon^2(q)[\exp(\hbar\omega_\mathbf{Q}/k_B T) - 1]} dq\, dq_z. \tag{6}$$

When the lattice temperature is $T_{/\!/}$ and the electron temperature is T_e the loss rate is $F(T_e) - F(T_{/\!/})$. In the Bloch limit we make the same approximations and substitutions as before and find that

$$F(T) = \frac{m^{*2} (k_B T)^7 \Xi_u^2}{2\pi^2 k_F^3 \hbar^9 \varrho Q_s^2} \times \sum_i \frac{1}{v_i^6} \int_{-\infty}^\infty dw \int_0^\infty du\, \frac{a_i^2 u^2 (u^2 + w^2)^{3/2}}{\exp(\sqrt{u^2 + w^2}) - 1}. \tag{7}$$

The expressions for S^g and $F(T)$ are derived with equivalent basic assumptions and their common key element is e-p coupling. The averages over the phonon spectrum are different in the two cases of S^g and $F(T)$, but the low-temperature equations have many common factors and we can write

$$F(T) = -\zeta S^g \frac{v e T}{\Lambda}, \tag{8}$$

where v is a suitable average velocity and ζ a numerical constant of order unity. Because v_i appears to a high power in Eqs. (4) and (7), the dominant contribution in each case is made by transverse modes, particularly at low temperatures, and v is essentially an average only over these modes. In the same limit, using the value of a_t appropriate to MOSFET's, $\zeta = 0.46$. If a_i is replaced by unity, $\zeta = 0.38$. Even when the scattering potential is unscreened so that the integrands in the above equations no longer contain the factor $(q/Q_s)^2$ we find that ζ only changes to 0.80. In other words, at low temperatures the value of ζ is not very sensitive to the precise form of the deformation potential used and it is essentially independent of the magnitude providing coupling to one of the phonon polarizations is dominant, as is the case here. Equation (8) provides a useful semiquantitative way of predicting either one of S^g or $F(T)$ given the other for any 2DEG and clearly reveals the common link of e-p scattering in both of these quantities.

III. EXPERIMENTS

The MOSFET was grown on a crystallographic plane of Si oriented nominally perpendicular to [001]; the Si was B doped and had a nominal resistivity of 20 Ω cm. The substrate of the sample used here had dimensions $4 \times 8 \times 0.5$ mm^3 and the gated region was 2.5×0.25 mm^2 with the long axis parallel to [100] and the direction of the temperature gradient. Electron densities in the range $n = (0-10) \times 10^{15}$ m^{-2} were available. At 4 K the mobility was ~ 1.4 m^2 V^{-1} s^{-1} in the density range of interest [$\sim (4-10) \times 10^{15}$ m^{-2}] and rose to ~ 1.75 m^2 V^{-1} s^{-1} at low temperatures.

Temperatures in the range 0.3–4.2 K were obtained with a ^{3}He cryostat. A matched pair of 5.1 kΩ Dale surface-mounted resistors, with good sensitivity over the whole temperature range,[16] was epoxied to the substrate to measure the temperature and temperature gradient. Temperature calibrations were made with a commercially calibrated germanium resistor (which agreed with temperatures measured by vapor pressure to $\leqslant 5$ mK in the ^{4}He superfluid region of about 1.4–2.1 K).

All measurements used dc techniques. Potential differences were determined by an EM type N11 nanovoltmeter, manufactured by EM Electronics, England, with a resolution of a few nV. At temperatures below ~ 1 K each pair of sample contacts was found to have a temperature-dependent output voltage even at zero-temperature gradient. The origin of this voltage is unknown, but its magnitude varied with cooldown procedure and was different for each contact pair. It was very important to allow for this voltage since it became of the order of the thermopower signal at ~ 0.8 K. We did this by fixing the average temperature of the sample at the same value both with and without the temperature gradient so that the differences accurately gave the thermoelectric signal.

The absolute accuracy of the thermopower is expected to be about 20%, the error mainly resulting from the measurement of the distance between the thermometers. However, the self-consistency between the values of p measured by different methods[17] (and the thermal conductivity of the two substrates discussed below) suggests that the total error is probably no more than 5%. The relative accuracy should be at the level of 1–2%. Measurements of the thermopower using different contacts on the MOSFET yielded the same values within 2–3%, suggesting that the distance between the contacts did not contribute a significant error and that the thermoelectric properties were uniform over the sample.

