Theory of photoluminescence from an interacting two-dimensional electron gas in strong magnetic fields

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A theory of photoluminescence from an interacting two-dimensional electron gas in strong magnetic fields in the vicinity of integer filling factors is developed. The recombination spectrum turns out to be a discontinuous function of the filling factor, related to spectral functions of the final-state excitations. The electron-electron interactions are shown to lead to splitting and oscillations of the recombination line. The splittings, associated with odd filling factors, are analogs of Anderson-Fano resonances, but in the Fock space of the interacting system. The oscillations are related to the competition between the electron-valence hole and electron-electron interactions. [S0163-1829(97)04343-9]

I. INTRODUCTION

Interband magneto-optics is a contactless probe of two-dimensional electron gas (2DEG) in the integer[1–5] and fractional[6,7] quantum Hall regime. In the recombination process an electron recombines with a valence hole, emitting a photon. This creates a hole in the electron gas. The hole is an elementary excitation, the spectral function of which carries information about excitations of the electron system. In Fermi liquids the hole weakly interacts with excitations of the electron gas, and is a well-defined quasiparticle.[8] The calculation of the broadening of the spectral function can be done by perturbative methods. In strongly correlated systems, the spectral function of a hole need not resemble the spectral function of a hole in a noninteracting system. Recent photoluminescence experiments in a high-mobility 2DEG in strong magnetic fields have shown discontinues jumps and splitting of the recombination line at integer filling factors.[9] We relate these observations to the nonquasiparticle behavior of the hole spectral function.

The recombination spectrum and the hole spectral function are investigated in the vicinity of integer filling factors. In strong magnetic fields and integer filling factors, electrons condense into Hartree-Fock ground states, separated from excited states by either cyclotron or Zeeman gaps. Starting from Hartree-Fock ground states, we work out a theory of interband recombination and of the spectral function of the optically created final states. The final states are calculated using exact numerical diagonalization techniques. In particular, we show that in the vicinity of odd filling factors $\nu > 2$, the spectral function of the hole in the lowest Landau level splits into two peaks due to resonant many-body interaction with a continuum of quasidegenerate excited states of the 2DEG. Spin plays an important role in this recombination process. The effect appears to be a many-body analog of the Anderson-Fano resonance known from solid-state and atomic physics.[8] A related problem of localization in Fock space and its effect on the spectral function of finite fermion systems was recently investigated theoretically by Altshuler *et al.*[10] The similarities illustrate the role of confinement and magnetic field in the quantization of the kinetic energy of electrons. The calculated splitting, after broadening by disorder, leads to filling-factor-dependent oscillations of the energy of the magnetophotoluminescence line. Such oscillations have previously been observed in strongly disordered samples,[2–5] and related predominantly to properties of the valence hole. The disorder broadens Landau levels, and electrons are capable of screening Coulomb interactions, renormalizing both their energy and interactions with valence holes. Neglecting the Zeeman splitting, diagrammatic calculations[3] of the electron-valence hole correlation function identified oscillations of photoluminescence energy primarily with oscillations of the valence hole self-energy[3] due to the smooth behavior of the electron energy.[4] The valence hole self-energy is due to interaction of a valence hole with a finite density of valence holes. The hole-hole interaction is screened by a finite density of electrons. A commonly used, but untested in a magnetic field, quasi static approximation is to split the self-energy into Coulomb hole and screened exchange part.[3,11] The screened exchange is proportional to the density of holes, and becomes negligible in the low-density limit. The Coulomb hole term, on the other hand, reflects the energy associated with correlations among valence holes. The correlations are due to the valence holes with parallel spin avoiding each other due to the Pauli exclusion principle. This creates a depleted charge around the valence hole, the electrostatic energy of which depends on the screening provided by electrons. The Coulomb hole self-energy survives in the limit of vanishing valence hole density due to the finite density of the electrons, and is responsible for the filling-factor-dependent oscillations of the energy of the emission line.

The diagrammatic theory, applicable to compressible systems, appears to explain the oscillations of photolumines-

cence in disordered systems. However, it clearly cannot be applied to a very high-mobility 2DEG, where both cyclotron and Zeeman gaps exist. In clean samples an approximation starting from Hartree-Fock ground states and treating the electron-electron interaction first and disorder later appears to be appropriate. This Hartree-Fock picture has already been successfully applied to the theory of cyclotron resonance in the self-consistent excitonic approximation,[12] to the recombination spectrum at filling factor $\nu=1$,[13–15] and to the filling factor dependence of the donor-related far infrared absorption.[16,17]

II. MODEL

The eigenstates $|m,n,\sigma\rangle$ and eigenenergies $E_{mn,\sigma}=\omega_c(n+\frac{1}{2})+g\mu_B B\sigma$ of an electron moving in an (x,y) plane in the presence of the perpendicular magnetic field B, in a symmetrical gauge, are that of two harmonic oscillators. Here n labels different Landau levels, m labels degenerate intra-Landau-level states, and $m-n$ is the angular momentum. The cyclotron energy is $\omega_c=(eB/m^*c)$, $l_0=1/(m^*\omega_c)^{1/2}$ is the magnetic length, m^* is the effective mass, g is an effective g factor, μ_B is the Bohr magneton, and $g\mu_B B$ is the Zeeman energy. The Coulomb energy is measured in units of exchange energy $E_0=\mathrm{Ry}\sqrt{2\pi}a_0/l_0$, where Ry is the effective Rydberg and a_0 is the effective Bohr radius. The same applies to photoexcited valence-band holes with appropriate valence-band parameters. Since holes have the opposite charge to electrons, the angular momentum of holes is opposite to the angular momentum of electrons.

After denoting the creation (annihilation) operators for electrons (holes) in states $|m,n\rangle$ and spin $|\sigma\rangle$ ($|i\rangle=|m,n,\sigma\rangle$) by $c_i^+(c_i),h_i^+(h_i)$ the Hamiltonian can be written as

$$H=\sum_i E_i c_i^+ c_i+E_i^h h_i^+ h_i + \tfrac{1}{2}\sum_{i_1 i_2 i_3 i_4}\langle i_1,i_2|V_{ee}|i_3,i_4\rangle c_{i_1}^+ c_{i_2}^+ c_{i_3} c_{i_4} + \sum_{i_1 s_2 s_3 i_4}\langle i_1,s_2|V_{eh}|s_3,i_4\rangle c_{i_1}^+ c_{i_4} h_{s_2}^+ h_{s_3}, \quad (1)$$

where $\langle i_1,i_2|V_{ee}|i_3,i_4\rangle$ are the electron-electron Coulomb matrix elements,[16,18,19] and $\langle i_1,s_2|V_{eh}|s_3,i_4\rangle$ are electron-hole Coulomb matrix elements.[20] The positive background is included in a standard way. The conservation of angular momentum in the Coulomb scattering of electrons guarantees that $m_1+m_2-n_1-n_2=m_3+m_4-n_3-n_4$. The electron-valence hole matrix elements[20] can be related to the electron-electron matrix elements as $\langle m_1,s_2|V_{\mathrm{eh}}|s_3,m_4\rangle=-\alpha\langle m_1,s_3|V_{\mathrm{ee}}|s_2,m_4\rangle$, where $\alpha=|V_{\mathrm{eh}}|/V_{\mathrm{ee}}$ measures the ratio of the electron-valence hole to electron-electron interaction. The parameter α in one-side modulation-doped quantum well depends on the well width and carrier concentration.

