Charging ultrasmall tunnel junctions in an electromagnetic environment

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(Received 23 April 1997; revised manuscript received 15 July 1997)

We have investigated the quantum admittance of an ultrasmall tunnel junction with arbitrary tunneling strength under an electromagnetic environment. Using the functional-integral approach a close analytical expression of the quantum admittance is derived for a general electromagnetic environment. We then consider a specific controllable environment where a resistance is connected in series with the tunneling junction, for which we derived the dc quantum conductance from the zero-frequency limit of the imaginary part of the quantum admittance. For such an electromagnetic environment the dc conductance has been investigated in recent experiments, and our numerical results agree quantitatively very well with the measurements. Our complete numerical results for the entire range of junction conductance and electromagnetic environmental conductance confirmed the few existing theoretical conclusions. [S0163-1829(97)02843-9]

I. INTRODUCTION

In semiclassical theory of Coulomb blockade,[1,2] electron tunneling is controlled by the charging energy, which is determined by the junction capacitance. While the existence of this elementary charging effect is clearly seen in multijunction configurations,[3] in a single junction, because of the stray capacitance, only partial blockade has been observed.[4] The question is then, including the electromagnetic environment, what is the effective junction capacitance? The work of Devoret *et al.*[5] has shown that external electrical circuit has strong impact on the effect of Coulomb blockade in a single junction, and under standard experimental conditions where the external resistance is much smaller than the resistance quantum $R_K = h/e^2$, the Coulomb gap is totally smeared out at low voltages and at low temperature. A similar result was also obtained by Girvin *et al.*[6] In both works, as well as in most existing studies on charging ultrasmall tunnel junctions, the tunnel Hamiltonian was treated perturbatively[1,7] using the Fermi golden rule. This lowest-order perturbative approach is valid for weak tunneling strength.

With increasing tunneling strength, one must go beyond the Fermi golden rule to include higher-order tunneling processes. The conventional perturbative calculation is very tedious, and has been used to study the statistical properties of a single electron box (SEB).[8–10] In a SEB an island is formed between a tunnel junction and a gate capacitor. This is the simplest structure exhibiting the charging effect. Even for such a simplest system, it is very difficult to derive the partition function beyond the second order of the dimensionless tunnel conductance R_K/R_T, where R_T is the tunnel resistance.[10] Therefore, an entirely different approach is required to investigate the physics associated to strong tunneling strength.

The functional-integral method has been used to study an open, undriven tunnel junction.[11–13] As an extension of this work, the relevant properties of a SEB have been investigated with the path-integral representation of the partition function.[13–18] It has been proved[19] that a formal perturbation expansion of the partition function results in exactly the same form as that derived with the path-integral method, provided that the channel number of the tunnel junction is very large, which is the case for metallic tunnel junctions. Furthermore, based on the functional-integral formulism, a Monte Carlo simulation can be constructed for the entire regime from weak to strong tunneling strength. The Monte Carlo simulation has been performed for the SEB and a very smooth interpolation between the weak-tunneling and the strong-tunneling regime has been obtained.[18] Therefore, functional integral is a reliable powerful tool to study ultrasmall metallic tunnel junctions with arbitrary tunneling rate.

Since most experimental investigations on ultrasmall tunnel junctions measure the current response to applied voltage, a relevant theoretical study must take into account the influence of the electromagnetic environment. This is the goal of the present work, which will use the function-integral approach. In Sec. II we introduce the model Hamiltonian of a single tunnel junction under an electromagnetic environment in a general form, and perform a theoretical analysis for arbitrary tunneling rate. In Sec. III we continue to derive the quantum admittance of such a system in a close analytical form. Then, in Sec. IV we restrict ourselves to the simple controllable electromagnetic environment, which allows us to compare our theoretical results with the recent measurements.[20,21] For this case we only need the dc conductance, which is the zero-frequency limit of the imaginary part of the quantum admittance. The numerical results will be presented in Sec. V, which agree well with experiments.[20,21] Finally, in Sec. VI we give a concluding remark.