Energy loss rates were determined by measuring the electron temperature as a function of Joule heating. Temperatures were deduced using the amplitude of Shubnikov–de Haas oscillations in the resistivity at low magnetic fields (up to 2–3 T). The amplitude was calibrated at low excitation current (down to 50 nA) where no change in amplitude with current was visible, particularly at the lowest temperatures. The sample was then held at 0.3 K and various currents up to ~ 15 μA provided electron heating. Above about 3 K the oscillations became too few and too small to be useful. The electron temperature was determined by analyzing the amplitude at many different values of magnetic field. Typically 3–10 values of the temperature were obtained at each excitation current with differences among the various determinations usually being $\leqslant 1\%$. At the highest values of Joule heating the temperature of the substrate increased to about 0.36 K, but in this case $F(T_e) \gg F(T_{/\!/})$ and this has no significant effect on the interpretation in the next section.

IV. RESULTS AND DISCUSSION

A. Thermopower

In order to compare experiment with calculation, we need the phonon mean free path Λ. This is obtained from the thermal conductivity λ of the substrate shown by circles in Fig. 1. The 2DEG plays no significant part in the magnitude of λ, which is completely dominated by phonons. The **Q** and i dependences of Λ are ignored and we write

$$\lambda = \frac{1}{3} \Lambda \sum_i C_i v_i, \tag{9}$$

where C_i is the phonon specific heat for each phonon branch in the Debye approximation. With acoustic velocities of longitudinal and transverse modes of $v_{/\!/} = 8834$ ms^{-1} and $v_t = 5269$ ms^{-1}, respectively (which are $\langle 1/v^2 \rangle$ averages of the longitudinal and transverse sound velocities over the three high-symmetry directions using data from Huntington[18]) we expect $\lambda = 1145 \Lambda T^3$ W m^{-1} K^{-1}. If boundary scattering were dominant, then Λ would be independent of temperature. The data are given in the form λ/T^3 to show that there are deviations from this behavior. Initially, Λ weakly decreases with T from 0.90 mm at $T = 4.5$ K to 0.78 mm at $T = 1.5$ K and then begins to increase again, reaching 1.85 mm at $T = 0.26$ K. The dip in Λ is attributed to phonon scattering by the boron acceptor impurities that have

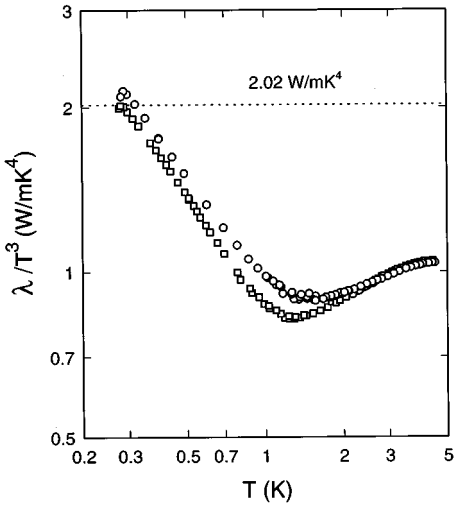

FIG. 1. Thermal conductivity λ of the substrates, plotted in the form λ/T^3, as a function of temperature T. The circles are for the first substrate (with the MOSFET used in all other measurements); the squares are for the second substrate. The horizontal line at 2.02 W m^{-1} K^{-4} is the calculated value of λ/T^3 (see the text).

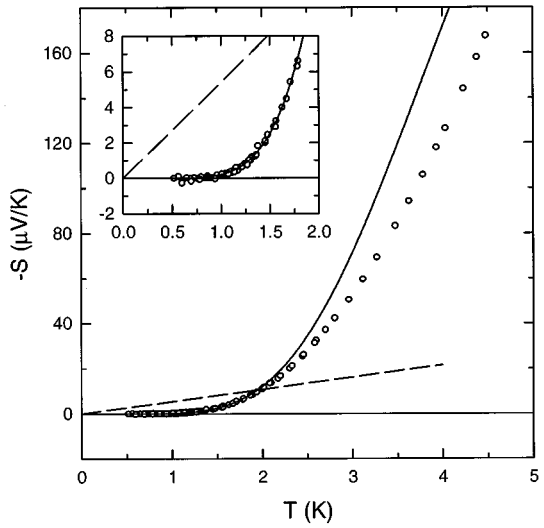

FIG. 2. Thermopower S as a function of temperature T. The circles are experimental data and the solid line is the calculated S^g. The dashed line is the calculated entropy contribution to the diffusion thermopower $S^d=-\pi^2k_B^2T/3e\varepsilon_F=-5.4T$ μV/K. Above 1.5 K the temperature dependence of S tends to $\sim T^3$. The sample density $n=8.5\times10^{15}$ m^{-2} and is also the same value for the following figures.

a resonance behavior.[19] A more highly B-doped substrate (10 Ω cm) measured by Gallagher *et al.*[4] showed a rather different behavior. We also examined a second substrate with a MOSFET from another, nominally identical wafer. The results are also shown in the same figure (squares). The two sets of data are in good agreement.