III. EMISSION SPECTRUM

Let us consider the recombination spectrum from filled Landau levels $|\nu\rangle$. The filled Landau level ν is a single Slater determinant of occupied states $|m,n\rangle$, e.g., $|\nu=3\rangle=\Pi_{m=0}^N c_{m,1,-}^+ c_{m,0,+}^+ c_{m,0,-}^+|0\rangle$, where N is the number of occupied orbitals in a given Landau level. We consider recombination from two initial states, $|i^+\rangle$ and $|i^-\rangle$, which differ by changing one flux quantum. The $|i^-\rangle=h_{m,0,+}^+|\nu\rangle$ state consists of a filled Landau level and a valence hole in orbital m of the lowest spin-up valence-band Landau level. This configuration is illustrated in Fig. 1(a) for a filling factor $\nu=3$. The total angular momentum $R=m-\Sigma_i m_i-n_i$ of the electron-valence hole system is a good quantum number. We can label our initial many electron states by m, the angular momentum of the valence hole. There are N degenerate initial states, occupied with probability P_i. It is important that Coulomb interactions do not mix different $|i^-\rangle=|m\rangle$ states. From Fermi's golden rule the recombination spectrum $E^-(\omega)$ is given in terms of the initial $|i^-\rangle$ and final $|f\rangle$ states by

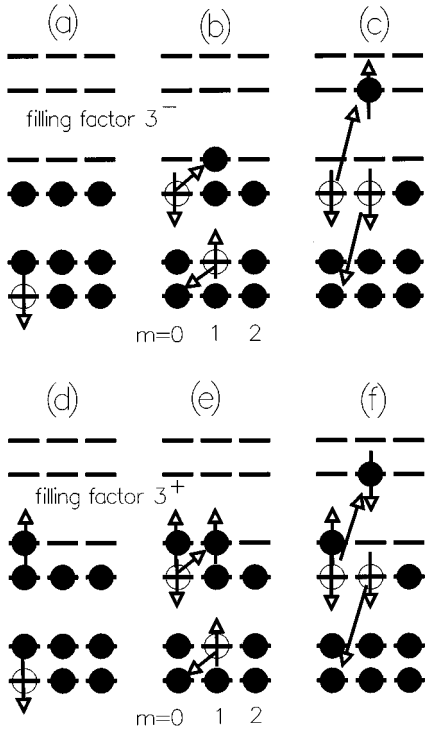

FIG. 1. A schematic picture of resonant electronic configurations for filling factor 3^- (a)–(c) and 3^+ (d)–(f). The black dots correspond to electrons occupying states $|m,n,\sigma\rangle$. For each m, states with increasing Zeeman and cyclotron energy are shown.

$$E^-(\omega)=\sum_{f,i} P_i|\langle f|P^-|i^-\rangle|^2\delta(E_f+\omega-E_i), \quad (2)$$

where the luminescence operator $P^-=\Sigma_{m,n}h_{m,n,-}c_{m,n,-}+h_{m,n,+}c_{m,n,+}$ removes electron-hole pairs without changing total angular momentum or spin. Using the identity $\langle f|P^-|i^-\rangle=\langle f|c_{m,0,-}|\nu\rangle$, the recombination spectrum reduces to

$$E^-(\omega)=\sum_{f,m} P_m|\langle f|c_{m,0,-}|\nu\rangle|^2\delta(E_f+\omega-E_i). \quad (3)$$

For a translationally invariant system, the recombination spectrum should not depend on m, and therefore it reduces to a spectral function of a hole created in the center of a filled disk:

$$E^-(\omega)=\sum_f |\langle f|c_{0,0,-}|\nu\rangle|^2\delta(E_f+\omega-E_i). \quad (4)$$

This spectral function has been investigated numerically by a number of authors[21–25] in the fractional quantum Hall-effect regime.

Let us now reduce the number of flux quanta by one. Our initial state $|i^+\rangle$ consists of a filled Landau level, one electron in an empty $n=1$ Landau level, and a valence hole in the $n=0$ Landau level in the valence band. There are many such excitonic states mixed by Coulomb interactions. The initial state corresponds to the lowest state of this (01) exciton. Neglecting Landau-level mixing, the initial state can be written as $|i^+\rangle=\Sigma_{m,m'}X^{m',n'=1}_{m,n=0}h^+_{m,n=0,+}c^+_{m',n'=1,+}|\nu\rangle$, with $m-m'=1$, and $X^{m',n'}_{m,n}$ the amplitude of the ground state of the (01) exciton. The recombination spectrum from an exciton state can be expressed solely in terms of electronic operators, and for $\nu=3$ it is

$$P^-|i^+\rangle=\sum_{m',m,1=m'-m} X^{m',1}_{m,0}c_{m,0,-}c^+_{m',1,\sigma=+}|\nu\rangle. \quad (5)$$

The spin σ is $+/-$ for odd/even filling factors. The final states in the recombination spectrum are therefore projected on inter-Landau-level charge and/or spin excitations $c_{m,0,-}c^+_{m',n',\sigma}|\nu\rangle$, but weighted by the initial exciton amplitudes $X^{m',n'}_{m,n}$:

$$E^+(\omega)=\sum_f \left|\langle f|\sum_{m',m,1=m'-m} X^{m',1}_{m,0}c_{m,0,-}c^+_{m',1,\sigma}|\nu\rangle\right|^2 \times\delta(E_f+\omega-E_i). \quad (6)$$

The final states for ν^+ are shown in Fig. 1(d). These are inter-Landau-level spin-flip excitations. Hence there is a discontinuous change across the filling factor in the recombination spectrum, from the spectral function of a hole to the spectral function of collective excitations.

IV. HARTREE-FOCK EMISSION SPECTRUM

The energy of the removed hole, neglecting all final-state interactions, is the Hartree-Fock self-energy of the electron in the lowest Landau level. The Hartree-Fock self-energy is given by

$$\Sigma^{\rm HF}_{m,n,\sigma}=\sum_{m',n',\sigma'}(\langle m'n',mn|V_{ee}|mn,m'n'\rangle -\langle m'n',mn|V_{ee}|m'n',mn\rangle\delta_{\sigma,\sigma'})f(m',n',\sigma'), \quad (7)$$

where $f(m',n',\sigma')=1$ for filled levels and 0 for empty levels. For example, for filling factors $\nu=1$, 2, and 3, the self-energy of an optically created hole is $\Sigma=-1E_0$, $-1E_0$, and $-1.5E_0$. In Fig. 2 we show the calculated Hartree-Fock self-energies, measured from $1/2\omega_c$, for optically created holes in the spin-down lowest Landau level (down arrows) as a function of the filling factor. For comparison, the energies of holes with spin-up are also shown. One can think of these energies as magnetic-field-modulated band-gap renormalization. At even filling factors the energies are the same, but for odd filling factors there is a splitting due to finite magnetization of 2DEG. The finite magnetization causes oscillation of the energy of the optically created hole as a function of the filling factor. The energy of the optically inactive hole also oscillates but in opposite direction.