II. THEORY FOR MODEL HAMILTONIAN

The Hamiltonian $H_0 = H_s + H_t$ of an open, undriven tunnel junction consists of two parts. The first one,

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$$H_s = H_c + H_e, \tag{1}$$

represents the system in the absence of tunneling, where

$$H_c = Q^2/2C \tag{2}$$

is the Coulomb charging energy when a charge Q is added to the junction with capacitance C. The electrodes are modeled as free quasiparticle systems described by

$$H_e = \sum_{k,\sigma} \varepsilon_{k\sigma} c^+_{k\sigma} c_{k\sigma} + \sum_{q,\sigma} \varepsilon_{q\sigma} c^+_{q\sigma} c_{q\sigma}, \tag{3}$$

where the *channel index* σ includes both the transverse and the spin quantum numbers. The longitudinal wave vectors in the left electrode is labeled by k, and in the right electrode by q. $c^+_{k\sigma}$ (or $c^+_{q\sigma}$) is the creation operator of an electron with energy $\varepsilon_{k\sigma}$ (or $\varepsilon_{q\sigma}$) in the left (or right) electrode. The second part of H describes the tunneling processes of electrons,

$$H_t = \sum_{kq\sigma} [t_{kq\sigma} c^+_{k\sigma} c_{q\sigma} \exp(-i\varphi) + \text{H.c.}], \tag{4}$$

where $t_{kq\sigma}$ is the transition amplitude of an electron tunneling from the state $q\sigma$ in the right electrode to the state $k\sigma$ in the left one. During the tunneling the channel index σ is invariant. Here we have neglected the tunneling time, which is approprite for metallic tunnel junctions. Otherwise the tunnel Hamiltonian must be modified.[22–24] The operator φ is conjugate to Q via the commutation relation

$$[\varphi, Q] = -ie.$$

To avoid possible ambiguity in our theoretical analysis, before introducing the electromagnetice environment, let us first write down the functional-integral representation of the partition function of H_0[11,12]

$$Z_0 = \int D\varphi e^{-S_0[\varphi(\tau)]}. \tag{5}$$

In the action $S_0[\varphi] = S_c[\varphi] + S_t[\varphi]$,

$$S_c[\varphi] = \int_0^\beta d\tau \frac{\dot{\varphi}^2}{4E_c} \tag{6}$$

describes the kinetic part of the system, where $E_c = e^2/2C$ is the elementary charging energy, and

$$S_t[\varphi] = -\int_0^\beta d\tau \int_0^\beta d\tau' \, \alpha_t(\tau - \tau') \cos[\varphi(\tau) - \varphi(\tau')] \tag{7}$$

is resulted from the influence of tunneling processes. With the commonly used approximation that the transition amplitude $t_{kq\sigma}$ in (4) is a constant $t_{kq\sigma} = t$, the junction resistance R_T can be calculated as

$$R_T = (4\pi^2 |t|^2 \rho_l \rho_r N)^{-1},$$

where ρ_l (or ρ_r) is the density of state in the left (or right) electrode, and N is the number of channels. In this case the damping kernel $\alpha_t(\tau)$ can be expressed as

$$\alpha_t(\tau) = \frac{\alpha_t}{4\beta^2} \frac{1}{\sin^2(\pi\tau/\beta)}, \tag{8}$$

where the coefficient $\alpha_t = R_K/R_T$ is the dimensionless junction conductance. The Matsubara frequencies $\omega_l = 2\pi l/\beta$, which will be relevant to our future analysis and calculation, lie in the region much smaller than the bandwidth D of the metal. For such frequencies the Fourier transform of $\alpha_t(\tau)$ reduces to the simple form

$$\alpha_t(\omega_l) = -\frac{\alpha_t |\omega_l|}{4\pi}. \tag{9}$$

The electromagnetic environment of the single tunnel junction represented by H_0, caused by the on-chip leads and pads, etc., will be modeled by a transmission line,[6] which can be described by a Caldeira-Leggett Hamiltonian,[5,6,25–27]

$$H_{\text{ex}} = \sum_{n=1}^{\infty} \left[\frac{q_n^2}{2C_n} + \frac{\varphi_n^2}{2e^2 L_n} \right], \tag{10}$$

for an infinite series of LC circuits. In H_{ex} the first part is the charging energy of all capacitors and the second part is the magnetic energy of the corresponding inductors. The eigenfrequency of the nth LC circuit is given by $\nu_n = 1/\sqrt{L_n C_n}$, and the spectral density

$$J(\nu) = \pi \sum_{n=1}^{\infty} \frac{C_n \nu_n^3}{2e^2} \delta(\nu - \nu_n) \tag{11}$$

is determined by the external impedance. The operators φ_n and q_n obey the commutation relation

$$[\varphi_m, q_n] = -ie\delta_{m,n}.$$

The electromagnetic environment H_{ex} is coupled to the tunnel junction H_0 bilinearly,

$$H_{\text{int}} = -\sum_{n=1}^{\infty} \frac{\varphi_n \varphi}{e^2 L_n} + \sum_{n=1}^{\infty} \frac{\varphi^2}{2e^2 L_n}. \tag{12}$$

The second term at the right-hand side of the above equation is a physical counter term similar to the one introduced in the formulation of the dissipative quantum Brownian motion.[27]

