Anisotropic superconductivity in the two-dimensional Hubbard model

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We address the problem of superconductivity in a system where the only many-body interactions are repulsive. The Eliashberg equations have been generalized to account for possible pairing correlations in the two-dimensional Hubbard model. Details of the two-dimensional band structure have been explicitly taken into account when considering the symmetry of the superconducting state. The pairing kernels have been discussed at low doping and at temperatures close to the superconducting transition temperature. We have proved that local Coulomb repulsion leads to attractive pairing correlations in the d-wave channel. Extended s-wave superconductivity is less likely to occur within this purely electronic model, at least in the physically interesting region of doping. [S0163-1829(97)04042-3]

I. INTRODUCTION

The proximity of antiferromagnetic and superconducting phases indicates that Coulomb correlations can be considered as possible nonphononic mechanism of high-temperature superconductivity. A purely electronic pairing originates from exchange of antiferromagnetic spin fluctuations[1–7] and leads to d-wave superconductivity. This remains in agreement with increasing experimental evidence for the d-wave gap parameter in systems under consideration.[8–11] The two-dimensional Hubbard model has been explored within advanced approximation schemes.[4,5,12,13] A $d_{x^2-y^2}$ state originates from pairing between fermions on near-neighbor sites whereas on-site pairing is expected to be suppressed due to local repulsion. Here, Monte Carlo calculations only indicate that the effective interaction is attractive in the $d_{x^2-y^2}$ channel because the temperatures at which this method works are too high to be related to superconducting instability.[4,5] Self-consistent decoupling in equations of motion for the Green's function is free from this drawback.[13] However, even sophisticated decoupling can be only partially controlled, in particular when compared to results of the perturbation theory. Therefore, the approach based on the framework of Eliashberg equations[14] seems to be more reliable. This method has already been developed in Ref. 15 where strong local correlations ($U\rightarrow\infty$ limit of the Hubbard model) have been incorporated in terms of auxiliary boson fields.[16] The cooperation of electron-phonon and electron-phonon-boson interactions in the stabilization of superconductivity in a d-wave state has been demonstrated.[15] However, the method developed in Ref. 15 may lead to the overestimation of correlation effects at low doping and vertex corrections in the electron-boson channel should be taken into account.

Our aim is to reconsider the problem of superconductivity in the two-dimensional Hubbard model within self-consistent second-order perturbation theory with respect to the Coulomb correlation U. A similar approach has recently been applied to discussing spectral properties in the normal state.[17] Here, we generalize Eliashberg equations to account for on-site and intersite pairing. Details of the two-dimensional band structure are explicitly taken into account. In the vicinity of the superconducting transition temperature T_c and at low doping rigorous results concerning pairing kernels can be obtained. We find that the isotropic kernel is always repulsive, whereas the anisotropic kernel leads to attractive interaction. This result remains in agreement with Mahan's theorem.[18] One can see from the structure of resulting equations that the d-wave contribution plays a dominating role in the formation of the superconducting state.

II. ELIASHBERG EQUATIONS FOR THE TWO-DIMENSIONAL HUBBARD MODEL

We consider the two-dimensional Hubbard model with the Hamiltonian

$$H=-t\sum_{\langle i,j\rangle\sigma}c^{\dagger}_{i\sigma}c_{j\sigma}-\mu\sum_{i\sigma}n_{i\sigma}+U\sum_{i}n_{i\uparrow}n_{i\downarrow}, \quad (1)$$

where $c^{\dagger}_{i\sigma}$ ($c_{i\sigma}$) creates (annihilates) an electron with spin σ on site i. $n_{i\sigma}=c^{\dagger}_{i\sigma}c_{i\sigma}$, t is the nearest-neighbor hopping amplitude, U is the on-site Coulomb repulsion, and μ denotes the chemical potential. To formulate Eliashberg equations we transform Eq. (1) to the momentum representation making use of the Nambu notation $\Psi^{\dagger}_{\mathbf{k}}=(c^{+}_{\mathbf{k}\uparrow}\ c_{-\mathbf{k}\downarrow})$:[14]