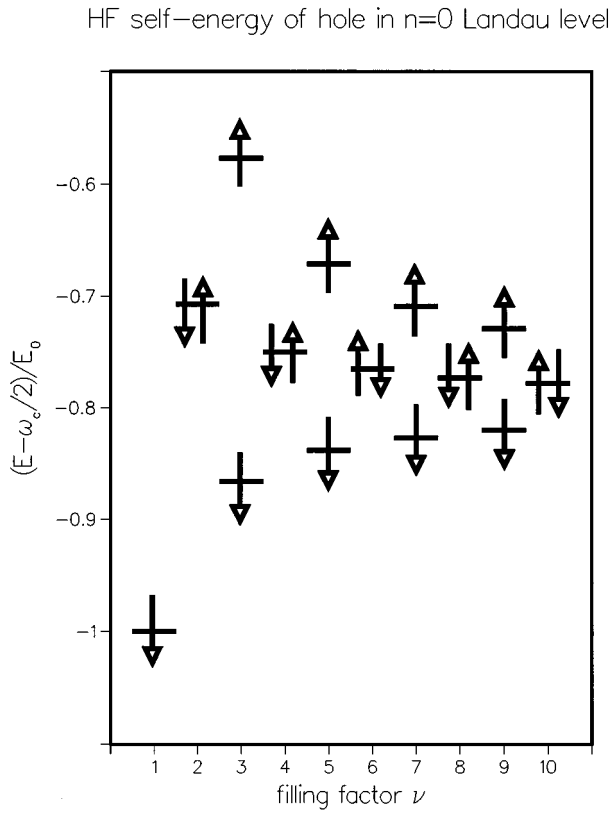

FIG. 2. Hartree-Fock energies of holes with spin-up and -down as a function of the filling factor.

V. SPECTRAL FUNCTION OF A HOLE AT $\nu=1,2$

The change of the recombination spectrum across the filling factor has important consequences at filling factors $\nu=1$ and 2, as pointed out in Refs. 13–15. At $\nu=1^-$ a recombination from the $|m=0,, n=0, \sigma=-\rangle$ state creates a final state with a single hole in the $n=0$ Landau level, as shown in Fig. 3. The completely filled Landau level of the initial state is negatively charged to compensate the positive charge of the valence hole. The transition energy (on a disk)

$$\omega(\nu=1)^-=E_g+\Sigma^{\rm HF}_{0,0}-\langle N0,00|V_{\rm ee}|N0,00\rangle + \sum_{m'=0}^{N}\langle 00,m'0|V_{\rm ee}|m'0,00\rangle\frac{(1-\alpha)}{(N+1)} \quad (8)$$

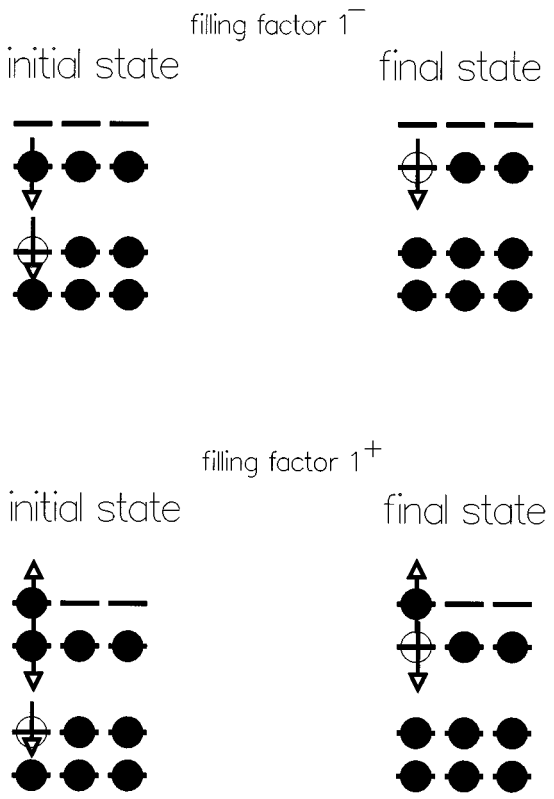

FIG. 3. A schematic picture of initial- and final-state electronic configurations for filling factor 1^- and 1^+ (d)–(f). The black dots correspond to electrons occupying states $|m,n,\sigma\rangle$ of the lowest Landau level in the valence and conduction band. For each m states with increasing Zeeman energy are shown.

can be expressed in terms of the self-energy of the $n=0$, spin-polarized lowest Landau level, $\Sigma_{0,0}^{\mathrm{HF}}=-E_0$, and finite-size corrections, which slowly decay to zero as the size of the system N increases to infinity. Therefore the transition energy at $\nu=1^-$, $\omega^-=E_g-E_0$ is lowered from the bare gap by the Hartree-Fock energy E_0. At $\nu=1^+$ the recombination takes place from an exciton initial state to a zero angular momentum spin-flip excitation as the final state, shown in Fig. 3. The energy of the zero angular momentum spin-flip excitation in the lowest Landau level is zero due to the invariance of the Hamiltonian to the rotation of the total spin. Therefore, the transition frequency is given simply by $\omega^+=E_g-E_{\mathrm{exc}}^{00}$, where E_{exc}^{00} is the (00) exciton binding energy. Since in a one-side modulation-doped quantum well $E_{\mathrm{exc}}<E_0$, we expect a discontinuous redshift $\Delta(\nu=1)=E_0-E_{\mathrm{exc}}$ of the recombination line at $\nu=1$.

At $\nu=2^-$ a recombination from the $|m=0,\ n=0,\ \sigma=-\rangle$ state creates a single hole in the $n=0$ spin down Landau level (LL). The completely filled spin-up and -down LL's of the initial state are negatively charged to compensate for the positive charge of the valence hole. After finite-size effects are neglected, the transition energy is $\omega(\nu=2)^-=E_g+\Sigma_{0,0}^{\mathrm{HF}}=E_g-E_0$, where E_0 is the exchange energy at the magnetic field corresponding to $\nu=2$. At $\nu=2^+$ the recombination takes place from the (01) exciton state involving an electron in the $n=1$ Landau level and a valence hole in the $n=0$ Landau level. In contrast to the (00) exciton involving only $n=0$ Landau levels, the lowest exciton state corresponds to a finite angular momentum $R=1$. The recombination from the finite angular momentum exciton creates the $R=1$ angular momentum inter-Landau-level excitation. This is a collective excitation, a cyclotron mode, which by Kohn's theorem has only cyclotron energy ω_c. The transition frequency is given by $\omega^+=E_g-E_{\mathrm{exc}}^{01}+\Sigma_{0,1}^{\mathrm{HF}}$, where E_{exc}^{01} is the inter-Lanadau-level $R=1$ exciton binding energy, and $\Sigma_{0,1}^{\mathrm{HF}}=-E_0/2$ is the $n=1$ Landau-level Hartree-Fock self-energy at $\nu=2$. We find that the (01) exciton binding is half of the (00) exciton binding energy. Therefore we expect a discontinuous redshift $\Delta=E_0-E_{\mathrm{exc}}^{01}-E_0/2=(E_0-E_{\mathrm{exc}})/2$ at $\nu=2$ which is half of the shift at $\nu=1$. If one includes the fact that $E_0(\nu=2)=E_0(\nu=1)/2^{1/2}$, the value of the discontinuity at $\nu=2$ is $2\times 2^{1/2}$ smaller than the value at $\nu=1$.