The partition function Z of the total Hamiltonian $H = H_0 + H_{\text{ex}} + H_{\text{int}}$ can be similarly obtained in the functional integral representation as

$$Z = \int D\varphi \prod_{n=1}^{\infty} \int D\varphi_n \exp\left\{ -S_0[\varphi] - \int_0^\beta d\tau \left[\frac{C_n \dot{\varphi}_n^2}{2e^2} + \frac{(\varphi - \varphi_n)^2}{2e^2 L_n} \right] \right\}. \tag{13}$$

The path integrals over the bath modes are Gaussian, and thus can be evaluated exactly. From the condition of vanishing first variation of the action with respect to φ_n, we obtain the classical equation of motion for the phase φ_n,

$$\ddot{\varphi}_n + \nu_n^2 (\varphi - \varphi_n) = 0.$$

The solution to this linear differential equation with the boundary conditions

$$\varphi_n(\beta)=\varphi_n(0)=\varphi_{n0}$$

is

$$\varphi_n^{(\mathrm{cl})}(\tau)=\frac{\sinh[\nu_n(\beta-\tau)]}{\sinh(\nu_n\beta)}\times\left[\varphi_{n0}+\frac{1}{\nu_n}\int_0^\tau d\tau'\ \sinh(\nu_n\tau')\varphi(\tau')\right]+\frac{\sinh(\nu_n\tau)}{\sinh(\nu_n\beta)}\left[\varphi_{n0}-\frac{1}{\nu_n}\int_\tau^\beta d\tau'\ \sinh[(\beta-\nu_n)\tau']\times\varphi(\tau')\right]. \tag{14}$$

The corresponding action is then calculated as

$$S[\varphi_n^{(\mathrm{cl})}]=S_{no}^{(\mathrm{cl})}+S_{\mathrm{ex}}[\varphi], \tag{15}$$

where $S_{no}^{(\mathrm{cl})}$ depends only on the boundary value φ_{n0}, and

$$S_{\mathrm{ex}}[\varphi]=\frac{1}{2}\int_0^\beta d\tau\int_0^\beta d\tau'\,\alpha_{\mathrm{ex}}(\tau-\tau')[\varphi(\tau)-\varphi(\tau')]^2. \tag{16}$$

The damping kernel of the electromagnetic environment $\alpha_{\mathrm{ex}}(\tau)$ can be expressed in a Fourier series

$$\alpha_{\mathrm{ex}}(\tau)=\frac{1}{\beta}\sum_{l=-\infty}^{\infty}\alpha_{\mathrm{ex}}(\omega_l)e^{i\omega_l\tau} \tag{17}$$

with the Fourier coefficients

$$\alpha_{\mathrm{ex}}(\omega_l)=-\int_0^\infty\frac{d\nu}{\pi}\frac{J(\nu)}{\nu}\frac{\omega_l^2}{\omega_l^2+\nu^2} \tag{18}$$

related to the spectral density $J(\nu)$. $\alpha_{\mathrm{ex}}(\omega_l)$ is thus proportional to the Fourier tramsform of the admittance $\mathcal{Y}_e(\omega)$ of the electromagnetic environment.[26] In terms of the Matsubara frequences, this relation is simply

$$\alpha_{\mathrm{ex}}(\omega_l)=-\frac{R_K\mathcal{Y}_e(-i|\omega_l|)|\omega_l|}{4\pi}. \tag{19}$$

Both $S_{no}^{(\mathrm{cl})}$ in Eq. (15) and the contribution to action from the fluctuations around classical paths are independent of φ. Therefore, they will generate an irrelevant prefactor in the partition function Z. We can neglect this factor and obtain from Eq. (13) the final result

$$Z=\int D\varphi e^{-S[\varphi]}, \tag{20}$$

with the total action

$$S[\varphi]=S_0[\varphi]+S_{\mathrm{ex}}[\varphi]. \tag{21}$$

We should point out that although $S_0[\varphi]$ is a periodic function of φ, the total action $S[\varphi]$ is not. The physical origin of the nonperiodic feature of $S[\varphi]$ is the continuous charge transfer between the two electrodes via the external circuit, which suppresses the discrete nature of the change of charge due to the tunneling processes. It is this modification of the system geometry that can smear out dramatically the Coulomb charging effect, as will be shown in the following sections.

