Flux penetration in slab-shaped type-I superconductors

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(Received 12 May 1997; revised manuscript received 20 June 1997)

We study the problem of flux penetration into type-I superconductors with a high demagnetization factor (slab geometry). We analyze the process by which flux invades the sample as the applied field is increased slowly from zero, concentrating on the role of demagnetization. We find that flux does not penetrate gradually. Rather there is an instability in the process and the flux penetrates from the boundary in a series of bursts, accompanied by the formation of isolated droplets of the normal phase, leading to a multiply connected flux-domain structure similar to what is seen in experiments. [S0163-1829(97)06238-3]

When a type-I superconductor is placed in a magnetic field less than the bulk upper critical field it exhibits a phase with interpenetrating domains of the normal and superconducting state called the intermediate state. Despite over 50 years of work, a complete description of the physics of this phenomenon has been elusive. The earliest theoretical work on this problem goes back to Landau[1] who studied the problem of the equilibrium configuration of an infinite superconducting slab of thickness d placed in an applied magnetic field B_{app}. When d is much larger than the penetration depth λ and the coherence length ξ, he showed that the Meissner state is unstable to a configuration of alternating superconducting and normal laminae. The gross physical picture of this phenomenon is that the screening currents set up by the superconducting sample generate magnetic fields that extend into the regions above and below the sample. The Meissner state occurs when the reduction in free energy due to the formation of the superconducting condensate is greater than the increase in the free energy due to the energy stored in the magnetic field. Since for a slab-shaped sample a large amount of magnetic energy is stored outside the sample, whereas the superconducting condensate only exists inside the sample, the Meissner state is less favorable. On the other hand, if the sample has normal regions with high magnetic field interspersed with superconducting regions (with the average magnetic field being equal to the applied magnetic field), the field will approach its externally applied value only a short distance above and below the sample, resulting in a large reduction in the magnetic field energy, with a relatively small decrease in the superconducting condensate.

As outlined in the above discussion, it has been clear since the work of Landau that the interpenetrating domains seen in experiments on slab-shaped type I superconductors can be explained as arising from a competition between the magnetic field energy and the condensation energy of the superconducting regions. (There is also a surface energy associated with the interfaces between the normal and superconducting regions, which needs to be included in any quantitative calculation of domain widths.) On the other hand, it is apparent why the actual shapes of the domains are very hard to calculate theoretically. The screening supercurrents in one superconducting region give rise to magnetic fields over the entire sample. In calculating the energy of a configuration, there is effectively a long-range interaction between the domains. This is unlike the case for a cylindrical sample, where (apart from small fringing effects) screening currents in a superconducting domain only affect the field inside that domain. Landau restricted his analysis to a comparison between the free energy of the Meissner state and the free energy of a configuration with long straight laminar domains, alternating between superconducting and normal. After Landau's work, other configurations of flux domains, sample geometries, etc., have been studied experimentally and theoretically[2,3] (and references therein). In fact, in most experiments the regular structures envisioned by Landau are rarely seen. Instead one sees complicated patterns which are strongly dependent on the sample temperature, disorder, and field history.

In the last few years considerable progress has been made in understanding a related problem: flux penetration into (or expulsion from) long cylindrical samples oriented parallel to the magnetic field.[4,5] Frahm, Ullah, and Dorsey[4] and Liu, Mondello, and Goldenfeld[5] studied the motion of an interface between normal and superconducting regions in cylindrical samples, following a field quench. When the magnetic field varies in time (as it does in this case), it can be shown[4,5] from Maxwell equations that the magnetic field satisfies the diffusion equation with the diffusion constant $D=c^2/4\pi\sigma_n^2$. Here c is the speed of light and σ_n is the normal-state conductivity. It can also be shown[4,5] that the appropriate boundary conditions for solving the diffusion equation reduce this problem to essentially that of solid growth into undercooled liquids.[6] Much of the work on solid growth then carries over into this case. In particular, when the superconducting phase grows into the normal phase (as would occur when the field quench is to a field $B_{\mathrm{app}}\leqslant H_c$, with H_c being the bulk upper critical field), it can be shown that a planar interface is unstable to long-wavelength perturbations. In contrast, the reverse process of the normal phase growing into the superconducting phase (as would occur when the field quench is to a field $B_{\mathrm{app}}\geqslant H_c$) is completely stable. Although interesting flux patterns can be seen as *transients* in the response to a quench for a cylindrical sample, this is different from the steady-state interpenetrating domains seen in the intermediate state of slab-shaped samples: Steady-state patterns arise due to demagnetization, which does not exist for cylindrical samples.

