

Real-space renormalization-group study of the two-dimensional Blume-Capel model with a random crystal field

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(Received 7 April 1997)

The phase diagram of the two-dimensional Blume-Capel model with a random crystal field is investigated within the framework of a real-space renormalization-group approximation. Our results suggest that, for any amount of randomness, the model exhibits a line of Ising-like continuous transitions, as in the pure model, but no first-order transition. At zero temperature the transition is also continuous, but not in the same universality class as the Ising model. In this limit, the attractor (in the renormalization-group sense) is the percolation fixed point of the site diluted spin-1/2 Ising model. The results we found are in qualitative agreement with general predictions made by Berker and Hui on the critical behavior of random models. [S0163-1829(97)03942-8]

I. INTRODUCTION

The Blume-Capel (BC) model is a spin-1 Ising model, originally proposed to study first-order magnetic phase transitions:¹ its phase diagram presents a line of continuous transitions and a line of first-order transitions, separated by a tricritical point. The Hamiltonian of the model is given by

$$\mathcal{H}_{\text{BC}} = -J \sum_{\langle i,j \rangle} S_i S_j + \Delta \sum_i S_i^2, \quad (1)$$

where the first sum is over all nearest-neighbor pairs on a lattice and the last one is over all sites, J is the exchange constant, Δ is the crystal field, and $S_i = \pm 1, 0$. Later, a generalization of the BC model was introduced, the Blume-Emery-Griffiths (BEG) model:² it has been used to study a rich variety of physical systems, among them ³He-⁴He mixtures. Its Hamiltonian reads $\mathcal{H} = \mathcal{H}_{\text{BC}} - K \sum_{\langle i,j \rangle} S_i^2 S_j^2$ and the parameters J , K , and Δ were originally related to the energy interactions between the constituents of the system. In ³He-⁴He mixtures, the state $S=0$ represents a ³He atom while ⁴He atoms are denoted by $S = \pm 1$ states: the superfluid transition corresponds to the symmetry breaking between the ± 1 states.

More recently, the critical behavior of ³He-⁴He mixtures in random media (more precisely, in aerogel) has been modeled by a BC model with a random crystal field (RFBC). The presence of the porous media is taken into account by the introduction of a site-dependent crystal field, which follows the probability distribution

$$\mathcal{P}(\Delta_i) = p \delta(\Delta_i - \Delta_1) + (1-p) \delta(\Delta_i - \Delta_2), \quad (2)$$

where Δ_1 is the field at the pore-grain interface and Δ_2 is a bulk field that controls the concentration of ³He atoms.^{3,4} More precisely, the BEG model has been used to describe ³He-⁴He mixtures, and the biquadratic exchange parameter K is related to the interaction energy between α He- β He atoms, $V_{\alpha\beta}$, through $V_{33} + V_{44} - 2V_{34}$. Since $V_{\alpha\beta}$ is nearly

independent of α and β , one can assume that K is zero, regaining the Blume-Capel model.

From the theoretical point of view, the presence of randomness may affect the critical behavior of systems in a drastic way. Random bonds⁵ and random fields⁶ effects on phase transitions have been studied for a long time. Briefly, the effect of random fields on multicritical phase diagrams is the following: the presence of an infinitesimal amount of randomness eliminates non-symmetry-breaking first-order transitions and replaces symmetry-breaking first-order transitions by continuous ones in two dimensions ($d=2$), while for $d>2$ tricritical points and critical end points are depressed in temperature and first-order phase transitions are suppressed only at a finite amount of disorder.⁷ Whether the first-order transition in two dimensions is replaced by a continuous transition in the same universality class as the spin-1/2 Ising model, or otherwise, is still an open problem.⁴

To the best of our knowledge, theoretical works with the RFBC model have used some sort of mean-field-like approximation.^{3,4,8,9} These approximations describe correctly the behavior of high-dimensional systems and, even in those cases, a meaningful discussion on universality classes is not possible. The so-called effective-field approximation,⁹ for instance, cannot describe first-order phase transitions; on the other hand, standard mean-field approaches (which assume that each spin interacts with all other spins in the system) do not lead to a correct discussion of percolation effects.