Because the heat flow is parallel to [100], both substrates should show substantial phonon focusing effects. Assuming only diffuse boundary scattering, the results of McCurdy[20] enable us to estimate $\lambda=2.02T^3$ W m^{-1} K^{-1} for these samples, which is close to the measured low-temperature values. However, with GaAs samples specular reflection increases λ by typically 50% and the factor is likely similar here, so it is possible that these samples are still not boundary limited even at 0.3 K.

The thermopower S of the 2DEG with a carrier density of $n=8.5\times10^{15}$ m^{-2} is shown by circles in Fig. 2. The solid line in the same figure is the theoretical S^g obtained from Eq. (1) and the temperature dependent Λ. The values of the parameters used are[3] $g_v=2$, $\Xi_u=9.0$ eV, and $\Xi_d=-6.0$ eV; $m^*=0.1905m_e$ and the mass in the z direction is $0.916m_e$; $\kappa_{ox}=3.9$ and $\kappa_{sc}=11.7$. The sound velocities used here are $v_\ell=8861$ ms^{-1} and $v_t=5331$ ms^{-1}, which are, respectively, simple averages of the longitudinal and transverse sound velocities over the three high-symmetry directions.[18] Also given in Fig. 2 is the calculated entropy part of S^d from Eq. (5), i.e., when $p=0$ (dashed line), and this shows that $1+p$ must be very small for these data. Thus practically all the measured S is due to S^g and this is seen to agree well with the calculation, especially at low temperatures.

The results obtained in Sec. II imply that at low temperatures $S=\beta T+\eta T^6$, where the terms proportional to β and η are, respectively, the diffusion and phonon-drag contributions to the thermopower. In Fig. 3 the circles show the experimental data of S/T as a function of T^5 at $T\leqslant1.35$ K; a straight line is indeed obtained. Other integer power laws for S^g gave poorer fits. Fortunately, Λ is fairly constant in the temperature range of interest for S^g ($\sim0.5-1.4$ K) so that this does not play a significant role in determining the temperature dependence. The measured slope of the best straight-line fit (dashed line in Fig. 3) is $\eta=-0.23\pm0.02$ μV K^{-7}, where the probable error includes all sources except the systematic error in the thermometer spacing. The thermometer spacing is an important source of experimental uncertainty, but in comparing experiment and theory it seems likely that this error would largely disappear

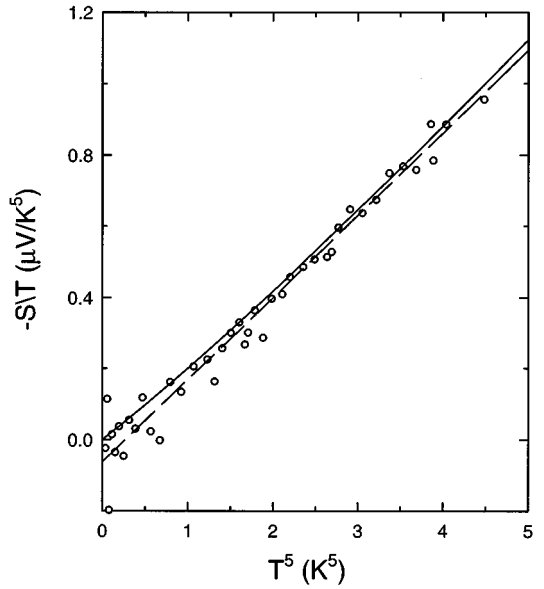

FIG. 3. Thermopower S plotted in the form $-S/T$ as a function of T^5. The symbols are the measured data and the dashed line is the best fit given by $S=0.06T-0.23T^6$ μV K^{-1}. The solid line is the calculated phonon-drag part S^g and is approximately given by $S^g=-0.22T^6$ μV K^{-1}. The data cover the range $\sim0.5-1.4$ K.

because the calculation of η takes Λ from λ, which involves the same thermometers as S^g. The cancellation of this error is not necessarily exact since different phonon averaging is involved in λ and S^g.