$$\begin{aligned}H &= \sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}\Psi^{+}_{\mathbf{k}}\tau_3\Psi_{\mathbf{k}}\\ &\quad+\frac{U}{N}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\Psi^{+}_{\mathbf{k}}\tau_{+}\Psi_{-\mathbf{k}'}\Psi^{+}_{-\mathbf{k}'-\mathbf{q}}\tau_{-}\Psi_{\mathbf{k}-\mathbf{q}}\\ &\equiv H_0+H_U, \end{aligned}\quad (2)$$

where $\varepsilon_{\mathbf{k}}=-t\gamma(\mathbf{k})-\mu$ with $\gamma(\mathbf{k})=2(\cos k_x a+\cos k_y a)$. $\tau_0\cdots\tau_3$ stand for the Pauli matrices. In our notation

$$\tau_{\pm}=\tfrac{1}{2}(\tau_1\pm i\tau_2). \quad (3)$$

The matrix Dyson equation is of the form[14]

$$\Sigma_{\mathbf{k}}(i\omega_l)=G^{-1}_{0\mathbf{k}}(i\omega_l)-G^{-1}_{\mathbf{k}}(i\omega_l), \quad (4)$$

where $G_{\mathbf{k}}(i\omega_l)$ stands for the Matsubara Green's function

$$G_{\mathbf{k}}(i\omega_l)=\langle\langle\Psi_{\mathbf{k}}|\Psi_{\mathbf{k}}^{+}\rangle\rangle = \begin{pmatrix} \langle\langle c_{\mathbf{k}\uparrow}|c_{\mathbf{k}\uparrow}^{+}\rangle\rangle & \langle\langle c_{\mathbf{k}\uparrow}|c_{-\mathbf{k}\downarrow}\rangle\rangle \\ \langle\langle c_{-\mathbf{k}\downarrow}^{+}|c_{\mathbf{k}\uparrow}^{+}\rangle\rangle & \langle\langle c_{-\mathbf{k}\downarrow}^{+}|c_{-\mathbf{k}\downarrow}\rangle\rangle \end{pmatrix}, \quad (5)$$

and $G_{0\,\mathbf{k}}(i\omega_l)$ denotes unperturbed ($U=0$) propagator

$$G_{0\,\mathbf{k}}^{-1}(i\omega_l)=i\omega_l\tau_0-\varepsilon_{\mathbf{k}}\tau_3. \quad (6)$$

ω_l is the Matsubara frequency $\omega_l=(\pi/\beta)(2l+1)$; $\beta=(kT)^{-1}$. Note that possible superconducting correlations will show up in the nondiagonal elements of $G_{\mathbf{k}}(i\omega_l)$. As the local repulsion acts destructively on the formations of local Cooper pairs one should distinguish between on-site and off-site pairing. Here we restrict ourselves to local and first-nearest-neighbor pairing. Then, the nondiagonal elements of the Green's function read

$$\begin{aligned}\langle\langle\Psi_{\mathbf{k}}|\Psi_{\mathbf{k}}^{\dagger}\rangle\rangle_{a\neq b}&=\frac{1}{N}\sum_{i,j} e^{i\mathbf{k}(\mathbf{R}_i-\mathbf{R}_j)}\langle\langle\Psi_i|\Psi_j^{+}\rangle\rangle_{a\neq b}\\ &\simeq\frac{1}{N}\sum_{i,j} e^{i\mathbf{k}(\mathbf{R}_i-\mathbf{R}_j)}\delta_{ij}\langle\langle\Psi_i|\Psi_j^{+}\rangle\rangle_{a\neq b}\\ &\quad+\frac{1}{N}\sum_{\langle i,j\rangle} e^{i\mathbf{k}(\mathbf{R}_i-\mathbf{R}_j)}\langle\langle\Psi_i|\Psi_j^{+}\rangle\rangle_{a\neq b}.\end{aligned} \quad (7)$$

Here, we have omitted the summation over second and higher-order nearest neighbors. Then, the momentum dependence of the nondiagonal elements of $G_{\mathbf{k}}(i\omega_l)$ is determined by

$$[G_{\mathbf{k}}(i\omega_l)]_{a\neq b}\simeq\frac{1}{N}\sum_{\mathbf{p}}[1+\gamma(\mathbf{k}-\mathbf{p})][G_{\mathbf{p}}(i\omega_l)]_{a\neq b}. \quad (8)$$