Very recently, Goldstein, Jackson, and Dorsey[7] (GJD) attempted to analyze the influence of demagnetization on the formation of flux structures for slablike type-I superconductors. They assumed that transient currents in the normal domains decay very fast. From the expression quoted in the previous paragraph for the diffusion constant for the magnetic field in the normal regions, this is equivalent to the assumption that the normal-state conductivity is negligible. In such a case, specifying the shape and location of the superconducting regions is enough to completely determine the free energy of a configuration. When a superconducting domain changes its shape, the normal regions respond almost instantaneously, and the shape change occurs if it leads to a lowering of the free energy. Thus GJD argued that one can then think of the dynamics as a simple gradient descent of the free energy. In calculating the free energy of a particular domain configuration, things are simplified if the scale of the domains is much larger than the superconducting coherence length ξ (as is indeed true in experiments). In this case, the interface between the normal and superconducting regions is quite sharp. The free energy for a sample in an applied field B_{app} can then be expressed as the sum of three terms:

$$\mathcal{F}(B_{\text{app}},\Delta)=\mathcal{F}_B+\mathcal{F}_c+\mathcal{F}_s\,. \tag{1}$$

Here the first term denotes the magnetic free energy, the second term the condensation energy which is $-H_c^2/8\pi$ per unit volume in the superconducting regions, and the third the interfacial energy which is $H_c^2A\Delta/8\pi$. (Δ is the surface energy parameter which is of the order of the coherence length ξ, H_c is the bulk upper critical field, and A is the area of the interfaces between the normal and superconducting regions.) Both the second and third terms in Eq. (1) are easy to calculate for any pattern of domains. On the other hand, the magnetic field has to be determined by the conditions that the field inside the superconducting regions be zero and that there should be no current flowing (in steady state) in the normal regions. Although these conditions are enough to completely determine the magnetic field, from which $\mathcal{F}_B$ can in principle be calculated, as discussed earlier this is a lengthy nonlocal computation.

GJD (Ref. 7) *assumed* that (in addition to a bulk term) the magnetic part of the free energy could be written as a long-range interaction between current loops localized on the interfaces between the normal and superconducting regions. This made the problem very similar to the problem of the dynamics of two-dimensional ferrofluid droplets in a magnetic field. For the ferrofluid case it is known[8] that regular shapes evolve *continuously* into labyrinthine patterns when the applied field is increased adiabatically. In the case of superconductors, GJD showed that a circular flux droplet in a sea of superconducting material (with an area much larger than the equilibrium area) changes into a many-armed structure with threefold-coordinated nodes. Such convoluted structures are indeed seen in some experiments.[2,3] Although they were unable to derive their approximate form for the magnetic free energy from the basic Ginzburg-Landau description, GJD (Ref. 7) argued that their assumption was a good approximation by computing the equilibrium periodicity for a laminar structure within their description, and comparing with the exact result due to Landau. For all values of the externally applied magnetic field, they obtain results numerically close to Landau's, which they argue suggests that their approximation is a reasonable one. With their approximation, it is possible to calculate the free energy of complicated patterns that are not amenable to Landau's approach.