VI. SPECTRAL FUNCTION OF A HOLE AT $\nu=3^-$

Let us consider in detail the spectral function of a hole at $\nu=3$. The hole in the $m=0$, $n=0$, $\sigma=-$ state is highly excited. It can therefore be degenerate with other excited states involving more than one quasiparticle. If many-particle configurations, which are degenerate with the single hole, exist, they will mix via Coulomb interaction in a non-perturbative fashion. These configurations are shown in Figs. 1(a)–1(c). Figure 1(b) shows configurations which have a hole with spin-up in the lowest Landau level, while Fig. 1(c) shows Auger-like configurations with holes in the second Landau level. Configurations (b) are only possible in the vicinity of odd filling factors, since only then can the spin of the hole be compensated for by a spin-flip excitation. The final states can be expanded in terms of all possible degenerate configurations with the same total angular momentum $R=0$ and spin $\sigma=-$. These noninteracting configurations can be classified by their kinetic energy. States with a finite kinetic-energy (multiples of ω_c) give rise to a shake-up effect.[2,26] They can be treated perturbatively if $E_0/\omega_c \ll 1$. Retaining only configurations with the same kinetic energy as the single hole in the lowest Landau level, shown in Figures 1(a), 1(b), and 1(c), the final states can be written as a sum of three contributions:

$$
\begin{aligned}
|f\rangle = A_0^f \cdot c_{0,-}|\nu=3\rangle &+ \sum_{\substack{m_1,m_2,m_3 \\ m_1+m_2-m_3=0}} B^f_{m_1,m_2,m_3}\cdot c^+_{m_3,1,+} \\
&\times c_{m_2,1,-}c_{m_1,0,+}|\nu=3\rangle \\
&+ \sum_{\substack{m_1,m_2,m_3 \\ m_1+m_2-m_3=0}} C^f_{m_1,m_2,m_3}\cdot c_{m_3,2,-} \\
&\times c_{m_2,1,-}c_{m_1,1,-}|\nu=3\rangle. \qquad (9)
\end{aligned}
$$

The first term, (a), is just a state of a hole with spin down. The second term, (b), illustrated in Fig. 1(b), involves a hole with spin-up (+) in the lowest Landau level, and a spin-flip excitation in the second Landau level. The last term, shown in Fig. 1(c), involves two holes in the second Landau level

$n=1$, and an electron in the third Landau level $n=2$. The configuration involving a hole with spin opposite to the one created optically is possible only in the vicinity of odd filling factors.

The calculations of the hole spectral function involves a diagonalization of the electron-electron Hamiltonian in the space of one- and three-particle excitations. Excitations (b) and (c) do not mix, and the Hilbert space can be divided into blocks corresponding to (a), (b), and (c) configurations. State (a) mixes with (b) and (c). The matrix elements of the electron-electron Coulomb interaction can be divided into different categories. The $\langle b|H|b\rangle$ matrix elements define three-body scattering involving spin-flip excitations:

$$\begin{aligned}
&\langle m_1' m_2' m_3'|H|m_3 m_2 m_1\rangle \\
&\quad=\delta_{m_1' m_1}\delta_{m_2' m_2}\delta_{m_3' m_3}\{\Sigma^{\mathrm{HF}}_{m_3,n=1}-\Sigma^{\mathrm{HF}}_{m_2,n=1}-\Sigma^{\mathrm{HF}}_{m_1,n=0}\} \\
&\qquad+\delta_{m_1' m_1}(-\langle m_3' 1 m_2 1|V_{ee}|m_2' 1 m_3 1\rangle) \\
&\qquad+\delta_{m_2' m_2}(-\langle m_3' 1 m_1 0|V_{ee}|m_1' 0 m_3 1\rangle \\
&\qquad+\langle m_3' 1 m_1 0|V_{ee}|m_3 1 m_1' 0\rangle) \\
&\qquad+\delta_{m_3' m_3}(+\langle m_2' 1 m_1' 0|V_{ee}|m_1 0 m_2 1\rangle). \qquad (10)
\end{aligned}$$

These matrix elements are a sum of four different processes: (1) Hartree-Fock self-energies of one electron and two holes, plus (2) attractive interaction of spin-flip excitations in the $n=1$ Landau level in the presence of a hole in the $n=0$ Landau level, plus (3) attractive (direct) and repulsive (exchange) inter Landau-level charge excitation in the presence of a hole in the Landau level $n=1$, plus (4) repulsive scattering of holes, one in the $n=0$ and one in the $n=1$ Landau levels.

The Hartree-Fock self-energy of an optically created hole is $\Sigma(0,0,-)=-1.5E_0$, the self-energy of a hole with spin-up is $\Sigma(0,0,+)=-1.0E_0$, the self-energy of a hole in the $n=1$ filled Landau level is $\Sigma(0,1,-)=-1.25E_0$, and the self-energy of an electron in an empty $n=1$ Landau level is $\Sigma(0,1,+)=-0.5E_0$. The energy of the final state with a single spin-down hole is $E_f=-\Sigma(0,0,-)=1.5E_0$, and the energy of the noninteracting configuration (b) is $\Sigma^{\mathrm{HF}}_{m_3}-\Sigma^{\mathrm{HF}}_{m_2}-\Sigma^{\mathrm{HF}}_{m_1}=1.75E_0$.

The $\langle c|H|c\rangle$ matrix elements define three-body scattering involving configurations $c^{+}_{m_3,2,-}c_{m_2,1,-}c_{m_1,1,-}|\nu=3\rangle$ with two holes in the $n=1$ Landau level, and one electron in the $n=2$ Landau level:

$$\begin{aligned}
\langle m_1' m_2' m_3'|H|m_3 m_2 m_1\rangle=&\,\delta_{m_1' m_1}\delta_{m_2' m_2}\delta_{m_3' m_3}\{\Sigma^{\mathrm{HF}}_{m_3,n=2}-\Sigma^{\mathrm{HF}}_{m_2,n=1}-\Sigma^{\mathrm{HF}}_{m_1,n=1}\}+\delta_{m_1' m_1}(\langle m_3' 2 m_2 1|V_{ee}|m_3 2 m_2' 1\rangle \\
&-\langle m_3' 1 m_2 1|V_{ee}|m_2 1' m_3 2\rangle)-\delta_{m_2' m_1}(\langle m_3' 2 m_2 1|V_{ee}|m_3 2 m_1' 1\rangle-\langle m_3' 1 m_2 1|V_{ee}|m_1 1' m_3 2\rangle) \\
&+\delta_{m_2' m_2}(\langle m_3' 2 m_1 1|V_{ee}|m_3 2 m_1' 1\rangle-\langle m_3' 2 m_1 1|V_{ee}|m_1 1' m_3 2\rangle)-\delta_{m_1' m_2}(\langle m_3' 2 m_1 1|V_{ee}|m_3 2 m_2' 1\rangle \\
&-\langle m_3' 2 m_1 1|V_{ee}|m_2 1' m_3 2\rangle)+\delta_{m_3' m_3}(+\langle m_2' 1 m_1' 0|V_{ee}|m_1 0 m_2 1\rangle-\langle m_2' 1 m_1' 0|V_{ee}|m_2 1 m_1 0\rangle). \qquad (11)
\end{aligned}$$

These matrix elements are the sum of three different processes:

(1) Hartree-Fock self-energies of one electron and two holes plus (2) attractive (direct) and repulsive (exchange) interaction of $n=1$ to $n=2$ charge excitations in the presence of a hole in the $n=1$ Landau level, including exchange of holes with parallel spin, plus (3) repulsive (direct) and attractive (exchange) scattering of holes in the $n=1$ Landau level in the presence of an electron in the $n=2$ Landau level.