The solid line in Fig. 3 was calculated using Eq. (1) and the material parameters given above. The average slope of this line is $\eta = -0.22\ \mu\mathrm{V\,K}^{-7}$, which is in excellent agreement with the experimental value. To obtain an accurate theoretical value of η it was necessary to use Eq. (1). The approximate formula (4) gives $\eta = -0.10\ \mu\mathrm{V\,K}^{-7}$ using $\Lambda = 0.8$ mm, which is a reasonable average for this temperature range. The reason for the difference is that the approximations made to obtain Eq. (4) are valid only at very low T for very thin 2DEG's and constant Λ. In the experimental temperature range the approximation that introduces the largest error in Eq. (4) is that of $F_s(q) = 1$. Because the dominant contribution to the phonon-drag integrals occurs for $\overline{q} \sim 5k_BT/\hbar v_i$,[21] we see from Eq. (2) that $F_s(q)$ is a decreasing function of T. For the 2DEG considered here the variational factor b that describes the spatial extent of the electron wave function in the z direction is $b = 0.89\ \mathrm{nm}^{-1}$. At $T = 1$ K, $F_s(q) \approx 0.7$ and since $F_s^2(q)$ appears in the denominator of Eq. (1) the calculated S^g using the above approximation is underestimated by a factor of ~ 2. The ratio $\Lambda/F_s^2(q)$ is a smooth function of T and one can see a slight curvature of the theoretical line in Fig. 3, but on average S^g still follows a power law close to T^6.

We should point out that the agreement of the experimental and calculated η is perhaps better than we might have expected. Equation (4) shows that at low temperatures $S^g \propto v_i^{-7}$ and consequently the calculated value of η is sensitively dependent on the average velocities used, especially for the transverse modes. The acoustic velocities show relatively strong anisotropy and one should include this in the calculation at a fundamental level. To acquire a rough idea of how the anisotropy can affect the results we used $\langle 1/v^7 \rangle$ averages of the longitudinal and transverse sound velocities and found a difference of $\sim 20\%$ for η. Nevertheless, the excellent agreement suggests that the theory is basically sound and that the deformation potentials are a good representation of the real situation.

The intercept in Fig. 3 is $\beta = 0.06 \pm 0.03\ \mu\mathrm{V\,K}^{-2}$, which, using Eq. (5), gives $1 + p = -0.01$. As mentioned above, calculation[1] predicts that p will pass through -1 in the present region of carrier density n and in fact n was adjusted by the gate voltage to give this rather precise cancellation in $1 + p$. However, p turned out to be surprisingly independent of n over the available range. Even at $n = 4.7 \times 10^{15}\ \mathrm{m}^{-2}$ we find $p = -0.97 \pm 0.05$.[17] The calculation predicts a more rapid change with density, but it seems that the present results simply reflect a preponderance of interface roughness scattering in this sample, even at low carrier densities where remote impurities began to dominate in the calculation. Had the value of $1 + p$ been of the order of unity, it is clear from Fig. 2 that S^g would have been very difficult to measure accurately in this low-temperature region.

We have also measured the thermopower at other carrier densities. At $n \sim 4.7 \times 10^{15}\ \mathrm{m}^{-2}$ the Bloch region moves down in temperature to below 1 K, where accuracy is low, though the rapid falloff of S^g was still visible in the data.

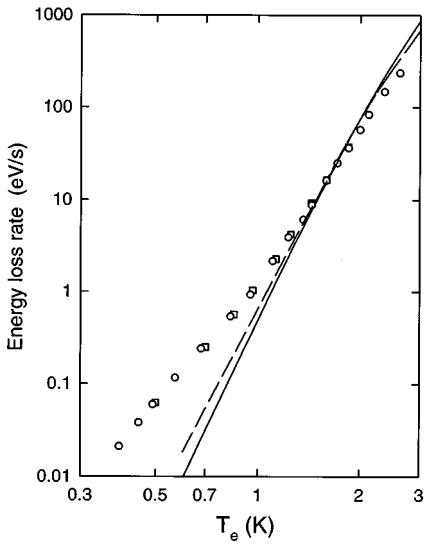

FIG. 4. Energy loss rate per electron as a function of electron temperature T_e at a lattice temperature $T_\ell = 0.3$ K. The symbols are the measurements from two independent runs. The solid line is the calculation using Eq. (6). The dashed line uses Eq. (8) with the calculated ζ as a function of T and with the measured S^g.

Higher densities would be an advantage if $1 + p$ remains small, but a significant improvement, say a factor of 2, was not available with the present sample.