Our approach is similar to that of GJD in that we also assume that (i) the transient currents in the normal regions decay very fast, so that the dynamics is a simple gradient descent in free energy, and (ii) the free energy can be written as Eq. (1). However, we performed a careful analysis of the magnetic part of the free energy, and as we show below, this analysis indicates that the interaction between superconducting domains *cannot* be expressed in terms of current carrying loops (at least for low fields), and results in qualitatively different behavior. (Late stages of flux invasion, when the supercondcting domains are tall and thin, are discussed later.) It is not difficult to see why superconductors and ferrofluids are so different. When a thin layer of a ferrofluid of thickness d and a characteristic transverse linear extent of L is placed in an applied magnetic field B_{app}, to leading order in d/L the B field inside (and outside) the ferrofluid is equal to B_{app}. This uniform field induces a uniform magnetization within the sample, which [using the equation $\nabla\times B=(4\pi/c)j_{\text{ext}}+4\pi\nabla\times M$] gives rise to a small fringing B field near the sample edges. (The fringing field in turn induces a higher-order nonuniform magnetization.) To leading order in d/L the induced currents consist of a ribbon flowing around the boundary of the ferrofluid, causing a long-range interaction between different parts of the boundary. On the other hand, for a thin superconductor in the intermediate phase, B is not even approximately equal to B_{app} near the superconducting regions. Outside the sample, just above or below a superconducting domain, B is *parallel* to the surface, while inside the domain B is zero. In addition to ribbons of current along the side walls of the superconducting regions, there are also large current sheets on the top and bottom surfaces, dominating the interdomain interaction. Thus while the basic GJD idea of long-range interactions destabilizing regular patterns is correct, the actual description of the experimental patterns is more complicated. The numerics we report in this paper show flux invading in bursts, pinching off from the boundaries to form droplets. This is qualitatively different from the continuous evolution of GJD.[7]

We now consider $\mathcal{F}_B$ in detail. For an arbitrary sample placed in an applied magnetic field B_{app}, the magnetic free energy is[9]

$$\mathcal{F}_B=\frac{1}{8\pi}\int d^3x(B^2-2\vec{B}\cdot\vec{H}). \tag{2}$$

Here B is the magnetic induction, H is the magnetic field, and the integral is over all of three-dimensional space. It is convenient to cast this equation in a slightly different form. Since $\nabla\times\vec{H}=\nabla\times\vec{B}_{\text{app}}$, the difference between $\vec{H}$ and $\vec{B}_{\text{app}}$ is a purely longitudinal field. On the other hand, $\vec{B}$ is a transverse field, so that $\vec{B}\cdot(\vec{H}-\vec{B}_{\text{app}})$ integrated over all space is zero, and in Eq. (2) one can replace $\vec{H}$ with $\vec{B}_{\text{app}}$. Adding a B-independent term $B_{\text{app}}^2/8\pi$ to the free-energy density yields

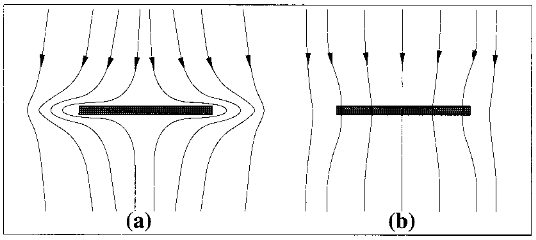


FIG. 1. B field for a slab-shaped (a) superconducting domain and (b) ferrofluid in a vertical applied field. The distortion of the field is small only for (b).

$$\mathcal{F}_B=\frac{1}{8\pi}\int d^3x(B-B_{\rm app})^2. \tag{3}$$

In order to evaluate $\mathcal{F}_B$, one has to find the magnetic induction B for a given applied field $B_{\rm app}$. For the superconducting regions, where $B=0$, the contribution to $\mathcal{F}_B$ is simple. In cylindrical samples the magnetic field is equal to applied field everywhere except in the superconducting regions so that the calculation of $\mathcal{F}_B$ is trivial. However, this is not the case in a slab geometry (as indicated in Fig. 1). This is the key difference between cylindrical and slab geometries and it precludes the existence of an intermediate state for the former. We now discuss the procedure for finding the magnetic energy in the slab geometry. Outside the superconducting regions, one can define a magnetic scalar potential ϕ by $B-B_{\rm app}=\nabla\phi$, satisfying $\nabla^2\phi=0$. Given any configuration of superconducting and normal regions we then have to solve Laplace's equation outside the superconducting regions. Since the normal component of B is zero at the boundaries, for a flat superconducting slab in a vertical field $B_{\rm app}$ pointing upwards, the boundary conditions are (i) $\partial_n\phi=\pm B_{\rm app}$ on the top and bottom surfaces of the superconducting regions and (ii) $\partial_n\phi=0$ on the interfaces between the normal and superconducting regions, where ∂_n is the normal derivative. (We have assumed that the interfaces are vertical, ignoring fanning out of flux at the top and bottom surfaces. The validity of this is discussed later in this paper, when we turn to the numerical calculations and the parameters used therein.)