A simple identity

$$\begin{aligned}\gamma(\mathbf{k}\pm\mathbf{p})=\tfrac{1}{4}[&\gamma(\mathbf{k})\gamma(\mathbf{p})+\eta(\mathbf{k})\eta(\mathbf{p})\mp\xi(\mathbf{k})\xi(\mathbf{p})\\ &\mp\zeta(\mathbf{k})\zeta(\mathbf{p})],\end{aligned} \quad (9)$$

where

$$\begin{aligned}\eta(\mathbf{k})&=2(\cos k_x a-\cos k_y a),\\ \xi(\mathbf{k})&=2(\sin k_x a+\sin k_y a),\\ \zeta(\mathbf{k})&=2(\sin k_x a-\sin k_y a),\end{aligned} \quad (10)$$

allow one to distinguish between extended s- $[\gamma(\mathbf{k})]$ and d-wave $[\eta(\mathbf{k})]$ contributions to the singlet superconductivity. Triplet superconductivity (represented by ξ and ζ), unlikely to occur in high-temperature superconductors,[19] will be left out of consideration in the present paper. The procedure described by Eqs. (7) and (8) can be easily generalized to nearest neighbors of arbitrary order and leads to serious modification of kernels in Eliashberg equations for the electron-phonon problem.[15,20–22] The identity

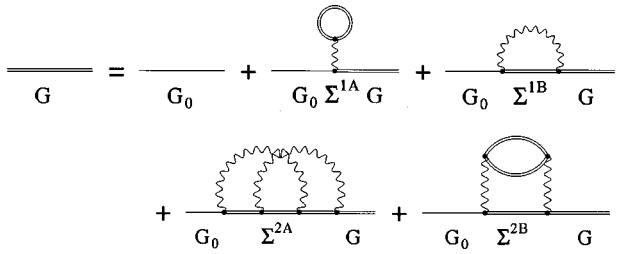

FIG. 1. The skeleton diagrammatic representation of the matrix Dyson equations. See Eqs. (15)–(21) for the notation.

$$\begin{aligned}\frac{1}{N}\sum_{\mathbf{p}}&[1+\gamma(\mathbf{k}-\mathbf{p})][G_{\mathbf{p}}(i\omega_l)]_{a\neq b}\\ &=\frac{1}{N^2}\sum_{\mathbf{p},\mathbf{p}'}[1+\gamma(\mathbf{k}-\mathbf{p}')][1+\gamma(\mathbf{p}'-\mathbf{p})]\\ &\quad\times[G_{\mathbf{p}}(i\omega_l)]_{a\neq b}\end{aligned} \quad (11)$$

indicates that one deals with a projectionlike procedure that selects given types of symmetry of the order parameter.

In the present notation the usual ansatz for $\Sigma_{\mathbf{k}}$ is of the form[14,15,20,22]

$$\Sigma_{\mathbf{k}}(i\omega_l)=[1-Z_{\mathbf{k}}(i\omega_l)]i\omega_l\tau_0+\phi_{\mathbf{k}}(i\omega_l)\tau_1+\chi_{\mathbf{k}}(i\omega_l)\tau_3, \quad (12)$$

with the momentum-dependent order parameter $\phi_{\mathbf{k}}(i\omega_l)$,

$$\phi_{\mathbf{k}}(i\omega_l)=\phi_0(i\omega_l)+\gamma(\mathbf{k})\phi_\gamma(i\omega_l)+\eta(\mathbf{k})\phi_\eta(i\omega_l), \quad (13)$$

where $\phi_0,\phi_\gamma,\phi_\eta$ correspond to the s-wave, extended s-wave, and d-wave components of the singlet pairing state, respectively.

A system of self-consistent equations for the matrix self-energy $\Sigma_{\mathbf{k}}$ can be found either from the equations of motion for $G_{\mathbf{k}}(i\omega_l)$ or from the diagrammatic analysis.[23,14] The equations of motion lead to the formally exact expression

$$\begin{aligned}G_{\mathbf{k}}(i\omega_l)=\tau_0 G_{0\,\mathbf{k}}(i\omega_l)+G_{0\,\mathbf{k}}(i\omega_l)(&\langle[[\Psi_{\mathbf{k}},H_U]_-,\Psi_{\mathbf{k}}^{+}]_+\rangle\\ &-\langle\langle[\Psi_{\mathbf{k}},H_U]_-|[\Psi_{\mathbf{k}}^{+},H_U]_-\rangle\rangle)G_{0\,\mathbf{k}}(i\omega_l),\end{aligned} \quad (14)$$