In the present work, we employ a real-space renormalization-group (RSRG) approximation to discuss the two-dimensional RFBC model. Our approximation takes into account spin correlations at all levels, and allows for the discussion of first-order transitions,¹⁰ universality classes, multicritical points,¹¹ etc. The crystal field probability distribution we chose is slightly different from Eq. (2), namely,

$$\mathcal{P}(\Delta_i) = p \delta(\Delta_i + \Delta) + (1-p) \delta(\Delta_i - \Delta). \quad (3)$$

We believe that the important physical ingredient lies on the presence of randomness and not on the exact form of the

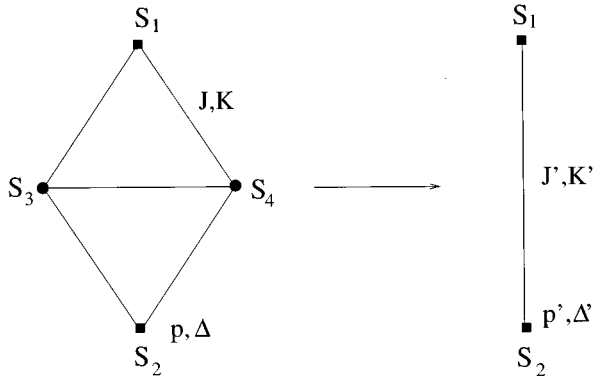


FIG. 1. Construction of a hierarchical lattice adequate to simulate the square lattice. S_1 and S_2 denote terminal spins while S_3 and S_4 denote internal spins. The original lattice is depicted on the left-hand side of the figure, with parameters p , J , K , and Δ ; after doing a partial trace over spins S_3 and S_4 we are left with the ‘‘new’’ lattice (right-hand side), with renormalized parameters p' , J' , K' , and Δ' . Note that the construction of the hierarchical lattice is done by reversing the length scale transformation.

probability distribution. In fact, we performed a mean-field calculation using Eq. (3) as the probability distribution and the results are qualitatively equivalent to those obtained using different distributions and the same mean-field approximation.

The remainder of this paper is organized as follows. In Sec. II we outline the formalism and discuss some technical points, in Sec. III we present the results, and in the last section we summarize our main conclusions.

II. FORMALISM

We approximate the Bravais lattice, which in our case, is the square lattice, by an appropriate hierarchical lattice. We chose one of the simplest cells, depicted in Fig. 1; despite its simplicity, this cell has been used with success in the study of many ferromagnetic systems. We note that the results obtained are *exact* on the chosen hierarchical lattice but only approximate on the square lattice. In particular, one does not expect to obtain results as precise as those from Monte Carlo simulations or conformal invariance arguments. Nevertheless, universality classes and the order of the transitions are very well described by RSRG approximations, particularly in two dimensions.

We then impose that the correlation function between the two terminal sites of the original and renormalized graphs are preserved:¹²

$$\exp(-\beta\mathcal{H}_{12}) = \text{Tr} \exp(-\beta\mathcal{H}_{1,2,3,4}), \quad (4)$$

where Tr means a partial trace over the internal sites of the cell (S_3 and S_4 in Fig. 1). We rewrite the cell Hamiltonians as a sum of ‘‘bond’’ terms (from now on, the factor β will be absorbed into the interaction parameters)

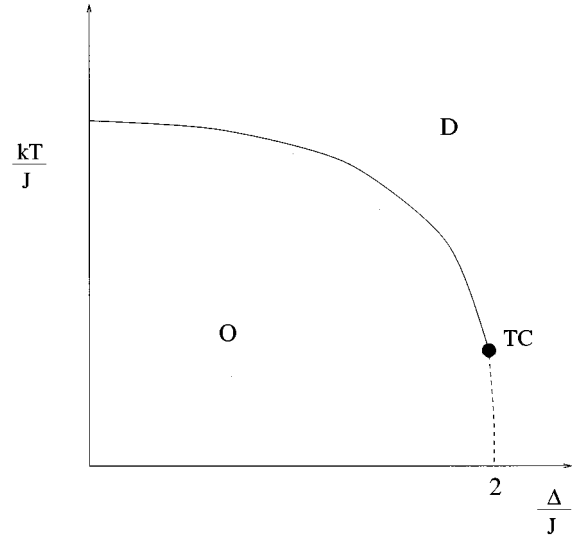


FIG. 2. Phase diagram of the pure Blume-Capel model, where k is the Boltzmann constant and T is the temperature. The first-order (dashed) line flows to a zero-temperature fixed point, where the largest eigenvalues for odd and even sectors of the RGT matrix equal b^d . TC is the tricritical point, which flows to $J^* = 1.51, K^* = 0.051, \Delta^* = 3.01$ (within the present approximation). The continuous line, to the left of TC, is attracted to the spin-1/2 Ising fixed point. O (D) stands for ordered (disordered) phase.