III. QUANTUM ADMITTANCE

The quantum admittance $Y(\omega)$ of a single tunnel junction under a general electromagnetic environment can be calculated conveniently with the path-integral approach by generalizing the Kubo formula for this system[12,28–30]

$$Y(\omega)=\omega^{-1}\left\{\lim_{i\omega_l\to\omega+i\delta}\int_0^\beta d\tau e^{i\omega_l\tau}\langle\hat{T}_\tau I(\tau)I(0)\rangle\right\}, \tag{22}$$

where $\hat{T}_\tau$ is the time-ordering operator in imaginary time. The correlation function is calculated as

$$\langle I(\tau)I(\tau')\rangle=Z^{-1}\int D\varphi e^{-S[\varphi]}\{2e^2\alpha_t(\tau-\tau')\times\cos[\varphi(\tau)-\varphi(\tau')]+I_T[\varphi,\tau]I_T[\varphi,\tau']\}, \tag{23}$$

with the current functional

$$I_T[\varphi,\tau]=-2e\int_0^\beta d\tau'\,\alpha_t(\tau-\tau')\sin[\varphi(\tau)-\varphi(\tau')]. \tag{24}$$

At very low temperature one can use the instantonlike or the renormalization-group techniques to evaluate the partition function,[15–17,31,32] but not the correlation function nor the quantum conductance, which are usually calculated numerically.[30] However, if the temperature is not very low, with a semiclassical approximation we can evaluate analytically all path integrals to obtain a systematic treatment of the influence of fluctuations in the form of a series expansion of βE_c. Another advantage of the semiclassical method is that the result is stable, because the fluctuation modes are of large eigenvalues.[16] Such relatively-high-temperature semiclassical results of ultrasmall tunnel junctions turn out to be very meaningful, in view of the recent measurements of Coulomb blockade effects performed at not-very-low temperature.[20,21,33,34] In the following theoretical analysis, we will use the semiclassical method.

Let us at first evaluate the partition function. The total action is now approximated by a Gaussian form,

$$S_{\mathrm{semi}}[\varphi]=\int_0^\beta d\tau\frac{\dot{\varphi}^2}{4E_c}+\frac{1}{2}\int_0^\beta d\tau\times\int_0^\beta d\tau'\,\alpha_w(\tau-\tau')[\varphi(\tau)-\varphi(\tau')]^2, \tag{25}$$

where

$$\alpha_w(\tau)=\alpha_t(\tau)+\alpha_{\mathrm{ex}}(\tau). \tag{26}$$

The above approximation means that the contribution of the tunneling resistance in the action is replaced by an equivalent Ohmic resistance. Higher-order variations of the action beyond the Gaussian approximataion can be calculated in the form of a power series of βE_c. However, when calculating the current correlation function, which is normalized by Z, the power series correction factor in the denominator Z is cancelled by similar factor in the numerator. Hence, $S_{\rm semi}[\varphi]$ given by Eq. (25) turns out to be the approprite form for calculating the quantum admittance as a series of βE_c. We notice that an arbitrary path $\varphi(\tau)$ may be expressed as a Fourier series,

$$\varphi(\tau)=\sum_{l=-\infty}^{\infty}\varphi_l e^{i\omega_l\tau}, \tag{27}$$

with the complex amplitudes

$$\varphi_l=\varphi_l'+i\varphi_l''$$

under the condition $\varphi_{-l}=\varphi_l^*$. Hence, the semiclassical action may be evaluated accordingly to give

$$S_{\rm semi}[\varphi]=\sum_{l=1}^{\infty}\lambda_l(\varphi_l'^2+\varphi_l''^2), \tag{28}$$

where the eigenvalues λ_l are

$$\lambda_l=\frac{\omega_l^2}{2E_c}-2[\alpha_t(\omega_l)+\alpha_{\rm ex}(\omega_l)]. \tag{29}$$

Now the partition function can be directly calculated according to the formula

$$Z_{\rm semi}=\prod_{l=1}^{\infty}\frac{\pi}{\beta\lambda_l}. \tag{30}$$

The current autocorrelation function (23) will be evaluated in the same way. Let $\langle I(\tau)I(\tau')\rangle_1$ represent the first term at the right-hand side of Eq. (23). By performing a series expansion in powers of $[\varphi(\tau)-\varphi(\tau')]$, we can express $\langle I(\tau)I(\tau')\rangle_1$ as

$$\langle I(\tau)I(\tau')\rangle_1=2e^2\alpha_t(\tau-\tau')Z_{\rm semi}^{-1}\int D\varphi e^{-S_{\rm semi}[\varphi]}\times\left[1-\frac{[\varphi(\tau)-\varphi(\tau')]^2}{2}+\cdots\right]. \tag{31}$$