where $[A,B]_{\pm}$ denotes an anticommutator $(+)$ or a commutator $(-)$. Then, the first-order contribution to $\Sigma_{\mathbf{k}}$ comes from the thermal average $\langle\ \rangle$ and the second-order contribution from the propagator in angular brackets, provided that only connected diagrams are taken into account. The skeleton diagrammatic representation of the self-consistent matrix equation for $G_{\mathbf{k}}$ is shown in Fig. 1. One can see that even to write down an explicit form of all second-order contributions to $\Sigma_{\mathbf{k}}$ is a very tedious task. However, when considering temperatures close to T_c only terms linear in the superconducting order parameter $\phi_{\mathbf{k}}$ are of importance. Therefore, products of nondiagonal elements of $G_{\mathbf{k}}$ can be neglected. In our notation

$$\Sigma^1=\Sigma^{1A}+\Sigma^{1B}, \quad (15)$$

where

$$\Sigma^{1A}=\frac{1}{\beta}\sum_n \frac{U}{N}\sum_{\mathbf{k}} \frac{1}{4}[(\tau_0+\tau_3)G_{\mathbf{k}}(i\omega_n)(\tau_0-\tau_3) + (\tau_0-\tau_3)G_{\mathbf{k}}(i\omega_n)(\tau_0+\tau_3)], \quad (16)$$

and

$$\Sigma^{1B}=\frac{Un}{2}\tau_3, \quad (17)$$

where n is the average number of electrons per lattice site: $n=n_\uparrow+n_\downarrow$, $n_\sigma=1/N\Sigma_{\mathbf{k}}\langle c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}\rangle=n_{-\sigma}$. To make things more transparent, in the conventional notation

$$\Sigma^{1}=U\begin{pmatrix} n/2 & \Delta \\ \Delta^* & -n/2 \end{pmatrix}. \quad (18)$$

Here, Δ denotes usual isotropic BCS order parameter, $\Delta=(1/N)\Sigma_{\mathbf{k}}\langle c_{-\mathbf{k}\downarrow}c_{\mathbf{k}\uparrow}\rangle$. One can note that Σ^{1B} can be incorporated into inversion of the unperturbed Green's function $G^{-1}_{0\,\mathbf{k}}(i\omega_l)$ and leads to the renormalization of the chemical potential $\varepsilon_{\mathbf{k}}\rightarrow\overline{\varepsilon}=\varepsilon_{\mathbf{k}}+Un/2$. The second-order contribution at temperatures close to T_c reads

$$\Sigma^{2}_{\mathbf{k}}(i\omega_l)=\Sigma^{2A}_{\mathbf{k}}(i\omega_l)+\Sigma^{2B}_{\mathbf{k}}(i\omega_l), \quad (19)$$

where

$$\Sigma^{2A}_{\mathbf{k}}(i\omega_l)=\frac{1}{\beta^2}\sum_{n,m}\left(\frac{U}{N}\right)^2\sum_{\mathbf{p},\mathbf{q}}\sum_{\alpha=\pm}\tau_\alpha G_{\mathbf{p}}(i\omega_l+i\omega_n-i\omega_m)\tau_{-\alpha}G_{\mathbf{p}+\mathbf{q}}(i\omega_n)\tau_{-\alpha}G_{\mathbf{k}+\mathbf{q}}(i\omega_m)\tau_\alpha, \quad (20)$$

and

$$\Sigma^{2B}_{\mathbf{k}}(i\omega_l)=-\frac{1}{\beta^2}\sum_{n,m}\left(\frac{U}{N}\right)^2\sum_{\mathbf{p},\mathbf{q}}\{[G_{\mathbf{p}}(i\omega_n+i\omega_m-i\omega_l)]_{22}\tau_+G_{\mathbf{p}+\mathbf{q}}(i\omega_n)\tau_-[G_{\mathbf{k}-\mathbf{q}}(i\omega_m)]_{11}+[G_{\mathbf{p}}(i\omega_n+i\omega_m-i\omega_l)]_{11} \times\tau_-G_{\mathbf{p}+\mathbf{q}}(i\omega_n)\tau_+[G_{\mathbf{k}-\mathbf{q}}(i\omega_m)]_{22}\}. \quad (21)$$