$$\begin{aligned} \mathcal{H}_{1,2,3,4} = & -J(S_1S_3 + S_1S_4 + S_3S_4 + S_2S_3 + S_2S_4) \\ & -K(S_1^2S_3^2 + S_1^2S_4^2 + S_3^2S_4^2 + S_2^2S_3^2 + S_2^2S_4^2) \\ & + \left(2\frac{\Delta_1}{4}S_1^2 + 3\frac{\Delta_3}{4}S_3^2 + 3\frac{\Delta_4}{4}S_4^2 + 2\frac{\Delta_2}{4}S_2^2 \right), \quad (5) \end{aligned}$$

where the crystal field Δ_i follows the probability distribution given by Eq. (3), and

$$\mathcal{H}_{12} = -J'S_1S_2 - K'S_1^2S_2^2 + \left(\frac{\Delta'_1}{4}S_1^2 + \frac{\Delta'_2}{4}S_2^2 \right) + G', \quad (6)$$

where primed quantities are renormalized parameters and G' is a constant, generated by the renormalization procedure. We comment below on the renormalized probability distributions.

Note that this way to write the cell Hamiltonians [Eqs. (5) and (6)] is equivalent to attributing weights to the sites in the one-site (crystal-field Δ) interaction, according to their coordination number. This is necessary for finite lattices in order to approximate correctly the infinite lattice behavior (see, for instance, Ref. 13). By using the above procedure we obtain the exact value for the point where both ferromagnetic and paramagnetic phases coexist at zero temperature for the pure ($p=0$) Blume-Capel model on the square lattice, namely, $(\Delta/J)_c = 2$ (see Fig. 2).

Some points are worth stressing at this stage. First, we comment on the presence of the biquadratic interaction K in our formalism. Although we are treating the Blume-Capel model ($K=0$), the parameter K is generated by the renormalization procedure and it must be taken into account to follow the renormalization path. To restrict oneself to a subspace that is not invariant usually leads to spurious results.

Second, the renormalization procedure will introduce randomness in all renormalized quantities (J' , K' , and Δ'). One possible approach would be to follow the successive renormalized distributions of these parameters in order to study the phase diagram. We adopted an alternative way that forces the renormalized distributions to be the same as the initial ones, but with renormalized parameters, namely, $\mathcal{P}'_{\text{ap}}(J) = \delta(J - J')$, $\mathcal{P}'_{\text{ap}}(K) = \delta(K - K')$, and $\mathcal{P}'_{\text{ap}}(\Delta_i) = p' \delta(\Delta_i + \Delta') + (1 - p') \delta(\Delta_i - \Delta')$. The values of J' and K' are obtained by imposing that the first moment of the actual distributions for J and K and of $\mathcal{P}'_{\text{ap}}(J)$ and $\mathcal{P}'_{\text{ap}}(K)$ are equal, respectively. The values p' and Δ' are calculated imposing that the two lowest moments of $\mathcal{P}'_{\text{ap}}(\Delta)$ match those of the real distribution. This procedure has to be used with some care: in some systems where the random-field mechanism is important and the initial randomness is on the interaction (J , for instance), forcing the field back into a uniform distribution leads to incorrect results. In Ref. 14, for instance, the crystal-field probability distribution is maintained uniform throughout the renormalization procedure. Consequently, the random model critical behavior is characteristic of a high-dimensional system: the critical temperature of the tricritical point diminishes as randomness is increased but only reaches the zero-temperature axis at a finite value of the disorder. As thoroughly discussed in Ref. 7, the mechanism responsible for the lack of first-order phase transitions in two-dimensional random systems is the disorder in the crystal field, which is not taken into account by approximations such as the one used in Ref. 14. In the model we study in this paper, however, the important physical ingredient is the disorder in the field, which is not approximated away by our RSRG procedure. Finally, we would like to mention that the way we treated the random field distribution is not unique. In this work we assume that only one field acts on each site and a weight is associated to the fields (this weight is the ratio between the coordination number of the site in the cell and the coordination number of the site on the square lattice). Conversely, one could also take the number of fields acting in a given site as equal to the coordination number of the site in the cell. We performed calculations using both procedures. The results do not vary qualitatively (and some times quantitatively) from one approach to the other. The approach we chose, however, leads to simpler expressions, which are easier to deal with analytically.