B. Energy loss rate

Figure 4 shows data on the energy loss rate of this sample with the substrate maintained at 0.3 K. As the temperature is lowered from 3 K the slope of measured data increases until about 1.5 K, where it decreases again. The solid line is the calculated function $F(T_e) - F(T_\ell)$ using Eq. (6) and the same material parameters as above. The agreement is good above about 1.5 K in both magnitude and trend, which shows that the same mechanism is responsible for both $F(T)$ and S^g. We have also calculated $F(T_e) - F(T_\ell)$ using Eq. (8), with the measured S^g and the calculated ζ as a function of T; this is shown by the dashed curve. Below about 1 K we have used $S^g = \eta T^6$ with the experimental value of η to extrapolate this curve. Over the temperature range of the experimental data (0.4–2.3 K) ζ only varies from about 0.46–0.58, but it has increased to 1.05 by 4.5 K. Clearly Eq. (8) gives an excellent representation of the measured $F(T)$ above 1.5 K using the measured S^g and the curve hardly changes if one uses a simple constant value for ζ. We should mention that the approximate Eq. (7) underestimates the values of $F(T)$ by factor similar to that found with S^g using Eq. (4).

Recently, Stöger *et al.*[22] measured loss rates for 2DEG's in $Si/Si_{1-x}Ge_x$ heterostructures above about 1.5 K. Their calculations used screened deformation potential coupling, but, following Stern and Laux,[23] assumed that only longitudinal acoustic phonons couple to the electrons with a deformation potential of $\Xi_\ell = 9$ eV and this gave satisfactory results at the temperatures of interest. The same coupling should also be appropriate to Si MOSFET's, but using it in Eq. (1) we obtain $\eta = -0.042\ \mu\mathrm{V\,K}^{-7}$, which is about a factor of 5 smaller than the experimental value and our estimate, the decrease mainly being caused by the use of v_ℓ rather than

v_t. We also find a correspondingly small energy loss rate in the Bloch limit. This suggests that one must be very careful in assuming that the e-p coupling is accurately known when good agreement is found between calculation and experiment.

Below 1.5 K the measured loss rate becomes larger than that calculated, with a continuously increasing separation of the curves. At low temperatures the observed variation of $F(T)$ is roughly T^4 instead of the expected T^7. Even at 1 K the difference is about a factor of 2, but Fig. 3 for S^g shows no obvious anomalies in this region, which implies that the observed extra energy loss rate at low temperatures is not caused by bulk acoustic phonons.

As mentioned in the Introduction, previous work on the loss rate in Si inversion layers[6–9] noted serious disagreement between calculation and experiment. Some of this can be traced to the absence of screening in the calculations. Thus it was predicted that $F(T) \propto T^5$ instead of the correct $F(T) \propto T^7$ in the Bloch limit. Phonon-drag measurements, including those in this paper, conclusively show that screening must be included. Because of this it is not clear whether any of the previous experiments found discrepancies similar to those discussed here. Hönlein and Landwehr[9] did find anomalously high loss rates in high-density samples at low temperatures. The extra loss rate was attributed to localized excitations in the amorphous SiO_2 insulating layer, though a detailed model was not developed. Such localized excitations would not contribute to S^g.

Chow *et al.*[24] reported an anomalous energy loss rate for a 2DEG in a $GaAs/Ga_{1-x}Al_xAs$ in the quantum Hall regime. At zero fields they observed $F(T) \propto T^5$, as expected for screened piezoelectric scattering, but between the Hall plateaus this changed to $F(T) \propto T^4$. The difference was ascribed to the effect of impurities on e-p coupling, which was shown to be much more important in the quantum Hall region than at zero field. With deformation coupling the authors predicted $F(T) \propto T^6$ rather than T^7 as calculated here. The transition from clean to dirty limits should occur when $\overline{q}\ell_e \sim 1$, where ℓ_e is the electronic mean free path. Using our previous value $\overline{q} \sim 5k_BT/\hbar v_i$,[21] this corresponds to $T \sim 0.04$ K in our sample. Thus, this effect occurs at too low a temperature and is also too weak to explain our data.

In 3D metals it has been known for many years that, even in the clean limit $\overline{Q}\ell_e > 1$, interference effects between electron-impurity and e-p scattering changes the effective temperature-dependent momentum scattering rate from T^5 to T^2 at low temperatures (for recent calculations and experiments see Refs. 25 and 26, respectively). This change in power law is similar to that seen here. If this effect is also present in 2DEG's, one would expect to see it in low-mobility samples at low temperatures, but we are not aware of any other relevant experiments.

V. CONCLUSION

We have obtained good agreement between the temperature dependence and magnitude of the phonon-drag thermopower in a Si inversion layer in the Bloch limit. In the case of the energy relaxation rate the agreement is good at high temperatures but poor at low temperatures. Possible sources for the discrepancy are localized excitations in the SiO_2 or interference effects between electron-impurity and electron-phonon scattering. In principle, S^g would be insensitive to the former but sensitive to the latter. Because S^g shows no pronounced anomalies we expect that the effect of impurities on the electron-phonon coupling is not very important here.

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