The system of Eliashberg equations can be obtained from

$$\Sigma_{\mathbf{k}}(i\omega_l)=\Sigma^{1A}+\Sigma^{2A}_{\mathbf{k}}(i\omega_l)+\Sigma^{2B}_{\mathbf{k}}(i\omega_l), \quad (22)$$

where $\Sigma_{\mathbf{k}}$ is defined by Eq. (12) and one has to substitute for $G_{\mathbf{k}}(i\omega_l)$

$$G_{\mathbf{k}}(i\omega_l)=-\{Z_{\mathbf{k}}(i\omega_l)i\omega_l\tau_0+[\overline{\varepsilon}_{\mathbf{k}}+\chi_{\mathbf{k}}(i\omega_l)]\tau_3 + \phi_{\mathbf{k}}(i\omega_l)\tau_1\}D^{-1}_{\mathbf{k}}(i\omega_l), \quad (23)$$

with

$$D_{\mathbf{k}}(i\omega_l)=[Z_{\mathbf{k}}(i\omega_l)\omega_l]^2+[\overline{\varepsilon}_{\mathbf{k}}+\chi_{\mathbf{k}}(i\omega_l)]^2, \quad (24)$$

in Eqs. (18), (20), and (21). In what follows we neglect the momentum dependence of the wave-function renormalization factor, $Z_{\mathbf{k}}\rightarrow Z$, and band energy shift, $\chi_{\mathbf{k}}\rightarrow\chi$. This approximation, commonly used when considering isotropic superconductivity (''the local approximation''),[14] works well in the present case. Here, the momentum dependence of the order parameter is given by form factors $\gamma(\mathbf{k})$ and $\eta(\mathbf{k})$. At $T\rightarrow T_c$ it does not restore the momentum dependence of Z and χ and the local approximation remains consistent when discussing properties of the pairing kernels. We get

$$Z(i\omega_l)=1+\frac{1}{\omega_l}\frac{U^2}{\beta}\sum_n K^I(i\omega_l+i\omega_n)d_Z(i\omega_n), \quad (25)$$

$$\chi(i\omega_l)=\frac{U^2}{\beta}\sum_n K^I(i\omega_l+i\omega_n)d_\varepsilon(i\omega_n), \quad (26)$$

$$\phi_{\mathbf{k}}(i\omega_l)=\frac{1}{\beta}\sum_n [-U+U^2K^I(i\omega_l+i\omega_n)] \times\frac{1}{N}\sum_{\mathbf{p}}\phi_{\mathbf{p}}(i\omega_n)D^{-1}_{\mathbf{p}}(i\omega_n) + U^2\frac{1}{\beta}\sum_n K^A(i\omega_l+i\omega_n)\frac{1}{N}\sum_{\mathbf{p}}\gamma(\mathbf{k}+\mathbf{p}) \times\phi_{\mathbf{p}}(i\omega_n)D^{-1}_{\mathbf{p}}(i\omega_n). \quad (27)$$

Here, we distinguish between two types of kernels:

$$K^I(i\omega_l+i\omega_n)=\frac{1}{\beta}\sum_m [d_Z(i\omega_l+i\omega_n-i\omega_m)d_Z(i\omega_m) + d_\varepsilon(i\omega_l+i\omega_n-i\omega_m)d_\varepsilon(i\omega_m)], \quad (28)$$

$$K^A(i\omega_l+i\omega_n)=\frac{1}{\beta}\sum_m [d_{Z\gamma}(i\omega_l+i\omega_n-i\omega_m)d_{Z\gamma}(i\omega_m) + d_{\varepsilon\gamma}(i\omega_l+i\omega_n-i\omega_m)d_{\varepsilon\gamma}(i\omega_m)], \quad (29)$$

where

$$d_Z(i\omega_n)=Z(i\omega_n)\omega_n\frac{1}{N}\sum_{\mathbf{k}} D^{-1}_{\mathbf{k}}(i\omega_n), \quad (30)$$

$$d_{Z\gamma}(i\omega_n)=\frac{1}{4}Z(i\omega_n)\omega_n\frac{1}{N}\sum_{\mathbf{k}}\gamma(\mathbf{k})D^{-1}_{\mathbf{k}}(i\omega_n), \quad (31)$$

$$d_{\varepsilon}(i\omega_n)=\frac{1}{N}\sum_{\mathbf{k}}\ [\overline{\varepsilon}_{\mathbf{k}}+\chi(i\omega_n)]D_{\mathbf{k}}^{-1}(i\omega_n), \tag{32}$$

$$d_{\varepsilon\gamma}(i\omega_n)=\frac{1}{4}\frac{1}{N}\sum_{\mathbf{k}}\ \gamma(\mathbf{k})[\overline{\varepsilon}_{\mathbf{k}}+\chi(i\omega_n)]D_{\mathbf{k}}^{-1}(i\omega_n). \tag{33}$$