Making use of the Fourier series of paths $\varphi(\tau)$, we obtain

$$\langle I(\tau)I(\tau')\rangle_1=2e^2\alpha_t(\tau-\tau')\left[1-2\sum_{n=1}^{\infty}\frac{1}{\beta\lambda_n}+2\sum_{n=1}^{\infty}\frac{\cos\omega_n(\tau-\tau')}{\beta\lambda_n}\right]+\mathcal{O}[(\beta E_c)^2]. \tag{32}$$

Substituting this expression of $\langle I(\tau)I(\tau')\rangle_1$ into Eq. (22), we obtain the contribution to the admittance from the first term at the right-hand side of Eq. (23),

$$Y_1(\omega)=2e^2\omega^{-1}\left\{\lim_{i\omega_l\to\omega+i\delta}\left[\left(1-2\sum_{n=1}^{\infty}\frac{1}{\beta\lambda_n}\right)\alpha_t(\omega_l)+\sum_{n=1}^{\infty}\frac{\alpha_t(\omega_l+\omega_n)+\alpha_t(\omega_l-\omega_n)}{\beta\lambda_n}\right]\right\}+\mathcal{O}[(\beta E_c)^2]. \tag{33}$$

The second term on the right part of Eq. (23) can be evaluated in the same way, although the algebra is more complicated. In terms of the eigenvalues of the fluctuation modes and the Fourier tramsform of the damping kernel, its contribution to the admittance is derived as

$$Y_2(\omega)=4e^2\omega^{-1}\left\{\lim_{i\omega_l\to\omega+i\delta}\left[\left(1-4\sum_{n=1}^{\infty}\frac{1}{\beta\lambda_n}\right)\frac{\alpha_t^2(\omega_l)}{\lambda_l}+\frac{2\alpha_t(\omega_l)}{\lambda_l}\times\sum_{n=1}^{\infty}\frac{\alpha_t(\omega_l+\omega_n)+\alpha_t(\omega_l-\omega_n)-2\alpha_t(\omega_n)}{\beta\lambda_n}+\frac{8\alpha_t^3(\omega_l)}{\lambda_l^2}\sum_{n=1}^{\infty}\frac{1}{\beta\lambda_n}\right]\right\}+\mathcal{O}[(\beta E_c)^2]. \tag{34}$$

So far our theoretical analysis is for a general electromagnetic environment. In the rest of this paper we will present a detail quantitative study on dc conductance $G\equiv\lim_{\omega\to 0}\{{\rm Im}\,Y(\omega)\}$ for a specific case that the electromagnetic environment contains only an Ohmic resistance. Such electromagnetic environment is relevant to recent experiments[20,21] with which our theoretical result can be compared.

IV. SIMPLE ELECTROMAGNETIC ENVIRONMENT

We will investegate the dc conductance of a single tunnel junction connected in series with a resistance $R_{\rm ex}$. In a real sample this external resistance can be varied in a controllable way. In this case we then have $\mathcal{Y}_e(-i|\omega_l|)=1/R_{\rm ex}$, and thus Eq. (19) becomes linear in frequency

$$\alpha_{\rm ex}(\omega_l)=-\frac{\alpha_{\rm ex}|\omega_l|}{4\pi}, \tag{35}$$

where $\alpha_{\rm ex}=R_K/R_{\rm ex}$ is the dimensionless conductance contributed by the electromagnetic environment. Substituting into Eq. (29) this $\alpha_{\rm ex}(\omega_l)$ and the $\alpha_t(\omega_l)$ given by Eq. (9), we obtain the eigenvelue λ_l for the simple resistance environment. If l is very large, λ is dominated by the term $\omega_l^2/2E_c$, and is insensitive to the tunneling terms represented by $\alpha_{ex}(\omega_l)$ and $\alpha_t(\omega_l)$. Hence, the corresponding effect turns out to be an uninteresting prefactor and contributes nothing to the conductance. For ω_l much less than the bandwidth D, the eigenvalues reduce to

$$\lambda_l = \frac{\omega_l^2}{2E_c} + \frac{\alpha_w \omega_l}{2\pi}, \tag{36}$$

where $\alpha_w = \alpha_t + \alpha_{ex}$. Inserting the eigenvalues into Eq. (33), after a straightforward algebra, we obtain

$$\begin{aligned} G_1 &= \lim_{\omega\to 0}\{\mathrm{Im}\ Y_1(\omega)\} \\ &= 2e^2\left\{\left[1 - \frac{2}{\alpha_w}[\Psi(\mu_w+1) - \Psi(1)]\right]\right. \\ &\quad \left.\times \frac{\alpha_t}{4\pi} - \frac{\mu_t \Psi'(\mu_w+1)}{2\pi}\right\} + \mathcal{O}[(\beta E_c)^2], \end{aligned} \tag{37}$$