Unfortunately, it is very hard to solve the above boundary-value problem. Even though the boundary conditions are quite simple, the Neumann Green's function (appropriate for these boundary conditions) is very difficult to compute even for simple shapes. Instead, at this stage, we use the quasi-two-dimensional nature of the problem to reduce its computational complexity. For a thin-sample, most of $\mathcal{F}_B$ is stored outside the sample (and in the normal regions). In the thin sample approximation, the magnetic free energy is then $(1/8\pi)\int d^3x(\nabla\phi)^2$, where the integral excludes the superconducting regions. Integrating by parts then gives

$$\mathcal{F}_B=\frac{1}{8\pi}\int_S ds\,\phi\partial_n\phi, \tag{4}$$

where the integral runs over the top and bottom surfaces of the superconducting regions. (On the side walls, the normal derivative of ϕ is zero.) In evaluating the surface integral, for a thin sample the top and bottom surfaces of the superconducting regions can be treated as approximately coincident. In this limit the slab-shaped sample reduces to a plate, which we choose to lie in the $z=0$ plane.

The potential ϕ is continuous (and satisfies Laplace's equation) everywhere except across the parts of the plate that correspond to the superconducting regions. The boundary condition (i) above must be applied even in this two-dimensional representation. If $\phi(x,y,z)$ satisifies Laplace's equation with the boundary condition, so does $-\phi(x,y,-z)$, from which it is clear that $\phi(x,y,z=0)=0$ except for the superconducting regions. [Thus, the normal parts of the $z=0$ plane are equipotential, and by symmetry, we can choose that potential to be zero. See Fig. 1(a).] One can thus obtain the potential ϕ by solving Laplace's equation in the upper half plane, with the boundary condition $\partial_z\phi=-B_{\rm app}$ in the superconducting regions and $\phi=0$ everywhere else, and then using $\phi(x,y,z)=-\phi(x,y,-z)$. The resulting potential is discontinuous across the superconducting regions, with $\phi(z=0^+)=-\phi(z=0^-)$. In fact, it is only necessary to solve for ϕ for $z=0^+$ rather than in the entire upper half plane in order to compute $\mathcal{F}_B$. Using the boundary condition $\partial_z\phi(z=0^\pm)=-B_{\rm app}$ for the superconducting regions and the condition $\phi(z=0^+)=-\phi(z=0^-)$ in Eq. (4) yields

$$\mathcal{F}_B=\frac{B_{\rm app}}{4\pi}\int d\vec{r}\,\phi(z=0^+,\vec{r}), \tag{5}$$

where $\vec{r}$ is a two-dimensional vector. The integral is performed only over the superconducting regions.

To obtain $\phi(z=0^+)$ we must solve Laplace's equation in the upper half-space with the mixed boundary conditions given in the previous paragraph. While such problems are not as familiar as the Dirichlet-Neumann boundary-value problems, they occur quite naturally in some engineering problems.[10] We now sketch how we can solve this boundary-value problem in our case. If one chooses the Green's function to vanish on the $z=0$ plane (Dirichlet boundary conditions), Laplace's equation can be written in integral form as

$$\phi(z,\vec{r})=\frac{1}{4\pi}\int d\vec{r}\,'\phi(z'=0^+,\vec{r}\,')\partial_{z'}G(z-z',\vec{r}-\vec{r}\,')\Big|_{z'=0}, \tag{6}$$

where G is the Green's function (vanishing on $z=0$). The integral on the right-hand side is over only the superconducting regions since $\phi=0$ in the normal regions. Now, differentiating this equation with respect to z (and taking the limit $z\to0^+$), it is straightforward to show that $\phi(z=0^+,r)$ and $\partial_z\phi(z=0^+,r')$ are related through the following integral equation:

$$\partial_z\phi(z=0^+,\vec{r})=-\int d\vec{r}\,'K(\vec{r}-\vec{r}\,')\phi(z=0^+,\vec{r}\,'). \tag{7}$$