The self-energy of a hole with spin down in the $n=1$ filled Landau level $\Sigma(0,1,-)=-1.25E_0$, and the self-energy of an electron in an empty $n=2$ Landau level is $\Sigma(0,2,-)=-0.8125E_0$. Therefore the energy of the final state with a single spin-down hole is $1.5E_0$, and the energy of the noninteracting configuration (c) is $1.6875E_0$.

The $\langle a|H|b\rangle$ matrix elements describe a decay of a hole with spin-up plus spin excitation into a single hole with spin-down via repulsive exchange scattering

$$\langle a|H|b\rangle=\langle m_1 0 m_2 1|V_{\mathrm{ee}}|m 0 m_3 1\rangle. \qquad (12)$$

The $\langle a|H|c\rangle$ matrix elements describe a decay of a hole with spin-down in the $n=1$ Landau level, plus an inter-Landau-level charge excitation into a single spin-down hole in the $n=0$ Landau level via repulsive exchange and attractive direct scattering,

$$\langle a|H|c\rangle=\langle m_1 1 m_2 1|V_{\mathrm{ee}}|m 0 m_3 2\rangle-\langle m_1 1 m_2 1|V_{\mathrm{ee}}|m_3 2 m 0\rangle. \qquad (13)$$

Spin-flip processes at $\nu=3^{-}$

Let us consider the Hamiltonian describing three-body scattering processes corresponding to configurations (b).

These processes have been divided into three channels: (1) $n=0$ hole plus spin-flip excitation in the $n=1$ Landau level, (2) $n=1$ hole plus inter-Landau-level spin-flip excitation, and (3) electron plus two holes. While all channels are important, on the basis of numerical calculations we find channel (1) to be the most important for the calculation of the hole spectal function. These states, of the form $c^{+}_{m_3,1,+}c_{m_2,1,-}c_{m_1,0,+}|\nu=3\rangle$, can be thought of as a product of a state of a hole with angular momentum m_1 and a spin-flip excitation with angular momentum m_2-m_3. The coulomb interaction leads to the scattering of the hole into a new angular momentum state by the spin-flip excitation conserving total momentum. Because we are interested in the process in which the hole flips its spin and becomes an optically active hole, we can force the hole to a $c_{0,0,+}|\nu=3\rangle$ state. Only the zero angular momentum spin-flip excitations contribute and our states are of the form $c^{+}_{m,1,+}c_{m,1,-}c_{0,0,+}|\nu=3\rangle$. There are N such states mixed by Coulomb interactions. The lowest-energy state corresponds to a zero angular momentum spin-flip mode $S^{+}=(1/\sqrt{N})\Sigma_{m=0}^{N-1}c^{+}_{m,1,+}c_{m,1,-}$, and a hole $|S\rangle=S^{+}c_{0,0,+}|\nu=3\rangle$. The vertex correction to the energy of this state is

$$\langle S|V|S\rangle=\frac{1}{N}\sum_{m}\left[\sum_{m'}(-)\langle m'1,m1|V|m'1,m1\rangle\right]+\frac{1}{N}\sum_{m}\langle m1,00|V|m1,00\rangle. \tag{14}$$

From the fact that $\Sigma_{m'}-\langle m'1,m1|V|m'1,m1\rangle=\Sigma_{11}^{\mathrm{HF}}$ and $\Sigma_{m}-\langle m1,00|V|m1,00\rangle=\Sigma_{01}^{\mathrm{HF}}$, we obtain $\langle S|V|S\rangle=\Sigma_{11}^{\mathrm{HF}}-(1/N)\Sigma_{01}^{\mathrm{HF}}$. In the thermodynamic limit $\langle S|V|S\rangle=\Sigma_{11}^{\mathrm{HF}}=-0.75E_0$. The total self-energy of this configuration is $1.75E_0$, so the total energy is $\langle S|H|S\rangle=1.75E_0-0.75E_0=1.0E_0$. The energy of this state is just the energy of the spin-up hole in the $n=0$ Landau level, since the lowest-energy spin-flip state carries zero energy. In other words, the vertex correction completely cancelled the self-energy contribution to restore the spin symmetry of the system.

We can now calculate the matrix element between our collective state $|S\rangle$ and a single state (a) of an optically active hole: $\langle a|H|S\rangle=(1/\sqrt{N})\Sigma_{m=0}^{N-1}\langle 00,m1|V|m1,00\rangle$. The sum over orbitals m is finite and this matrix element scales as $\approx 1/\sqrt{N}$ with the size of the system. Hence the interaction of a hole with individual collective states vanishes in the limit of the infinite system. The finite response comes from the growing number of collective states. The situation described here is analogous to the Anderson-Fano problem of a localized state interacting with a continuum of states.[8] For a one-dimensional problem each state of the continuum is normalized over the size L of the system and the hopping matrix element between the continuum and the localized state vanishes as $1/\sqrt{(L)}$. Yet a finite and dramatic modification of the density of states of the localized state is found. The analogy between an Anderson-Fano resonance and our problem becomes apparent when we identify the extended states of the Anderson-Fano problem with the collective excited states. This analogy will be exploited later.

VII. SPECTRAL FUNCTION OF A SPIN-FLIP EXCITATION AT $\nu=3^{+}$

The final state for a completely filled Landau-level at $\nu=3$ is the inter-Landau-level spin-flip excitation shown in Fig. 1(d). This excitation is degenerate with configurations shown in Figs. 1(e) and 1(f). Figure 1(f) shows configurations similar to those in Fig. 1(b), which have a hole with spin-up in the lowest Landau level. Configurations (e) are only possible in the vicinity of odd filling factors, since only then can the spin of the hole be compensated for by a spin-flip excitation. The final states can be expanded in terms of all possible degenerate configurations with the same total angular momentum $R=1$ and spin. Retaining only the lowest number of degenerate configurations, the final states can be written as a sum of three contributions:

$$\begin{aligned}|f\rangle=&\sum_{\substack{m_1,m_2\\ m_1-m_2=1}}D^{f}_{m_1,m_2}c^{+}_{m_2,1,+}c_{m_1,0,-}|\nu=3\rangle\\&+\sum_{\substack{m_1,m_2,m_3,m_4\\ m_1+m_2-m_3-m_4=1}}E^{f}_{m_1,m_2,m_3,m_4}c^{+}_{m_4,1,+}c^{+}_{m_3,1,+}\\&\times c_{m_2,1,-}c_{m_1,0,+}|\nu=3\rangle\\&+\sum_{\substack{m_1,m_2,m_3,m_4\\ m_1+m_2-m_3=1}}F^{f}_{m_1,m_2,m_3,m_4}\\&\times c^{+}_{m_4,2,-}c^{+}_{m_3,1,+}c_{m_2,1,-}c_{m_1,1,-}|\nu=3\rangle.\end{aligned} \tag{15}$$

The excitations (e) can be thought of as the sum of the $n=0$ to $n=1$ charge ($n=0$ hole with spin-up and $n=1$ electron with spin-up) and the $n=1$ spin-flip excitation. These excitations are degenerate with inter-Landau-level spin-flip excitations.