Equations (25)–(27) constitute Eliashberg equations for the two-dimensional Hubbard model. We have generalized the Eliashberg scheme to the case of many-body electron-electron interaction represented by local Coulomb repulsion. K^I is the kernel that determines normal-state properties and gives rise to the isotropic pairing. K^A describes correlations that lead to anisotropic pairing. It is useful to rewrite Eq. (27) in terms of amplitudes defined in Eq. (13):

$$\phi_0(i\omega_l)=\frac{1}{\beta}\sum_{n}\ [-U+U^2K^I(i\omega_l+i\omega_n)]\frac{1}{N}\sum_{\mathbf{k}}\ [\phi_0(i\omega_n)+\gamma(\mathbf{k})\phi_{\gamma}(i\omega_n)]D_{\mathbf{k}}^{-1}(i\omega_n), \tag{34}$$

$$\phi_{\gamma}(i\omega_l)=\frac{U^2}{4\beta}\sum_{n}\ K^A(i\omega_l+i\omega_n)\frac{1}{N}\sum_{\mathbf{k}}\ [\gamma(\mathbf{k})\phi_0(i\omega_n)+\gamma^2(\mathbf{k})\phi_{\gamma}(i\omega_n)]D_{\mathbf{k}}^{-1}(i\omega_n), \tag{35}$$

$$\phi_{\eta}(i\omega_l)=\frac{U^2}{4\beta}\sum_{n}\ K^A(i\omega_l+i\omega_n)\frac{1}{N}\sum_{\mathbf{k}}\ \eta^2(\mathbf{k})\phi_{\eta}(i\omega_n)\times D_{\mathbf{k}}^{-1}(i\omega_n). \tag{36}$$

Therefore, at $T\rightarrow T_c$, the s- and extended s-wave channel separates from the d-wave channel. Equations (34)–(36) determine superconducting properties of the model and will be discussed in Sec. III.

III. PAIRING KERNELS AND SYMMETRY OF THE ORDER PARAMETER

The resulting Eliashberg equations for the Hubbard model impose a tremendous numerical problem. This needs the thorough analysis and is beyond the scope of the present paper. Nevertheless, to some extent, the rigorous discussion of the pairing correlations is possible.

For $n\rightarrow 1$ the effective chemical potential $(\mu-Un/2)$ goes to zero and it is obvious [from Eqs. (26) and (32)] that $\chi(i\omega_l)=0$ fulfills the Eliashberg equations. Therefore, for $n\rightarrow 1$, $d_{Z\gamma}=0$, $d_{\varepsilon}=0$, and $d_{\varepsilon\gamma}<0$. This can be inferred from Eqs. (30)–(33). One can note that the anisotropic pairing kernel is attractive, $K^A>0$, for any value of Matsubara frequency. An important observation is that this is not the case when considering K^I, the isotropic pairing kernel. These functions have been plotted in Figs. 2 and 3 for different temperatures. As we are not able to solve the Eliashberg equations for the wave function renormalization factor $Z(i\omega_l)$, we have assumed $Z=1$ when evaluating $K^{A(I)}$. This corresponds to the standard (non-self-consistent) perturbative expansion and does not influence the sign of pairing kernels. One can also see that for $n\rightarrow 1$

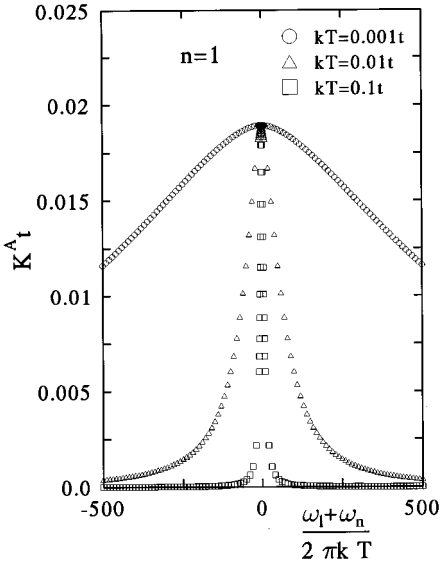

FIG. 2. The anisotropic pairing kernel K^A as a function of reduced Matsubara frequency for different temperature values.