where $\Psi(z)$ and $\Psi'(z)$ are, respectively, the digamma and trigamma functions. The parameters μ_t and μ_w are defined as

$$\mu_i = \alpha_i \beta E_c / 2\pi^2; \quad i = t, w. \tag{38}$$

Now we calculate from Eq. (34) the second part of the dc conductance $G_2 = \lim_{\omega\to 0}\{\mathrm{Im}\ Y_2(\omega)\}$. When carrying out the limits at the right-hand side of Eq. (34), the orders in the series of βE_c become mixed. This makes the calculation of G_2 very complicated, yet straightforward. The final result is

$$\begin{aligned} G_2 = &- \frac{\alpha_t^2}{R_K \alpha_w}\{\alpha_w - 4[\Psi(\mu_w+1) - \Psi(1)]\} \\ &+ \frac{4\mu_t\alpha_t}{R_K\alpha_w}\Psi'(\mu_w+1) - \frac{4\alpha_t^3}{R_K\alpha_w^3}[\Psi(\mu_w+1) - \Psi(1)] \\ &+ \mathrm{O}[(\beta E_c)^2]. \end{aligned} \tag{39}$$

Consequently, the dc conductance of a single tunneling junction under the resistance environment is derived as

$$\begin{aligned} G = \frac{1}{R_{ex}+R_T}&\left\{1 - \frac{\beta E_c}{\pi^2}\frac{\alpha_{ex}}{\alpha_w}\left[\frac{\Psi(\mu_w+1) - \Psi(1)}{\mu_w}\right.\right. \\ &\left.\left. + \Psi'(\mu_w+1)\right] + \mathrm{O}[(\beta E_c)^2]\right\}. \end{aligned} \tag{40}$$

In the high-temperature limit, the tunneling system exhibits the conventional Ohmic behavior $G_{\mathrm{Ohm}} = 1/(R_T + R_{ex})$ as expected.

V. RESULTS AND DISCUSSION

To demonstrate the influence of electromagnetic environment on Coulomb charging effects, let us introduce the dimensionless conductance correction,

$$\begin{aligned} \Delta g &\equiv 1 - G(R_{ex}+R_T) \\ &= \frac{\beta E_c}{\pi^2}\frac{\alpha_{ex}}{\alpha_w}\left[\frac{\Psi(\mu_w+1) - \Psi(1)}{\mu_w} + \Psi'(\mu_w+1)\right] \\ &\quad + \mathrm{O}[(\beta E_c)^2], \end{aligned} \tag{41}$$

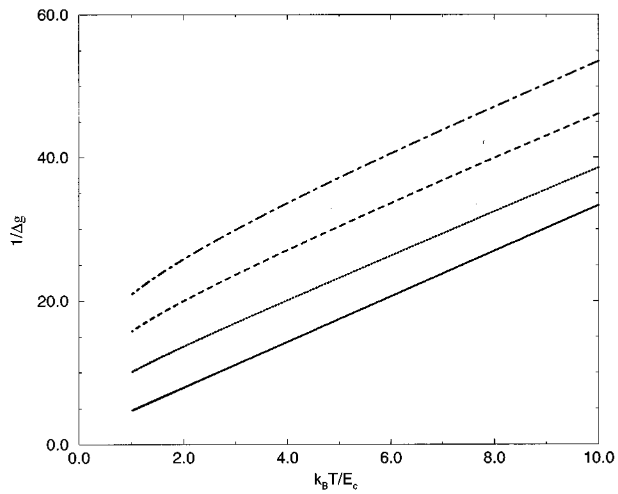

FIG. 1. $1/\Delta g$ as a function of the normalized temperature k_BT/E_c for the dimensionless junction conductance $\alpha_t = 0.5$, with various values of the dimensionless external conductance $\alpha_{ex} = 10$ (solid curve), 50 (dotted curve), 100 (dashed curve), and 150 (dot-dashed curve). The cases $\alpha_{ex} = 10$ and 100 correspond closely to the two measured samples in Ref. 20 and Ref. 21, respectively.