The expressions connecting renormalized and original parameters are easily obtained following the procedure outlined above but are too lengthy to be explicitly written here. Formally, they can be expressed as

$$\begin{aligned} J' &= J'(p, J, K, \Delta), & K' &= K'(p, J, K, \Delta), \\ \Delta' &= \Delta'(p, J, K, \Delta), & p' &= p'(p, J, K, \Delta). \end{aligned} \quad (7)$$

Critical points are then evaluated as nontrivial fixed points of the above relations; phases are identified according to the attractor of their points. The order of the transition is obtained through the study of the largest eigenvalue of the renormalization-group transformation (RGT) matrix.¹⁰ More precisely, a first-order phase transition such that $m \equiv \langle S \rangle$ is discontinuous at the transition point is signaled by the presence of an eigenvalue equal to b^d in the odd sector of the

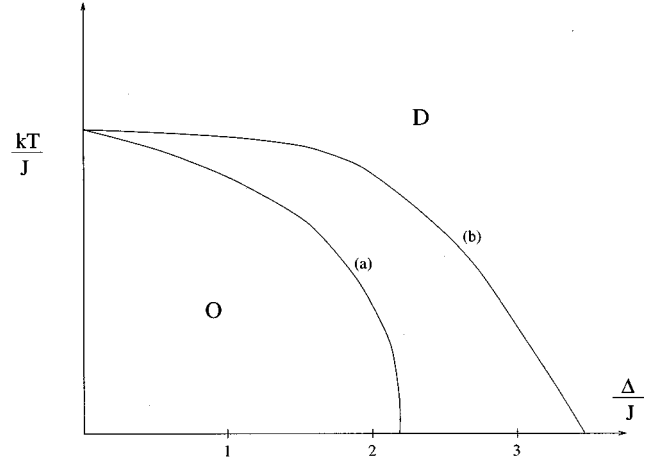


FIG. 3. Phase diagrams of the RFBC model for $p < p_c$ (see text). (a) represents $p=0.1$ and (b) represents $p=0.3$. There is no first-order transition and both the continuous lines belong to the pure Ising model universality class. The frontier at zero temperature is attracted to the random fixed point (see text). O (D) stands for ordered (disordered) phase.

linearized RGT matrix, where b is the length-scaling parameter and d is the dimension of the system. If $q \equiv \langle S^2 \rangle$ is discontinuous, the b^d eigenvalue belongs to the even sector of the RGT matrix. In the present case, $b^d = N/N' = 5$, where N is the number of bonds of the original cell and N' is the number of bonds of the renormalized one.

III. RESULTS

In Fig. 2 the pure ($p=0$) phase diagram is depicted for completeness. We would like to stress that the dashed line (and its zero-temperature point) is attracted to a fixed point (which depends on the approximation), where the largest eigenvalue for both even and odd sections of the RGT matrix equals b^d , indicating a first-order phase transition in m and in q . Note that the $K=0$ plane is *not* an invariant one and the biquadratic interaction K is generated by the renormalization transformation.

Following Ref. 7, the first-order transition should vanish for $p > 0$ (random model). This is actually the behavior we observe. In fact, the first-order fixed-point attractor of the dashed line in Fig. 2 is found to be unstable along the p direction. This is the expected physical behavior when randomness is introduced. On the other hand, the attractor of the pure Ising model transition line, namely, ($p^* = 0, J^* = 0.4407, K^* = -0.0731, \Delta^* = -\infty$), is stable along the same direction. There are still two possibilities for the random model critical behavior: either the whole line of continuous transition belongs to the universality class of the spin-1/2 Ising model or an unstable fixed point at finite temperature separates the Ising critical line from another continuous line, which belongs to a new universality class. Our results support the first option: the Ising critical line extends down to the zero-temperature point (see Fig. 3, where typical phase diagrams for $p < p_c = 1/2$ are depicted). Here there are still two possible scenarios. The continuous transition for $p \neq 0$ or 1 belongs either to the pure or to the disordered Ising model