$$\frac{1}{N}\sum_{\mathbf{k}}\ \gamma(\mathbf{k})D_{\mathbf{k}}^{-1}(i\omega_n)=0. \tag{37}$$

Therefore, one gets the separation of s-wave and extended s-wave channels at $T\rightarrow T_c$. This (due to $K^I<0$) in turn implies that one can eliminate the isotropic pairing from the scenario of superconductivity in the Hubbard model, as observed by Mahan.[18] It is also clear that d-wave symmetry will always dominate over the extended s-wave symmetry due to the fact that in the vicinity of the Fermi energy $(n\rightarrow 1)$

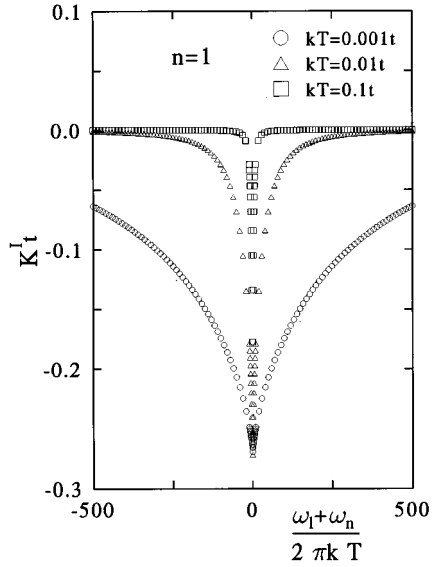

FIG. 3. The same as in Fig. 2 but for the isotropic pairing kernel K^I. Note the negative sign, which means that one deals with repulsive channel.

$$\sum_{\mathbf{k}} \eta^2(\mathbf{k}) D_{\mathbf{k}}^{-1} \simeq \sum_{\mathbf{k}} \langle \eta^2(\mathbf{k}) \rangle_{\mathrm{FS}} D_{\mathbf{k}}^{-1} > \sum_{\mathbf{k}} \langle \gamma^2(\mathbf{k}) \rangle_{\mathrm{FS}} D_{\mathbf{k}}^{-1}$$

$$\simeq \sum_{\mathbf{k}} \gamma^2(\mathbf{k}) D_{\mathbf{k}}^{-1}. \tag{38}$$

What one can conclude from the analysis at $n \to 1$ is that certain admixture of extended s-wave symmetry is possible when considering purely electronic pairing within the Hubbard model. To prove that this type of solution always exists for finite transition temperature let us make a quite general and physically justified assumption that the wave-function renormalization factor remains finite for any value of the Matsubara frequency. We introduce $Z = \max Z(i\omega_l) < \infty$. The equation for T_c corresponding to d-wave symmetry reads

$$\det[M_{ln} - 1] = 0, \tag{39}$$

where

$$M_{ln} = U^2 [K^A(i\omega_l + i\omega_n) + K^A(i\omega_l - i\omega_n)] d_\eta(i\omega_n) \tag{40}$$

and

$$d_\eta(i\omega_n) = \frac{1}{\beta} \frac{1}{4N} \sum_{\mathbf{k}} \eta^2(\mathbf{k}) D_{\mathbf{k}}^{-1}(i\omega_n). \tag{41}$$

In Eq. (40) $\omega_l > 0$, $\omega_n > 0$. It is clear that if there is a solution of Eq. (39) calculated with reduced pairing kernel $M_{ln} \to M'_{ln} \leqslant M_{ln}$, then one can expect a finite superconducting transition temperature. After rather straightforward analysis for $T \to 0$ and $l, n \leqslant n_{\max}$ one gets

$$M_{ln} \geqslant M'_{ln} = a(2n+1)^{-1}, \tag{42}$$

where a is finite, positive, quantity and

$$n_{\max} = \mathrm{Int}\left[\frac{\delta\beta}{4\pi Z} - \frac{1}{2} \right] \leqslant \frac{\delta}{4\pi k T Z} - \frac{1}{2}. \tag{43}$$