which can be considered as a measure of the effective Coulomb charging energy: smaller Δg corresponds to weaker effective Coulomb charging energy. In recent experiments[20,21] Pekola and co-workers have manufactured a single tunnel junction with $\alpha_t = R_K/R_t = 0.5$ and measured $1/\Delta g$ as a function of normalized temperature k_BT/E_c for different values of $\alpha_{ex} = R_K/R_{ex}$. R_{ex} contains two contributions $R_{ex} = R_{ex,in} + R_{ex,con}$, where $R_{ex,in}$ is the intrinsic value of the circuit, which is of the order of the free space impedance $\sqrt{\mu_0/\varepsilon_0} \simeq 377\ \Omega$ and $R_{ex,con}$ is experimentally controlable. At high-temperature $1/\Delta g$ is found to be linear in k_BT/E_c. Extrapolating this high temperature straight line to zero temperature, it was found that the zero-temperature offset $(1/\Delta g)_0$ is positive and decreases with diminishing α_{ex}. Our theory has reproduced these observed features as shown in Fig. 1 for $\alpha_t = R_K/R_t = 0.5$. For high temperature, $1/\Delta g$ can be readily derived as

$$\frac{1}{\Delta g} = \frac{3\alpha_w}{\alpha_{ex}}\frac{k_BT}{E_c} + \frac{(0.6 + 0.2\alpha_w)\alpha_w}{\alpha_{ex}}. \tag{42}$$

Thus, the offset is simply $(1/\Delta g)_0 = (0.6 + 0.2\alpha_w)\alpha_w/\alpha_{ex}$. For the case that the tunnel conductance α_t is much smaller than the electromagnetic environmental conductance α_{ex}, and in the vicinity of $\mu_w \ll 1$, the offset can be well approximated by the very simple form $(1/\Delta g)_0 = 0.6 + 0.2R_K/R_{ex}$. Hence, if we take $R_{ex,in} = 377\ \Omega$, we obtain $(1/\Delta g)_0 \simeq 14$ for $R_{ex,con} = 0$ and $(1/\Delta g)_0 \simeq 2$ for $R_{ex,con} = 3\ k\Omega$, which agree exactly with the observed values.[20,21] An effective-capacitance model has been proposed[20,21] to explain these measured offset values. The so-calculated $(1/\Delta g)_0$ is smaller than the experimental value by a factor 5 for $R_{ex,con} = 0$, and fails to explain the case of $R_{ex,con} = 3\ k\Omega$.

For a fixed value of βE_c, Δg is a function of the junction conductance α_t and the electromagnetic environmental conductance α_{ex}. Using a standard formalism, it has been proven[34] that for high temperature, in the plane of α_t and

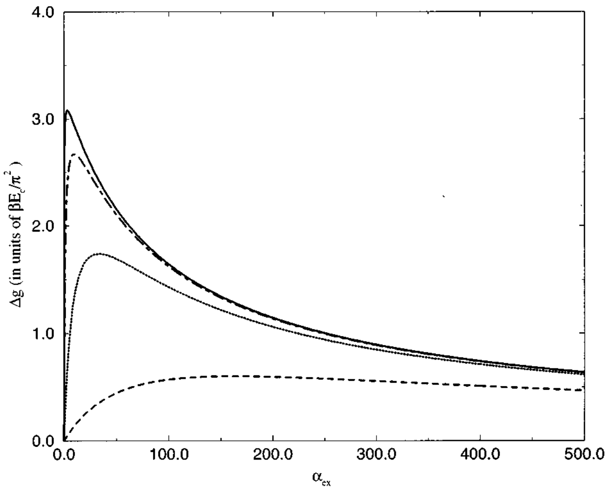

FIG. 2. Δg in units of $\beta E_c/\pi^2$ as a function of the dimensionless external conductance α_{ex} for $\beta E_c = 0.2$, with various values of the dimensionless junction conductance $\alpha_t = 0.1$ (solid curve), 1.0 (dot-dashed curve), 10.0 (dotted curve), and 100.0 (dashed curve).

α_{ex}, the ratio $\Delta g/\beta E_c$ has an absolute maximum value 1/3. However, the functional dependence of $\Delta g/\beta E_c$ on α_t and α_{ex} has never been obtained. Our theory allows us to calculate such important results, which are shown in Fig. 2 for $\beta E_c = 0.2$ and in Fig. 3 for $\beta E_c = 0.02$. In each figure the ratio $\pi^2 \Delta g/\beta E_c$ is plotted as a function of α_{ex} for $\alpha_t = 0.1$ (solid curve), 1.0 (dot-dashed curve), 10.0 (dotted curve), and 100.0 (dashed curve). It is clear that in the α_t-α_{ex} plane, the absolute maximum value of $\Delta g/\beta E_c$ lies in the region where $R_T \gg R_{ex} \gg R_K$. A careful numerical search reveals that the value of this absolute maximum is just 1/3, corresponding to the charging effects due to the geometric capacitance of the ultrasmall tunnel junction.