Here, K is given by

$$K(\vec{r}-\vec{r}\,')=\lim_{z\to0}\frac{1}{2\pi}\frac{2z^2-|\vec{r}-\vec{r}\,'|^2}{[|\vec{r}-\vec{r}\,'|^2+z^2]^{5/2}} \tag{8}$$

and its (two-dimensional) Fourier transform is given by $|k|/2\pi$. Now, since we know that $\partial_z\phi(z=0^+,r)=-B_{\rm app}$ in the superconducting regions (and since, as stated above, the integral on the right is only over the superconducting regions), we can in principle solve for $\phi(z=0^+,r)$ in the superconducting regions, and obtain the magnetic part of the free energy by using Eq. (5). Note that since K is convoluted with ϕ, which is a smooth function, it can be replaced by

$$K(\vec{r}-\vec{r}\,')=-1/[2\pi|\vec{r}-\vec{r}\,'|^3],|\vec{r}-\vec{r}\,'|>\epsilon, \qquad (9)$$

with a compensating δ function of strength $1/\epsilon$ at the origin, in the limit $\epsilon\to 0$. (For the numerics presented in this paper, the cutoff ϵ is effectively the lattice size.)

In the thin-sample limit, we have thus reduced the calculation of the magnetic part of the free energy to a computation that can be performed entirely in two dimensions (both finding ϕ and then evaluating $\mathcal{F}_B$), and can in fact be restricted further to the superconducting regions. Unfortunately, no further simplifications are possible, like reducing the expression to integrals over domain boundaries. For an arbitrary pattern of superconducting and normal regions, one can only evaluate $\mathcal{F}_B$ numerically using the prescription above. The special case of a single circular superconducting region can be solved analytically.[11] For a sample of radius R one finds that the potential $\phi(z=0^+)$ is given by $(2B_{\rm app}/\pi)\sqrt{(R^2-r^2)}$, for $r\leqslant R$, leading to a magnetic free energy $B_{\rm app}R^3/3\pi$. (We used these results as a check on the numerics.)

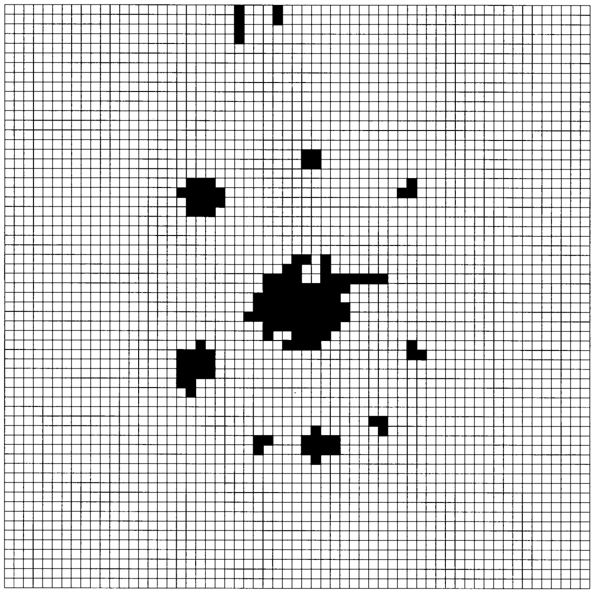
We now discuss the numerics. Since the experimental patterns depend a great deal on the field history of the sample, we concentrate on the following question: If one slowly increases the field from zero, how does flux penetrate the sample? Note that the flux will enter the sample from the edges since the magnetic pressure is the highest there (Fig. 1). In fact, in the limit where the sample thickness is much less than its cross-sectional dimension, the magnetic fields and the magnetic pressure on the side walls becomes infinite, whereas the magnetic fields on the top and bottom surfaces remain finite. [This is implicit in the two-dimensional representation we use, because the condensation energy per unit area is $H_c^2 d/(8\pi)$, so that all externally applied fields scale as $\sqrt{d/L}$, where L is the linear dimension of the sample.] We choose a sample with a thickness $d\simeq 12\Delta$ and linear dimension $L=10d$ (as stated before Δ is essentially the coherence length ξ). In experiments the flux domains branch near the surfaces for $d>O(800\Delta)$, and type-II behavior is seen for $d<O(\Delta)$. Our choice of d avoids both these regimes. Because $L>>d$, we can use the two-dimensional formulation of the magnetic boundary value problem to find $\mathcal{F}_B$. A typical value for type-I superconductors, $\Delta\simeq 1500$ Å, corresponds to $L\simeq 15\mu$m. Although this is much smaller than experimental sample sizes, qualitative aspects of our results should apply to larger samples as well, since we see isolated droplets much smaller than the sample size.