Here, δ denotes some non-negative number. Then,

$$\det[M'_{ln} - 1] = \det \begin{pmatrix} a-1 & \frac{a}{3} & \cdots & \frac{a}{2n_{\max}+1} \\ a & \frac{a}{3}-1 & \cdots & \frac{a}{2n_{\max}+1} \\ \vdots & \vdots & \vdots & \vdots \\ a & \frac{a}{3} & \cdots & \frac{a}{2n_{\max}+1}-1 \end{pmatrix}$$

$$= \left(1 - a \sum_{l=0}^{n_{\max}} \frac{1}{2l+1} \right) (-1)^{n_{\max}+1}. \tag{44}$$

For $T \to 0$ $n_{\max} \to \infty$, which implies that the determinant $\det[M_{ln} - 1]$ diverges to $(-1)^{n_{\max}\infty}$. On the other hand, for $T \to \infty$, the determinant of the same matrix equals $(-1)^{n_{\max}+1}$ and is of the opposite sign. This proves that one gets finite transition temperature in the anisotropic d-wave channel. Nevertheless, the admixture of extended s-wave symmetry is possible.

IV. CONCLUDING REMARKS

The model that has been used most frequently to discuss the possibility for superconductivity in strongly correlated systems is the Hubbard model. There have been proposals that strong local correlations, usually thought to lead to magnetic order and metal-insulator transition,[24] may also be responsible for the superconductivity. The Monte Carlo calculations give strong support for this point of view[4,5] and suggest that the channel of d-wave symmetry plays the dominant role in the formation of the superconducting state. It is believed that the isotropic, s-wave superconductivity is very unlikely to occur in the standard Hubbard model. There are rigorous results concerning that problem.[18] In the present paper the Eliashberg-type approach to superconductivity in the two-dimensional Hubbard model has been developed. This is the most rigorous scheme when considering perturbative treatment of superconductivity, independently of the underlying pairing mechanism. The challenge is that, besides U and n, there are no free parameters when considering purely electronic superconductivity in the Hubbard model. On one hand, this clearly restricts the ambiguity of results. On the other hand, to make the results conclusive an advanced many-body technique is needed. We have proved in a general way that d-wave symmetry dominates in superconductivity originating from purely repulsive local correlations. However, also for this type of pairing some admixture of extended s-wave symmetry is possible. We have not taken into account the electron-phonon channel. Nevertheless, our previous results obtained for $U \to \infty$ (Refs. 15, 20, and 21) indicate that also in this case one ends up with almost d-wave symmetry at low doping. Both s- and d-wave solutions have been found when considering electron-phonon interaction in a strong coupling scenario where the Coulomb pseudopotential provided the repulsion needed to favor d-wave symmetry.[25]

One should bear in mind that our formulation is valid for moderate values of U. For the case of genuinely strongly correlated systems ($U/t \gg 1$) one should consider higher-order contributions to the pairing kernels and discuss convergence of the self-consistent perturbation series. The open problem (strongly correlated case) is the stability of the superconducting phase with respect to the Mott-Hubbard transition (antiferromagnetic phase) in the $n = 1$ limit. The antiferromagnetic superexchange correlations contained in the Hubbard model will stabilize the antiferromagnetic order at half filling. Therefore, one should reformulate the strong coupling description including the antiferromagnetic order parameter. This is a highly nontrivial problem that needs a separate study. Nevertheless, it is d-wave superconductivity (with possible admixture of s wave) that will be replaced by antiferromagnetism at half filling.

Complete numerical treatment of resulting equations is a separate problem, in particular if effects corresponding to the Mott-Hubbard transition and antiferromagnetic order at $n = 1$ are taken into account. One faces a difficult task imposed by the coupled system of many integral equations. In particular, to get the satisfactory convergence the summation over a few hundred Matsubara frequencies probably will be necessary. Note that the generalized Eliashberg equations, as

given by Eqs. (25)–(27), have been derived at $T \to T_c$. It is clear that to go below T_c and evaluate thermodynamical quantities of physical interest is a very difficult problem due to the fact that many more diagrams containing nondiagonal elements of the Green's function should be taken into account. Nevertheless, our results demonstrate the possibility for phonon-free anisotropic superconductivity (being of predominantly d-wave symmetry) in the system where the only many-body interactions are repulsive.

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