The calculations of the spectral function of the spin-flip excitations involves the diagonalization of the electron-electron Hamiltonian in the space of two- and four-particle excitations. Excitations (e) and (f) do not mix, and the Hilbert space can be divided into blocks corresponding to the (d), (e), and (f) configurations. The matrix elements $\langle d|H|d\rangle$ describe spin-flip excitations:

$$\langle m_1'm_2'|H|m_2m_1\rangle=\delta_{m_1'm_1}\delta_{m_2'm_2}\{\Sigma^{\mathrm{HF}}_{m_2,n=1}-\Sigma^{\mathrm{HF}}_{m_1,n=0}\}-\langle m_2'1m_10|V_{\mathrm{ee}}|m_1'0m_21\rangle. \tag{16}$$

The equation for spin-flip excitation is identical to the equation for the (01) exciton.

The $\langle e|H|e\rangle$ matrix elements define four-body scattering involving spin flip and charge excitations,

$$\langle m_1' m_2' m_3' m_4' | H | m_4 m_3 m_2 m_1 \rangle = \delta_{m_1' m_1} \delta_{m_2' m_2} \delta_{m_3' m_3} \delta_{m_4' m_4} \{ \Sigma^{\mathrm{HF}}_{m_4,1} + \Sigma^{\mathrm{HF}}_{m_3,1} - \Sigma^{\mathrm{HF}}_{m_2,1} - \Sigma^{\mathrm{HF}}_{m_1,0} \} + \delta_{m_1' m_1}$$
$$\times [- \delta_{m_4' m_4} \langle m_3' 1 m_2 1 | V_{\mathrm{ee}} | m_2' 1 m_3 1 \rangle - \delta_{m_3' m_3} \langle m_4' 1 m_2 1 | V_{\mathrm{ee}} | m_2' 1 m_4 1 \rangle$$
$$+ \delta_{m_4' m_3} \langle m_3' 1 m_2 1 | V_{\mathrm{ee}} | m_2' 1 m_4 1 \rangle + \delta_{m_3' m_4} (\langle m_4' 1 m_2 1 | V_{\mathrm{ee}} | m_2' 1 m_3 1 \rangle] + \delta_{m_2' m_2}$$
$$\times \delta_{m_4' m_4} (- \langle m_3' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_3 1 \rangle + \langle m_3' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_3 1 \rangle) + \delta_{m_2' m_2}$$
$$\times \delta_{m_3' m_3} (- \langle m_4' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_4 1 \rangle + \langle m_4' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_4 1 \rangle)$$
$$+ \delta_{m_2' m_2} \delta_{m_4' m_3} (+ \langle m_3' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_4 1 \rangle - \langle m_3' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_4 1 \rangle)$$
$$+ \delta_{m_2' m_2} \delta_{m_3' m_4} (+ \langle m_4' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_3 1 \rangle - \langle m_4' 1 m_1 0 | V_{\mathrm{ee}} | m_1' 0 m_3 1 \rangle) + \delta_{m_3' m_3}$$
$$\times \delta_{m_4' m_4} (+ \langle m_2' 1 m_1' 0 | V_{\mathrm{ee}} | m_1 0 m_2 1 \rangle) + \delta_{m_1' m_1} \delta_{m_2' m_2} (+ \langle m_4' 1 m_3' 0 | V_{\mathrm{ee}} | m_3 1 m_4 1 \rangle$$
$$- \langle m_4' 1 m_3' 0 | V_{\mathrm{ee}} | m_4 1 m_3 1 \rangle). \tag{17}$$

These matrix elements are a sum of the following processes: (1) Hartree-Fock self-energies of two electrons and two holes; plus (2) attractive or repulsive interactions of spin-flip excitations in the $n=1$ Landau level in the presence of a hole in the $n=0$ Landau level and either one of the electrons in the $n=1$ Landau level; plus (3) attractive (direct) and repulsive (exchange) inter-Landau-level charge excitation in the presence of a hole in the Landau level $n=1$; plus (4) repulsive scattering of holes, one in Landau level $n=0$ and one in Landau level $n=1$; plus (5) direct and exchange scattering of electrons.

The self-energy of a hole with spin down in the $n=1$ filled Landau level $\Sigma(0,1,-)=-1.25E_0$, and the self-energy of an electron in an empty $n=2$ Landau level is $\Sigma(0,2,-)=-0.8125E_0$. Therefore the energy of the final state with a single spin-down hole is $1.5E_0$ and the energy of the noninteracting configuration (c) is $1.6875E_0$.

The $\langle d|H|e\rangle$ matrix elements describe a decay of a $n=0$ to $n=1$ charge ($n=0$ hole with spin-up and $n=1$ electron with spin-up) and a spin excitation into an inter-Landau-level spin-flip excitation $c_{l_2,1,+} c_{l_1,0,-} |\nu=3\rangle$,

$$\langle d|H|e\rangle = \delta_{l_2 m_4} \langle m_1 0 1 m_2 1 | V_{\mathrm{ee}} | l_1 0 m_3 1 \rangle - \delta_{l_2 m_3} \langle m_1 0 1 m_2 1 | V_{\mathrm{ee}} | l_1 0 m_4 1 \rangle. \tag{18}$$

In numerical calculations only configurations (d) and (e) were considered. The spectral function corresponds to a projection of the spin-flip inter-Landau-level excitation on the interband exciton amplitude

$$E^+(\omega) = \sum_f \left| \sum_{m',m,m'-m=1} X^{m',1}_{m,0} A^f_{m,m'} \right|^2 \delta(E_f + \omega - E_i). \tag{19}$$