We divide the sample into lattices of various sizes; for a 61×61 lattice the lattice constant is approximately 1.9Δ. The plaquettes in the lattice are either superconducting or normal. Apart from isolated normal droplets, the superconducting-normal boundary consists of an outer interface close to the sample edges. There are two basic moves in the numerics: changing any one superconducting plaquette on the outer interface to a normal plaquette or vice versa. (The dynamics of the droplets are different, due to flux conservation, and are discussed later.) Both possibilities need to be allowed for in order allow for droplet formation. A plaquette is flipped if there is a force that favors the move; the force is the gradient of the free energy, which is given by the sum in Eq. (1). The surface tension force (from $\mathcal{F}_s$) is calculated by smoothing the lattice interface and computing the local curvature.

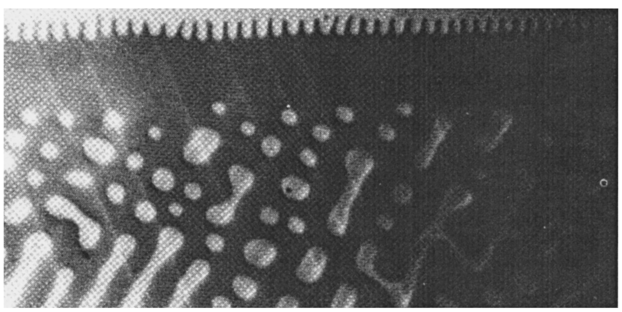
Trying single-plaquette moves requires a special treatment of surface tension in the dynamics, since flipping a single plaquette corresponds to a small sharp protrusion of the interface, with roughly the same forward and lateral extent. For a small lattice constant such a move will always cost a large surface energy compared to the energy gained from the magnetic and condensation terms. Thus in the lattice dynamics whenever the force on a segment of the interface favors flipping a plaquette, it is likely that at the next time step it will be favorable to revert the plaquette to its original state. This is because in a lattice approximation the interface is forced to make larger excursions than it would like to; it would be better for a segment of the interface to move forward by only a fraction of a plaquette, and then let neighboring segments catch up. We use a simple prescription to cure this lattice effect: When a plaquette is flipped, it is not allowed to flip back at the *very next* time step, although it can flip back thereafter. This should give rise to errors in the pattern only of the order of a lattice constant.

We start with $B_{\rm app}=0$, when the whole sample is superconducting. We raise the field until one plaquette on the boundary becomes normal. Flipping this plaquette can make it favorable to flip other plaquettes, in which case we let the system evolve until it reaches a stationary state. At this point the field is raised again. This process simulates the adiabatic increase of the magnetic field that we wish to study. Isolated droplets have to be handled differently, since the flux in them is conserved. In our simulations this constraint is obeyed approximately: Once a droplet is formed, we move it rigidly in the direction of the force on it until the force is zero. At this stage we adjust the number of plaquettes in the droplet so that the flux in it is as close as possible to its original value. As $B_{\rm app}$ is increased further, we continue to adjust the droplet size so that the flux in it stays constant.

We now discuss our results. When the field is raised just above the lower critical field, one plaquette becomes normal. If the penetration of flux were gradual, one would expect to have to raise the field further for more flux to enter. Instead we found that at a field just slightly above the lower critical field, the flux penetrates a distance into the sample of the order of 20 times the coherence length before the first droplet pinches off. Increasing the field further produces similar behavior: Much of the evolution is in the form of bursts of magnetic flux penetrating from the boundary which then pinch off to form droplets. This reflects an instability in the process of flux penetration and is the main result of our work. In Fig. 2(a) we show the patterns seen for $L=61$ at a field only moderately above the field of first flux penetration. On the coarser lattices we saw droplets with somewhat different detailed shapes, which we attribute to our approximate treatment of surface tension. However, the rough sizes of the droplets in the patterns and the magnetic fields at which they enter the sample are similar for the various lattices. The droplets form near the boundary of the sample and then move towards the center of the sample, leaving a region near



(a)



(b)

FIG. 2. Droplet state for (a) numerical simulations on a 61×61 lattice and (b) experiments on mercury (Ref. 13). Normal regions are black in (a) and white in (b). The flux front near the top in (b) is due to finite sample thickness.

the boundary flux free, similar to what is seen in experiments. We also saw that the droplets typically shrink when one puts in flux conservation, though this observation may not have much experimental significance in view of the fact that we deal with the collective motion of the droplet approximately. Finally, we should emphasize that the instability in the process of flux penetration is robust and is independent of the lattice size.