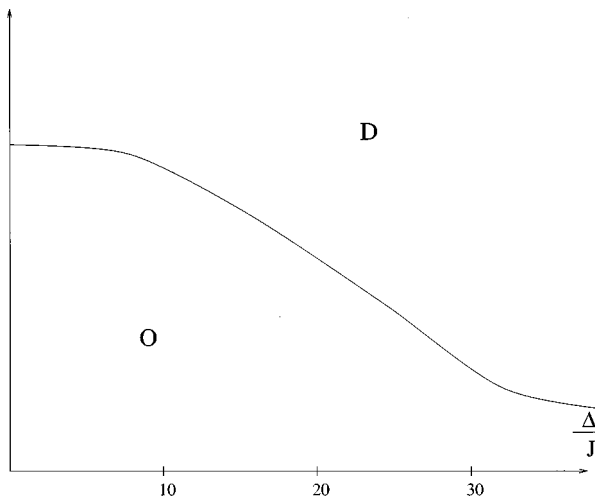


FIG. 4. Phase diagrams of the RFBC model for $p=1/2$, which is the value of p_c in our approximation. The critical line touches the zero-temperature axis at $\Delta/J=\infty$. O (D) stands for ordered (disordered) phase.

universality class. For the hierarchical lattice we use in this work, the specific-heat critical exponent of the pure Ising model α is negative and disorder is irrelevant, according to the Harris criterion.⁵ Therefore, the continuous transitions depicted in Figs. 3, 4, and 5 belong to the pure Ising model universality class. For the corresponding model on a two-dimensional Bravais lattice, where $\alpha=0$, the Harris criterion is inconclusive. The accepted behavior, when disorder is present, is the following: critical exponents of the random model retain the same values as their pure counterparts but logarithmic corrections are introduced by randomness.¹⁵ Experimental results also indicate the same critical exponents for the pure and random two-dimensional Ising model.¹⁶ On the other hand, when α is positive, as in the three-dimensional Ising model, disorder makes the system cross over to a new universality class.

Note that the critical value of Δ/J that separates the ordered and disordered phases, $(\Delta/J)_c$, increases as p in-

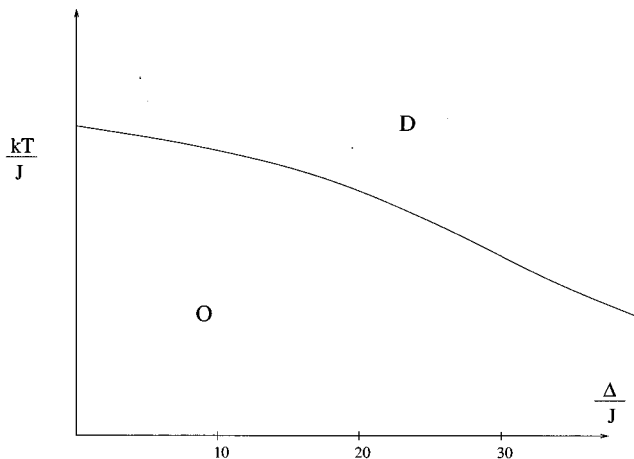


FIG. 5. Phase diagram of the RFBC model for $p=0.6 \geq p_c$. The critical line never touches the zero-temperature axis. O (D) stands for ordered (disordered) phase.

creases, in contrast to the result obtained by a cluster variational approach on a similar model,⁴ which leads to a constant value for $(\Delta/J)_c$ for any $p < p_c$. The latter result might be an artifact of the cluster variational approximation.

At zero temperature, points on the frontier between the disordered and ordered phases flow to a random fixed point [$p^*=1/2, J^*=\infty, K^*=\frac{3}{4}\ln(2)-J^*, \Delta^*=\infty$], such that $J^*/\Delta^*=0$. This is the percolation fixed point of the site-diluted spin-1/2 Ising model. In fact, for $\Delta=\infty$ the RFBC model is equivalent to the random site spin-1/2 Ising model, where sites are present or absent with probability p or $1-p$, respectively. This comes from the fact that, for $\Delta=\infty$, a $+\Delta$ crystal field acting on a given site forces that site to be in the $S=0$ state (absent), while a $-\Delta$ field forces the site to be either in the state $S=1$ or in the state $S=-1$ (both represent a present site). Thus, only for high enough p an infinite cluster of $S=\pm 1$ states will form and will be able to sustain order. Exactly at $p=p_c$, there is such an infinite cluster but its critical temperature is zero. Therefore, the critical parameter $(\Delta/J)_c$ reaches ∞ for $p=p_c$ (see Fig. 4). Our evaluation of p_c is $1/2$, while the accepted value for the site percolation critical probability on the square lattice is $p_c=0.5927$.¹⁷ It is not unusual that small-cell RSRG approximations fail to obtain a quantitatively precise value. Note, however, that we do obtain the correct qualitative behavior, i.e., a finite value of p_c [contrarily to the standard mean-field approximation, which predicts $p_c=0$ (Ref. 3)].