VIII. SPECTRAL FUNCTIONS AS ANDERSON-FANO RESONANCE

The spectral function of the hole

$$E^-(\omega) = \sum_f |A^f_{0,0,-}|^2 \delta(E_f + \omega - E_i), \tag{20}$$

describes an interaction of a spin-down hole with, e.g., a hole with spin-up plus a spin-flip excitation [configuration (b)]. There is a continuum of such excitations and the hole energy falls into this continuum. Let us label all states of the continuum by $|k\rangle$ and the state of a hole by $|0\rangle$. In Eq. (9) we expanded the eigenstates $|j\rangle$ of the interacting system $|j\rangle = A^j_0|0\rangle + A^j_k|k\rangle$ in terms of the localized and continuum states. The amplitudes A_0 and A_k satisfy the eigenvalue problem:

$$\epsilon_0 A^j_0 + \sum_k \langle 0|H|k\rangle A^j_k = E_j A^j_0,$$
$$\epsilon_k A^j_k + \sum_{k'} \langle k|H|k'\rangle A^j_{k'} + \langle k|H|0\rangle A^j_0 = E_j A^j_k, \tag{21}$$

where ϵ_0 and ϵ_k are energies of excitations and $\langle 0|H|k\rangle$ correspond to the Coulomb coupling of the localized hole state to the continuum of excitations. Because $\langle 0|H|k\rangle$ involves scattering in the lowest Landau level and $\langle k'|H|k\rangle$ involves either inter-Landau level scattering or scattering in the second Landau level, $\langle 0|H|k\rangle \gg \langle k'|H|k\rangle$. If we neglect the small matrix elements $\langle k'|H|k\rangle$, and assume $\langle 0|H|k\rangle = U$, the eigenvalue problem now reduces to the Anderson-Fano eigenvalue problem describing the interaction of a localized state with a continuum,[8]

$$U^2 \sum_k \frac{1}{(E_j - \epsilon_k)} = (E_j - \epsilon_0). \tag{22}$$

If the energy of a single bound state, ϵ_0, falls into the continuum, the interaction with the continuum leads to two bound states outside of the continuum.[8] The spectral function of the localized state shows two peaks associated with two bound states outside of the continuum. This is a general behavior associated with the structure of the Anderson-Fano Hamiltonian.

The diagonalization of the Hamiltonian for a single hole at $\nu=3^-$ and its spectral function shows a similar behavior. Figure 4 shows the calculated spectral function of the hole at

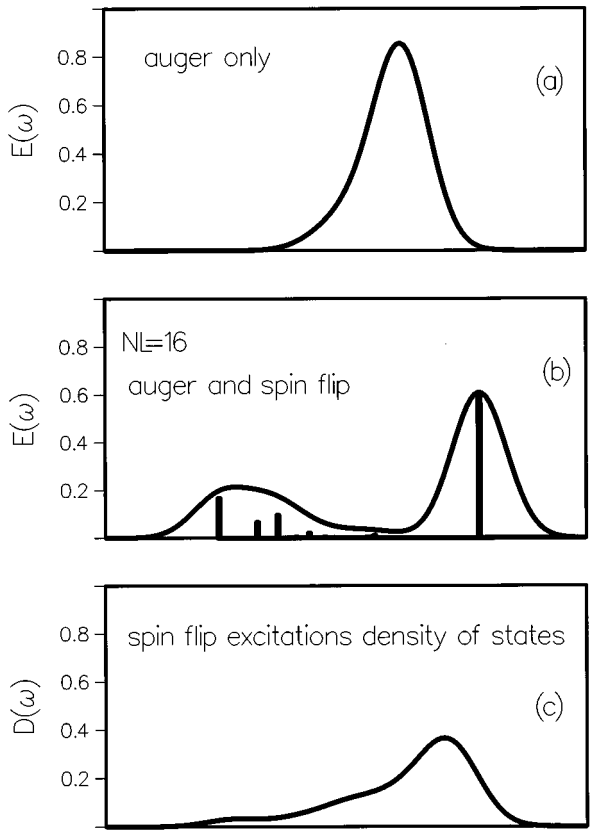

FIG. 4. Spectral function of final states of the recombination process at filling factor 3^-: (a) only auger processes, (b) auger and spin-flip scattering processes, and (c) density of states of spin flip excitations in the $n=1$ Landau level.

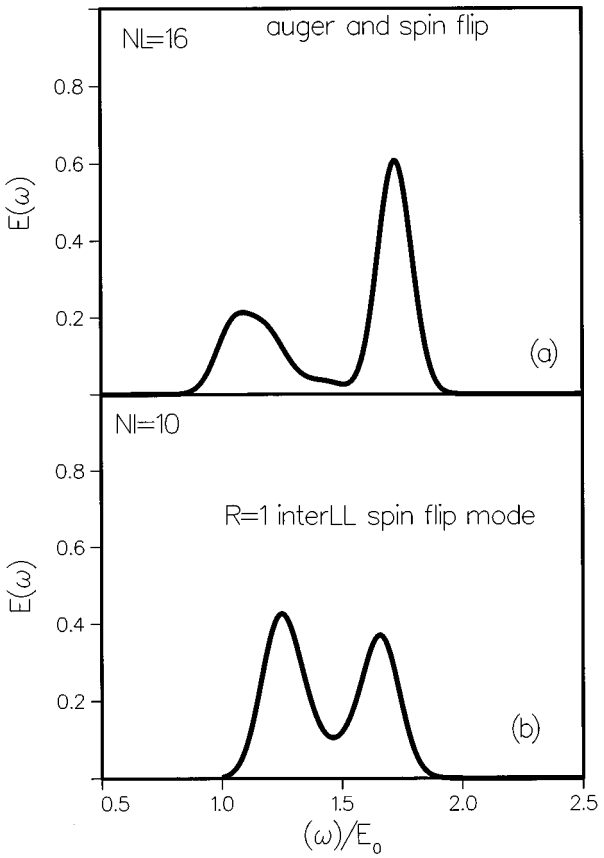

FIG. 5. Spectral function of final states of the recombination process: (a) hole at filling factor 3^-, (b) spin-flip inter-Landau-level excitation at 3^+.

$\nu=3^-$. The top part of Fig. 4 shows the spectral function of the hole with spin-down coupled only to Auger-like configurations (c) for a disk with 48 electrons ($NL=16$). The discrete levels of the finite-size system were broadened using a Gaussian. The spectral function shows a broadened and slightly shifted peak. However, when the Coulomb interaction between the single spin-down hole and a hole plus spin-flip excitation is allowed, configurations (b), the spectral function of the hole splits into two peaks of almost equal weight. The splitting is proportional to the strength of Coulomb interactions, E_0. The spectral function is shown in the second frame of Fig. 4. The position of two peaks is related to the position of the edges of the continuum of spin-flip excitations. The density of states of the spin-flip excitations is shown in the bottom frame of Fig. 4. The bottom edge corresponds to a hole with spin up and a $k=0$ spin-flip excitation. The total energy of the complex is just the energy of the hole with spin-up. The upper edge corresponds closely to the energy of spin-up hole plus the binding energy of $0.75E_0$ of the spin-flip exciton in the $n=1$ Landau level. This energy is close to the energy of a spin-down hole. Hence a rather complicated processes of a decay of a spin-down hole allows us to see both the spin-down and spin-up holes, and the splitting of energies between them as shown in Fig. 2.