At this point we would like to comment on why the quasistatic approximation should be valid in a situation where the flux penetrates in bursts The quasistatic approximation assumes that transient currents die out very fast, so that knowing which regions are normal and which superconducting completely determines the state of the system. The relevant time scales to compare the dynamics with are (i) the time it takes flux to diffuse through a normal region of size comparable to the droplets, (ii) the time it takes for superconducting shielding currents to be set up in response to a change in field, and (iii) the time it takes for the magnetic field to propagate outside the sample, from one region of the sample to another (thereby establishing the effectively nonlocal interaction). The first two are controlled by the normal-state diffusion constant for the flux,[12] the first being of the order of l^2/D and the second of the order of λ^2/D, where l is the typical linear extent of a droplet, λ is the penetration depth, and D the diffusion constant. With $l\simeq 10^{-4}$ m, $\lambda\simeq 500$ Å(Ref. 13) and $D\simeq 10^{-2}$ m^2/s,[4] typical values for type-I superconductors, we find the time scales to be $\simeq 10^{-6}$ and $\simeq 10^{-12}$ s. The time it takes for electromagnetic fields to propagate across the sample above or below it is of the order of 10^{-10} – 10^{-11} s. These three time scales have to be compared to the time scale of the flux invasion process. This is controlled by the time it takes to convert a superconducting region to normal (or vice versa), which we expect to be a much slower process.

The formation of droplets is promoted by the discontinuous nature of the flux invasion, which causes relatively large amounts of flux to enter the system at low magnetic fields. A large amount of flux penetration reduces the magnetic forces which tend to drive further flux into the system. It is then possible for a normal region to find it favorable to revert to being superconducting, which is how droplets are formed. At later stages when the applied field is higher such reversion to a superconducting state is less likely. Further flux coming in from the boundary would then probably coalesce with the existing droplets, leading to the labyrinthine patterns seen in some experiments, like the patterns obtained by GJD.[7] However, since the superconducting regions become thinner at higher fields, it is not clear whether quasi-two-dimensional descriptions are valid here.

Although the existence of droplets is fairly ubiquitous in the experiments, the actual shape of the droplets varies rather widely, from compact droplets for mercury in the early stages of flux penetration to long laminae for lead.[14] Farrell, Huebener, and Kampwirth[13] reported experiments in monocrystalline mercury plates of varying thicknesses and found patterns with compact droplets at lower fields which evolved to laminar structure at higher fields. Their photograph of the low-field patterns is shown in Fig. 2(b). This looks at least moderately similar to what we see in our simulations. The edge structure seen at top of the figure is attributed to the finite thickness of the sample.[2,15] Similar experiments by Kiendl and Kirchner[14] on lead foils of varying thickness also found isolated domains at low fields, although the domains were long and thin instead of being drop like. The details of the domain shapes in a particular sample will depend on its material parameters; moreover, a complete understanding of domain shapes would need a continuum description, which would allow a more accurate treatment of effects of surface tension and flux conservation within the droplets. There have also been experiments showing the influence of lattice structure, field history, disorder, etc., on the intermediate state patterns. Many of these have been discussed in Refs. 3 and 2. Once again, it would be necessary to develop a continuum description to understand these issues. Finally, there are experiments at low sample thickness which show flux spots growing into laminae and experiments at high thickness which show branching of domains.[3] As mentioned earlier, when we discussed the choice of parameters for the numerical simulations, both these regimes have been excluded from our analysis.

To conclude, we have developed a description of the problem of flux penetration into slab-shaped type-I superconductors based on the sharp interface approximation. Lattice simulations show that as the applied magnetic field is increased, flux enters in bursts, forming isolated normal droplets. While the multiply connected nature of the patterns has been emphasized in the literature,[3] the instability that we have noticed does not seem to have been reported so far. This instability should be apparent in real-time imaging of the process of flux penetration in all experiments, independent of the actual shape of the droplets.

We thank Alan Dorsey, Daniel Fisher, Tanya Kurosky, Carsten Wengel, and Peter Young for useful discussions. H. B. was supported by NSF Grant No. DMR-9411964 and O.N. in part by the A.P. Sloan Foundation.

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