For $p > p_c$, the critical line never touches the (Δ/J) axis. Even at $\Delta/J=\infty$ the infinite cluster of $S=\pm 1$ spins is, on a large scale, a two-dimensional object and its critical temperature is finite (see Fig. 5).

At this point, it is worthwhile to compare our results with those from mean-field calculations (see Refs. 3,4,8,9). Standard mean-field analysis leads to a first-order transition inside the ordered phase, ending in a critical end point, a reentrant behavior in the $kT/J \times \Delta/J$ diagram and a physically incorrect value for p_c . We have already commented on this last feature. Concerning the first-order transition inside the ordered phase, it has been shown that it is unstable against randomness in two dimensions.⁷ Thus, it is expected that a reliable approximation to a two-dimensional system will not find such a transition. Finally, we found no reentrance in our results; actually, in some other models reentrant behavior has been found for $d=3$ systems, but not in their two-dimensional counterpart (see, for example, Ref. 18). We should also point out that more sophisticated mean-field-like approximations have been applied to the RFBC model. They lead to a finite value of p_c but still predict the existence of a first-order transition in the random model as well as a reentrant behavior. Hence, the results shown in this work reflect the correct qualitative behavior of the RFBC model in two dimensions.

IV. SUMMARY

A RSRG procedure is applied to the RFBC model in two dimensions. Our calculation recovers the correct phase diagram of the pure model and predicts that no first-order phase

transition is maintained when randomness is introduced. This is in accordance with general predictions for two-dimensional disordered models.⁷ We also obtain that the whole line of continuous transitions, for $p \neq 0$, belongs to the Ising universality class, discarding the existence of an unstable fixed point at finite temperature. The zero-temperature frontier between ordered and disordered phases, $(\Delta/J)_c$, is attracted to the percolation fixed point of the site diluted Ising model. Contrary to results from a cluster variational analysis,⁴ the value of $(\Delta/J)_c$ increases as p increases. Such

behavior is also predicted by standard mean-field approximations.

ACKNOWLEDGMENTS

This work was partially supported by the Brazilian agencies CNPq and FINEP. We would like to thank P. M. C. de Oliveira, S. M. de Oliveira, and S. L. A. de Queiroz for helpful discussions.

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¹M. Blume, Phys. Rev. **141**, 517 (1966); H. W. Capel, Physica **32**, 966 (1966).

²M. Blume, V. J. Emery, and R. B. Griffiths, Phys. Rev. A **4**, 1071 (1971).

³A. Maritan, M. Cieplak, M. R. Swift, F. Toigo, and J. R. Banavar, Phys. Rev. Lett. **69**, 221 (1992).

⁴C. Buzano, A. Maritan, and A. Pelizzola, J. Phys. Condens. Matter **6**, 327 (1994).

⁵A. B. Harris, J. Phys. C **7**, 1671 (1974).

⁶Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

⁷See, for instance, K. Hui and A. N. Berker, Phys. Rev. Lett. **62**, 2507 (1989); or A. N. Berker, J. Appl. Phys. **70**, 5941 (1991), and references therein.

⁸C. E. I. Carneiro, V. B. Henriques, and S. R. Salinas, J. Phys. A **23**, 3383 (1990), and references therein.

⁹T. Kaneyoshi and J. Mielnicki, J. Phys. Condens. Matter **2**, 8773 (1990).

¹⁰M. E. Fisher and A. N. Berker, Phys. Rev. B **26**, 2507 (1982).

¹¹R. R. Netz and A. N. Berker, Phys. Rev. B **47**, 15 019 (1993).

¹²J. W. Essam and C. Tsallis, J. Phys. A **19**, 409 (1986).

¹³R. R. dos Santos, J. Phys. C **18**, L1067 (1985).

¹⁴H. Dickison and J. Yeomans, J. Phys. C **16**, L345 (1983).

¹⁵F. D. A. Aarão Reis, S. L. A. de Queiroz, and R. R. dos Santos, Phys. Rev. B **54**, R9616 (1996).

¹⁶I. B. Ferreira, A. R. King, V. Jaccarino, J. L. Cardy, and H. J. Guggenheim, Phys. Rev. B **28**, 5192 (1983); M. Hagen, R. A. Cowley, R. M. Nicklow, and H. Ikeda, *ibid.* **36**, 401 (1987).

¹⁷D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, 2nd ed. (Taylor and Francis, London, 1992).

¹⁸W. Hoston and A. N. Berker, Phys. Rev. Lett. **67**, 1027 (1991); N. S. Branco, Physica A **232**, 477 (1996), and references therein.