A similar behavior, with a slightly smaller splitting is observed for $\nu=3^+$, as shown in Fig. 5(b). In Fig. 5(b) the

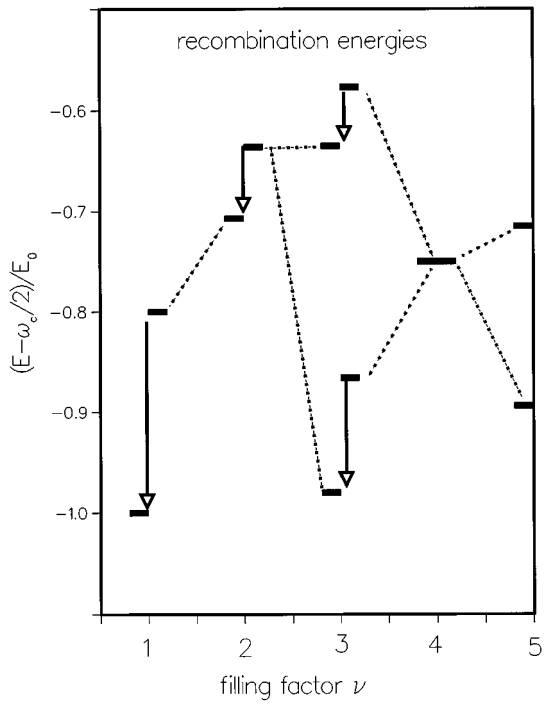

FIG. 6. Calculated energies of peaks and splittings of the recombination line as a function of the filling factor.

energy of the spin-flip mode has been shifted by the energy of the initial state $E_{\rm exc}^{01}+\Sigma_{01}^{\rm HF}=1E_0$.

We conclude that the splitting of the density of final states at filling factor $\nu=3$ is a robust phenomenon. This robust behavior of interacting electron systems is consistent with the physics of Anderson-Fano resonances. The origin of the splitting is the strong coupling of two holes with opposite spin in the lowest Landau level. Therefore the splitting is expected to occur in the vicinity of odd filling factors. In Fig. 6 we summarize the filling factor dependence of the calculated emission spectrum. There is a striking resemblance of the emission spectrum to the Hartree-Fock spectrum shown in Fig. 2. The many-body effects effectively ''break'' the spin selection rules, and allow us to see both the optically active spin-down and optically disallowed spin-up states of a hole in the lowest Landau level.

IX. CONCLUSIONS

In summary, the photoluminescence from an interacting two-dimensional electron gas in magnetic fields corresponding to vicinity of integer filling factors has been discussed. The recombination spectrum turns out to be a discontinuous function of the filling factor, related to spectral functions of the final-state excitations. The electron-electron interactions are shown to lead to large splitting and oscillations of the recombination line. The splitting, associated with odd filling factors, is an analog of Anderson-Fano resonances but in the Fock space of the interacting system. The electron-electron interactions effectively ''break'' the single-particle spin selection rules and make visible both the optically active and disallowed spin states of a hole in the lowest Landau level. The phenomena discussed here also apply to tunneling which also probes the spectral function of the hole. The extension of theory and experiment to partially filled Landau levels remains a challenge.

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[1] K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[2] M. S. Skolnick, K. J. Nash, S. J. Bass, P. E. Simmonds, and M. Kane, Solid State Commun. **67**, 637 (1988); M. S. Skolnick, D. M. Whittaker, P. E. Simmonds, T. A. Fisher, M. K. Saker, J. M. Rorison, R. S. Smith, P. B. Kirby, C. R. H. White, Phys. Rev. B **43**, 7354 (1991); M. S. Skolnick, K. J. Nash, M. K. Saker, and S. J. Bass, *ibid.* **50**, 11 771 (1994).

[3] S. Katayama and T. Ando, Solid State Commun. **70**, 97 (1989).

[4] K. Meimberg, M. Potemski, P. Hawrylak, Y-H. Zhang, and K. Ploog, Phys. Rev. B **55**, 7685 (1997).

[5] M. Potemski, R. Stepniewski, J. C. Maan, G. Martinez, P. Wyder, and B. Etienne, Phys. Rev. Lett. **66**, 2239 (1991).

[6] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982); R. B. Laughlin, *ibid.* **50**, 1395 (1983). J. Eisenstein, and H. L. Stormer, Science **248**, 1510 (1990).

[7] V. M. Apalkov and E. I. Rashba, Solid State Commun. **93**, 193 (1995); X. M. Chen and J. J. Quinn, Phys. Rev. B **50**, 2354 (1994); A. H. MacDonald *et al.*, Phys. Rev. Lett. **68**, 1939 (1992); H. Buchmann *et al.*, *ibid.* **65**, 1056 (1990); A. J. Turberfield *et al.*, *ibid.* **65**, 637 (1990); B. B. Goldberg *et al.*, *ibid.* **65**, 641 (1990).

[8] G. D. Mahan, *Many-Particle Physics*, 2nd ed. (Plenum, New York, 1990).

[9] L. Gravier, M. Potemski, P. Hawrylak, and B. Etienne, Phys. Rev. Lett. (to be published).

[10] Boris Altshuler, Yuval Gefen, Alex Kamenev, and Leonid S. Levitov, Phys. Rev. Lett. **78**, 2803 (1997).

[11] P. Hawrylak, Phys. Rev. Lett. **59**, 485 (1987).

[12] C. Kallin and B. I. Halperin, Phys. Rev. B **31**, 3635 (1985); J.-W. Wu, P. Hawrylak and J. J. Quinn, *ibid.* **31**, 6592 (1985).

[13] B. A. Muzykantskii, Zh. Eksp. Teor. Fiz. **101**, 1084 (1992) [Sov. Phys. JETP **74**, 897 (1992)].

[14] N. R. Cooper and D. B. Chklovskii, Phys. Rev. B **55**, 2436 (1997).

[15] T. Portengen, J. R. Chapman, V. Nikos Nicopoulos, and Neil F. Johnson, Phys. Rev. B **55**, 7367 (1997); Darren J. T. Leonard, Neil F. Johnson, V. Nikos Nicopoulos, and F. J. Rodriguez, Phys. Rev. B **55**, 7141 (1997).

[16] P. Hawrylak, Phys. Rev. Lett. **72**, 2943 (1994).

[17] J.-P. Cheng, Y. J. Wang, B. D. McCombe, and W. Schaff, Phys. Rev. Lett. **70**, 489 (1993).

[18] P. Hawrylak, Solid State Commun. **88**, 475 (1993).

[19] P. Hawrylak, A. Wojs, and J. A. Brum, Phys. Rev. B **54**, 11 397 (1996).

[20] A. Wojs and P. Hawrylak, Phys. Rev. B **51**, 10 880 (1995).

[21] V. M. Apalkov and E. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 420 (1991) [JETP Lett. **53**, 442 (1991)].

[22] E. H. Rezayi, in *Optical Phenomena in Semiconductor Structures of Reduced Dimension*, Vol. 343 of *NATO Advanced Study Institute, Series B: Physics*, edited by D. J. Lockwood and A. Pinczuk (Kluwer, New York, 1993), p. 79.

[23] Y. Hatsugai, P.-A. Bares, and X. G. Wen, Phys. Rev. Lett. **71**, 424 (1993).

[24] Song He, P. M. Platzmann, and B. I. Halperin, Phys. Rev. Lett. **71**, 777 (1993).

[25] A. Wojs and P. Hawrylak, Phys. Rev. B (to be published).

[26] P. Hawrylak, Phys. Rev. B **42**, 8986 (1990); **44**, 11 236 (1991).