We noticed that in both Fig. 2 and Fig. 3, for a fixed value of α_{ex}, the value of $\pi^2 \Delta g/\beta E_c$ decreases monotonically with α_t. This is understandable because an increase of the junction conductance (or the tunnel strength) leads to a reduction of the effective Coulomb charging energy. On the other hand, for a fixed value of the junction conductance α_t, $\pi^2 \Delta g/\beta E_c$ as a function of the external conductance α_{ex} exhibits a peak. Such an interesting feature has been observed very recently, but has not appeared in the literature yet.[21] The physics of this peak structure can be explained as follows. Once an electron tunnels through the junction from the left side to the right side, it can return to the left side via the external circuit. Therefore, the larger is the external conductance α_{ex}, the faster the tunnel junction can *relax* to its initial charge configuration. Consequently, the effective Coulomb charging energy is reduced and so is Δg. This conclusion then agrees with the one reached in Ref. 5 and Ref. 6. It is worthwhile to mention that in the limit of very high tunneling rate or very large α_{ex} such that $\mu_w \gg 1$, Eq. (41) is simplified to

$$\Delta g = \frac{2\alpha_{ex}}{\alpha_w^2}\left[\ln\left\{\frac{\alpha_w \beta E_c}{2\pi^2}\right\} + 1 - \Psi(1)\right], \tag{43}$$

which is not analytical in α_t. Therefore, the method fo calculating G in Ref. 23 based on the Fermi golden rule is no longer valid here.

The picture is entirely different at the other end of very small external conductance α_{ex}, or very large external resistance R_{ex}/R_K. In this case the external returning path from the right side of the junction to the left side is heavily blocked. Hence, once an electron tunnels through the junction, its probability to tunnel back is much larger than that to travel through the external circuit. Eventually, an electron spends most of the time tunneling back and forth, and so higher-order tunneling terms must be taken into account, even though each tunnel process is still incoherently sequential. Such tunneling processes cannot be treated with the Fermi golden rule either. In fact, the system is in a dynamical state with an equivalent reduced effective Coulomb charging energy. Each curve in Fig. 2 and Fig. 3 then drops down to zero as α_{ex} approaches zero, at which the external circuit is completely blocked. This point of view is similar to that appeares in the investigation of the statistical behavior of a single electron box.[10,14–19,32] Nevertheless, in the present problem the renormalization of the capacitance is dynamical rather than static. To our knowledge, such dynamical renormalization of the charging energy in the regime of very small external conductance α_{ex} is discovered for the first time.

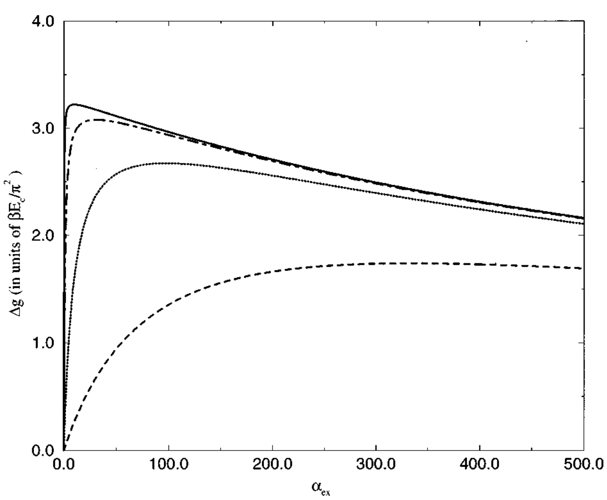

FIG. 3. The same as Fig. 2 but for $\beta E_c = 0.02$.

VI. REMARKS

We have used a nonperturbative approach to derive the quantum admittance of a single tunnel junction in the presence of an electromagnetic environment. Besides the case with an Ohmic environment, which has been investigated in the present work, our theoretical formulation is very general and can be conveniently used to study the effects of non-Ohmic environments. If the electromagnetic environment contains a frequency-dependent part described by a transmission line,[5,6,26,35] the quantum admittance of the system as a function of frequency, which is of great interest, can be analyzed with our theory. Such theoretical results can be compared with experiments in a very direct manner.[21,36] With a slight modification of the action, we can also investigate the behavior of an array of tunnel junctions. If the junction array has a pure Ohmic environment, the dc conductance of the system can be calculated by evaluating the mean value of the current through the external resistance.[37] Finally, we should mention that very recently a dissipation-driven superconductor-insulator transition in a Josephson junction array has been observed by changing the external impedance continuously.[38] Using the theoretical approach developed in the present work, this problem can be investigated theoretically at a quantitative level.

ACKNOWLEDGMENTS

This work was supported by the Swedish Natural Science Research Council. The authors would like to thank J. P. Pekola for providing preliminary experimental results and stimulating discussions, and Yu. M. Galperin and P. Johansson for fruitful discussions about the theoretical aspect of this work.

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